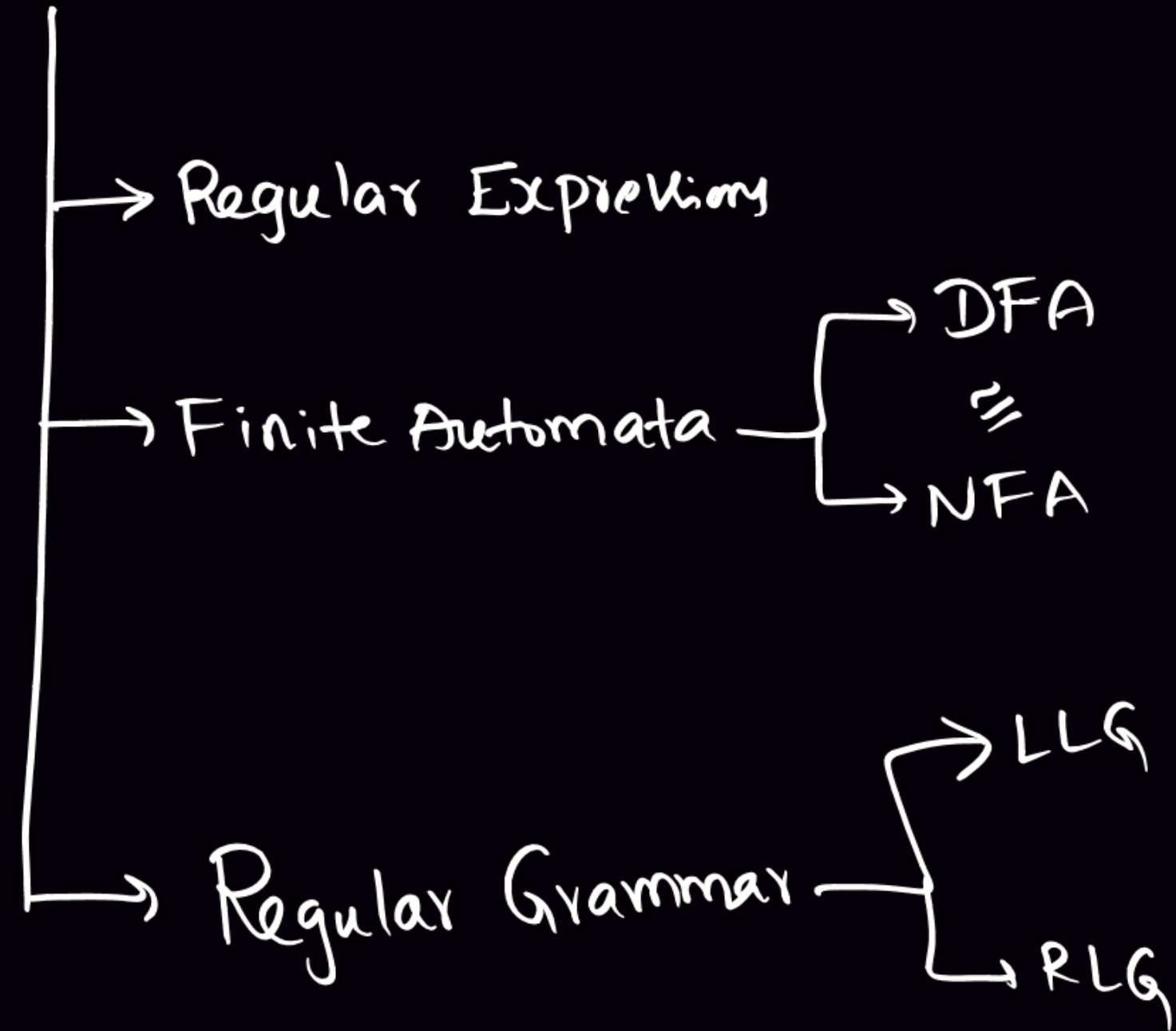
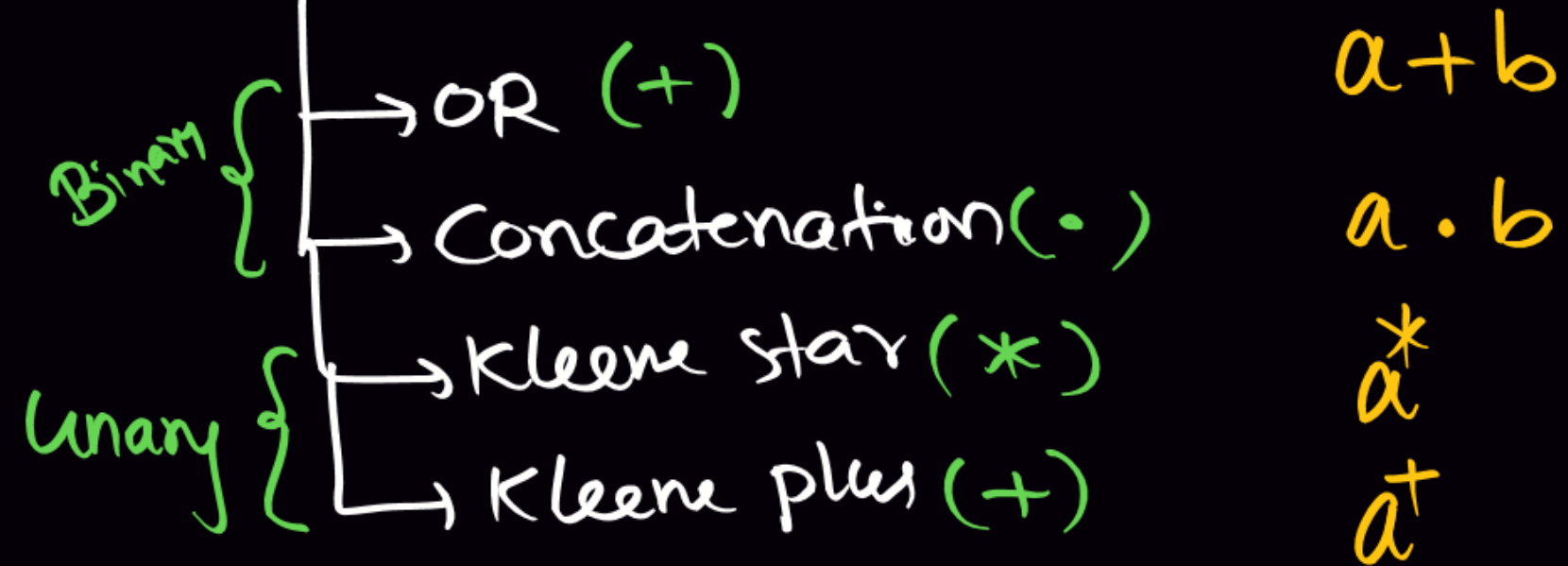


