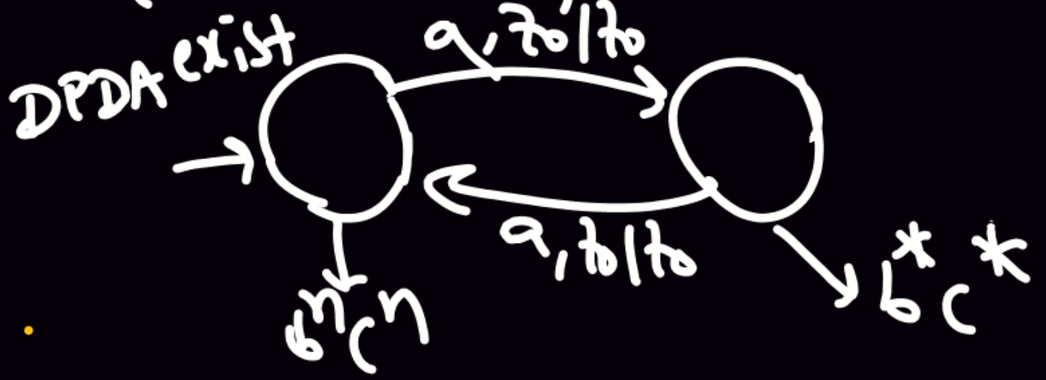
19) $\{a^m b^n c^k \mid \text{if } m=\text{even then } n=k\} \Rightarrow$ DCFL

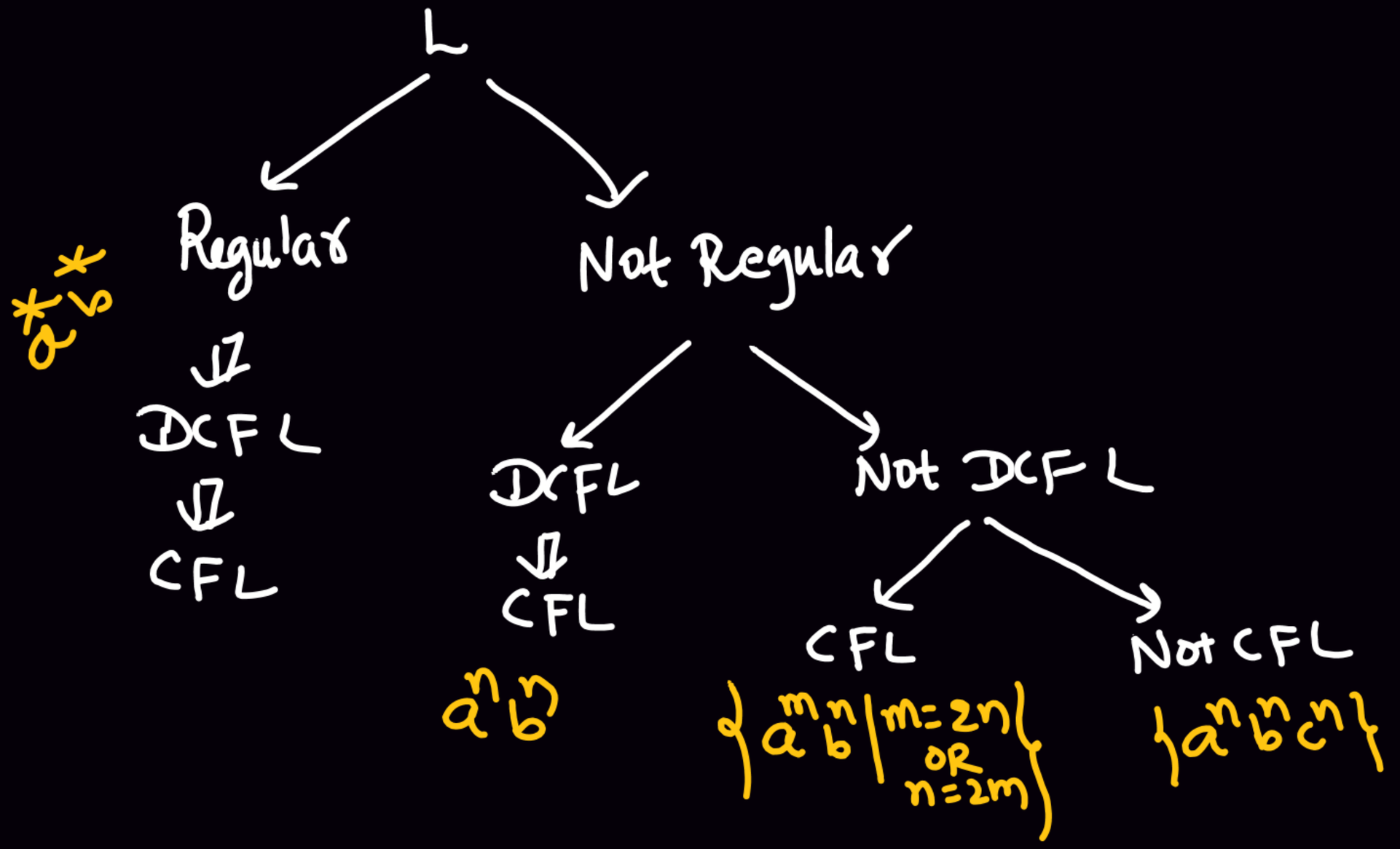
H.W.

