# Turing Machine & Undecidability [OM-3M]

1) TM

2) Recursive Vs RE

3) LBA Vs HTM Vs TM

4) Construction of TM

5) Closure properties

6) Undecidability

FA

$$(Q, \Sigma, \delta, q_0, F)$$

Read Head

Finite Control

I/P Tape

PDA

$$(Q, \Sigma, \delta, q_0, F, Z_0, \Gamma)$$

Read Head

FC

STACK

TM

$$(Q, \Sigma, \delta, q_0, F, B, \Gamma)$$

R/W head

Tape

FC

$\Sigma = \{a, b\}$

$\Gamma = \{a, b, B, x, \Gamma\}$

$\Sigma \subset \Gamma$

$B \in \Gamma$

FA	PDA	TM

$$\text{DFA} \cong \text{NFA}$$

$$\delta_{DFA} : Q \times \Sigma \to Q$$

$$\delta_{NFA} : Q \times \Sigma_\varepsilon \to 2^Q$$

$$\underset{\text{(NPDA)}}{\text{DPDA} < \text{PDA}}$$

$$\delta_{DPDA} : Q \times \Sigma \times \Gamma \to Q \times \Gamma^*$$

$$\delta_{PDA} : Q \times \Sigma_\varepsilon \times \Gamma^* \to 2^{Q \times \Gamma^*}$$

$$\text{DTM} \cong \text{NTM}$$

$$\delta_{DTM} : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$$

$$\delta_{NTM} : Q \times \Gamma \to 2^{Q \times \Gamma \times \{L, R\}}$$

TM
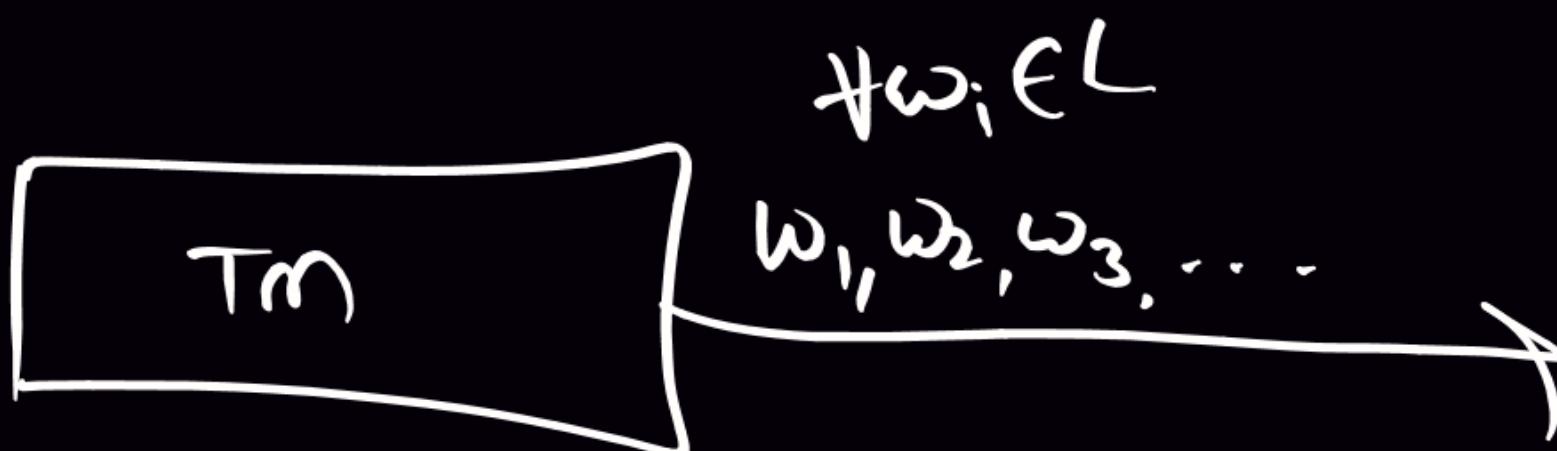
$\rightarrow$ It "accepts" Recursively Enumerable Language (REL) "
(recognizes)
(Enumerable Set)
(Turing Recognizable Set)
(Recognizable lang)
(Acceptable Set)
(Semi-decidable language)