Regular Languages:



Regular Expression

↳ It uses 4 operators to describe a Regular language.



OR

- 1) $+$, \cup , $|$
- 2) $R_1 + R_2$
 $R_1 \cup R_2$
 $R_1 | R_2$
- 3) either R_1 or R_2
- 4) $a+b$
- 5)
 - $\rightarrow a \checkmark$
 - $\rightarrow b \checkmark$
 - $\rightarrow ab \times$
 - $\rightarrow ba \times$

Concatenation

- 1) \cdot
- 2) $R_1 \cdot R_2$
 $R_1 R_2$
- 3) R_1 followed by R_2
- 4) $a \cdot b$
- 5)
 - $\rightarrow ab \checkmark$
 - $\rightarrow ax$
 - $\rightarrow bx$
 - $\rightarrow bax$

Kleene star

- 1) $*$
- 2) R^*
 $R^* = R^0 + R^1 + R^2 + \dots$
- 3) zero or more repetitions of R
- 4) $a^* = a^0 + a^1 + a^2 + a^3 + \dots$
- 5)
 - $\rightarrow a^0 = \epsilon \checkmark$
 - $\rightarrow a^1 = a \checkmark$
 - $\rightarrow a^2 = aa \checkmark$
 - \vdots $a^{\geq 0}$

