TM

It represents REL

HTM

It represents Decidable set (Recursive set)

HTM is a TM

LBA

It represents CSL.

It uses linealy bounded tape.

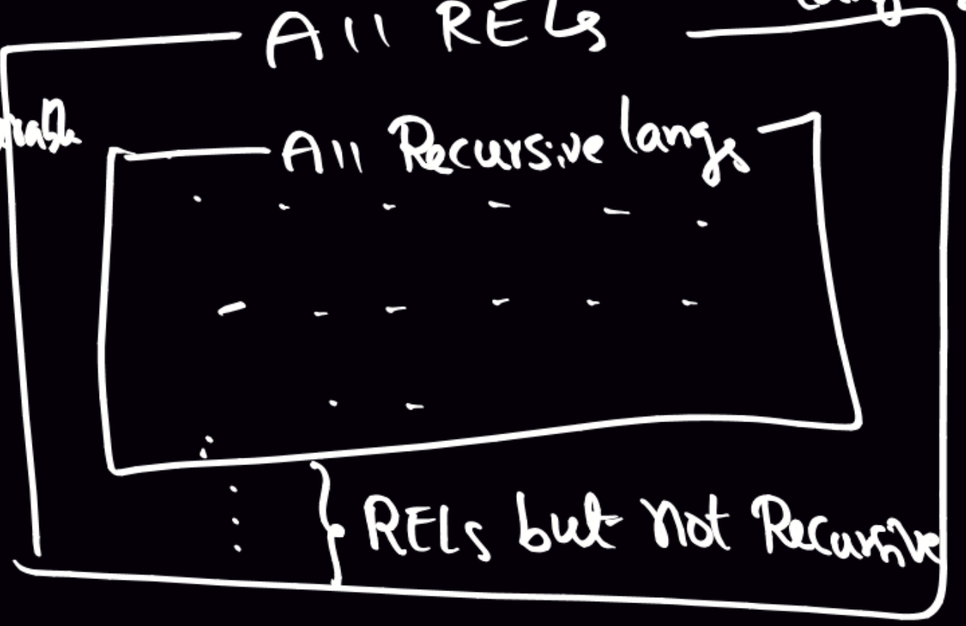
LBA is HTM
LBA is TM

Recursive Language (Rec)

1) Equivalent to HTM
2) Every Rec is REL
3) It is also called as "Turing Decidable lang"
4) TM enumerates in lexicographical order (effective)

Recursively Enumerable Language (REL)

1) Equivalent to Tm
2) REL need not be Rec
3) It is also called as "Turing Recognizable language".
4) Enumerable lang



Recursive Language

$\cong$

Decidable Language

$\cong$

HTM acceptable lang

$\cong$

Effectively enumerable set

$\cong$

Lexicographically enumerable set

$\cong$

Total Recursive function

$\cong$

Computable language

$\cong$

Algorithm exist

$\cong$

Halting program

RE L

$\cong$

Enumerable set

$\cong$

Turing Recognizable set
(acceptable)

