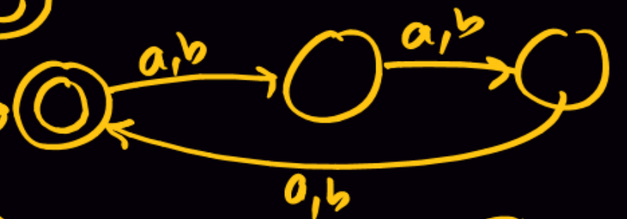
21) $\{w \mid w \in (a+b)^*,\ |w| = \text{even}\}$

22) $\{w \mid$ " , $|w| = \text{odd}\}$

23) $\{w \mid$ " , $|w|$ is divisible by 3$\}$



24) $\{w \mid$ " , $|w| = 2 \bmod 3\}$

H.W.

25) $\{w \mid$ " , $\#_a(w) = 2\}$

26) $\{w \mid$ " , $\#_a(w) \leq 2\}$

27) $\{w \mid$ " , $\#_a(w) \geq 2\}$

28) $\{w \mid$ " , $\#_a(w) = \text{even}\}$

29) $\{w \mid$ " , $\#_a(w) = \text{odd}\}$

30) $\{w \mid w \in (a+b)^*,$ 2nd symbol of w is 'a' $\} = (a+b)\,a\,(a+b)^*$

$= (aa+ba)(a+b)^*$

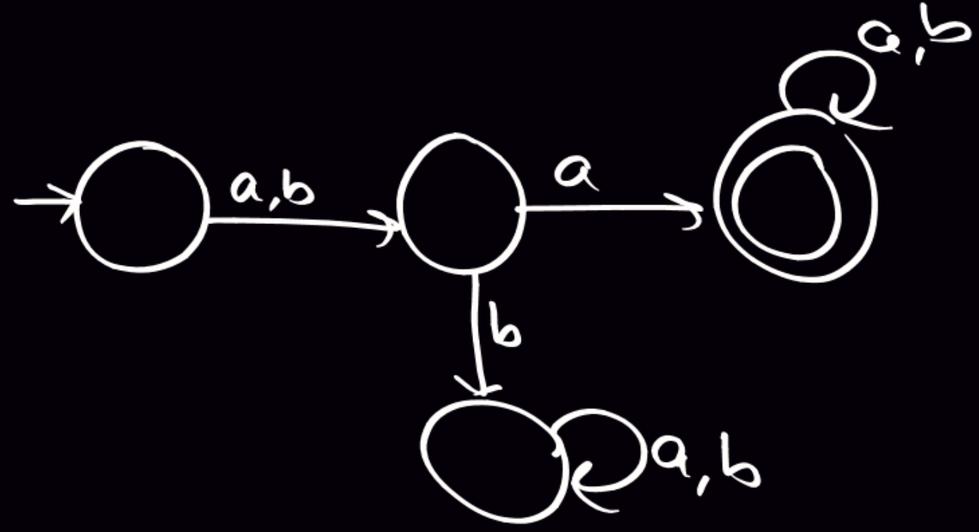
K^{th} Symbol is 'a'

