Find CFL generated by following CFGs:

1) $S \to a \mid \varepsilon$

$L = \{\varepsilon, a\}$

2) $S \to Sa \mid \varepsilon$

$L = a^*$

3) $S \to aS \mid \varepsilon$

$L = a^*$

4) $S \to Sa \mid a$

$L = a^+$

5) $S \to aS \mid a$

$L = a^+$

6) $S \to aS \mid bS \mid \varepsilon$

$L = (a+b)^*$

7) $S \to Sa \mid Sb \mid \varepsilon$

$L = (a+b)^*$

8) $S \to aS \mid bS \mid a \mid b$

$L = (a+b)^+$

9) $S \to Sa \mid Sb \mid a \mid b$

$L = (a+b)^+$

10) $S \to AB$

$A \to aA \mid \varepsilon$

$B \to bB \mid \varepsilon$

$L = AB = a^*b^*$

11) $S \to aSb \mid \varepsilon$

$L = \{a^n b^n \mid n \geq 0\}$

12) $S \to aSb \mid ab$

$L = \{a^n b^n \mid n \geq 1\}$

13) $S \to aSb \mid a$

$L = \{a^n a b^n \mid n \geq 0\} = \{a^{n+1} b^n\}$

14) $S \to aSb \mid b$

$L = \{a^n b^{n+1} \mid n \geq 0\}$

15) $S \to aSb \mid A$, $A = c^*$

$A \to cA \mid \varepsilon$

$L = \{a^n c^* b^n\}$

$$S \rightarrow aSb \mid A$$

$$A \rightarrow cA \mid \varepsilon$$

$$A = c^*$$

$$L = a^n S b^n$$

$$= a^n A b^n$$

$$= a^n c^* b^n$$

16) $S \to aSbb \mid \varepsilon \quad L = \{a^n b^{2n} \mid n \geq 0\}$

17) $S \to aaSb \mid \varepsilon \quad L = a^{2n} b^n$

18) $S \to aS \mid Sb \mid \varepsilon \quad L = a^* b^*$

19) $S \to bS \mid Sa \mid \varepsilon \quad L = b^* a^*$

20) $S \to aSb \mid aSbb \mid aSbbb \mid \varepsilon$

$L = \{a^m b^n \mid m \leq n \leq 3m\}$

21) $S \to aSb \mid aaSb \mid aaaSb \mid \varepsilon$