$\rightarrow$ It can enumerate "REL"

Acceptor:

$w \longrightarrow$

TM
accepts L

If $w \in L$, Halts at final

If $w \notin L$, either TM halts at nonfinal
or TM never halts

Enumerator:

TM

$\forall w_i \in L$
$w_1, w_2, w_3, \ldots \longrightarrow$

FA:

TO   Remember `a`:

Change state

PDA:

I) Change state

II) PUSH/POP

III) I & II

$a, \varepsilon / \varepsilon$

$a, \varepsilon / a$

Push a

TM:

I) Change state

Read Write Direction

a, a, R

$Q \times T \rightarrow Q \times T \times \{l, R\}$
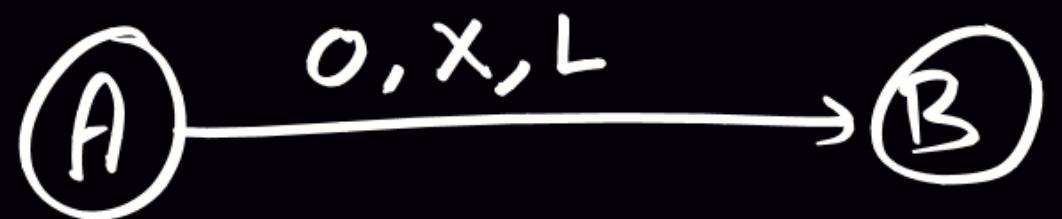
a, X, R


II) Write with new symbol
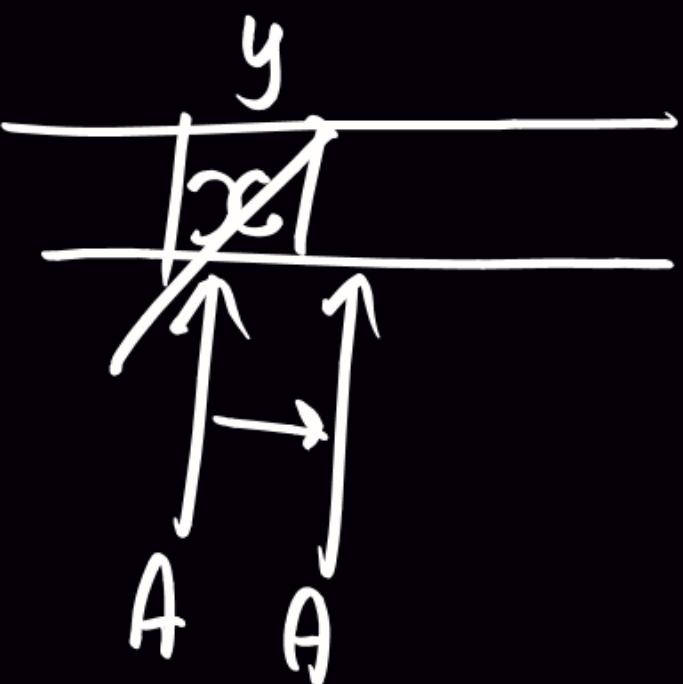
III) I & II

1) $\delta(A, 0) = (B, X, L)$

From state A, by reading 0, replaces with X and header moves left. (Write)

2) $\delta(A, x) = (A, y, R)$

(x, y, R)

Construction of Tm:

1) $L = \{\varepsilon\}$

$$\cdots\;|B|B|B|\;\cdots$$

$\xrightarrow{\quad}$ ◯ $\xrightarrow{\;B,B,R\;}$ ◎

2) $L = \{ab\}$

$\xrightarrow{\quad}$ (1) $\xrightarrow{\;a,a,R\;}$ (2) $\xrightarrow{\;b,b,R\;}$ (3) $\xrightarrow{\;B,\;\;\;}$ (4)

$$\cdots\;|B|a|b|B|\;\cdots$$

3) $L = a^*$

a, a, R

B, ′, ′

FA

a

4)

