

Data Structures & Algorithms

(PCC-CS 301)

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Topics Covered

1. Application of Stack
 - a. Conversion of Infix to Postfix expression
 - b. Evaluation of Postfix expression

Arithmetic Expression

- Different operations

- Binary operations

- Exponentiation
 - Multiplication , Division
 - Addition , Subtraction

- Precedence

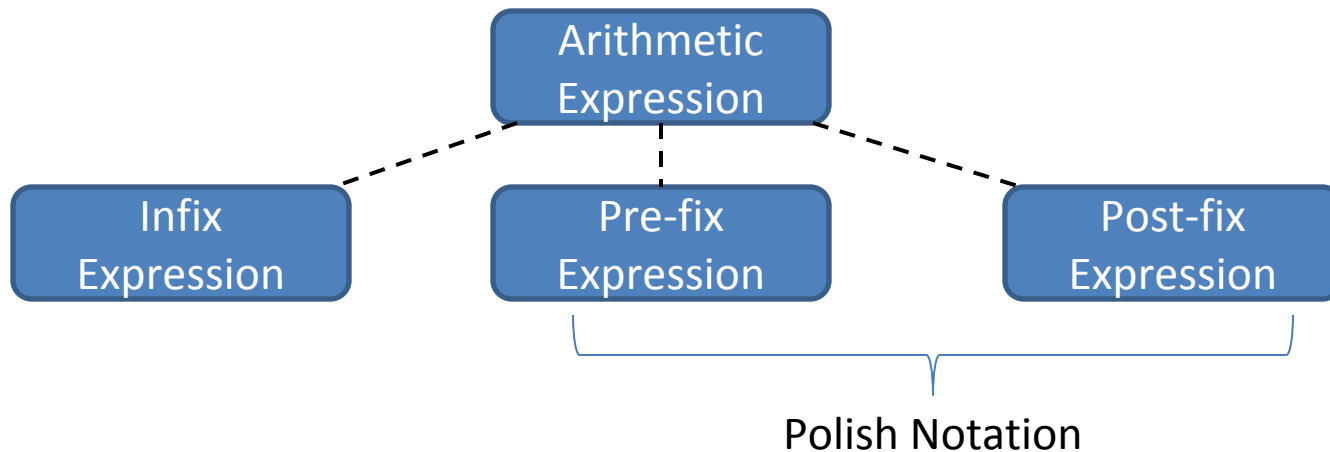
| Any operations inside parenthesis | Highest | ↑ |
|-----------------------------------|---------|---|
| Exponentiation | | |
| Multiplication , Division | | |
| Addition , Subtraction | Lowest | ↓ |

- Example

$$2^3 + 5 * 2^2 - 12 / 6 = 8 + 5 * 4 - 12 / 6 = 8 + 20 - 2 = 26$$

Arithmetic Expression

- Representation



□ Infix expression

- Operators should lie between two operands (for binary operations)

Ex. - $(A+B) / (C+D)$

Arithmetic Expression

- Representation

- Polish notation (proposed by Jan Lukasiewicz)

- Pre-fix expression

- Any operator will be placed before its associated operands

Ex. - $/+AB+CD$

- Post-fix expression

- An operator will be placed after its associated operands
 - Also termed as reverse polish notation
 - Used for arithmetic expression evaluation in computer

Ex. - $AB+CD+/$

There will be no parenthesis used in Polish notations

Arithmetic Expression

- Why Polish notation?

- Arithmetic expression (infix)

- It requires to express in proper parenthesization to evaluate

$(A+B)*C$ and $A+(B*C)$ are different

- Polish notation of expression

- Can be evaluated without parenthesis
 - Useful in evaluating arithmetic expression in computer system
 - Stack data structure is sufficient to perform the job

Postfix: $AB+C^*$ and ABC^*+

Polish Notation

- Infix to Prefix conversion (manually)

Infix Expression: $A + B * (C - D) / E + F ^ G + H$

Pre-fix conversion:

$$\begin{aligned} & A + B * (C - D) / E + F ^ G + H \\ &= A + B * [- C D] / E + F ^ G + H \\ &= A + B * [- C D] / E + [^ F G] + H \\ &= A + [* B - C D] / E + [^ F G] + H \\ &= A + [/ * B - C D E] + [^ F G] + H \\ &= [+ A / * B - C D E] + [^ F G] + H \\ &= [[+ + A / * B - C D E ^ F G] + H \\ &= + + + A / * B - C D E ^ F G H \end{aligned}$$