$L = \{a^m b^n \mid n \leq m \leq 3n\}$

22) $S \to aSa \mid bSb \mid \varepsilon$

$L = \{ww^R \mid w \in (a+b)^*\}$

23) $S \to aSa \mid bSb \mid a \mid b$

$L = \{w(a+b)w^R \mid w \in (a+b)^*\}$

24) $S \to aSa \mid bSb \mid \varepsilon \mid a \mid b$

$L = \{w \mid w \in (a+b)^*, w = w^R\}$

25) $S \to AB$

$A \to aA \mid \varepsilon \quad A = a^*$

$B \to aBb \mid \varepsilon \quad B = a^n b^n$

$L = \{a^i b^j \mid i \geq j\}$

26) $S \to AB$

$A \to aAb \mid \varepsilon \quad A = a^n b^n$

$B \to bB \mid \varepsilon \quad B = b^*$

$L = a^n b^n b^*$

$L = \{a^i b^j \mid j \geq i\}$

(20) $S \to aSb \mid aSbb \mid aSbbb \mid \varepsilon$

$$\{ a^m b^n \mid m \le n \le 3m \}$$

ε ✓

ab ✓
abb ✓
abbb ✓

a^1b^1
a^1b^2
a^1b^3

aabb
a^2b^3
a^2b^4
a^2b^5
a^2b^6

#a's ≤ #b's ≤ 3*#a's

27) $S \to aSbS \mid bSaS \mid \varepsilon$

28) $S \to SS \mid aSb \mid bSa \mid \varepsilon$

29) $S \to SaSbS \mid SbSaS \mid \varepsilon$

30) $S \to SaSb \mid SbSa \mid \varepsilon$

$L = \{ w \mid w \in (a+b)^* / n_a(w) = n_b(w) \}$

31) $S \to aS \mid \varepsilon$

$L = a^*S = a^*\varepsilon = a^*$

32) $S \to AS \mid \varepsilon$

$A \to aAb \mid \varepsilon$

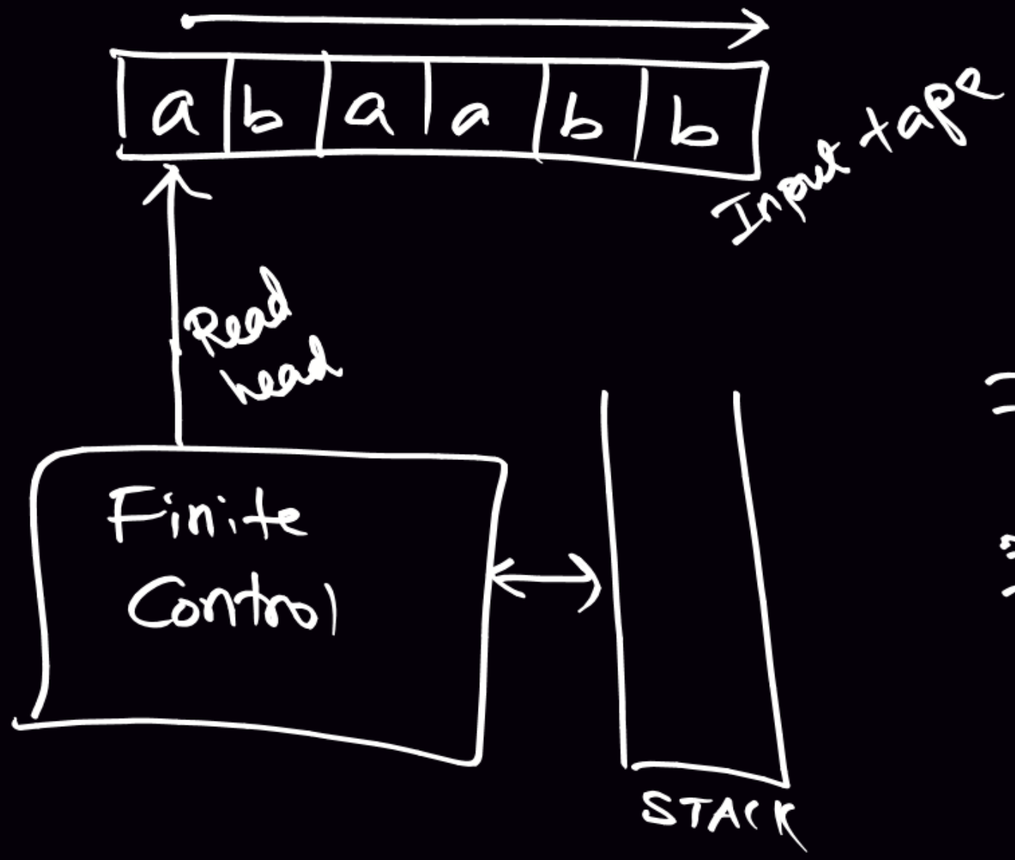
$L = A^*$

$= \{a^n b^n / n \geq 0\}^*$

$= \{a^{n_1} b^{n_1} \; a^{n_2} b^{n_2} \; a^{n_3} b^{n_3} \cdots a^{n_k} b^{k} \mid$

$k \geq 0, n_1, n_2, n_3, \ldots, n_k \geq 0\}$

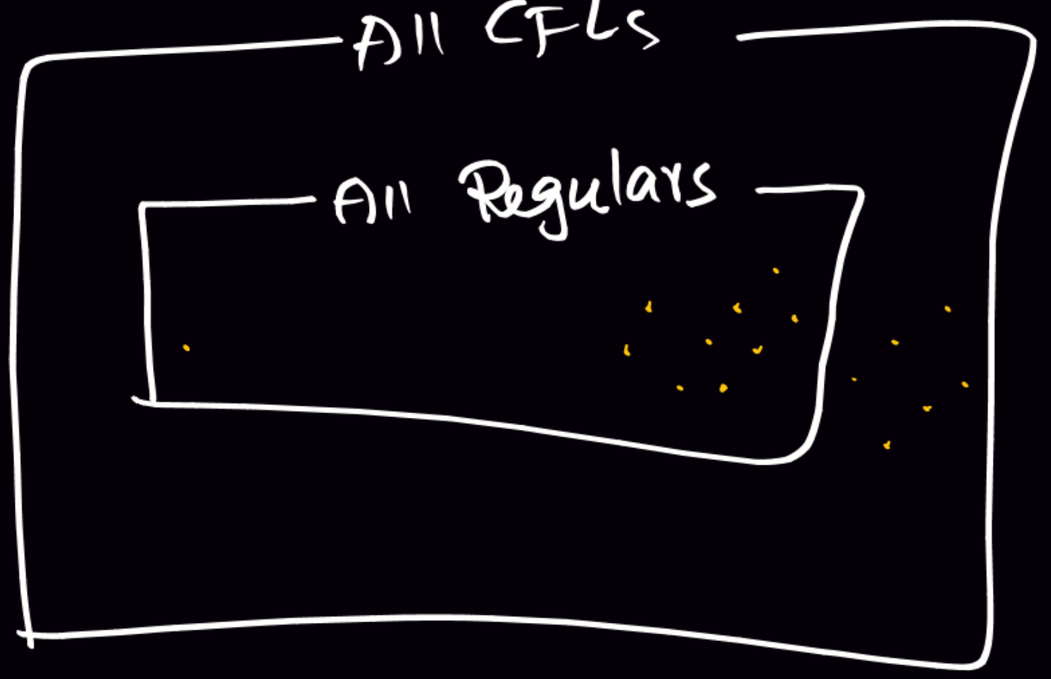
Push Down Automata (Non-deterministic PDA)