$\cong$

Semi-decidable set

$\cong$

Partial function

$\cong$

program exist

$\cong$

TM acceptable set

REL but not Recursive Languages

$\cong$

REL but not decidable

$\cong$

REL but UD

$\cong$

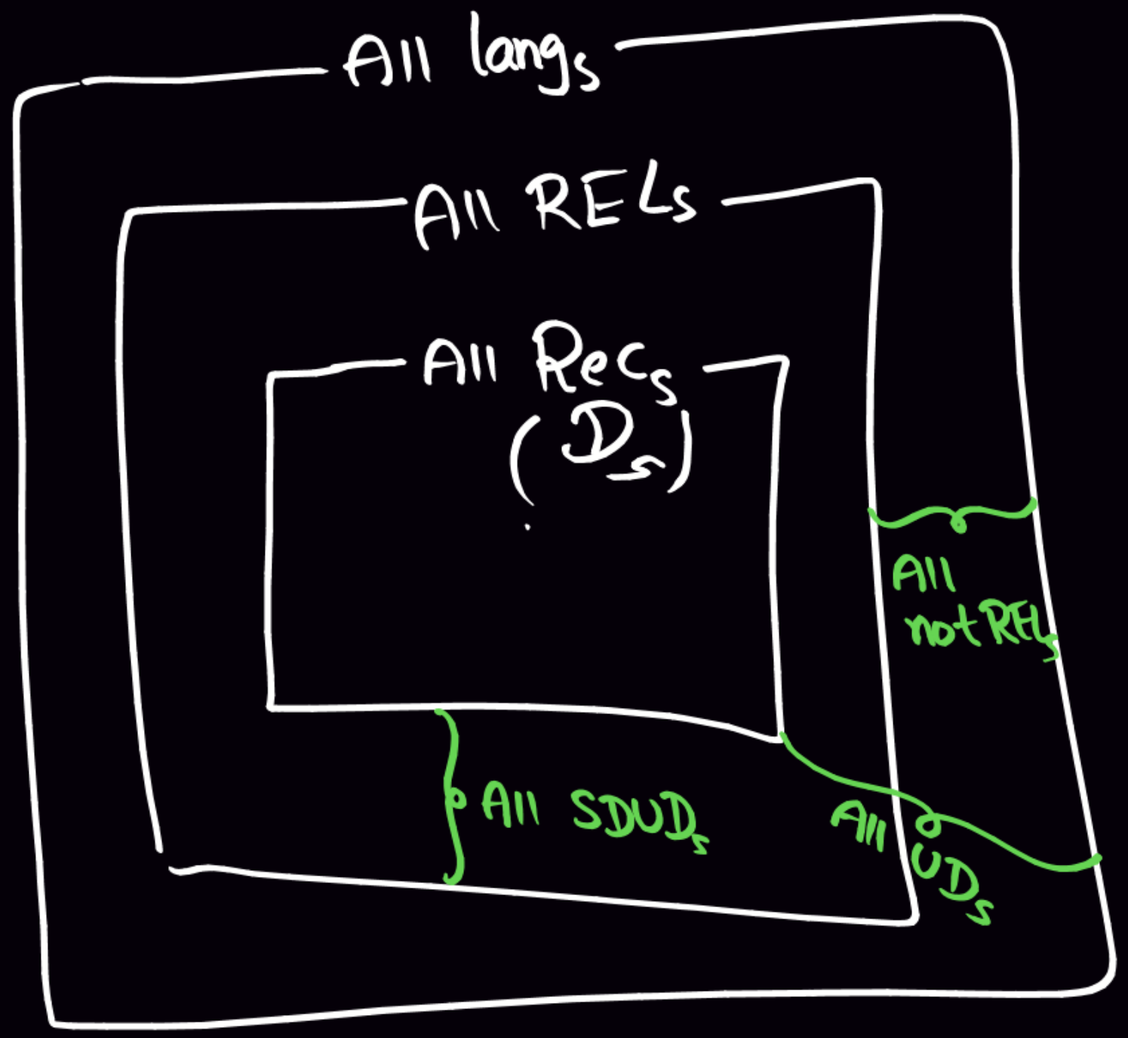
REL and UD

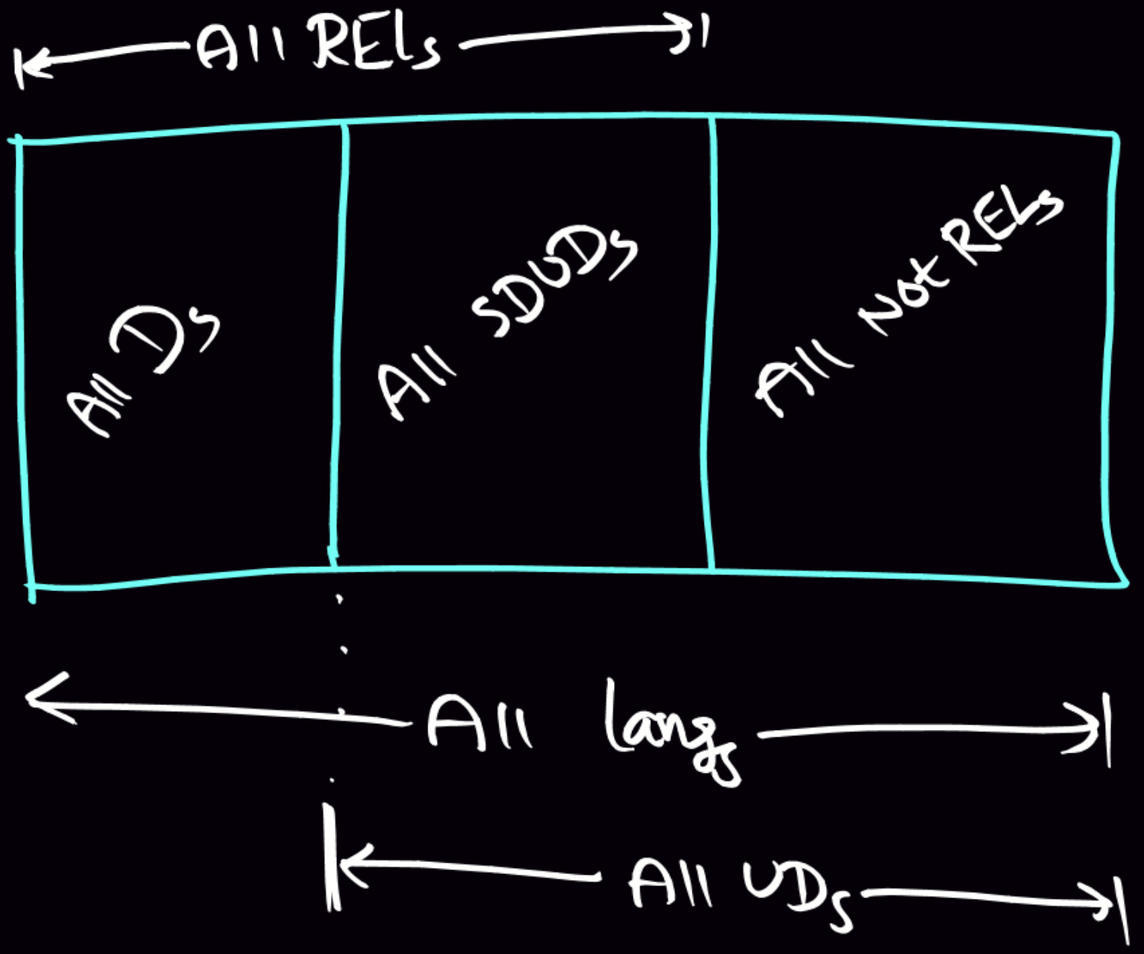
$\cong$

SD and UD

$\cong$

TM exist but no HTM


All langs
All RELs
All Recs
(Ds)
All not RELs
All SDUDs
All UDs


All RELs
All Ds
All SDUDs
All Not RELs
All langs
All UDs

Closure properties for RECURSIVE Languages:

1) $L_1 \cup L_2$
2) $L_1 \cap L_2$
3) $\overline{L}$
4) $L_1 - L_2$
5) $L_1 \cdot L_2$
6) L^{Rev}
7) L^*
8) L^+
X 9) subset(L)

10) prefix(L)
11) suffix(L)
12) substring(L)
13) Quotient (L_1, L_2)
X 14) $f(L)$
X 15) $h(L)$
16) ε-free $h(L)$
17) $h^{-1}(L)$

Remember NOT CLOSED:
$=, f, h$
(Subset Substitution Homomorphism)

Closure properties for RELs (SDs):

1) $L_1 \cup L_2$
2) $L_1 \cap L_2$
X 3) $\overline{L}$
X 4) $L_1 - L_2$
5) $L_1 \cdot L_2$
6) L^{Rev}
7) L^*
8) L^+
X 9) subset(L)

10) prefix(L)
11) suffix(L)
12) substring(L)
13) Quotient (L_1, L_2)
14) $f(L)$
15) $h(L)$
16) ε-free $h(L)$
17) $h^{-1}(L)$

Remember NOT CLOSED:

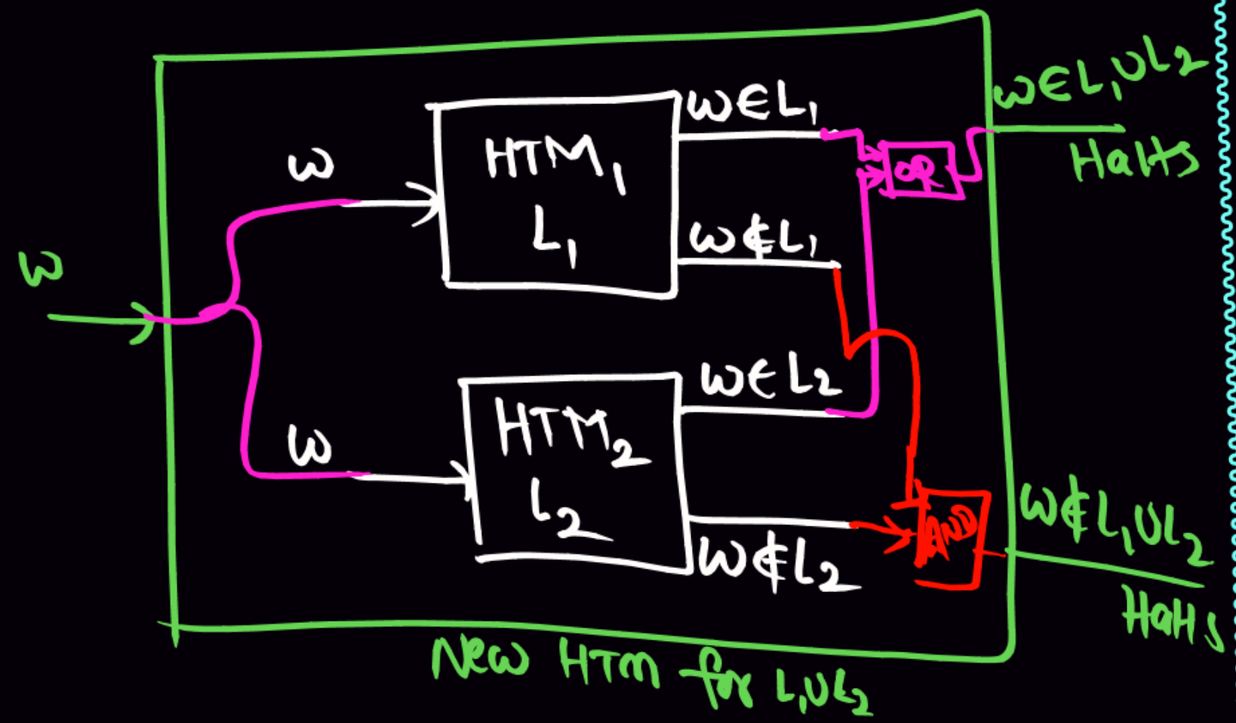
Complement
Difference
Subset

Union:

