Finite Automata (FA)

- Finite state machine (FSM)
- $FA = (\underbrace{Q}_{\{\dots\}}, \underbrace{\Sigma}_{\{\dots\}}, \underbrace{\delta}_{\{\dots\}}, q_0, \underbrace{F}_{\{\dots\}})$
- It represents (accepts) (recognizes) a REGULAR language.

Q = Set of states

Σ = Set of symbols [Input Alphabet]

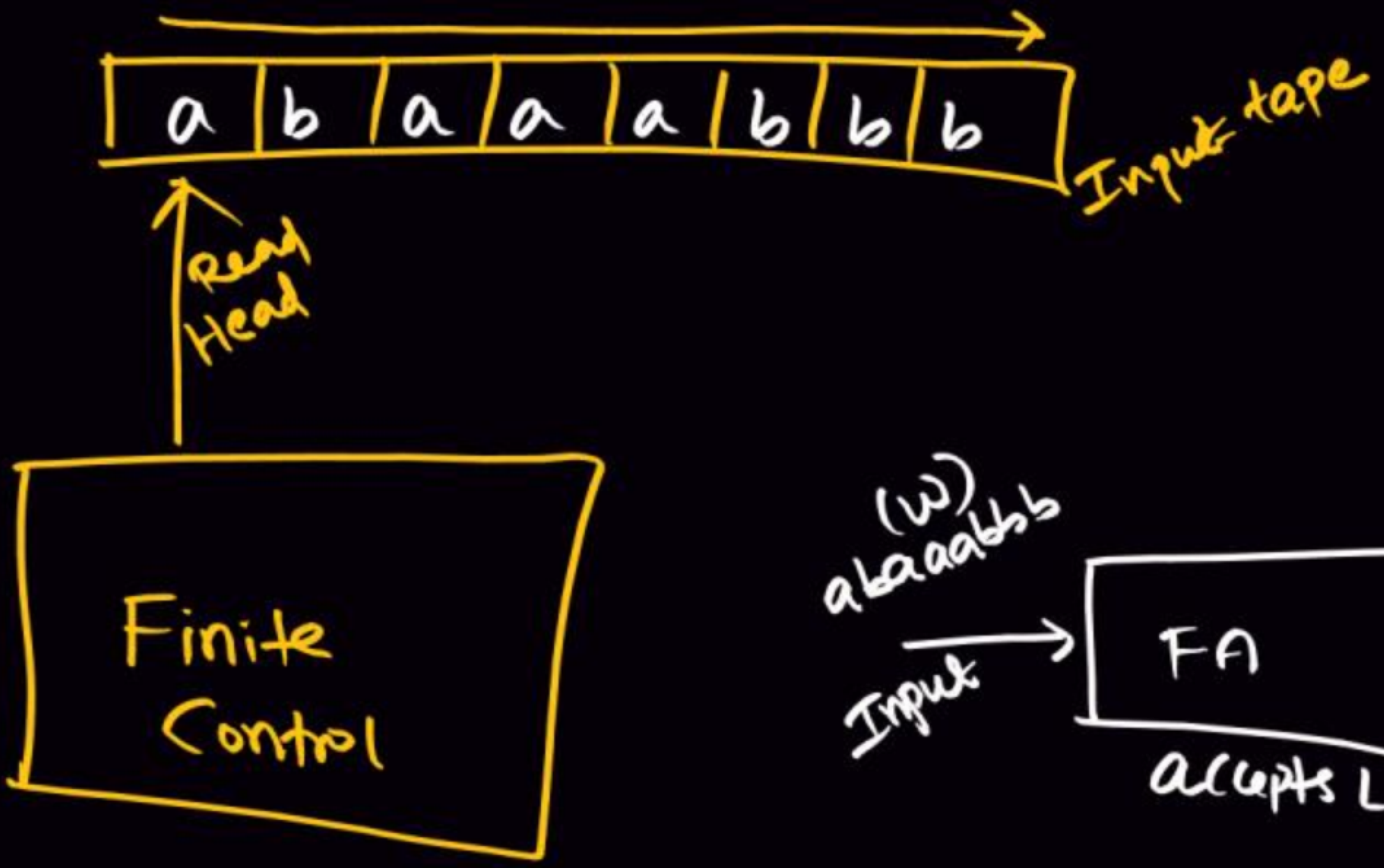
δ = Set of transitions [Transition Function]

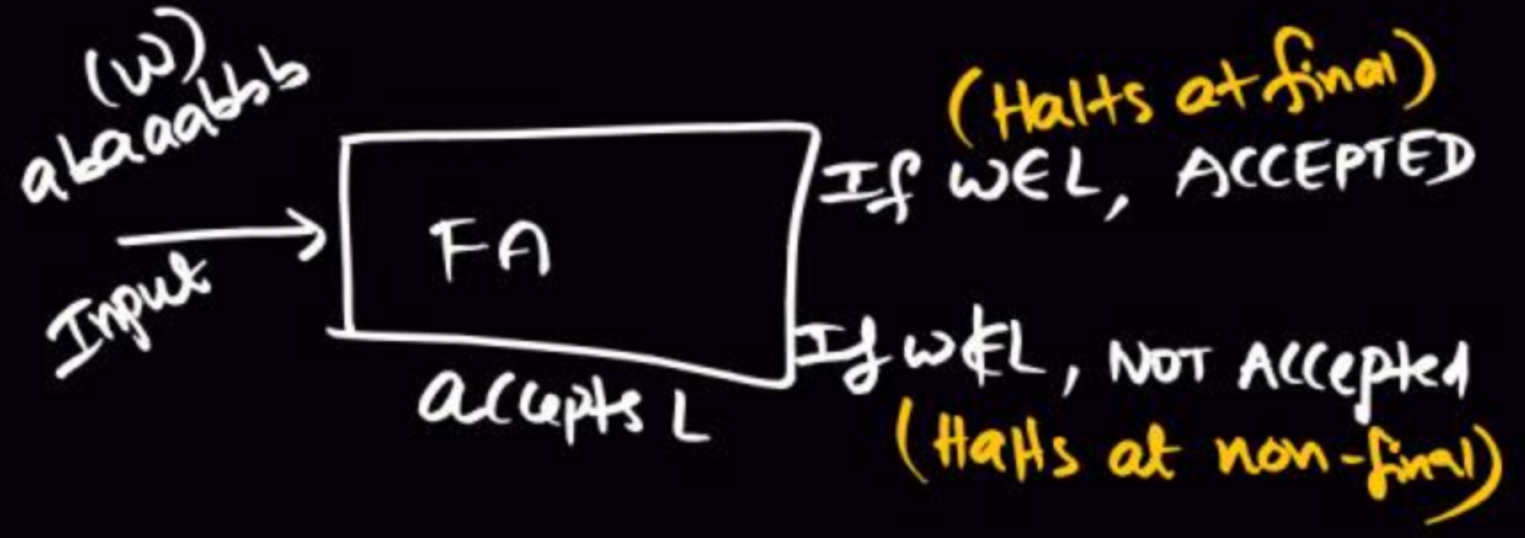
q_0 = Initial state

F = Set of Finals

FA

- Acceptor
  1) DFA
  2) NFA
     - without ε moves
     - with ε moves
- Transducer
  1) Moore M/c
  2) Mealy M/c