$\equiv$ PDA

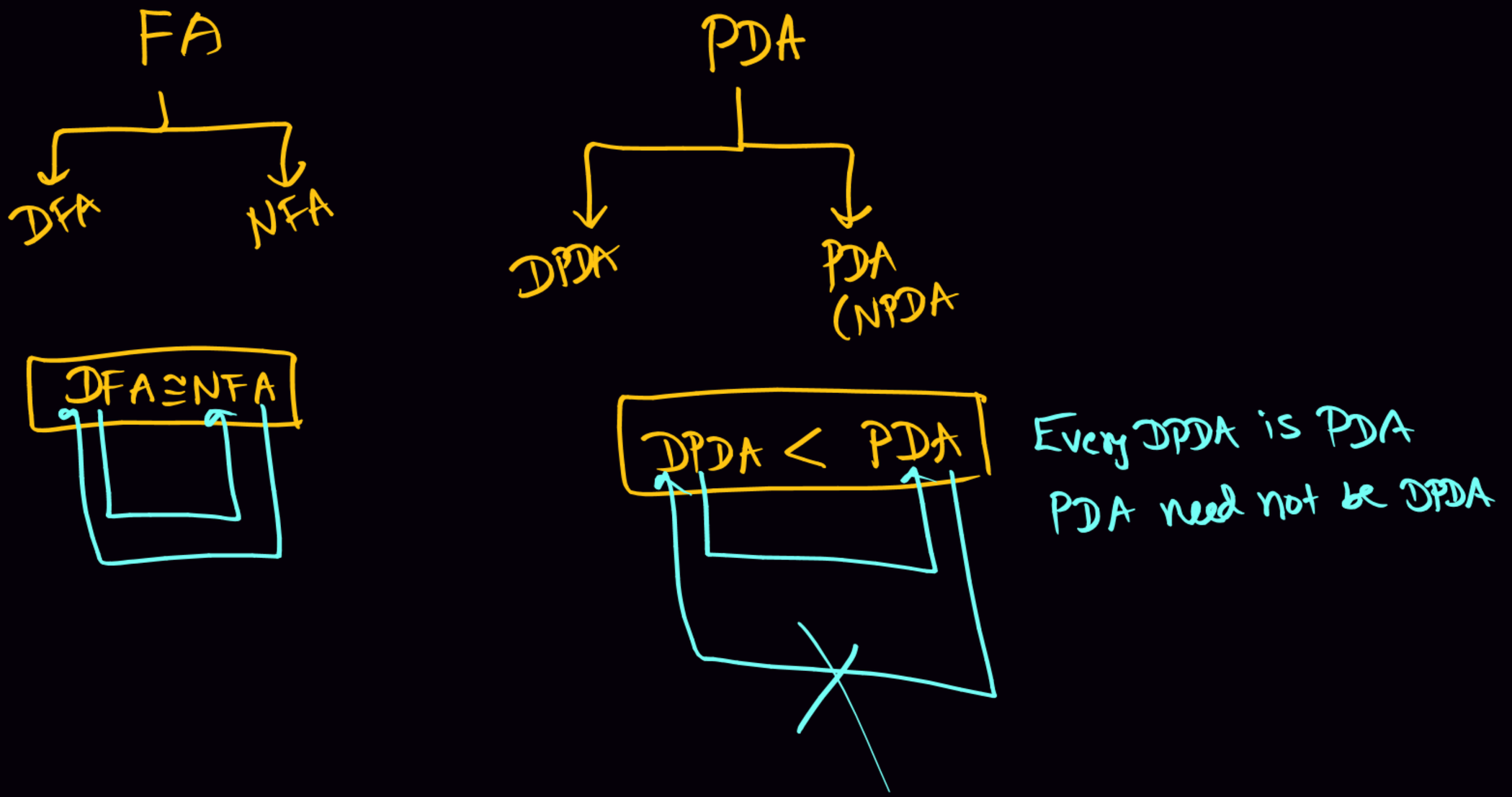
$\equiv$ FA + 1 stack

FA < PDA



Every Reg is CFL

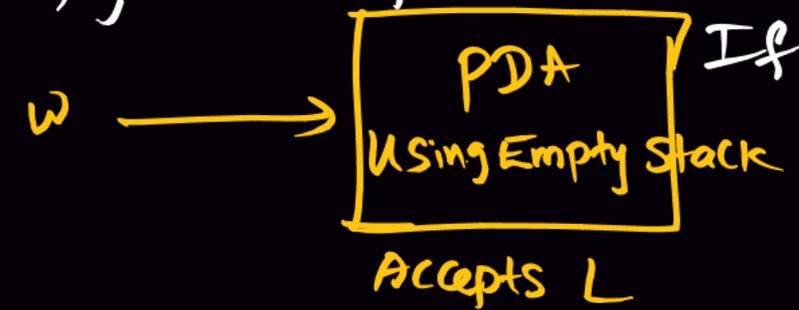
Some CFLs are not regulars
Some CFLs are regulars


FA
DFA
NFA
DFA ≅ NFA
PDA
DPDA
PDA
(NPDA
DPDA < PDA
Every DPDA is PDA
PDA need not be DPDA

PDA Using Final State Acceptance:

If $w \in L$ string is valid then
Atleast one path halts at final state

PDA Using Empty stack Acceptance:



If $w \in L$, atleast 1 path makes stack as empty.