Kleene plus

- 1) $+$
- 2) R^+
 $R^+ = R^1 + R^2 + R^3 + \dots$
- 3) one or more repetitions of R
- 4) $a^+ = a^1 + a^2 + a^3 + \dots$
- 5)
 - $\rightarrow a^0 \times$
 - $\rightarrow a^1 \checkmark$
 - $\rightarrow a^2 \checkmark$
 - \vdots

Regular Exp
 R

Regular Language
(Regular Set)

ϵ

$$L(\epsilon) = \{\epsilon\}$$

$\epsilon + a$

$$L(\epsilon + a) = \{\epsilon, a\}$$

a^*

$$L(a^*) = \{a^n \mid n \geq 0\} = \{\epsilon, a, a^2, a^3, \dots\}$$

ϕ

\hookrightarrow empty exp

$L(\phi)$

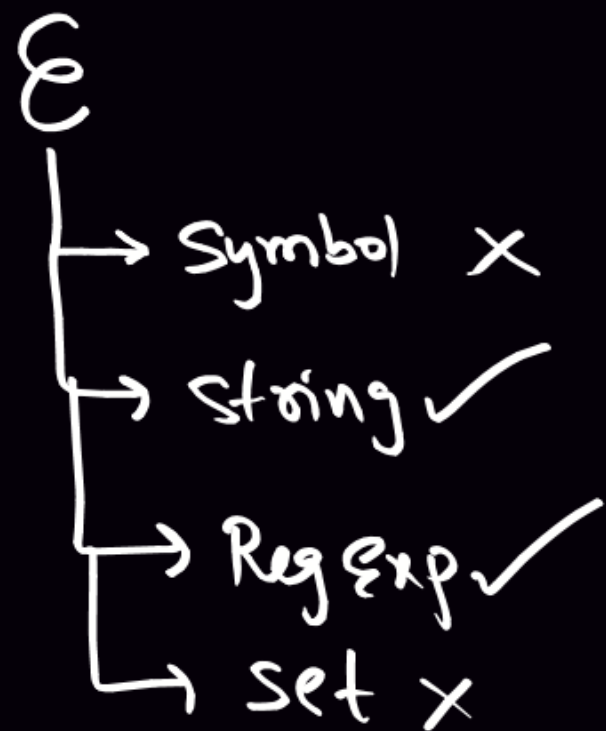
$$= \{\} = \phi$$

\hookrightarrow empty set

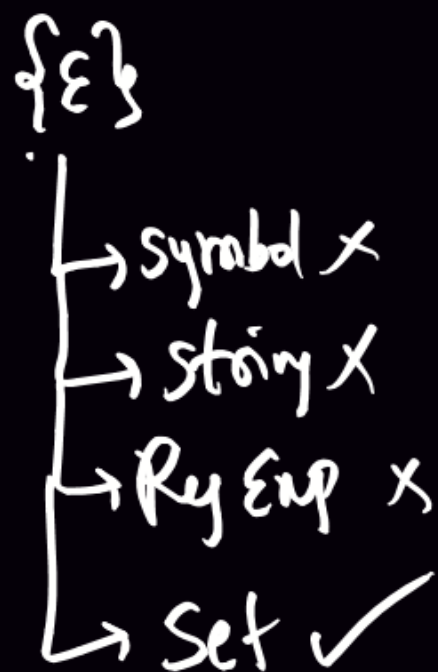
$a + b$

$$L(a + b) = \{a, b\}$$

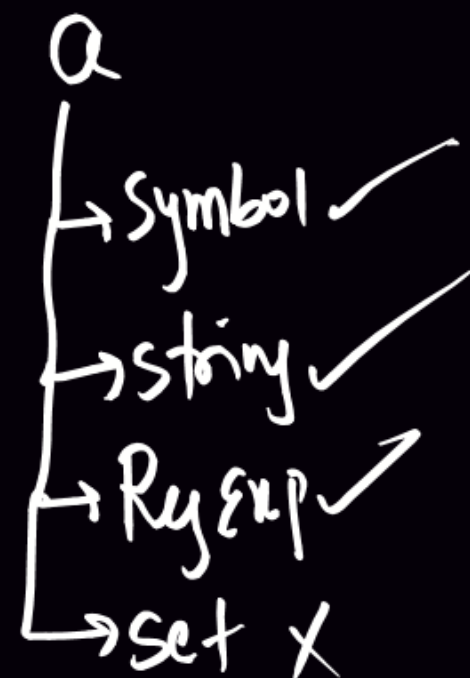
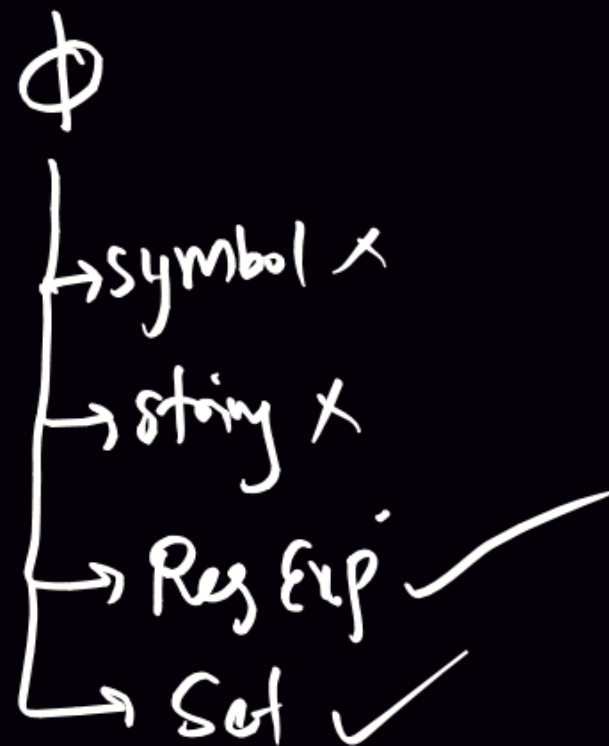
$$\Sigma = \{a, b\}$$



$$L(\varepsilon) = \{\varepsilon\}$$



$$L(\phi) = \phi = \{\}$$



$$1) \phi + \phi = \phi$$

$$2) \varepsilon + \varepsilon = \varepsilon$$