$\Downarrow$

$= (K+1)+1$

$= K+2$ states in min DFA

Min = aa or ba

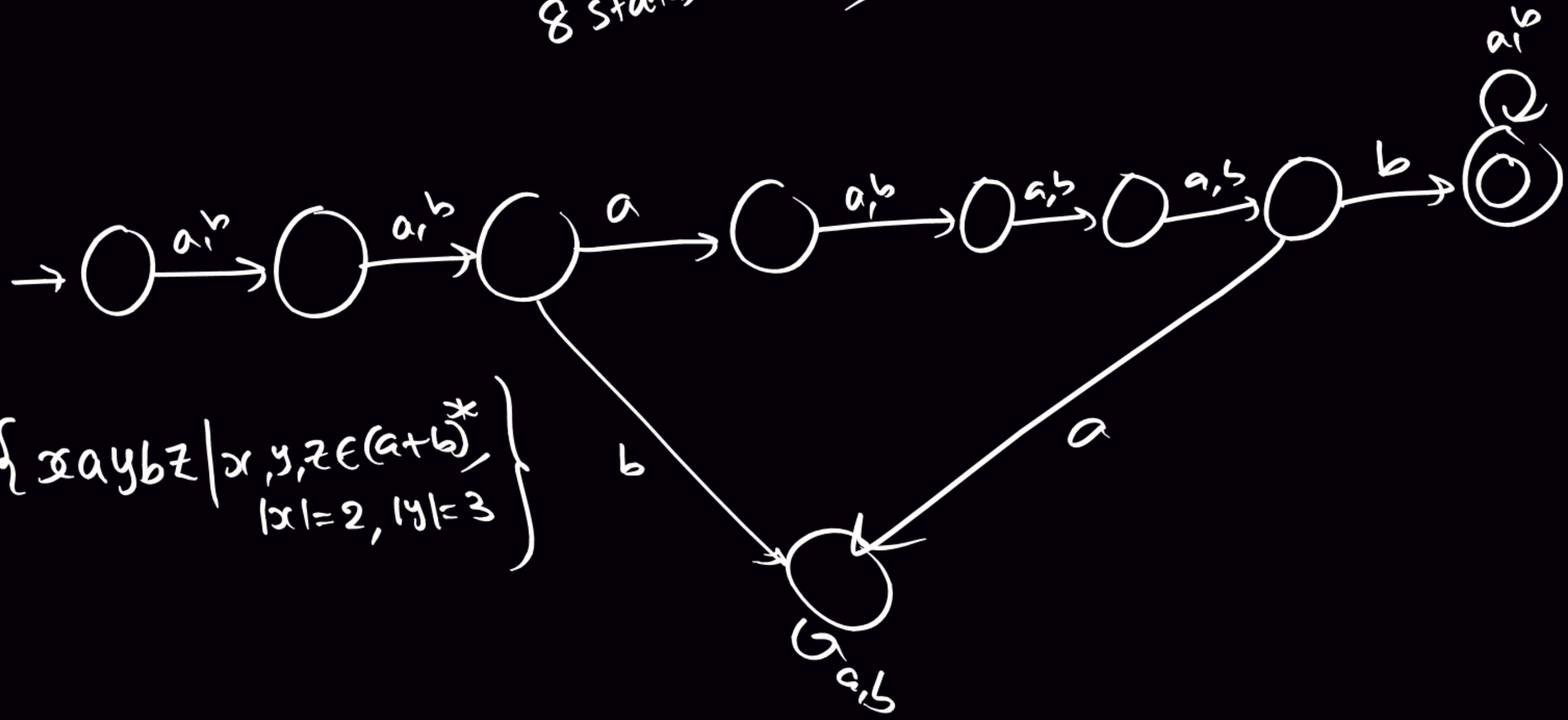


31) $L = (a+b)^2 a (a+b)^3 \underline{b} (a+b)^*$

$\neq^*$

$\Downarrow$

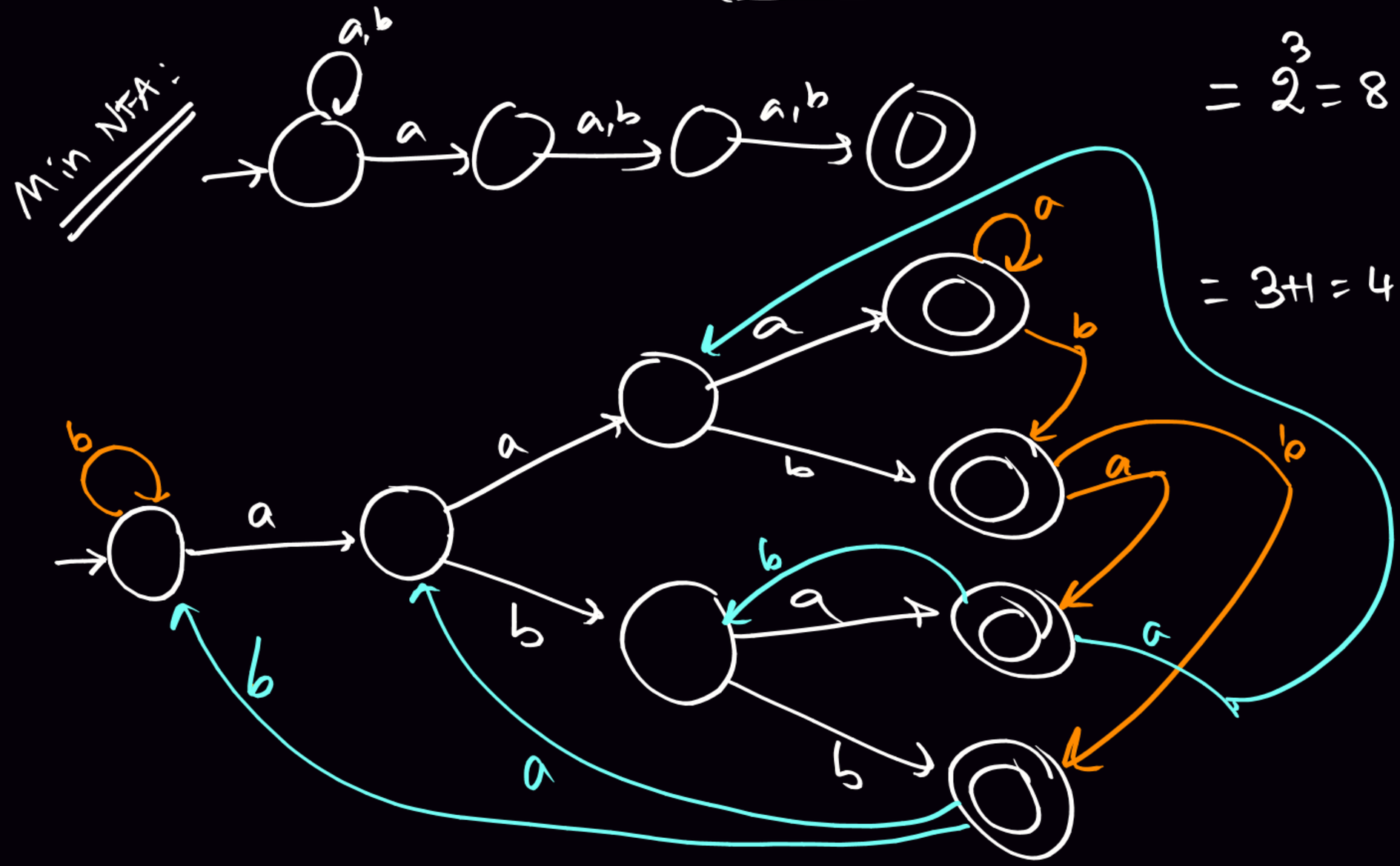
8 states + 1 = 9 states



$L = \{ xaybz \mid x,y,z \in (a+b)^*, |x|=2, |y|=3 \}$

32) $L = (a+b)^* a\,(a+b)^2$

3rd last is 'a'



$= 2^3 = 8$ states in min DFA

$= 3+1 = 4$ states in min NFA

$\Sigma = \{a\}$ 33) $L = \{a^{3n+2} \mid n \geq 0\} = \{w \mid w \in a^*, |w| = 2 \bmod 3\} \Rightarrow$ 3 States

$\Sigma = \{a\}$ 34) $L = \{a^{3n+7} \mid n \geq 0\} = \{a^7, a^{10}, a^{13}, a^{16}, \ldots\} \Rightarrow$ 8 states