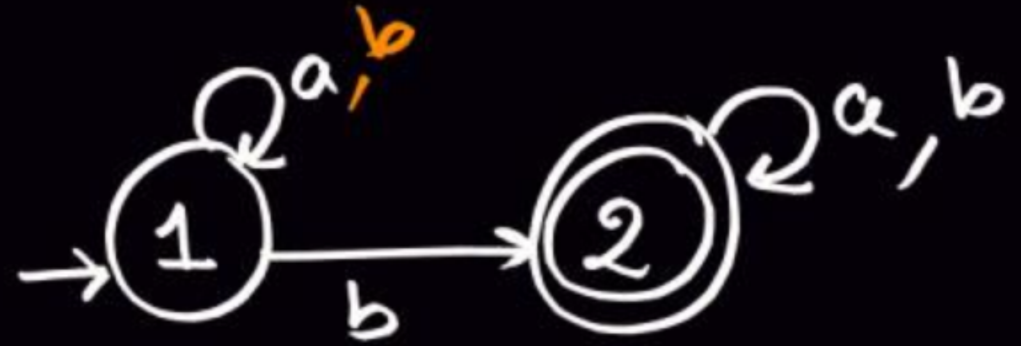
DFA $\simeq$ NFA

DFA: $\delta : Q \times \Sigma \rightarrow Q$

NFA: $\delta : Q \times \Sigma_\varepsilon \rightarrow 2^Q$

FA Representations:

1) State Diagram [Graph]:



$q_0 = 1$

$Q = \{1, 2\}$

$\Sigma = \{a, b\}$

$\delta(1,a) = 1, \quad \delta(1,b) = 2$

$\delta(2,a) = 2, \quad \delta(2,b) = 2$

$F = \{2\}$

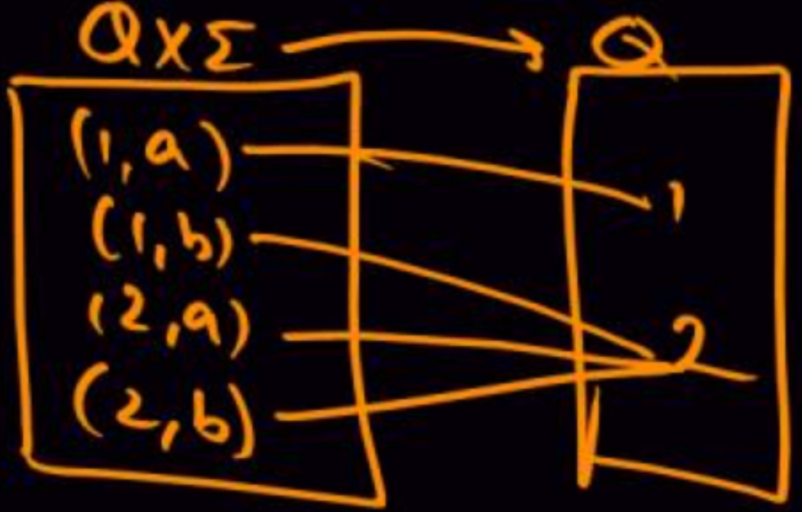
2) Transition Table

	a	b
$\rightarrow 1$	1	2
$*2$	2	2

3) Set / Function:

$$\delta = \{((1,a),1), ((1,b),2), ((2,a),2), ((2,b),2)\}$$

$$FA = (\underbrace{\{1,2\}}_{Q}, \underbrace{\{a,b\}}_{\Sigma}, \delta, \underbrace{1}_{q_0}, \underbrace{\{2\}}_{F})$$



FA

- DFA (Deterministic FA)
- NFA (NDFA) (Non-deterministic FA)

DFA:

$$Q \times \Sigma \rightarrow Q$$

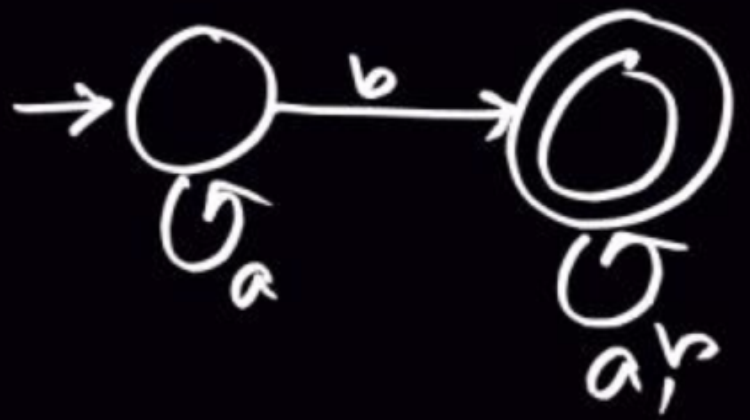
From each state in Q,
for every i/p symbol in Σ,
there is exactly one transition
to the state in Q.

NFA:

$$Q \times \Sigma_{\varepsilon} \rightarrow \text{Powerset}(Q) \quad 2^{Q}$$

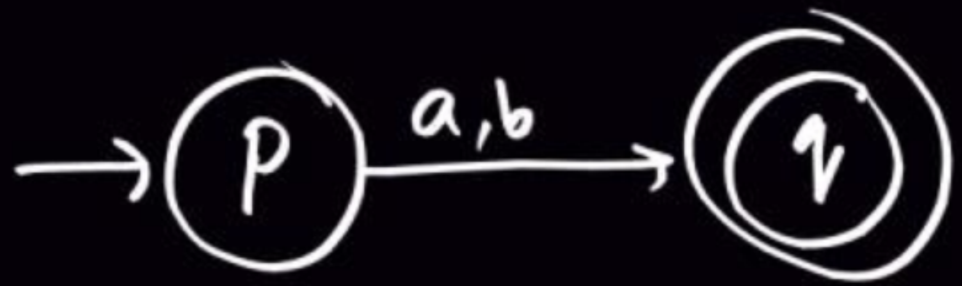
From each state in Q, for every i/p or no i/p,

<u>Zero or more</u> transitions possible

1) 

DFA ✓ , NFA ✓

Note: Every DFA is NFA.
NFA need not be DFA.

2) 

DFA ✗ NFA ✓

$\delta(q,a)=\phi$
$\delta(q,b)=\phi$

L(DFA) is Regular
L(NFA) is Regular

Accepts L

If $w \in L$, exactly one path halts at final

If $w \notin L$, exactly one path halts at non-final

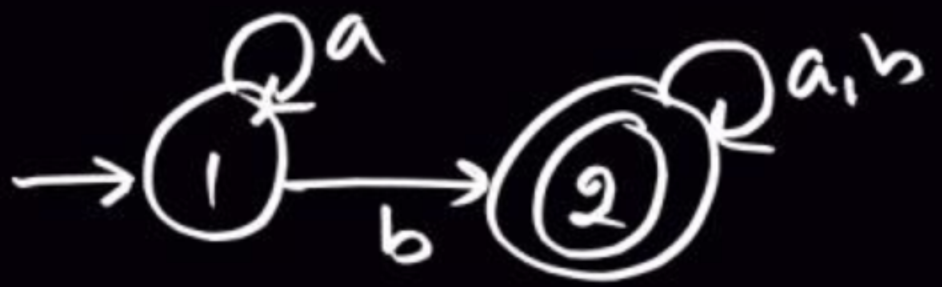
Accepts L

If $\underbrace{w \in L}_{\text{valid}}$, atleast one path halts at final

If $w \notin L$, either no path in NFA

or every path halts at non-final.