PDA Using F.S. Acceptance
$\cong$
PDA Using E.S. Acceptance
$\cong$
PDA

$\cong$ CFL

$PDA = (Q, \Sigma, \delta, q_0, F, Z_0, \Gamma)$

→ Stack Alphabet
(Set of all stack symbols)

→ Bottom of stack symbol
($\perp$)

DPDA:
→ $Q \times \Sigma \times \Gamma \to Q \times \Gamma^*$

→ PDA:
$Q \times \Sigma_\varepsilon \times \Gamma^* \to 2^{Q \times \Gamma^*}$

DPDA

$$Q \times \Sigma \times \Gamma \to Q \times \Gamma^*$$

PDA

$$Q \times \Sigma_\varepsilon \times \Gamma^* \to 2^{Q \times \Gamma^*}$$

$k \geq 1$
LR(k) parser
$\cong$
DPDA

$k \geq 1$
LR(k) language
$=$
DCFL

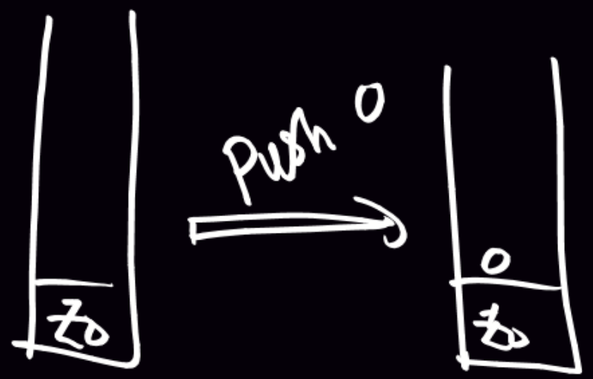
$Q \times \Sigma \times \Gamma \longrightarrow Q \times \Gamma^*$

1) $\delta(A, 0, Z_0) = (A, 0Z_0)$

State — i/p symbol — top of stack symbol — State — Sequence of stack symbols

Operation

Γ / Γ^*



Push

Pop

No operation

ε means zero symbols

2) $\delta(q, a, x) = (q', \varepsilon)$ Pop x

3) $\delta(q, a, x) = (q', x)$ No operation

4) $\delta(q, a, x) = (q', xxx)$ Push $\underbrace{xx}_{2\,x's}$

5) $\delta(q, \varepsilon, x) = (q', \varepsilon)$ pop x

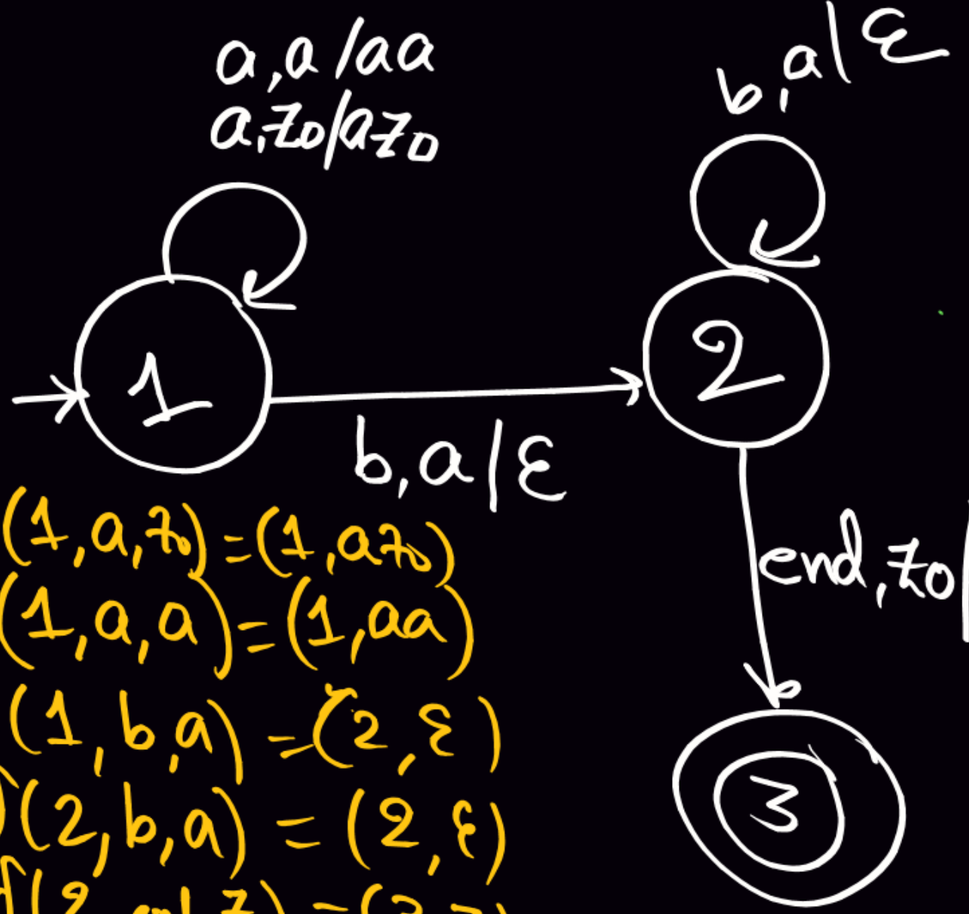
6) $\delta(q, \varepsilon, \varepsilon) = (q', x)$ Push x