$$3) a + a = a$$

$$4) \boxed{R + R = R}$$

$$5) \phi + \varepsilon = \varepsilon$$

$$6) \varepsilon + \phi = \varepsilon$$

$$7) \phi + a = a$$

$$8) a + \phi = a$$

same {

$$9) a + \varepsilon = \text{same}$$

$$10) \varepsilon + a = \text{same}$$

$$\boxed{a \neq a + \varepsilon}$$

$$11) \phi \cdot \phi = \phi$$

$$12) \varepsilon \cdot \varepsilon = \varepsilon$$

$$13) a \cdot a = aa = a^2$$

$$14) R \cdot R = RR = R^2$$

$$15) \phi \cdot \varepsilon = \phi$$

$$16) \varepsilon \cdot \phi = \phi$$

$$17) \phi \cdot a = \phi$$

$$18) a \cdot \phi = \phi$$

$$19) \varepsilon \cdot a = a$$

$$20) a \cdot \varepsilon = a$$

$$\text{I) } R + \phi = R$$

$$\text{II) } R \cdot \phi = \phi$$

$$\text{III) } R \cdot \varepsilon = R$$

$$21) \quad \phi^* = \phi^0 + \phi^1 + \phi^2 + \dots = \varepsilon + \underbrace{\phi + \phi + \phi + \dots}_{\phi} = \varepsilon + \phi = \boxed{\varepsilon}$$