1)



symbol

$\delta(1, a) =$

$\delta(1, b) =$

$\delta(2, a) =$

$\delta(2, b) =$

$\underbrace{\delta : Q \times \Sigma \to Q}_{\text{DFA}}$

string

$\hat{\delta}(1, aa) =$

$\hat{\delta}(1, b) =$

$\hat{\delta}(1, aba) =$

$\underbrace{\hat{\delta} : Q \times \Sigma^{*} \to Q}$

✗ ε : 1

✗ a : $1 \xrightarrow{a} 1$

✓ b : $1 \xrightarrow{b} \boxed{2}$

✗ aa : $\overbrace{1 \xrightarrow[\delta]{a} 1 \xrightarrow[\delta]{a} 1}^{\hat{\delta}}$ $\left[1 \stackrel{aa}{\Longrightarrow} 1\right]$

✓ ab : $1 \xrightarrow{a} 1 \xrightarrow{b} \boxed{2}$ $\left[1 \stackrel{ab}{\Longrightarrow} 2\right]$

✓ ba : $1 \stackrel{ba}{\Longrightarrow} \boxed{2}$

✓ bb : $1 \stackrel{bb}{\Longrightarrow} \boxed{2}$

✗ aaa : $1 \stackrel{aaa}{\Longrightarrow} 1$

✓ aab : $1 \stackrel{aab}{\Longrightarrow} \boxed{2}$

✓ aba : $1 \stackrel{aba}{\Longrightarrow} \boxed{2}$

⋮

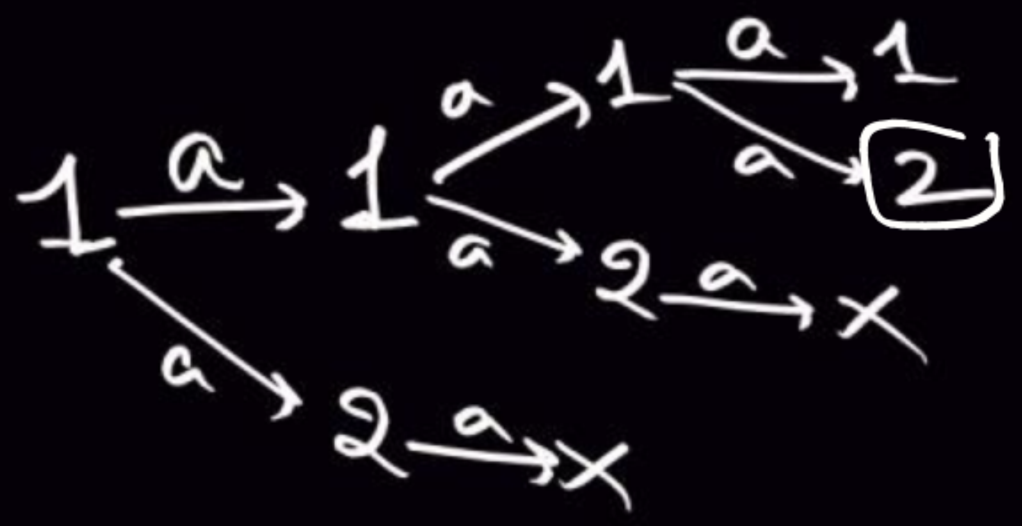
2)

$$1 \xrightarrow{aaa} ?$$

$$\hat{\delta}(1, aaa) = ?$$

$$= \{1, \underbrace{2}_{\text{final}}\}$$

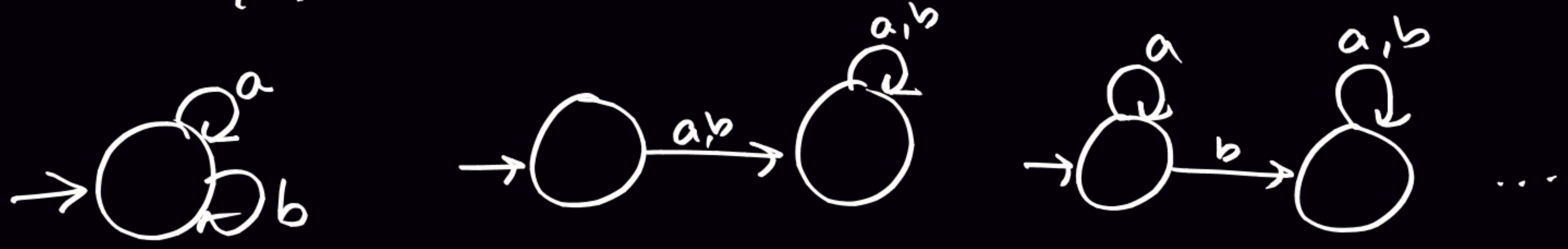
$$aaa \in L$$



$$1 \xrightarrow{aaa} 1$$

$$1 \xrightarrow{aaa} 2$$

I) $L = \phi$ over $\Sigma = \{a\}$



Min DFA DFA DFA

I) How many DFAs exist for any Regular language? $\Rightarrow$ Infinite

II) How many minimized DFAs exist for any Regular? $\Rightarrow$ Only one

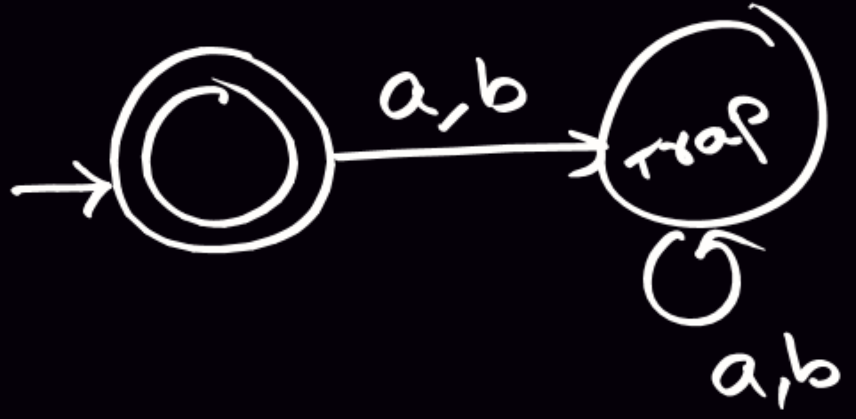
min NFA Min NFA Min NFA Min NFA

III) How many NFAs exist for any regular? $\Rightarrow$ Infinite

IV) How many Min NFAs exist for any regular? $\Rightarrow$ one or more (≥ 1)

2) $L = \{\varepsilon\}$ over $\Sigma = \{a, b\}$

Min DFA



If initial state is final, then it accepts string "ε".