35) $L = \{a^{100n+23} \mid n \geq 0\} \Rightarrow$ 100 states

36) $L = \{a^{100n+1234} \mid n \geq 0\} \Rightarrow$ 1235 states

$\{a^{K_1 n + K_2} \mid n \geq 0\}$

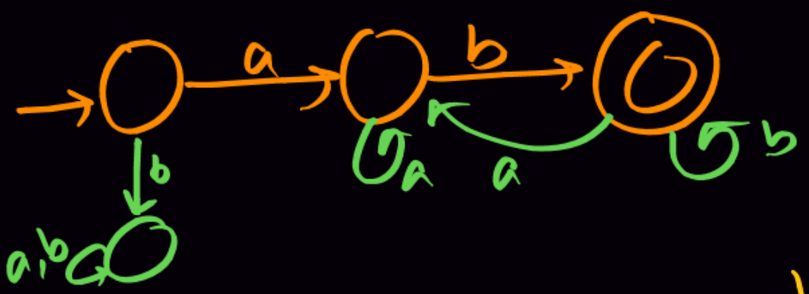
If $K_1 > K_2 \Rightarrow K_1$ states

Otherwise $K_2 + 1$ states

37) $L = a(a+b)^* b$

↳ 4 states in min DFA

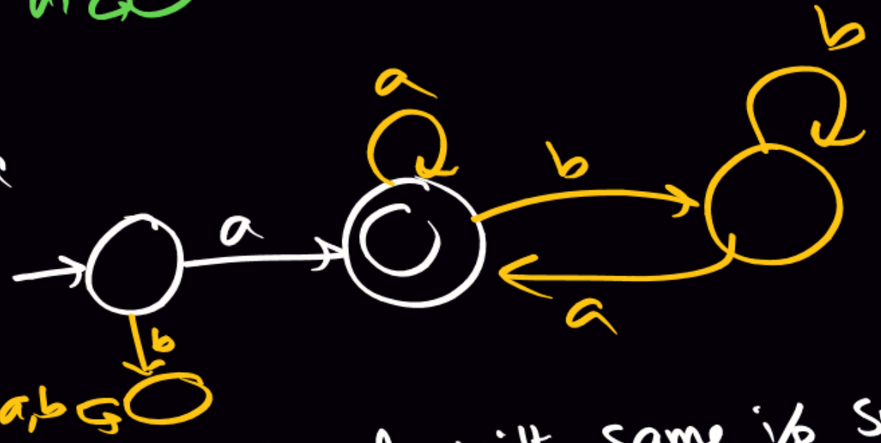
3 " " min NFA



38) $L = a + a(a+b)^* a$

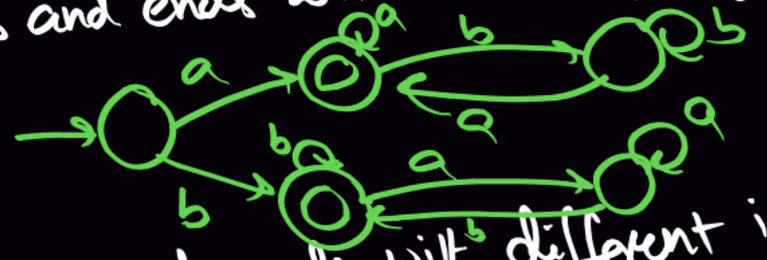
Min = a

↳ 4 states in min DFA



39) $L = \{w \mid w \in (a+b)^*$, w starts and ends with same i/p symbol$\}$

↳ 5 states $= a + a\Sigma^* a + b + b\Sigma^* b$

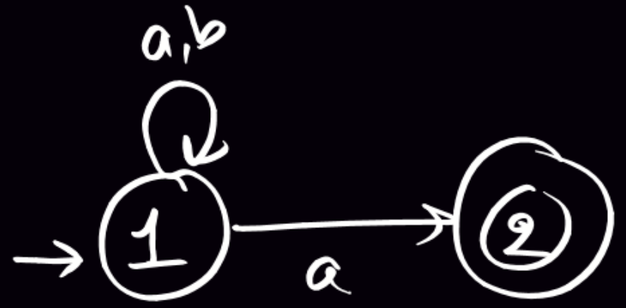


40) $L = \{w \mid w \in (a+b)^*$, w starts and ends with different i/p symbols$\}$