$$22) \quad \varepsilon^* = \varepsilon$$

$$23) \quad a^* = a^0 + a^1 + a^2 + \dots$$

$$24) \quad R^* = R^0 + R^1 + R^2 + R^3 + \dots$$

$$\boxed{R^0 = \varepsilon}$$

$$25) \quad \phi^+ = \phi$$

$$26) \quad \varepsilon^+ = \varepsilon$$

$$27) \quad a^+ = a + aa + aaa + \dots$$

$$28) \quad R^+ = R^1 + R^2 + R^3 + \dots$$

$$\varepsilon^0 = \varepsilon$$

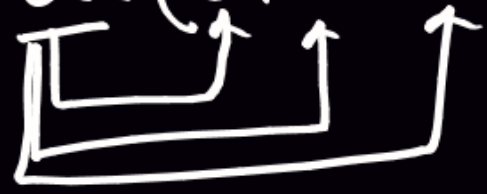
$$a^0 = \varepsilon$$

$$\phi^0 = \varepsilon$$

$$b^0 = \varepsilon$$

$$(ab)^0 = \varepsilon$$

$$(a+b)^0 = \varepsilon$$

$$29) a \cdot a^* = a \cdot (\epsilon + a + aa + \dots) = a\epsilon + a \cdot a + a \cdot aa + \dots$$


$$= a + a^2 + a^3 + \dots = \boxed{a^+}$$

$$30) a^* \cdot a = a^+$$

$$31) a + a^* = a + [\epsilon + a + aa + \dots] = \underline{a} + \epsilon + \underline{a} + a^2 + a^3 + \dots = a^*$$

$$32) a^* + a = a^*$$

$$33) \quad a^+ + a^+ = a^+$$

$$34) \quad a^+ \cdot a^+ = \underbrace{(a^1 + a^2 + a^3 + \dots)}_{\substack{\uparrow \\ a^1}} \cdot \underbrace{(a^1 + a^2 + a^3 + \dots)}_{\substack{\uparrow \quad \uparrow \quad \uparrow \\ a^2 \quad a^3 \quad a^4}} \\ = (a^+)^2$$

$$= a \cdot a + a a^2 + a \cdot a^3 + \dots + \underbrace{a^2 \cdot a}_{a^3} + \underbrace{a^2 \cdot a^2}_{a^4} + \underbrace{a^2 \cdot a^3}_{a^5} + \dots$$

$$= a^2 + a^3 + a^4 + \dots$$

$$= a^2 a^* = a \underbrace{a a^*} = a a^+ = a^+ a$$

$$35) (a^*)^{100} = \overset{*}{a} \overset{*}{a} \overset{*}{a} \dots 100 \text{ times} = \overset{*}{a}$$

$$36) (a^{100})^* = \text{same}$$

$$37) (a^*)^* = (a^*)^0 + (a^*)^1 + (a^*)^2 + \dots = \overset{*}{a}$$

$$38) (a^*)^+ = (\overset{*}{a})^1 + (\overset{*}{a})^2 + \dots = \overset{*}{a}$$

$$39) (a^+)^* = \underbrace{(a^+)^0 + (a^+)^1 + (a^+)^2 + \dots}_{\text{same}} = \overset{*}{a}$$

$$40) (a^+)^+ = \overset{+}{a}$$

$$\text{I)} \quad (a^*)^2 = a^* \cdot a^* = a^*$$

$$\text{II)} \quad (a^2)^* = (aa)^* = \varepsilon + aa + a^4 + a^6 + \dots$$

$$(a^*)^2 \neq (a^2)^*$$

Note:

$$\text{I) } a^* = \varepsilon + a^+ = (a + \varepsilon)^* = (a + \varepsilon)^+ = (a^*)^2 = (a^*)^* = (a^*)^+ = (a^+)^*$$

$$\begin{aligned} \text{II) } (a+b)^* &= \varepsilon + (a+b)^+ &= (a^*b)^*a^* &= (a+b)^*a^* \\ &= (\varepsilon + a + b)^+ &= (b^*a)^*b^* &= a^*(a+b)^* \\ &= (\varepsilon + a + b)^* &= a^*(ba^*)^* &= \\ &= (a^*b^*)^* &= b^*(ab^*)^* &= \\ &= (b^*a^*)^* &= (a^*+b^*)^+ &= (a^*+b)^* = (a^++b)^+ \\ &= (a^*+b^*)^* & & \end{aligned}$$