1 $\circlearrowright$ a, x, R

$\xrightarrow{}$ (1)

1 $\downarrow$ B, B, R

((2))
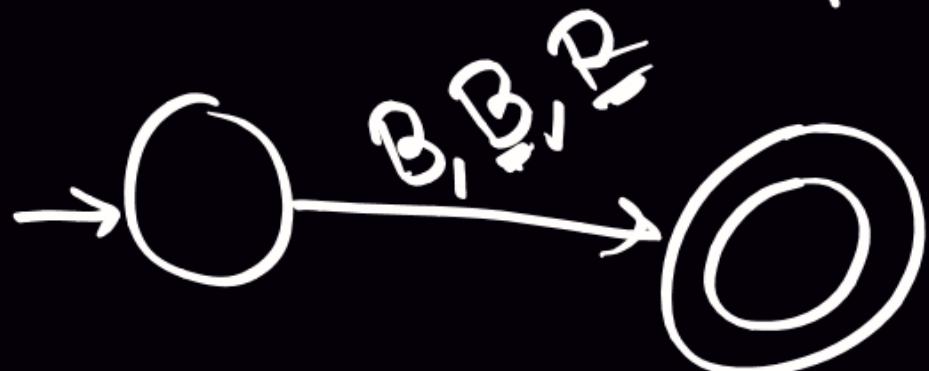