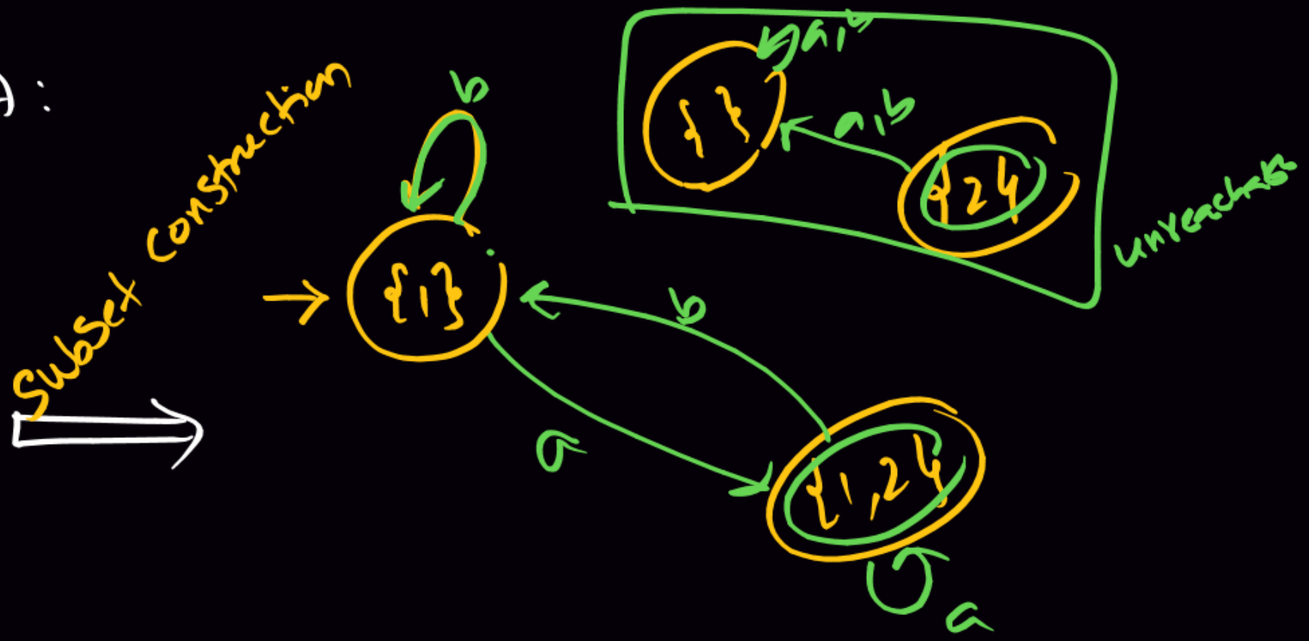
$= a\Sigma^* b + b\Sigma^* a$

↳ 5 states

Conversion From NFA to DFA:



Subset construction



$Q_{NFA} = \{1,2\}$

NFA

δ	a	b
→ 1	{1,2}	{1}
*2	{ }	{ }

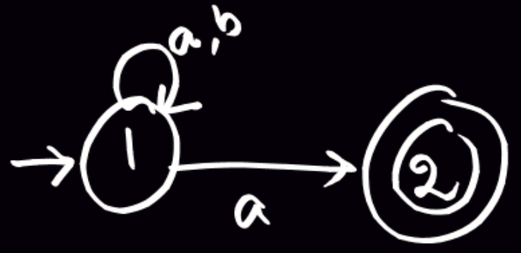
DFA

$Q_{DFA} = P(Q_{NFA})$

$= \{\{\}, \{1\}, \{2\}, \{1,2\}\}$

State of DFA

Subset of Q_{NFA}



	a	b
→ !	{1,2}	{1}
*2	{}	{}

⇒

	a	b
{}	{}	{}
→ {1}	{1,2} ✓	{1} ✓
* {1,2}	{}	{}
* {1,2}	{1,2} ✓	{1} ✓

$$\delta_D(\{1,2\}, a) = \delta_N(1,a) \cup \delta_N(2,a)$$

NFA ⟹ DFA ⟶ Min DFA

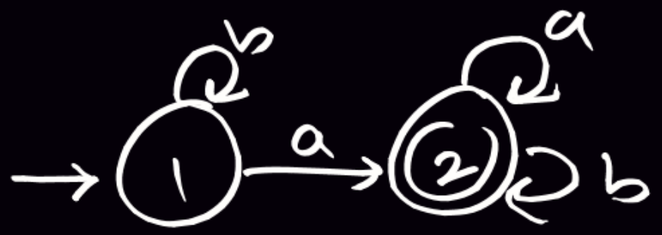
n states

2^n states

atmost 2^n states

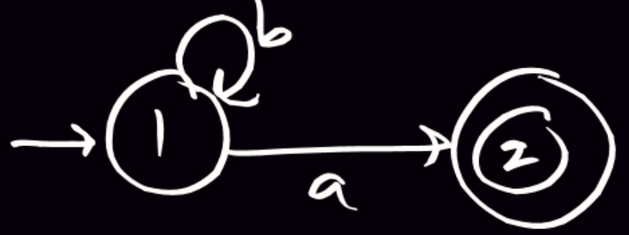
$(\leq 2^n$ states$)$

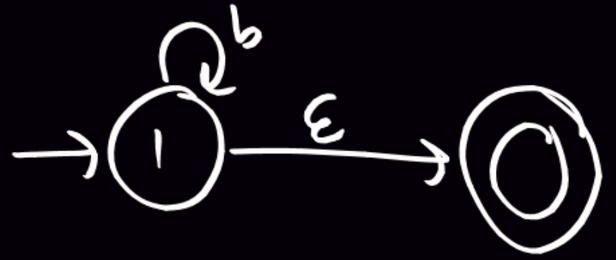
NFA with ε moves:



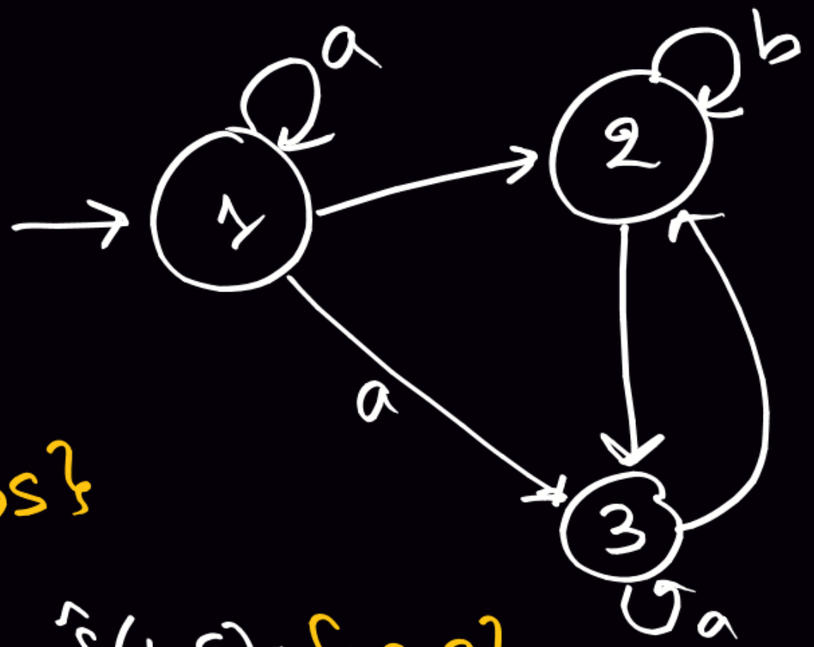
$\delta(1,\varepsilon) = \{\}$

$Q \times \Sigma_\varepsilon \to 2^Q$





	DFA	NFA without ε moves	NFA with ε moves
	✓	✓	✓
	✗	✓	✓
	✗	✗	✓
δ:	$Q \times \Sigma \to Q$	$Q \times \Sigma \to 2^Q$	$Q \times \Sigma_\varepsilon \to 2^Q$