Min NFAs

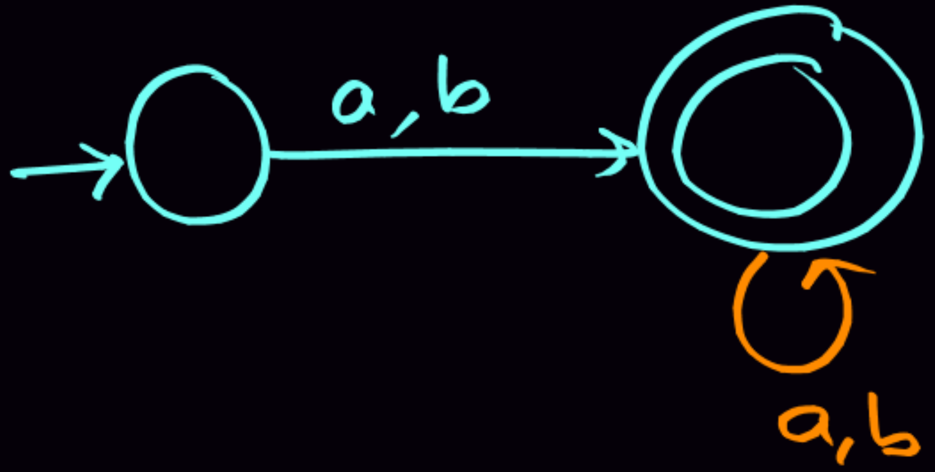
3) $L = (a+b)^*$

Min DFA

Min NFA

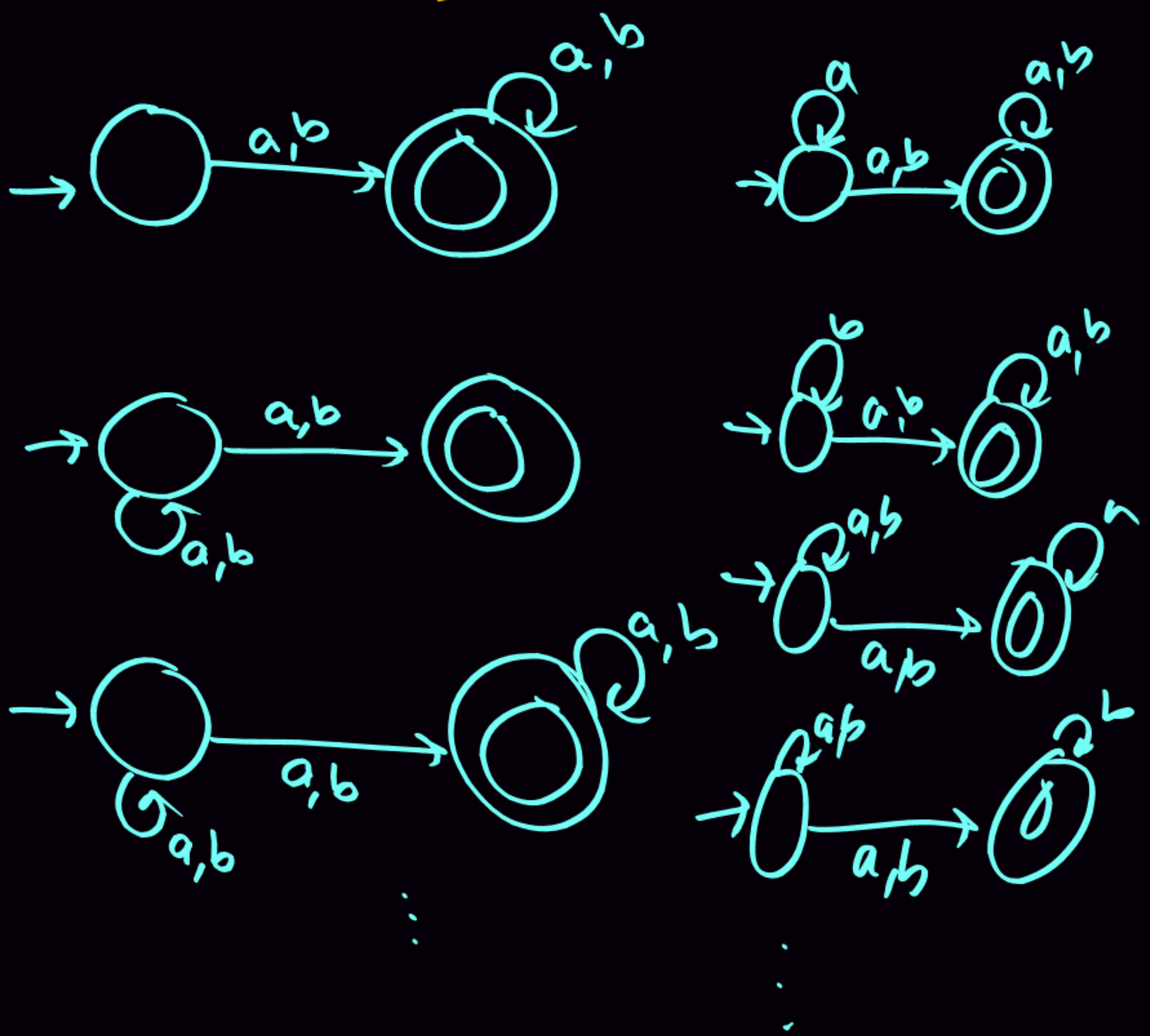
4) $L = (a+b)^+$

Min DFA



$= \{w \mid w \in (a+b)^*, |w| \geq 1\}$

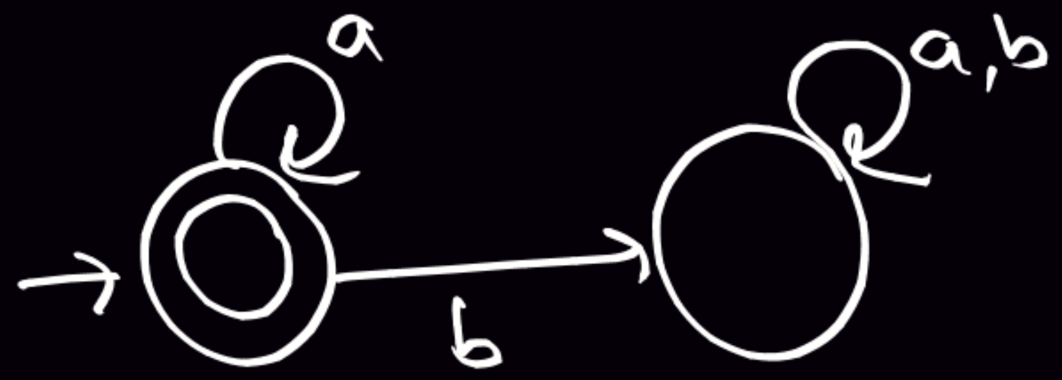
Min NFAs