7) $\delta(q, \varepsilon, \varepsilon) = (q', \varepsilon)$ No operation

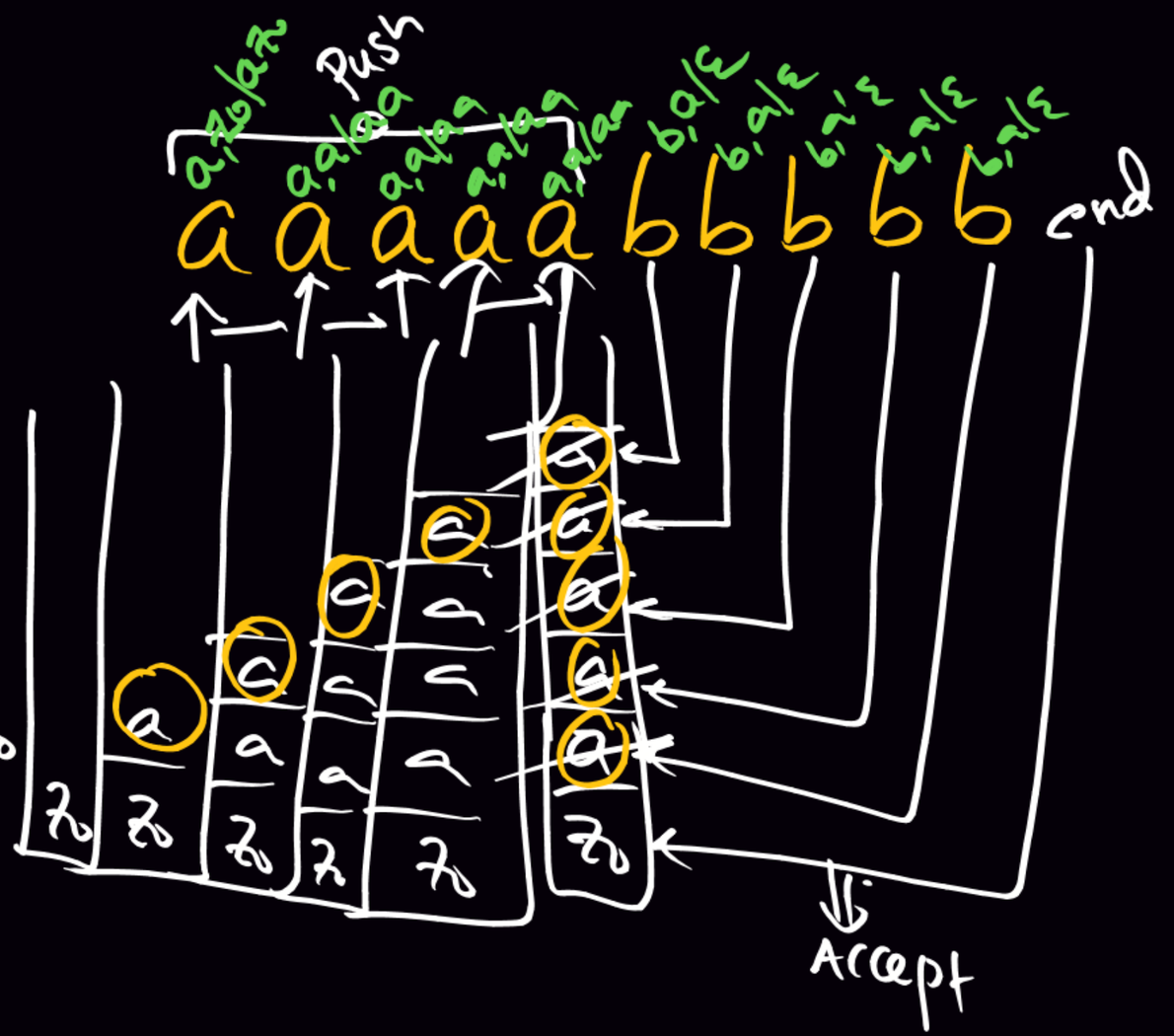
Construction:

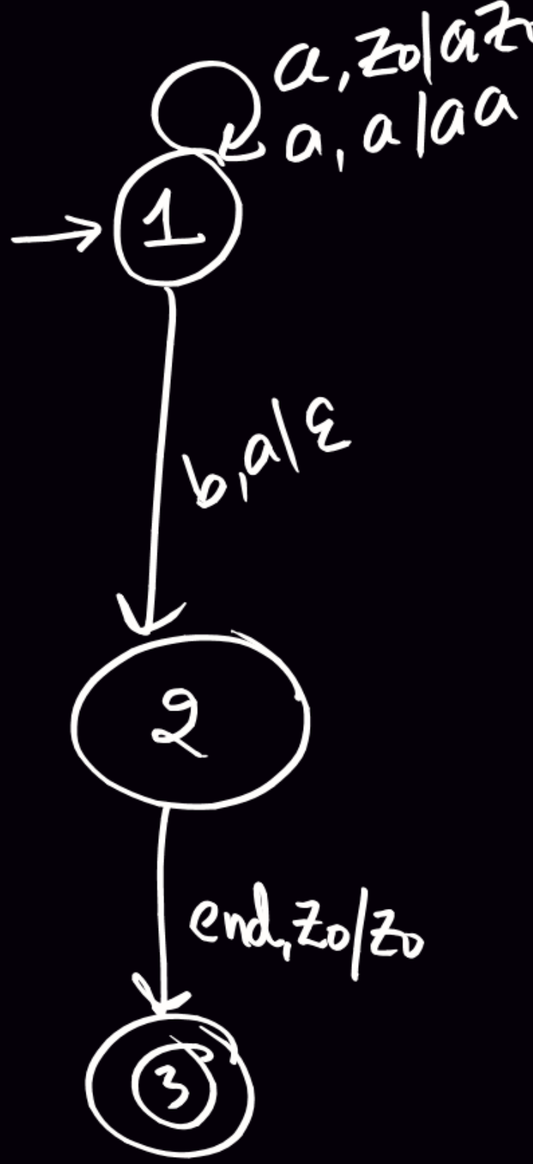
$Q \times \Sigma \times \Gamma \rightarrow Q \times \Gamma^*$