20) $\{a^m b^n \mid m=2n \text{ OR } n=2m\} = \{a^n b^{2n}\} \cup \{a^{2n} b^n\}$ CFL but not DCFL

$a^m b^n c^k \mid$ if $m=\text{even} \Rightarrow n=k$

$\Rightarrow \{a^{even} b^n c^n\} \cup \{a^{odd} b^* c^*\}$




L
Regular
Not Regular
$a^* b^*$
DCFL
CFL
DCFL
CFL
Not DCFL
$a^n b^n$
CFL
Not CFL
$\{a^m b^n \mid m=2n \text{ OR } n=2m\}$
$\{a^n b^n c^n\}$

1) $\{a^n b^n c^* \}$ ⇒ DCFL but not regular
Push a's, pop a's, skip c's

2) $\{a^* b^n c^n\}$
Skip, push, pop

3) $\{a^n b^* c^n\}$

4) $\{a^n \# a^{2n}\}$ ⟹ "
Push, skip, pop twice

5) $\{a^n b^{100n}\}$ ⟹ "

6) $\{w a^n b^n \mid w \in (a+b)^*, n \geq 0\} = w = (a+b)^*$ Regular

7) $\{w a^n b^n \mid w \in (a+b)^*, n \geq 1\}$ ⇒ CFL but not
Push w

8) $\{a^n w b^n \mid n \geq 0, w \in (a+b)^*\}$ ⇒ Reg

9) $\{a^n w b^n \mid n \geq 1, w \in (a+b)^*\}$ ⇒ CFL but not DCFL
Push a

10) $\{a^n b^n w \mid n \geq 0, w \in (a+b)^*\}$ ⇒ Reg

11) $\{a^n b^n w \mid n \geq 1, w \in (a+b)^*\}$ ⇒ DCFL
Push a, pop b, skip w

12) $\{w w^R x \mid w, x \in (a+b)^+\}$ ⇒ CFL not DCFL
Push w, pop w, skip x

13) $\{w x w^R \mid$ " $\}$ ⇒ Reg

14) $\{x w w^R \mid$ " $\}$ CFL not DCFL
Skip x, push w, pop w

15) $\{a^n b^n c^k d^k\}$ ⇒ DCFL

Push Push

16) $\{a^n b^k c^k d^n\}$ ⇒ DCFL

Push Push

17) $\{a^n b^k a^n b^k\}$ ⇒ not CFL

18) $\{a b^n c^n\} \cup \{b^n c^{2n}\}$ ⇒ DCFL

19) $\{b^n c^n a\} \cup \{b^n c^{2n}\}$ ⇒ CFL, not DCFL

20) $\{w \# w^R \# w \mid w \in (a+b)^*\}$ not CFL

21) $\{ww \mid w \in (a+b)^*\}$ not CFL

22) $\{w \# w \mid w \in (a+b)^*\}$ not CFL

23) $\{w w^R \mid w \in (a+b)^*\}$ CFL

24) $\{w \# w^R \mid w \in (a+b)^*\}$ DCFL

25) $\{wwwx \mid w, x \in (a+b)^*\}$ Reg

$= (a+b)^*$

26) $\{wwwwx \mid w, x \in (a+b)^+\}$

not CFL

27) $\{a^n b^n c^n\}$ not CFL

28) $\{a^n b^{2n} c^{3n}\}$ not CFL

29) $\{a^n b^{n^2}\}$ not CFL

30) $\{a^{n+1} b^{n+2} c^{n+3}\}$ not CFL

31) $\{a^{n^2}\}$ not CFL

32) $\{a^{2^n}\}$ not CFL

33) $\{a^{\text{Prime}}\}$ not CFL

34) $\{a^{n^n}\}$ not CFL

35) $\{a^{n!}\}$ not CFL

Note:

L over 1 Symbol:

PDA $\cong$ FA

# Closure Properties for DCFLs: [Closed: Complement, Prefix, $h^{-1}$, finite subset]

1) $L_1 \cup L_2$
2) $L_1 \cap L_2$
3) $\overline{L}$
4) $L_1 - L_2$
5) $L_1 \Delta L_2$
6) $L_1 \cdot L_2$
7) $L^{Rev}$
8) $L^*$
9) $L^+$
10) Subset(L)
11) Prefix(L)
12) Suffix(L)
13) Substring(L)
14) Quotient$(L_1, L_2)$
15) $f(L)$
16) $h(L)$
17) $h^{-1}(L)$
18) Finite Union
19) Finite Intersection
20) Finite Difference
21) Finite Concatenation
22) Finite Subset
23) Finite Substitution
24) Infinite Union
25) " Intersection
26) " Difference
27) " Concatenation
28) " Subset
29) " Substitution

# Closure Properties for CFLs:

[not closed: $\cap, \bar{L}$, Subset, Quotient, $\text{Inf}_{\text{All}}$]

1) $L_1 \cup L_2$
2) ~~$L_1 \cap L_2$~~
3) ~~$\bar{L}$~~
4) ~~$L_1 - L_2$~~
5) ~~$L_1 \Delta L_2$~~
6) $L_1 \cdot L_2$
7) $L^{Rev}$
8) $L^*$
9) $L^+$

10) ~~Subset(L)~~
11) Prefix(L)
12) Suffix(L)
13) Substring(L)
14) ~~Quotient$(L_1, L_2)$~~
15) $f(L)$
16) $h(L)$
17) $h^{-1}(L)$

Diff Δ
Fin ∩
Fin −

18) Finite Union
19) ~~Finite Intersection~~
20) ~~Finite Difference~~
21) Finite Concatenation
22) Finite Subset
23) Finite Substitution

24) ~~Infinite Union~~
25) ~~"~~ Intersection
26) ~~"~~ Difference
27) ~~"~~ Concatenation
28) ~~"~~ Subset
29) ~~"~~ Substitution

1) $L_1 = \{a^n b^n c^*\}$
$L_2 = \{a^* b^k c^k\}$
$\Rightarrow L_1 \cap L_2 = \{a^n b^n c^n\}$

2) $L_1 = \{a^n b^n\}$
$L_2 = \{a^n b^{2n}\}$
$\Rightarrow L_1 \cup L_2 = \{a^i b^j \mid i=j \text{ OR } j=2i\}$
$L_1 \cap L_2 = \{\varepsilon\}$

3) $L = \{a^n b^n\} \Rightarrow \overline{L} = (a+b)^* - L = (a+b)^* - \{a^n b^n\}$
$= \{a^m b^n \mid m \neq n\} \cup \Sigma^* ba \Sigma^*$

$$\overline{a^* b^*} = \Sigma^* b a \Sigma^*$$

$$\{a^* b^*\} = \{a^n b^n\} \cup \{a^m b^n \mid m \neq n\}$$

$$\{a^* b^*\} \cup \Sigma^* b a \Sigma^* = (a+b)^*$$

$$\{a^n b^n\} \cup \{a^m b^n \mid m \neq n\} \cup \Sigma^* b a \Sigma^* = (a+b)^*$$

4) $L = \{a^n b^n\}$

$\text{prefix}(L) = \{a^i b^j \mid i \geq j\}$

$\text{Suffix}(L) = \{a^i b^j \mid i \leq j\}$

$\text{Substring}(L) = a^* b^*$

$a^n b^n$ prefixi:

$\varepsilon \Rightarrow \varepsilon$

$ab \Rightarrow \varepsilon, a, ab$

$aabb \Rightarrow \varepsilon, a, aa, ab, aab$

$\vdots$

$\{a^i b^j \mid i \geq j\}$

Next: TM & undecidability