$(a^*b^*)^*$ \Rightarrow It will generate every string

$$\epsilon = ()^0 = (a^0b^0)^1 = (a^0b^0)^2 = \dots$$

$$a = (a^1b^0)^1 = (a\epsilon)^1 = a$$

$$b = (a^0b^1)^1$$

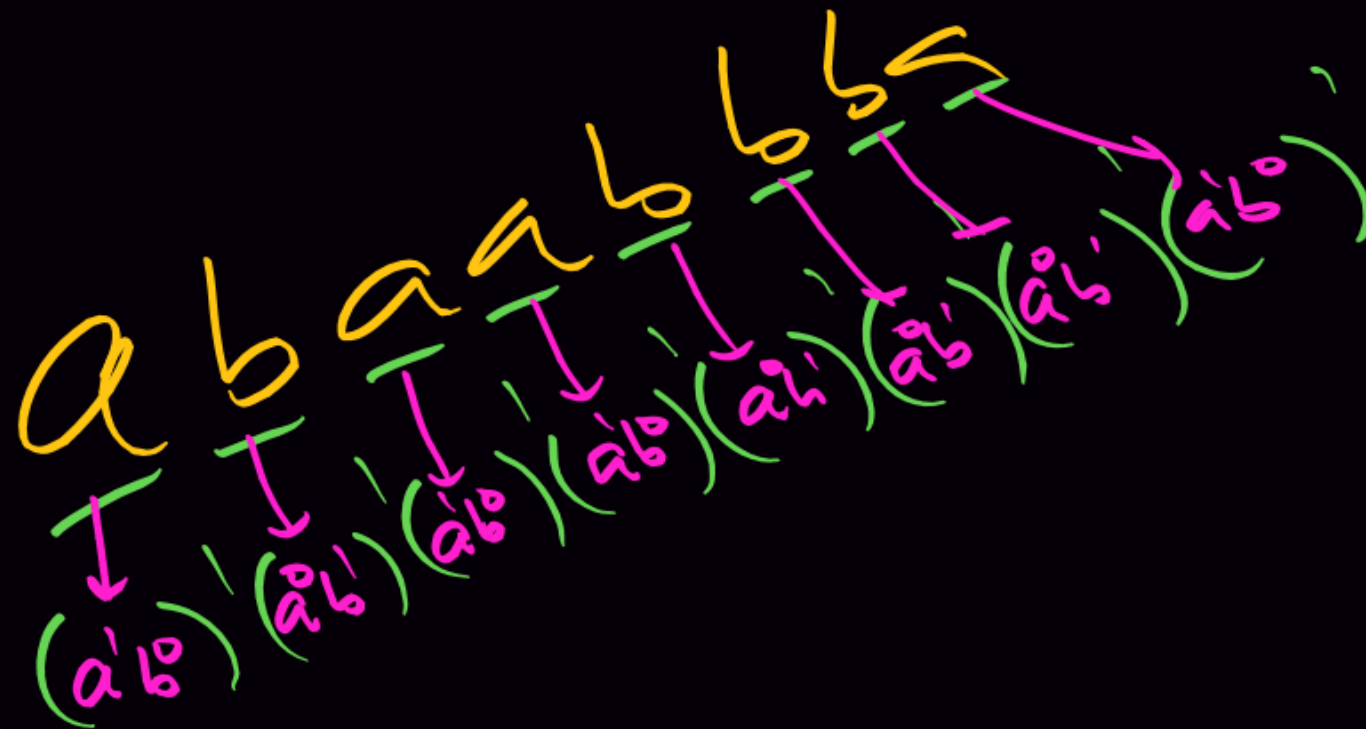
$$aa = (a^2b^0)^1$$

$$ab = (a^1b^1)^1$$

$$ba = (a^0b^1)^1(a^1b^0)^1$$

$$bb = (a^0b^2)^1$$

\vdots



$$(a^*b^*)^* \neq a^*b^*$$

$$(a+\varepsilon)^+ = (a+\varepsilon)^1 + (a+\varepsilon)^2 + (a+\varepsilon)^3 + \dots \left\{ \begin{array}{l} R^+ = R^1 + R^2 + R^3 + \dots \\ \dots \end{array} \right.$$