5) $L = a^*$ over $\Sigma = \{a\}$

min DFA
min NFA

6) $L = a^*$ over $\Sigma = \{a, b\}$



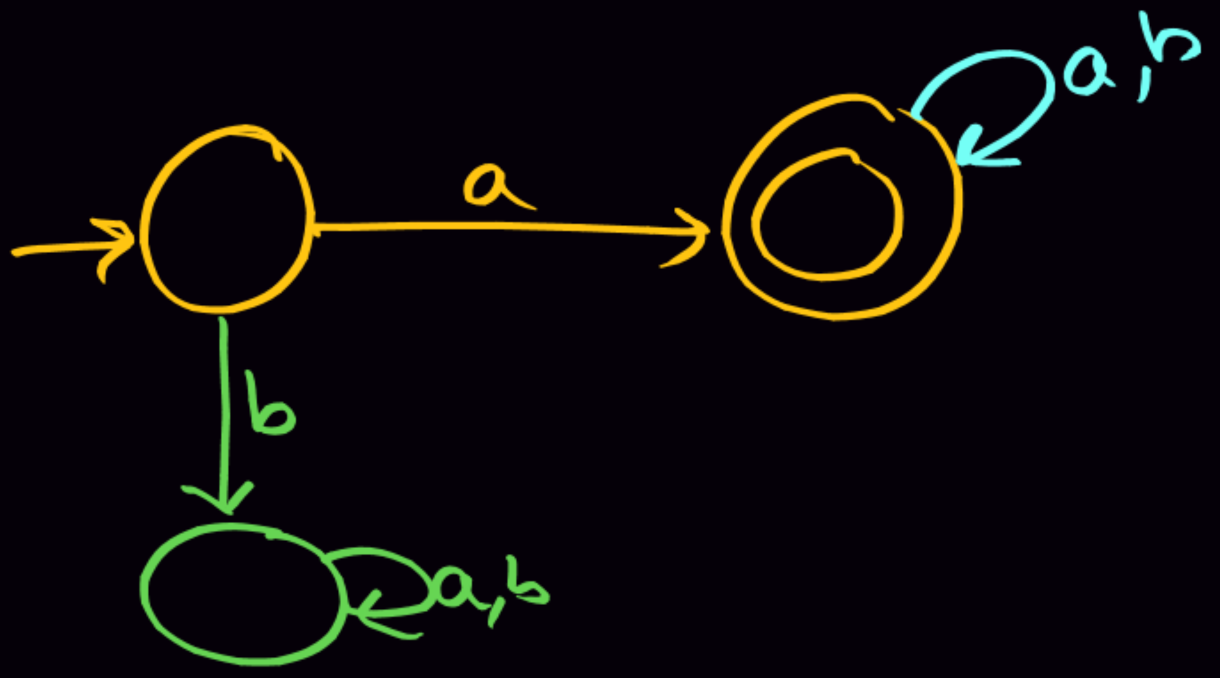
min DFA

min NFA

7) $L = a(a+b)^*$

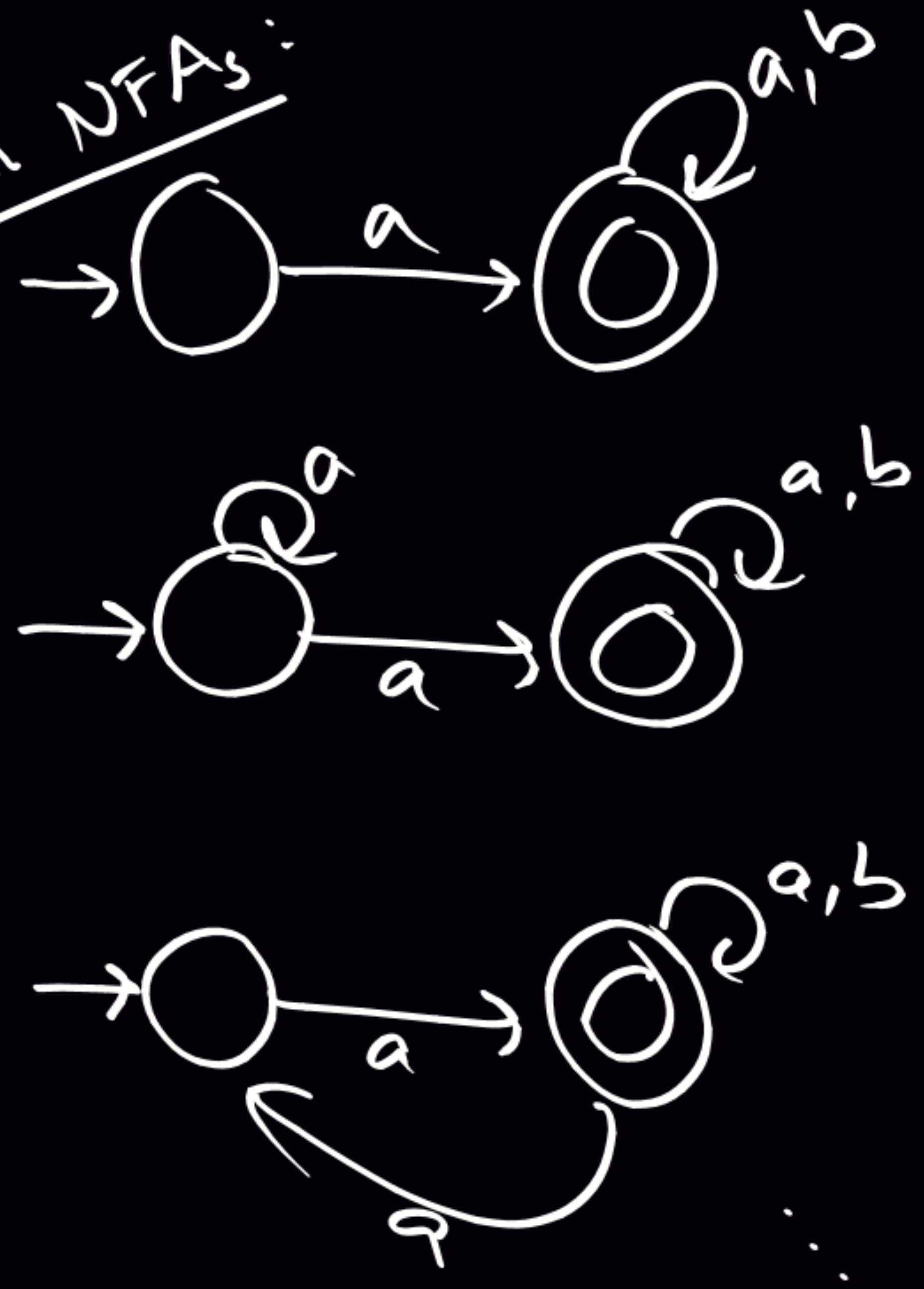
$L = \{a, aa, ab, \ldots\}$

Min string = a



Min DFA

Min NFAs:



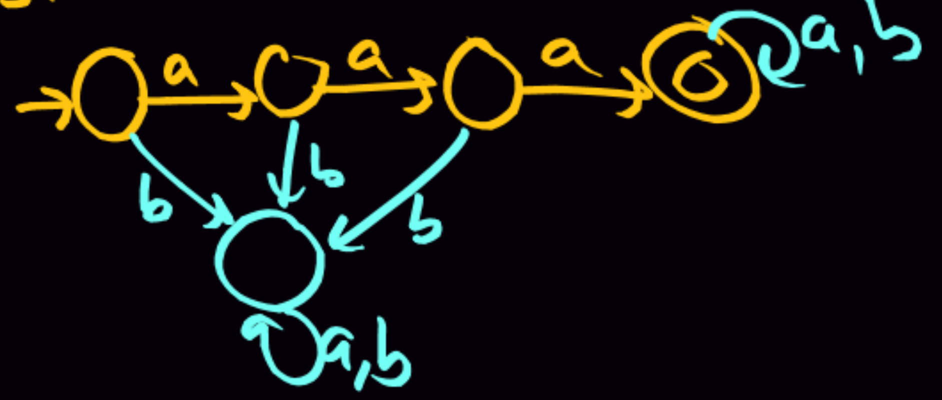
8) $L = aaa(a+b)^*$

No. of states in Min DFA $= (|aaa|+1)+1 = 5$

No. of states in Min NFA $= |aaa|+1 = 4$

Min string : $\underbrace{aaa}$

↳ 4 states needed just to remember aaa + 1 trap

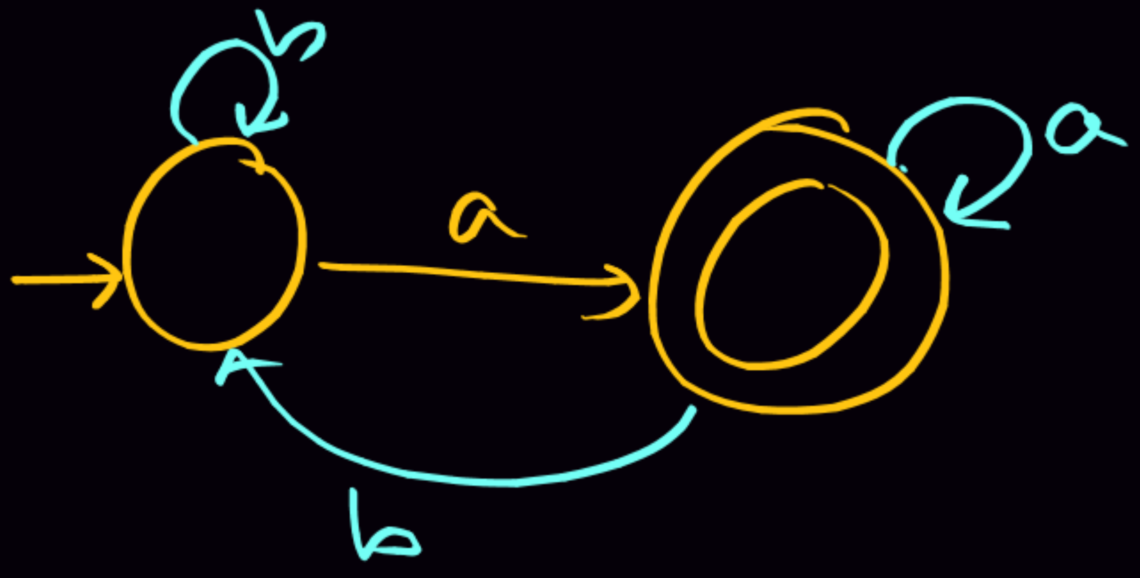


9) $L = (a+b)^*a$

Min = $\underline{a}$

Min DFA

Min NFA



= 2 states

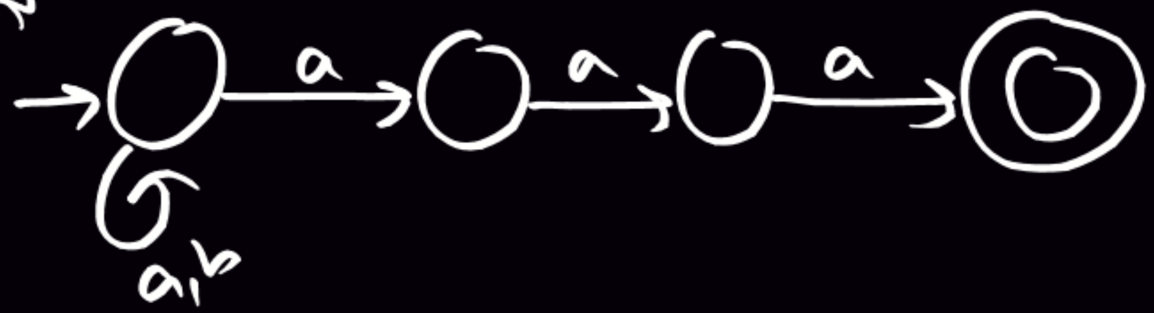
Min NFA

Min NFA

...