- closed for Recs
- closed for RELs

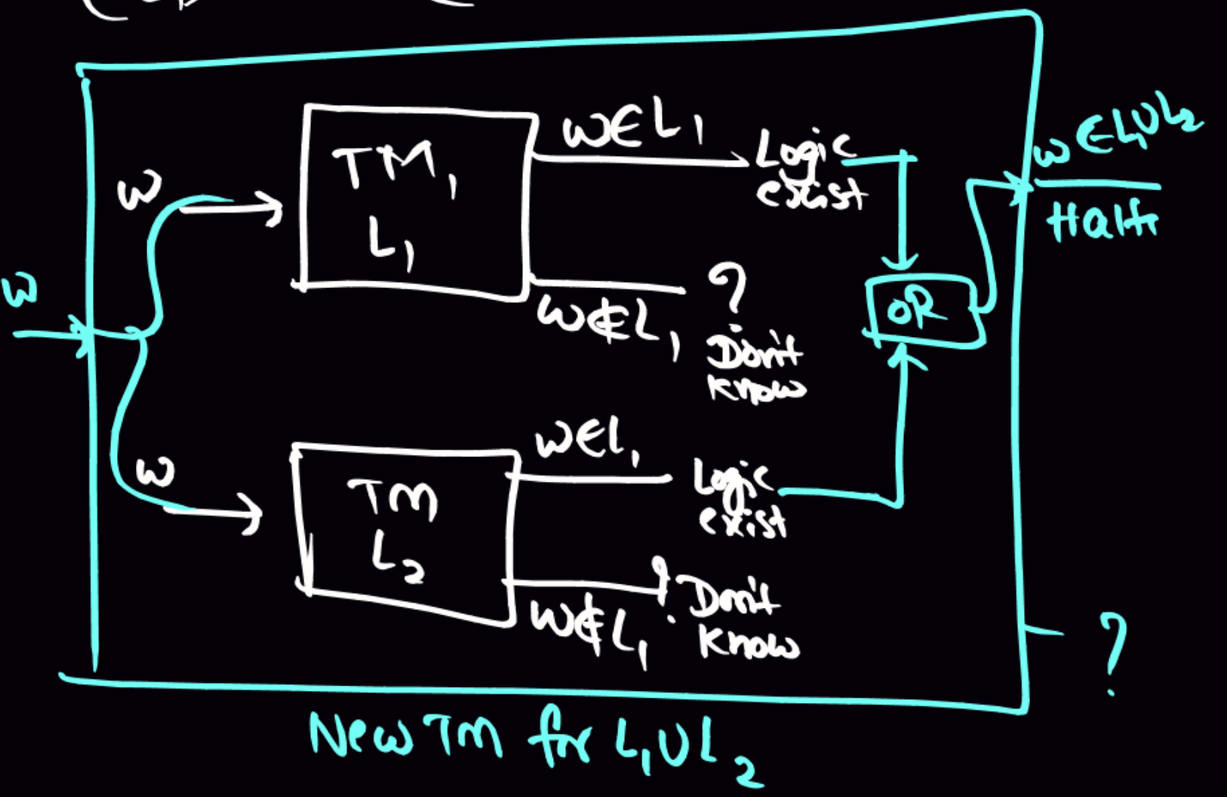
$Rec_1 \cup Rec_2 \Rightarrow Rec$

(L_1) (L_2)



$REL_1 \cup REL_2 \Rightarrow REL$

(L_1) (L_2)

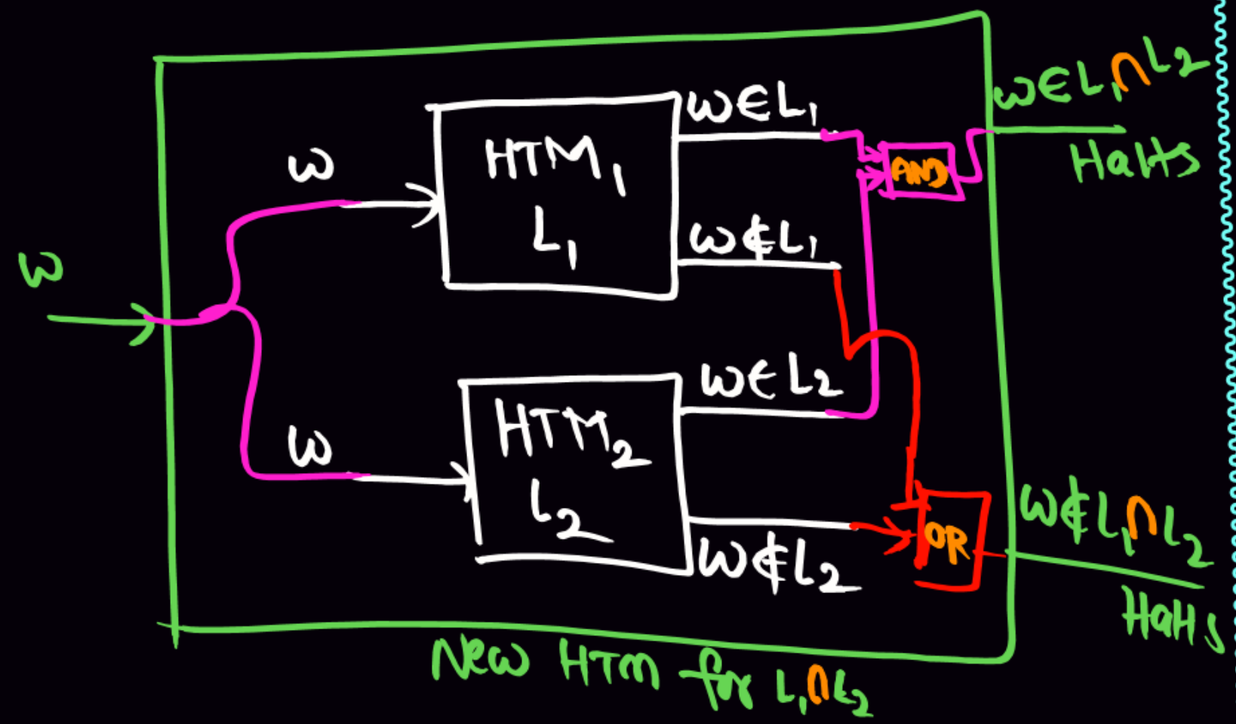


Intersection:

- closed for Recs
- closed for RELs

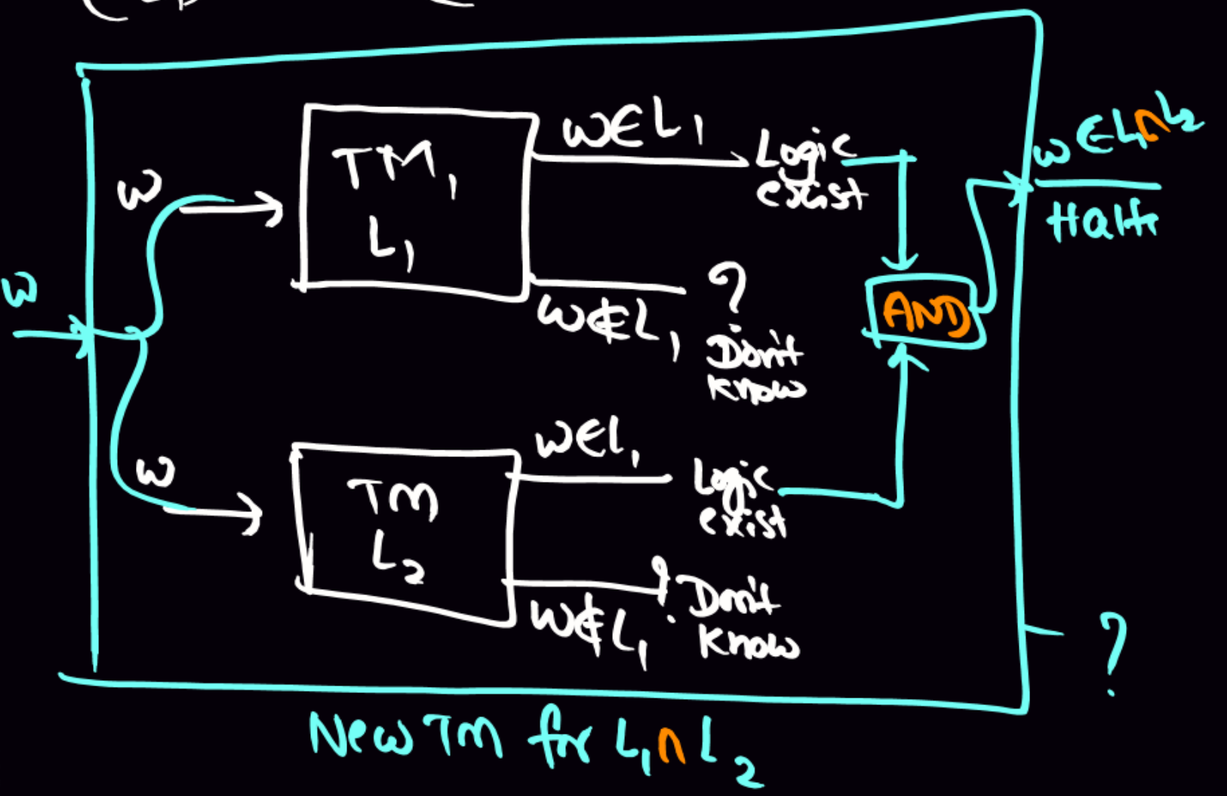
$Rec_1 \cap Rec_2 \Rightarrow Rec$

(L_1) (L_2)