$$= \underbrace{a+\varepsilon} + \underbrace{\varepsilon+a+a\varepsilon} + \underbrace{\varepsilon+a+a^2+a^3}_{\dots}$$

$$= \varepsilon + a + a^2 + a^3 + \dots$$

$$= a^*$$

$$a+\varepsilon \begin{cases} \rightarrow \varepsilon \checkmark \\ \rightarrow a \checkmark \end{cases}$$

$$(a+\varepsilon)^2 = (a+\varepsilon)(a+\varepsilon)$$

$$\begin{cases} \rightarrow \varepsilon \checkmark \\ \rightarrow a \checkmark \\ \rightarrow aa \checkmark \end{cases}$$

Note:

$$\text{I)} \quad a.b \neq b.a \quad [\cdot \text{ is not commutative}]$$

$$\text{II)} \quad a+b = b+a \quad [+ \text{ is commutative}]$$

$$\text{III)} \quad R+\phi = \phi+R = R \quad [\phi \text{ is identity exp for } +]$$

$$\text{IV)} \quad R.\varepsilon = \varepsilon.R = R \quad [\varepsilon \text{ is identity exp for } \cdot]$$

$$\text{V)} \quad R.\phi = \phi.R = \phi \quad [\phi \text{ is dominator for } \cdot \text{ (annihilator)}]$$

$$\text{VI)} \quad R+\Sigma^* = \Sigma^*+R = \Sigma^* \quad [\Sigma^* \text{ is dominator for } +]$$

$$\begin{array}{l} \text{VII)} \quad a \cdot (b+c) = ab+ac \\ \text{VIII)} \quad (b+c) \cdot a = ba+ca \end{array} \left. \vphantom{\begin{array}{l} \text{VII)} \\ \text{VIII)} \end{array}} \right\} \cdot \text{can be distributed over } +$$

$$\text{IX)} \quad a+(b \cdot c) \neq (a+b) \cdot (a+c)$$

$$\Sigma^* = \Sigma^0 + \Sigma^1 + \Sigma^2 + \Sigma^3 + \dots$$

$\Sigma = \{a\}$

$$a^* = a^0 + a^1 + a^2 + a^3 + \dots$$

$$= \epsilon + a + aa + aaa + \dots$$

generates all strings over $\Sigma = \{a\}$

$$(a+b)(a+b)$$

$$aa + ab + ba + bb$$

$\Sigma = \{a, b\}$

$$\Sigma^* = \{a, b\}^* = (a+b)^* = \underbrace{(a+b)^0}_{\epsilon} + \underbrace{(a+b)^1}_{a+b} + \underbrace{(a+b)^2}_{a+ab+ba+bb} + \dots$$

Write Regular Expression for following Regular Languages.

1) $L = \{ \}$ over $\Sigma = \{a\}$ 2) $L = \{ \}$ over $\Sigma = \{a, b\}$ 3) $L = \{ \}$ over $\Sigma = \{a, b, c\}$

$$\begin{aligned} R &= \phi \\ &= \phi.a \\ &= \phi.a^* \\ &= \phi.aa \\ &\vdots \end{aligned}$$

$$\begin{aligned} R &= \phi \\ &= \phi.a \\ &= b.\phi \\ &= a.\phi.b \\ &= a^*.\phi \\ &= b^*.\phi \\ &\vdots \end{aligned}$$