ε-closure$(q) = \hat{\delta}(q, \varepsilon)$

$= \{s \mid q \overset{\varepsilon}{\Rightarrow} s\}$

ε-closure$(1) = \hat{\delta}(1, \varepsilon) = \{1, 2, 3\}$

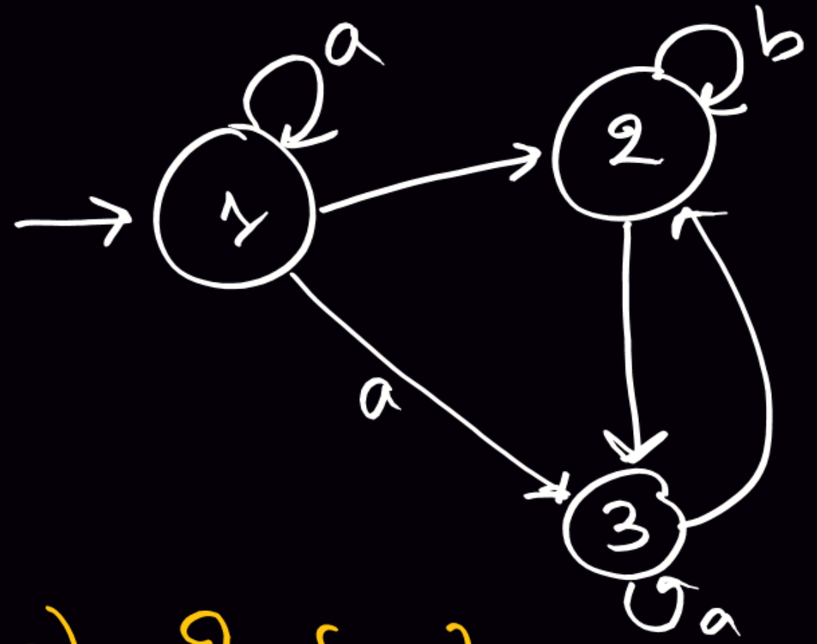
ε-closure$(2) = \hat{\delta}(2, \varepsilon) = \{2, 3\}$

ε-closure$(3) = \hat{\delta}(3, \varepsilon) = \{3, 2\}$

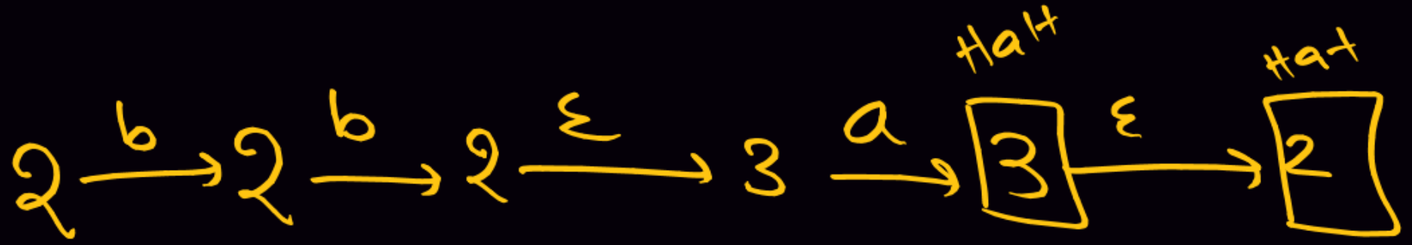
$1 \overset{\varepsilon}{\Rightarrow} 1$

$1 \overset{\varepsilon}{\Rightarrow} 2$

$1 \overset{\varepsilon}{\Rightarrow} 3$



$\hat{\delta}(2, bba) = ? = \{3, 2\}$



Regular Grammar (RG)

↳ It is Left Linear Grammar (LLG) OR Right Linear Grammar (RLG)