10) $L = (a+b)^* aaa$

Min NFA

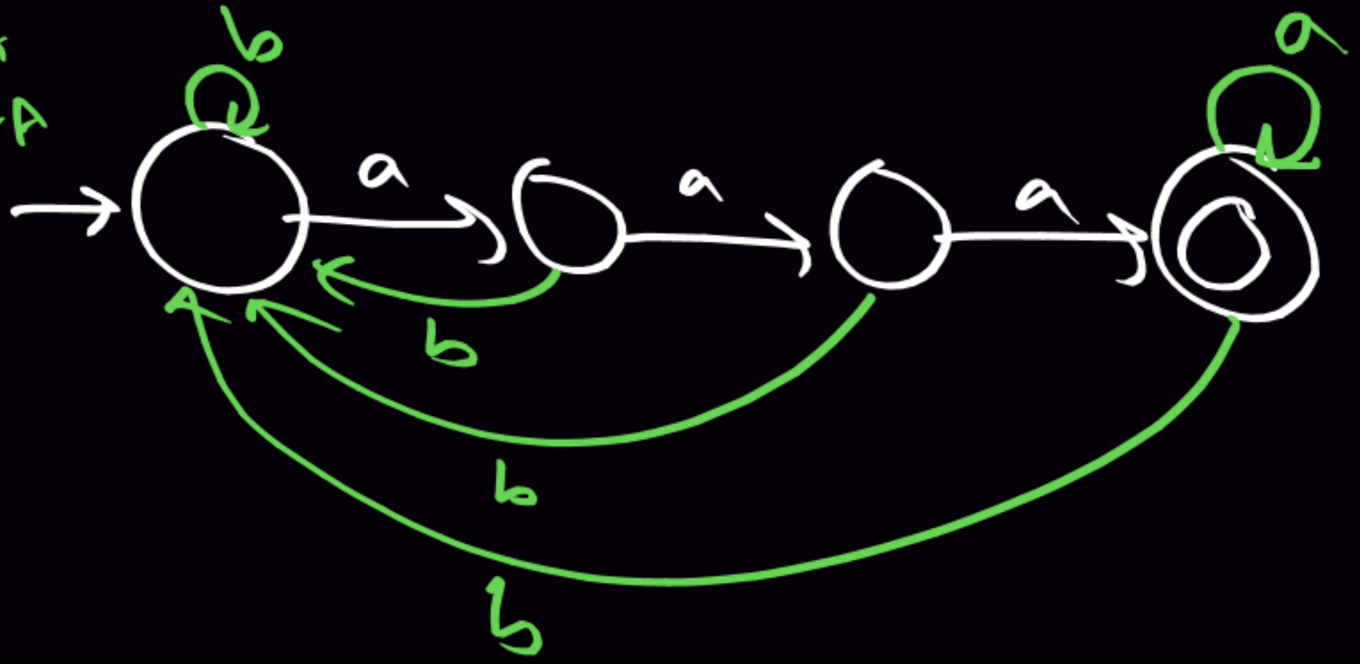


min = aaa

↓

4 states

Min DFA
min NFA

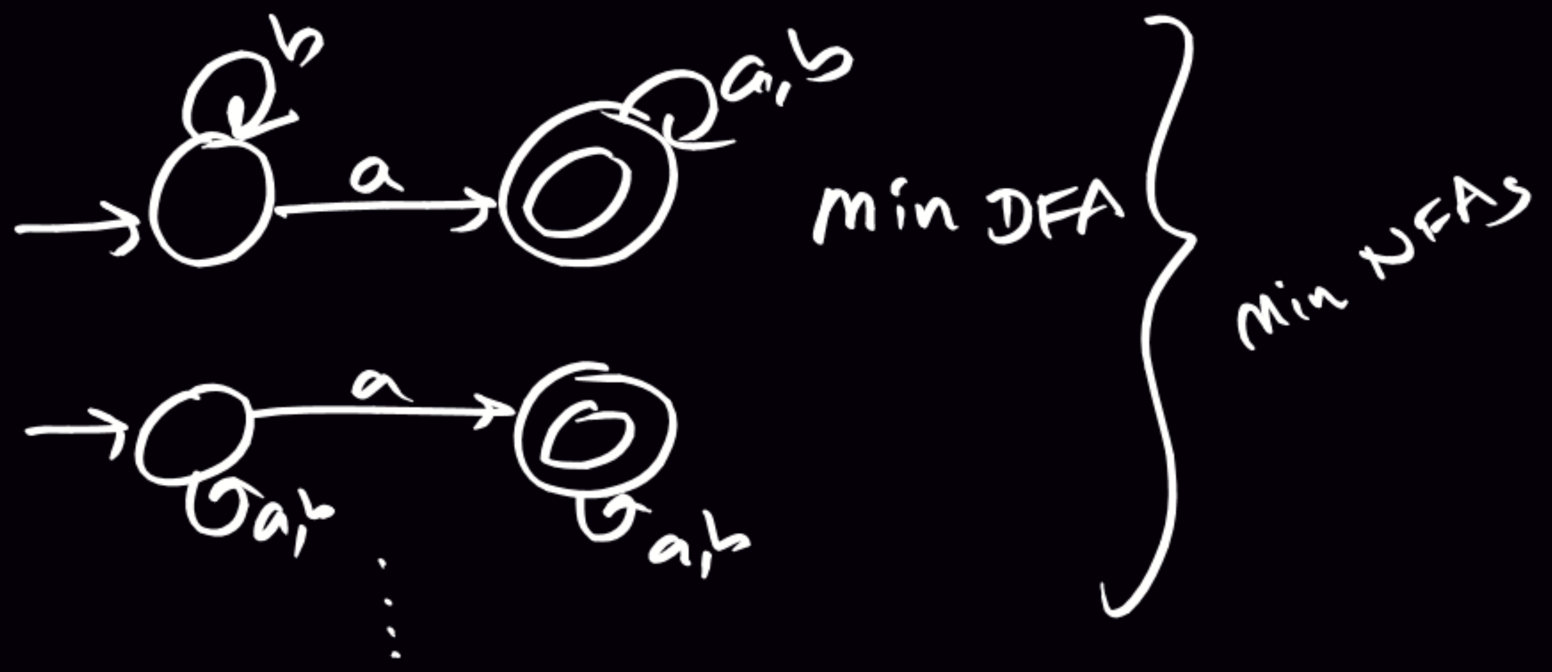


11) $L = (a+b)^* a (a+b)^*$

min = a

$\Downarrow$

2 states



12) $L = (a+b)^* aaa (a+b)^*$

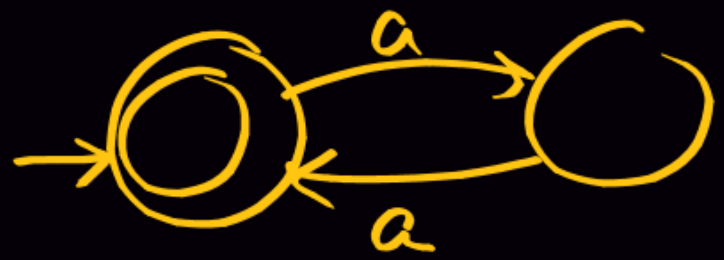
min = aaa

$\Downarrow$

4 states

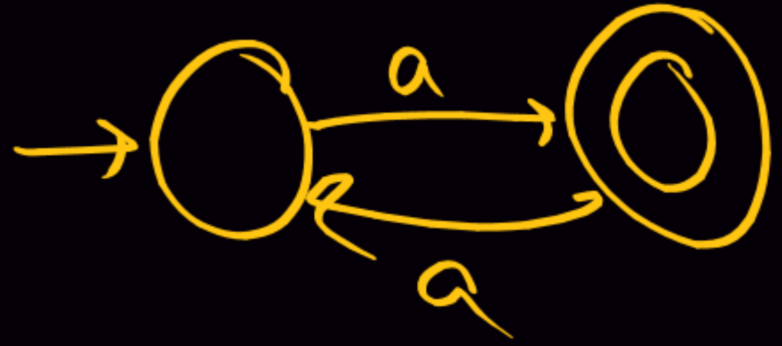
13) $L = (aa)^*$ over $\Sigma = \{a\}$

$= \{a^{2n}\} = \{\varepsilon, a^2, a^4, a^6, a^8, \dots\}$

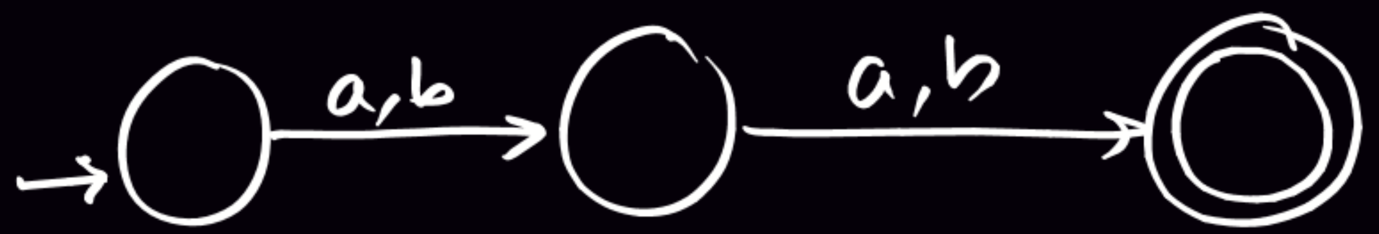


14) $L = a(aa)^*$ over $\Sigma = \{a\}$

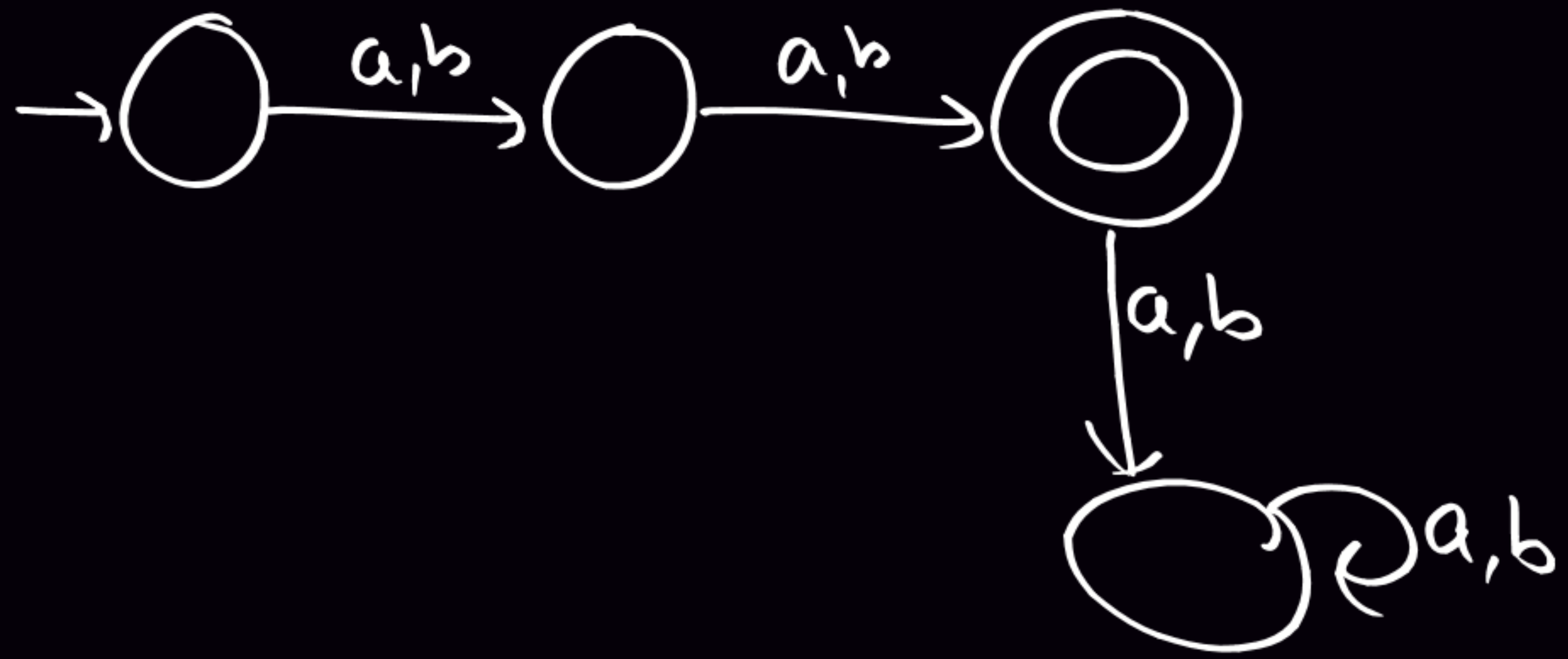
$= \{a^{2n+1} \mid n \geq 0\} = \{a, a^3, a^5, a^7, \dots\}$