1) $\{a^n b^n \mid n \geq 1\}$



$\delta(1,a,z_0) = (1,az_0)$
$\delta(1,a,a) = (1,aa)$
$\delta(1,b,a) = (2,\varepsilon)$
$\delta(2,b,a) = (2,\varepsilon)$
$\delta(2,end,z_0) = (3,z_0)$





$x\varepsilon$: 1

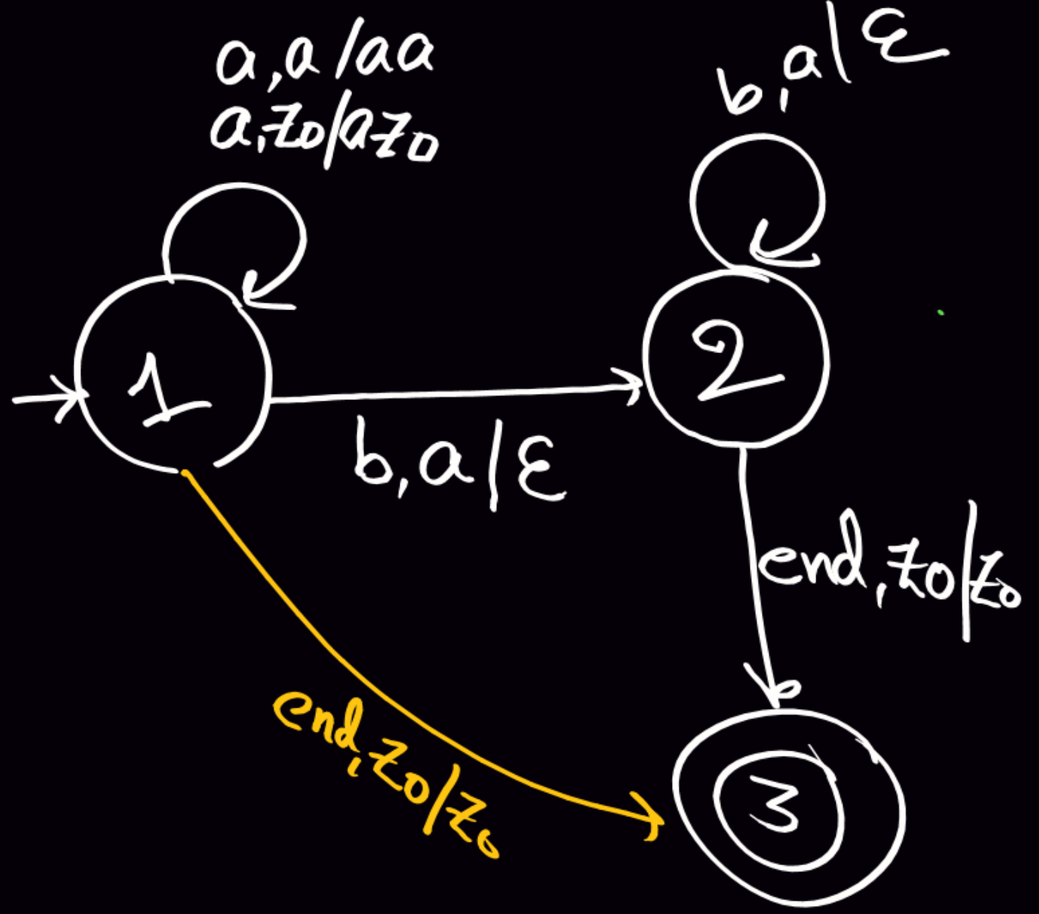
xa : $1 \xrightarrow{a, Z_0 / a Z_0} 1$

xb : $1 \xrightarrow{b, Z_0 /}$ no transition

xaa :