$G=(V, T, P, S)$

↳ set of non-terminals

$V \rightarrow VT^* \mid T^*$

$S \rightarrow Aab \mid Sa \mid bcd$

$A \rightarrow \varepsilon$

$V \rightarrow T^*V \mid T^*$

$S \rightarrow abS \mid aS \mid \varepsilon \mid abc$

	LLG	RLG	RG
1) $S \to \varepsilon$	✓	✓	✓
2) $S \to a \mid bc \mid \varepsilon$	✓	✓	✓
3) $S \to Aa \mid Bb$, $A \to \varepsilon$, $B \to ab$	✓	✗	✓
4) $S \to Sa \mid bS \mid \varepsilon$	✗	✗	✗
5) $S \to Sa \mid Sb \mid \varepsilon$	✓	✗	✓
6) $S \to A$, $A \to \varepsilon$	✓	✓	✓
7) $S \to AB$, $A \to a$, $B \to b$	✗	✗	✗

Find Language generated by following RG's.

1) $S \to \varepsilon$ $\quad L = L(S) = \{\varepsilon\}$

2) $S \to a \mid \varepsilon$ $\quad L = \{a, \varepsilon\}$

3) $S \to a \mid b \mid \varepsilon$ $\quad L = \{a, b, \varepsilon\}$ (OR, OR)

$S \Rightarrow a$ $\quad S \Rightarrow b$ $\quad S \Rightarrow \varepsilon$

4) $S \to ab \mid \varepsilon$ $\quad L = \{ab, \varepsilon\}$

5) $S \to ab \mid c \mid \varepsilon$ $\quad L = \{\varepsilon, c, ab\}$

6) $S \to Aa$, $A \to b$ ⇑ $A = b$ $\quad L = \{ba\}$

7) $S \to Aa \mid Bb$, $A \to c$, $B \to d$ ⇑ $A = c$, $B = d$ $\quad L = \{ca, db\}$

8) $S \to Aa$, $A \to \varepsilon \mid b$ ⇑ $L(A) = \{\varepsilon, b\}$ $\Rightarrow L = \{a, ba\}$

9) $S \to aB$, $B \to \varepsilon \mid a$ ⇑ $B = \varepsilon + a$ $\Rightarrow L = aB = \{a, aa\}$

10) $S \to Sa \mid b$

$L = ba^*$

b ✓
ba ✓
baa ✓
baaa ✓
⋮

$\Rightarrow ba^*$

11) $S \to Sa \mid \varepsilon$

$L = \varepsilon \cdot a^* = a^*$

12) $S \to Sa \mid a$

$L = aa^* = a^+$
$= a^*a$

13) $S \to Saa \mid \varepsilon$

$L = (aa)^*$

14) $S \to Sa \mid Sb \mid \varepsilon$

$S(a+b)$

$L = (a+b)^*$

15) $S \to Sa \mid Sb \mid a \mid b$

$S(a+b)$ | $a+b$

$L = (a+b)^+$

16) $S \to aS \mid \varepsilon$ $\quad L = a^*$

17) $S \to aS \mid bS \mid \varepsilon$ $\quad L = (a+b)^*$

18) $S \to aS \mid bS \mid a \mid b$ $\quad L = (a+b)^+$

19) $S \to aaS \mid bS \mid c$

$(aa+b)S$

$L = (aa+b)^*c$

20) $S \to aA$

$A \to aA \mid bA \mid \varepsilon \Rightarrow A = (a+b)^*$

$\Rightarrow L = a(a+b)^*$

H.W.

21) $S \to aS \mid bS \mid A$

$A \to aB$

$B \to aB \mid bB \mid \varepsilon$

22) $S \to Sa \mid Sb \mid A$

$A \to Ba$

$B \to Ba \mid Bb \mid \varepsilon$

23) $S \to aS \mid bS \mid a$

24) $S \to Sa \mid Sb \mid a$

25) $S \to aA \mid bB$

$A \to aA \mid \varepsilon$

$B \to bB \mid \varepsilon$

26) $S \to Aa \mid Bb$

$A \to Aa \mid \varepsilon$

$B \to Bb \mid \varepsilon$

27) $S \to abS \mid \varepsilon$

28) $S \to aA \mid bA$

$A \to a \mid b$