$$R = \phi$$

$$4) L = \{\epsilon\} \text{ over } \Sigma = \{a, b\} \implies R = \epsilon = \phi^* = a^0 = (ab)^0$$

$$5) L = \{w \mid w \in (a+b)^*, |w| \geq 1\} \implies (a+b)^+$$

$$6) L = \{w \mid w \in (a+b)^*, |w| \geq 0\} \implies (a+b)^*$$

$$7) L = \{w \mid w \in (a+b)^*, |w| = 2\} \implies (a+b)^2 = aa + ab + ba + bb$$

$$8) L = \{w \mid w \in (a+b)^*, |w| \leq 2\} \implies \epsilon + (a+b) + (a+b)^2 = (\epsilon + a + b)^2$$

$$9) L = \{w \mid w \in (a+b)^*, |w| \geq 2\} \implies (a+b)^2 (a+b)^*$$

$$10) L = \{w \mid w \in (a+b)^*, |w| \leq 100\} \implies (\epsilon + a + b)^{100}$$

$\epsilon \rightarrow \text{zero length}$

$a+b \rightarrow 1 \text{ len}$

$\epsilon+a+b \rightarrow \underbrace{0 \text{ or } 1 \text{ length}}_{\text{at most } 1 \text{ len}}$

$$(\epsilon+a+b)^2 = (\epsilon+a+b)(\epsilon+a+b)$$

$$= \underbrace{\epsilon + a + b + aa + ab + ba + bb}_{\text{at most } 2 \text{ length}}$$

$$(\epsilon+a+b)^{100} = \text{All strings upto } 100 \text{ length}$$

$$11) L = \{w \mid w \in (a+b)^*, w \text{ starts with 'a'}\} \Rightarrow a(a+b)^* = a\Sigma^*$$

$$12) L = \{w \mid w \in (a+b)^*, w \text{ starts with aa or bb}\} = (aa+bb)\Sigma^*$$

$$13) L = \{w \mid w \in (a+b)^*, w \text{ ends with 'a'}\} = \Sigma^*a = (a+b)^*a$$

$$14) L = \{w \mid w \in (a+b)^*, w \text{ ends with aa or bb}\} = \Sigma^*(aa+bb)$$

$$15) L = \{w \mid w \in (a+b)^*, w \text{ contains 'a'}\} = \Sigma^*a\Sigma^* = (a+b)^*a(a+b)^*$$

$$16) L = \{w \mid w \in (a+b)^*, w \text{ contains aa or bb}\} = \Sigma^*(aa+bb)\Sigma^*$$

$$17) \{a^m b^n \mid m, n \geq 0\} \Rightarrow a^* b^*$$

$$18) \{a^m b^n \mid m \geq 0, n \geq 1\} \Rightarrow a^* b^+$$

$$19) \{a^m b^n \mid m \geq 1, n \geq 0\} \Rightarrow a^+ b^*$$

$$20) \{a^m b^n \mid m \geq 1, n \geq 1\} \Rightarrow a^+ b^+$$

$$21) \{a^m \mid m = \text{even}\} \Rightarrow (aa)^*$$

$$22) \{a^m \mid m = \text{odd}\} \Rightarrow a(aa)^* = (aa)^* a$$

$$23) \{a^m b^n \mid m = \text{even}, n = \text{odd}\} \Rightarrow (aa)^* b(bb)^*$$

$$24) \{a^m b^n \mid m+n = \text{even}\} \Rightarrow \overset{\text{even}}{a} \overset{\text{even}}{b} + \overset{\text{odd}}{a} \overset{\text{odd}}{b} \Rightarrow (aa)^* (bb)^* + a(aa)^* b(bb)^*$$

$$25) \{a^m b^n \mid m+n = \text{odd}\} \Rightarrow \overset{\text{odd}}{a} \overset{\text{even}}{b} + \overset{\text{even}}{a} \overset{\text{odd}}{b} = a(aa)^* (bb)^* + (aa)^* b(bb)^*$$

$m+n = \text{even}$

\Downarrow

Both m & n are even

OR

Both m & n are odd