$+++A/*B-CDE^FGH$

Polish Notation

- Infix to Postfix conversion (manually)

Infix Expression: $A + B * (C - D) / E + F ^ G + H$

Pre-fix conversion:

$$\begin{aligned} & A + B * (C - D) / E + F ^ G + H \\ &= A + B * [C D -] / E + F ^ G + H \\ &= A + B * [C D -] / E + [F G ^] + H \\ &= A + [B C D - *] / E + [F G ^] + H \\ &= A + [B C D - * E /] + [F G ^] + H \\ &= [A B C D - * E / +] + [F G ^] + H \\ &= [A B C D - * E / + F G ^ +] + H \\ &= A B C D - * E / + F G ^ + H + \end{aligned}$$

$A B C D - * E / + F G ^ + H +$

Solving Arithmetic Expression

- Solve by computer (involves two steps)
 - Conversion of infix to postfix expression
 - Solve postfix expression



Both the methods require STACK to solve

Solving Arithmetic Expression

- Infix to Postfix conversion (algorithm)

```
Postfix ( Q ) // Q is the infix arithmetic expression
{
    step 1: Push "(" into the stack and add ")" to the end of Q
    step 2: Scan Q from left to right and repeat steps 3 to 6 for each element of Q
            until the Stack is empty
    step 3: If an operand is encountered, add it to P // P is output postfix expression
    step 4: If a left parenthesis is encountered, push it into Stack
    step 5: if an operator is encountered then
        5.1. Repeatedly pop from Stack and add to P each operator which has same or
              higher precedence than the encountered operator
        5.2. Push the encountered operator into Stack
    step 6: If a right parenthesis is encountered then
        6.1. Repeatedly pop from Stack and add to P each operator until a left
              parenthesis is encountered
        6.2. Remove the left parenthesis (but do not add to P )
    step 7: Exit
}
```

Solving Arithmetic Expression

- Infix to Postfix conversion (mechanism)

Infix expression: $A + (B * C - (D / E \wedge F) * G) * H$

| Symbol Scanned | STACK | Expression P |
|----------------|---------------|-------------------------------|
| (1) A | (| A |
| (2) + | (+ | A |
| (3) (| (+ (| A |
| (4) B | (+ (| A B |
| (5) * | (+ (* | A B |
| (6) C | (+ (* | A B C |
| (7) - | (+ (- | A B C * |
| (8) (| (+ (- (| A B C * |
| (9) D | (+ (- (| A B C * D |
| (10) / | (+ (- (/ | A B C * D |
| (11) E | (+ (- (/ | A B C * D E |
| (12) ↑ | (+ (- (/ ↑ | A B C * D E |
| (13) F | (+ (- (/ ↑ | A B C * D E F |
| (14)) | (+ (- | A B C * D E F ↑ / |
| (15) * | (+ (- * | A B C * D E F ↑ / |
| (16) G | (+ (- * | A B C * D E F ↑ / G |
| (17)) | (+ | A B C * D E F ↑ / G * - |
| (18) * | (+ * | A B C * D E F ↑ / G * - |
| (19) H | (+ * | A B C * D E F ↑ / G * - H |
| (20)) | | A B C * D E F ↑ / G * - H * + |

Solving Arithmetic Expression

- Evaluation of Postfix expression (algorithm)

```
Evaluate_Postfix ( P ) // P is the post-fix expression
{
  step 1: Add “)” at the end of P
  step 2: Scan P from left to right and repeat steps 3 and 4 for each element of P
           until “)” is encountered
  step 3: If an operand is encountered, push it into Stack
  step 4: if an operator is encountered then
           5.1. Pop two top elements from Stack, where A is the top element and B is the
                next-to-top element
           5.2. Evaluate B (operator) A
           5.3. Push the result of back to Stack
  step 5: Set VALUE equal to the top element on Stack
  step 6: Exit
}
```

Solving Arithmetic Expression

- Evaluation of Postfix expression (mechanism)

Post-fix expression: 5 6 2 + * 12 4 / -

| <i>Symbol Scanned</i> | | <i>STACK</i> |
|-----------------------|----|--------------|
| (1) | 5 | 5 |
| (2) | 6 | 5, 6 |
| (3) | 2 | 5, 6, 2 |
| (4) | + | 5, 8 |
| (5) | * | 40 |
| (6) | 12 | 40, 12 |
| (7) | 4 | 40, 12, 4 |
| (8) | / | 40, 3 |
| (9) | - | 37 |
| (10) |) | |

Queries?