ab : $1 \xrightarrow{a, Z_0 / a Z_0} 1 \xrightarrow{b, a / \varepsilon} 2$

2) $\{a^n b^n \mid n \geq 0\}$

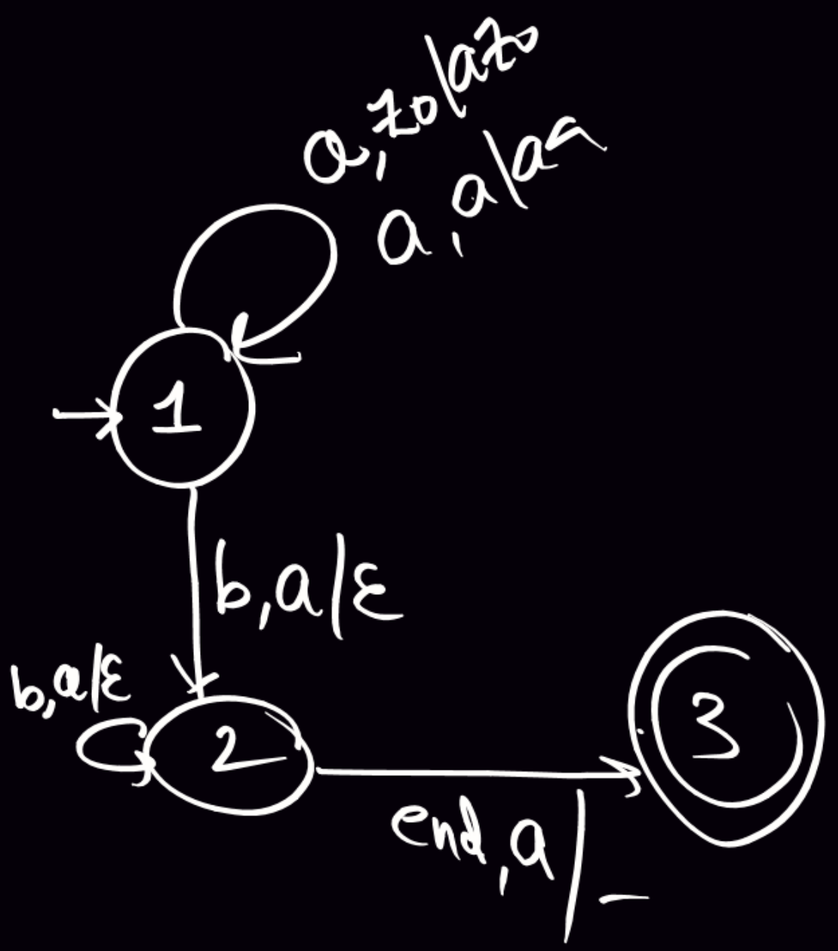


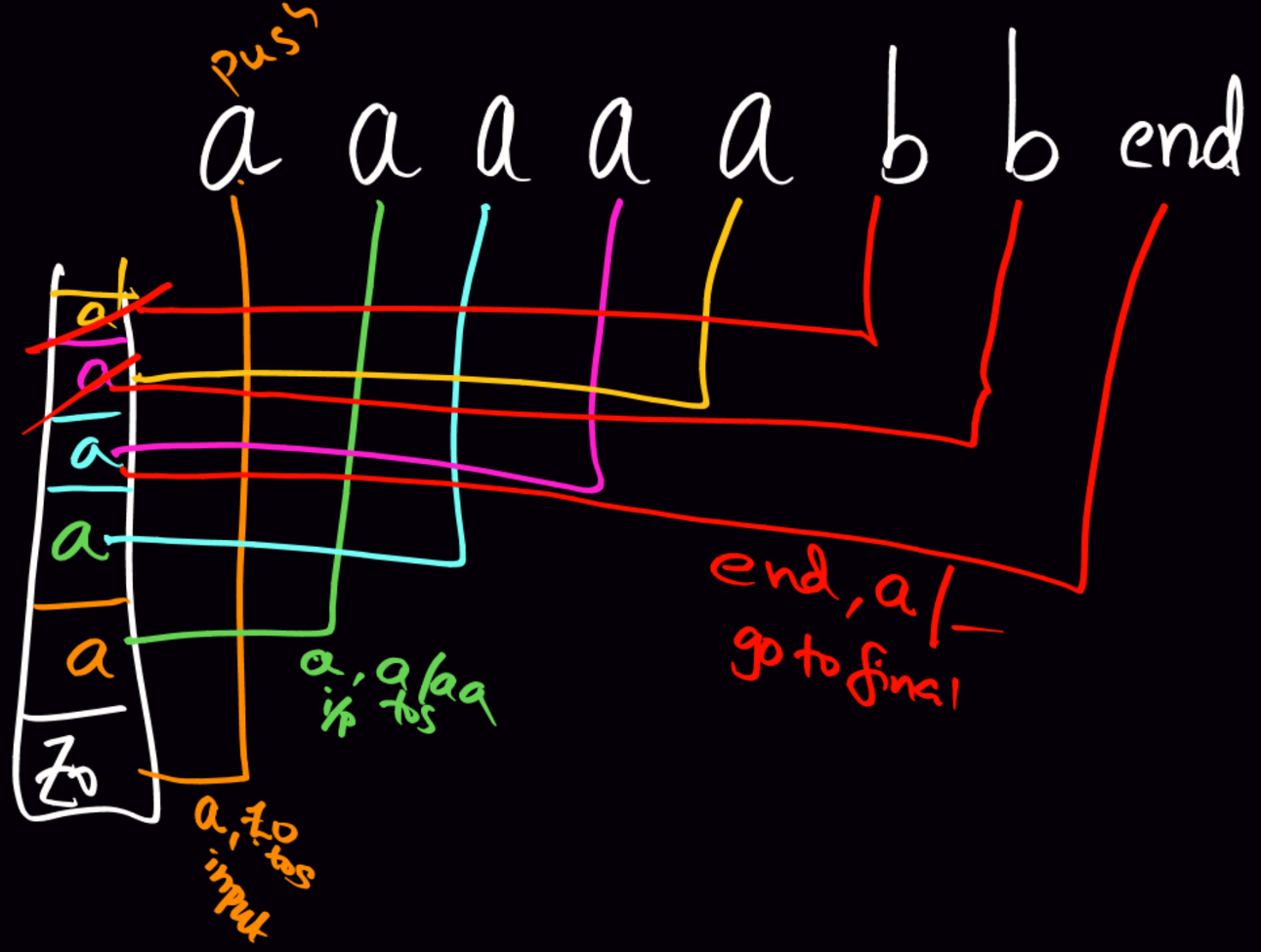
ab $\underline{\varepsilon}$

ab $\underline{end}$