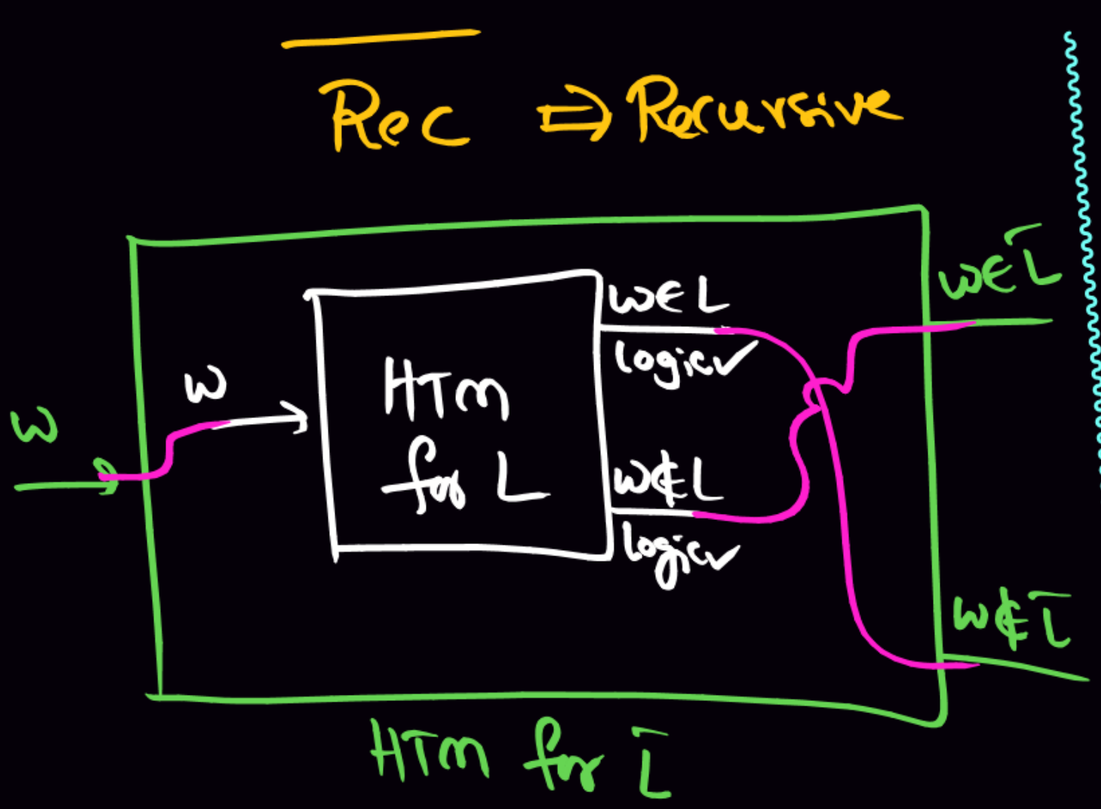
$REL_1 \cap REL_2 \Rightarrow REL$

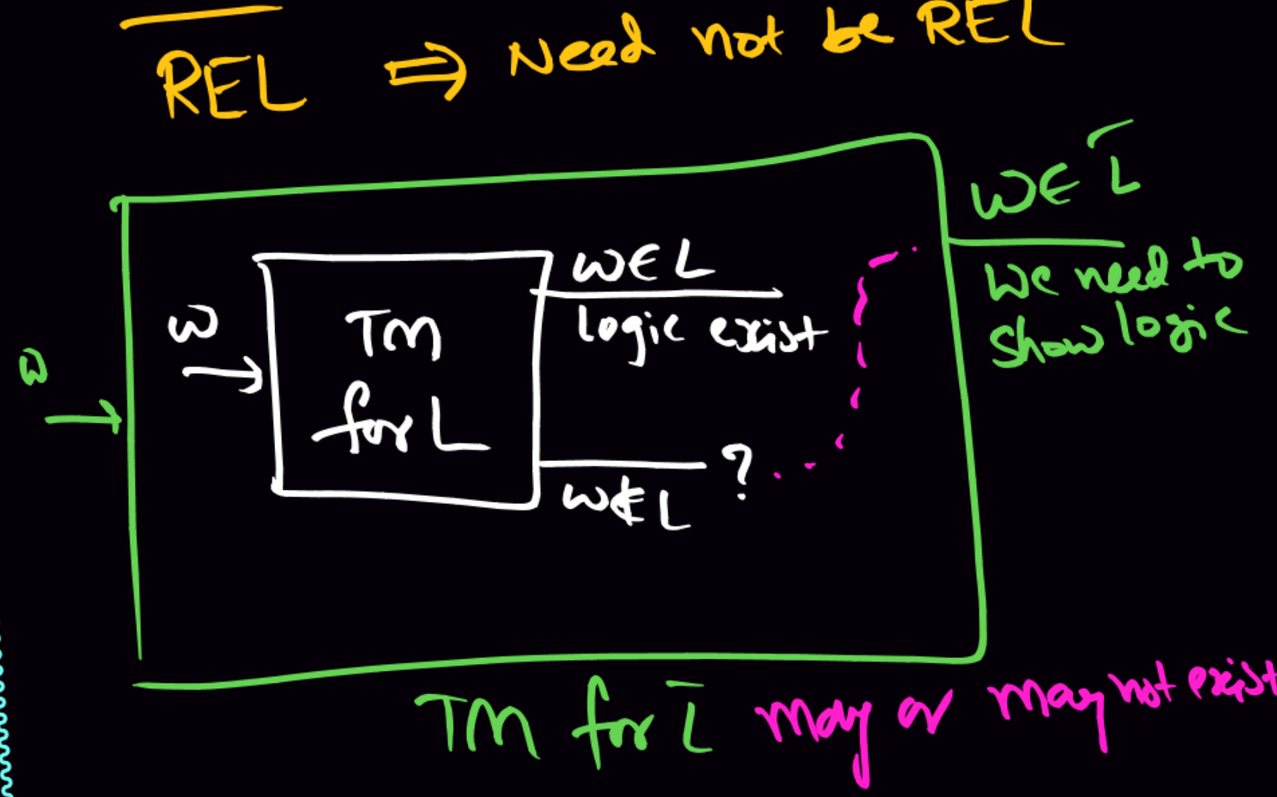
(L_1) (L_2)