15) $\{ w \mid w \in (a+b)^*, |w| = 2 \}$



Min NFA



Min DFA

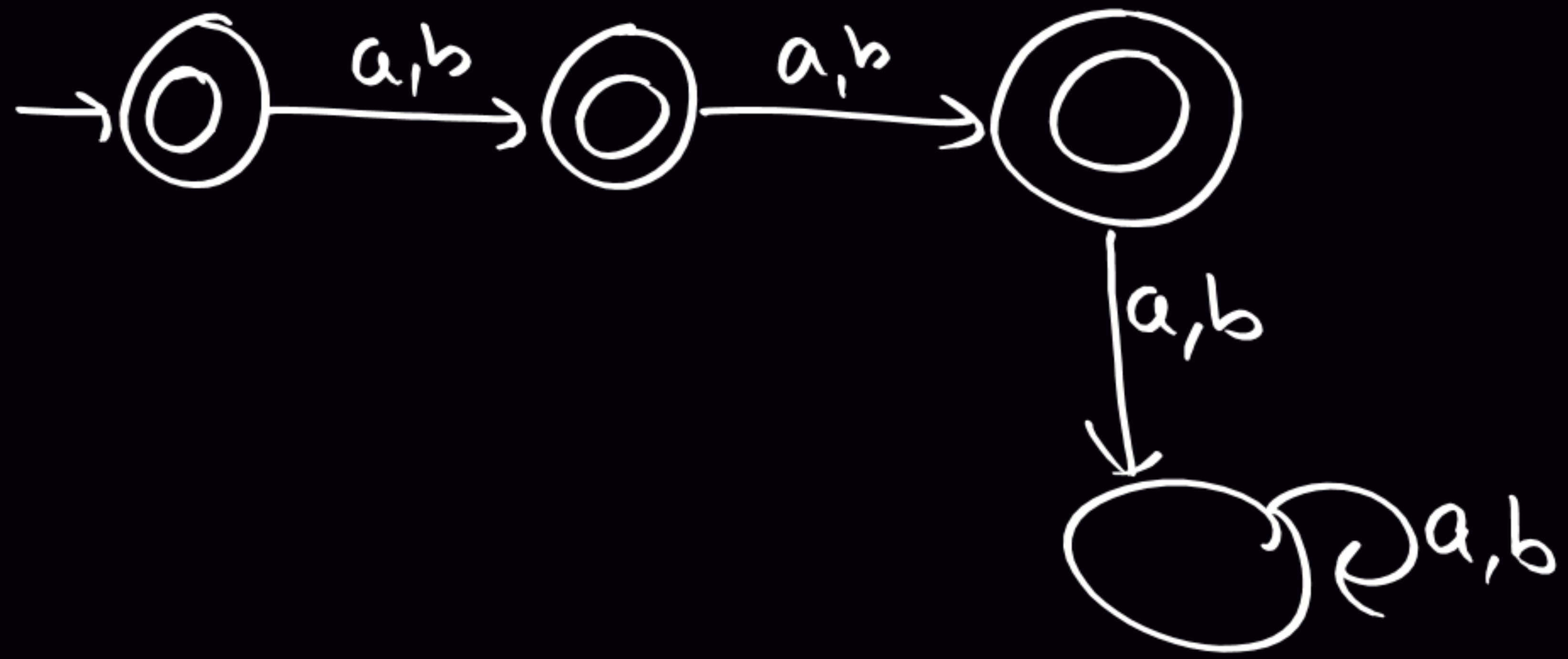
16) $\{w \mid w \in (a+b)^*, |w| = 100\}$

Min NFA $\Rightarrow$ 101 states

Min DFA $\Rightarrow$ 102 states

17) $\{ w \mid w \in (a+b)^*, |w| \leq 2 \}$

Min NFA



Min DFA

18) $\{ w \mid w \in (a+b)^*, |w| \leq 100 \} = (\varepsilon + a + b)^{100}$

101 states

Min NFA

102 states

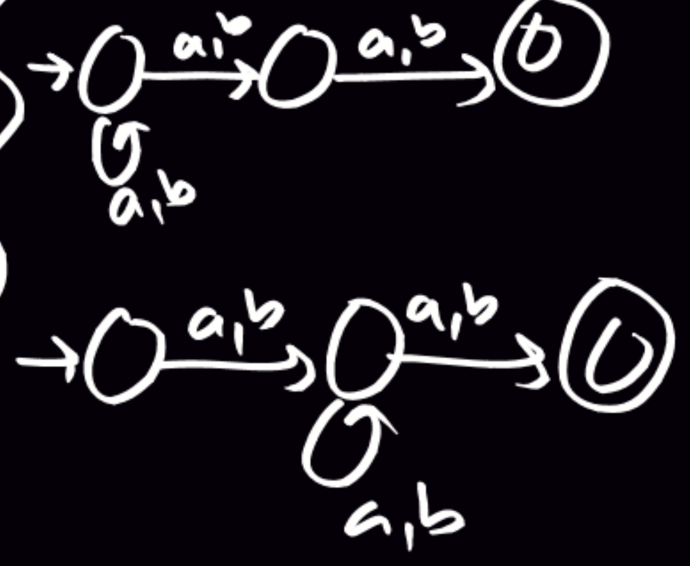
Min DFA

19) $\{w \mid w \in (a+b)^*, |w| \geq 2\} = (a+b)^2(a+b)^*$

min NFA
min DFA

3 states

Many Min NFAs



20) $\{w \mid w \in (a+b)^*, |w| \geq 100\}$

101 states

21) $\{w \mid w \in (a+b)^*,\ |w| = \text{even}\}$

22) $\{w \mid$ " $,\ |w| = \text{odd}\}$

23) $\{w \mid$ " $,\ |w|$ is divisible by $3\}$

24) $\{w \mid$ " $,\ |w| = 2 \bmod 3\}$

H.W.

25) $\{w \mid$ " $,\ \#_a(w) = 2\}$

26) $\{w \mid$ " $,\ \#_a(w) \leq 2\}$

27) $\{w \mid$ " $,\ \#_a(w) \geq 2\}$

28) $\{w \mid$ " $,\ \#_a(w) = \text{even}\}$

29) $\{w \mid$ " $,\ \#_a(w) = \text{odd}\}$