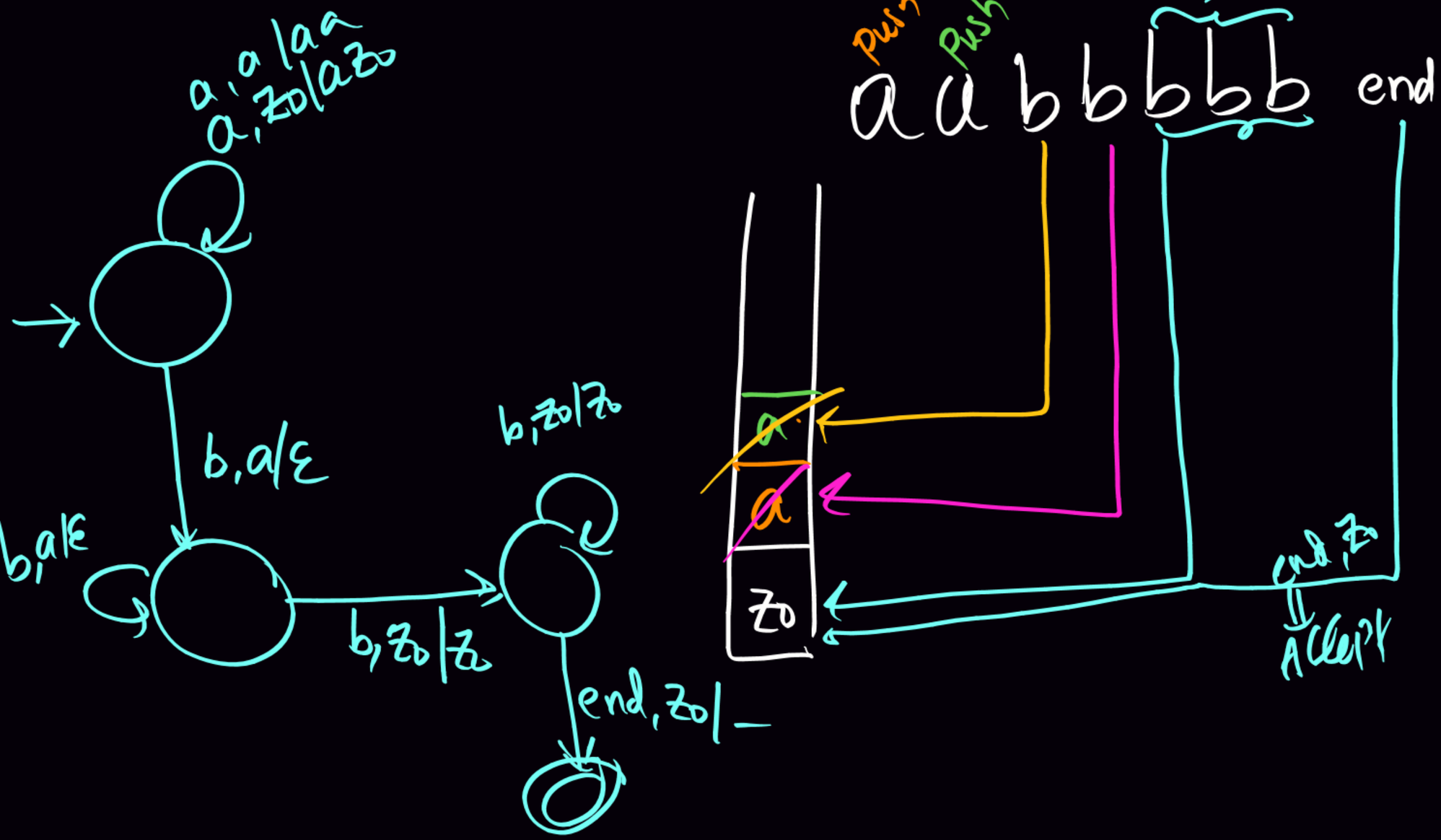
ab $\underline{\$}$

3) $\{a^m b^n \mid m>n>0\}$





4) $\{a^m b^n \mid m<n,\ m,n \geq 1\}$



5) $\{w \mid w \in (a+b)^*,\ n_a(w) = n_b(w)\}$

6) $\{b^n a^n \mid n \geq 1\}$

7) $\{a^n b^{2n}\}$

8) $\{a^{2n} b^n\}$

H.W.