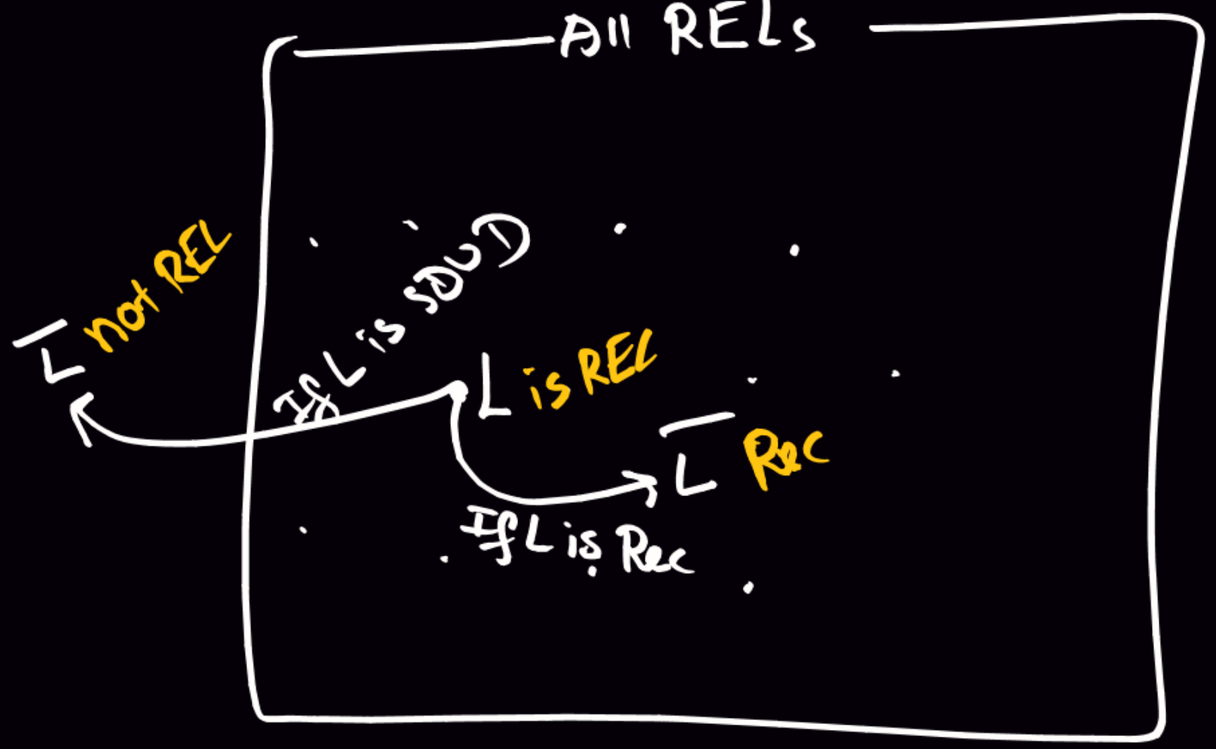


Complement:

- Closed for Recs
- Not closed for RELs







$\overline{REL}$ ⇒ Need not be REL

⇒ either Rec or not REL

⇒ Never be SDUD

D : Htm exist

SDUD : Tm exist but Htm not exist

Not RE : No Tm exist

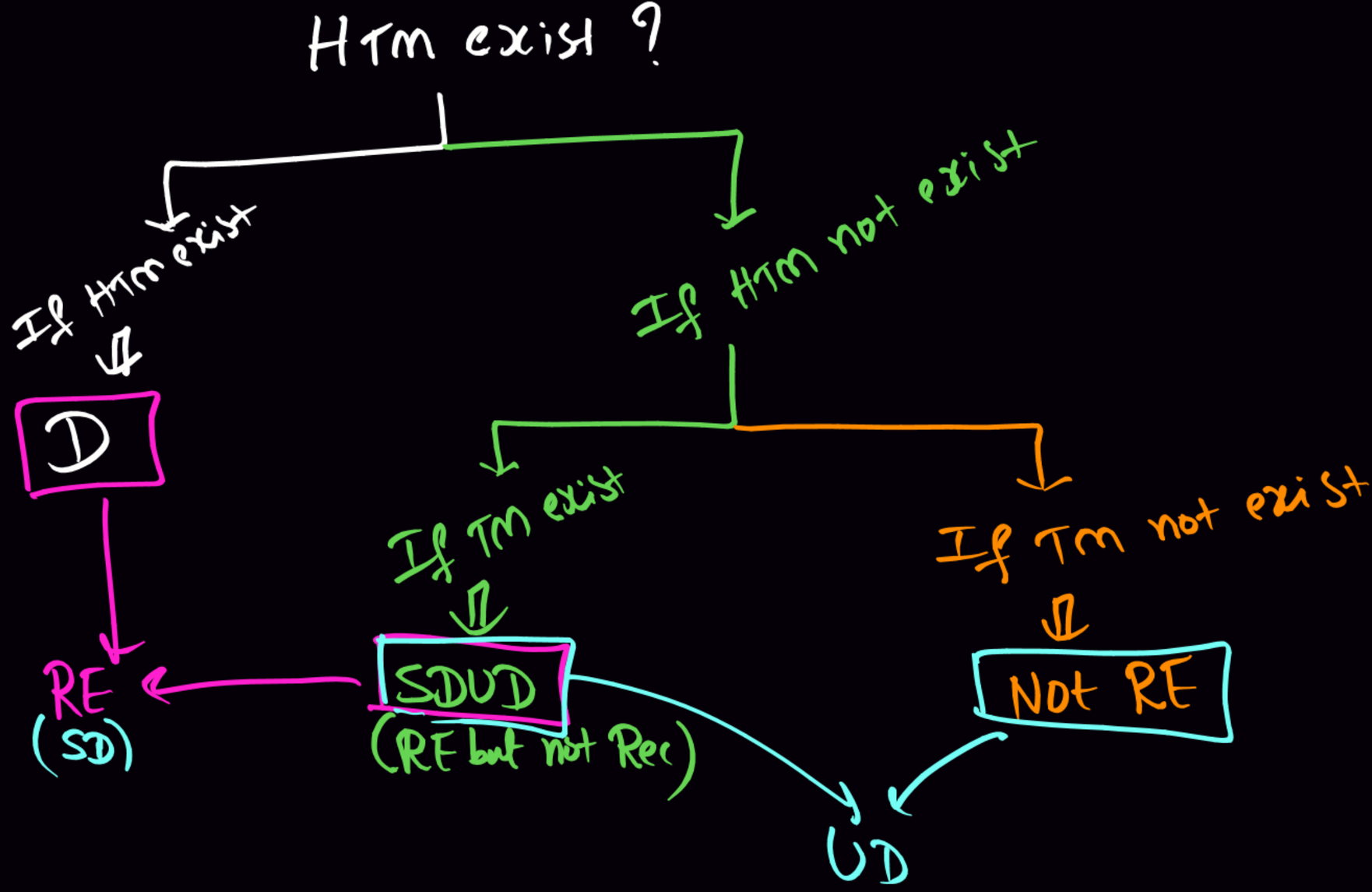
RE : either D or SDUD : Tm exist

UD : either SDUD or Not REL : No Htm exist

Undecidability:

1) Decidable, SDUD, Not RE, RE (SD), and UD Languages

2) " " " " " Problems

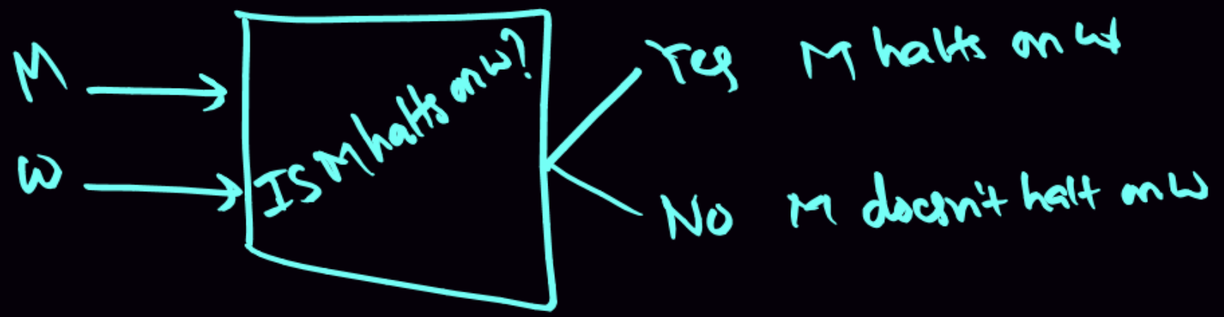

Htm exist ?
If Htm exist
D
RE
(SD)
If Htm not exist
If Tm exist
SDUD
(RE but not Rec)
If Tm not exist
Not RE
UD

Language { L = Set of CFGs = {CFG$_1$, CFG$_2$, CFG$_3$, ...}
= {S → a, S → AB, A → a, B → b, E → E+E, ...}

∈ L
Valid member

Invalid Member
∉ L

Problem { P : Is M halts on w ?



Decision properties

		FA Reg Exp Regulars RGs	DCFLs DPDA	CFGs CFLs PDA	CSLs LBAs	Recs TM	UGs REls TM
1.	Halting Problem	D	D	D	D	UD	
2.	Non-Halting	D	D	D	D	UD	
3.	Membership	D	D	D	D	UD	
4.	Non-Membership	D	D	D	D	UD	
5.	Emptiness	D	D	D	UD	UD	
6.	Non-Emptiness	D	D	D	UD	UD	
7.	Finiteness	D	D	D	UD	UD	
8.	Non-finiteness	D	D	D	UD	UD	
9.	Totality	D	D	UD	UD	UD	
10.	Non-totality	D	D	UD	UD	UD	
11.	Equivalence	D	D	UD	UD	UD	
12.	Non-equivalence	D	D	UD	UD	UD	
13.	Disjoint	D	UD	UD	UD	UD	
14.	Non-disjoint	D	UD	UD	UD	UD	
15.	Set Containment	D	UD	UD	UD	UD	
16.	Non-Set Containment	D	UD	UD	UD	UD	

Halting Problem

Is given M halts on given w ?

Is M halts on w ?

M halts on w

Membership

Is given M accepts given w ?

Is M accepts w ?

M accepts w

Emptiness

Is given M accepts nothing ?

Is $L(M) = \phi$?

M accepts nothing

Finiteness

Is M accepts finite language ?

Is $L(M)$ = finite set ?

M accepts FiniteSet

Totality

Is M accepts everything ?

Is $L(M) = \Sigma^*$?

M accepts Everything

Equivalence

Is $M_1 \cong M_2$?

Is $L_1 = L_2$?

Disjoint

Is $L_1 \cap L_2 = \phi$?

Set containment

Is $L_1 \subseteq L_2$?