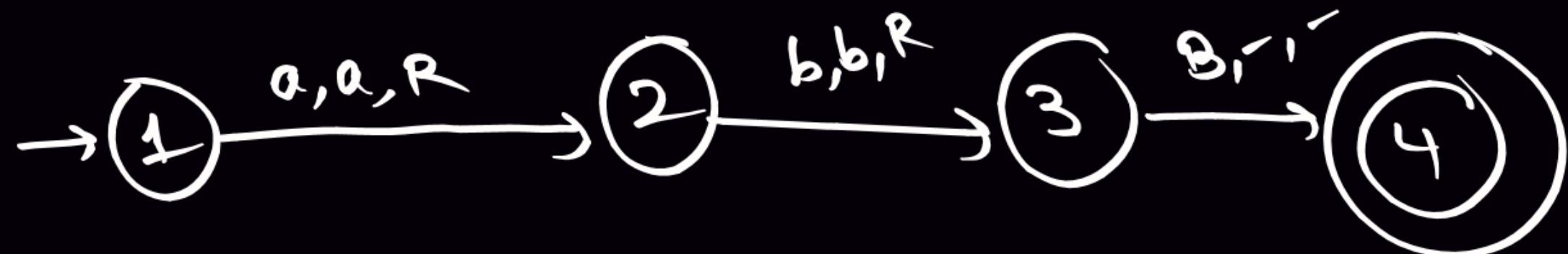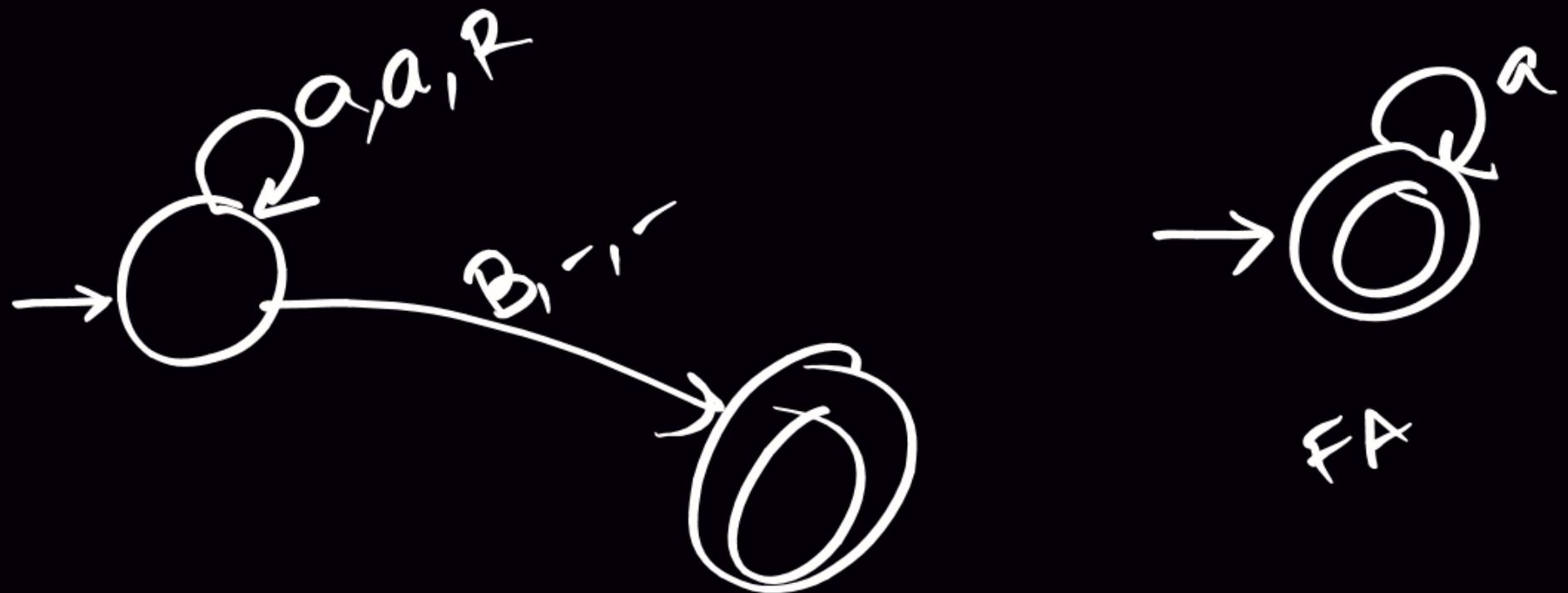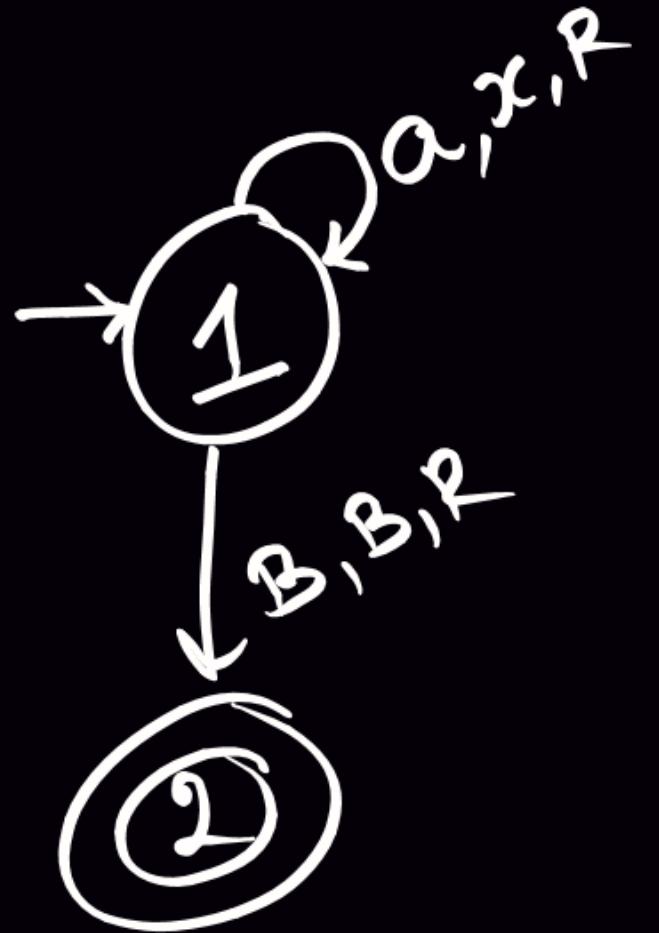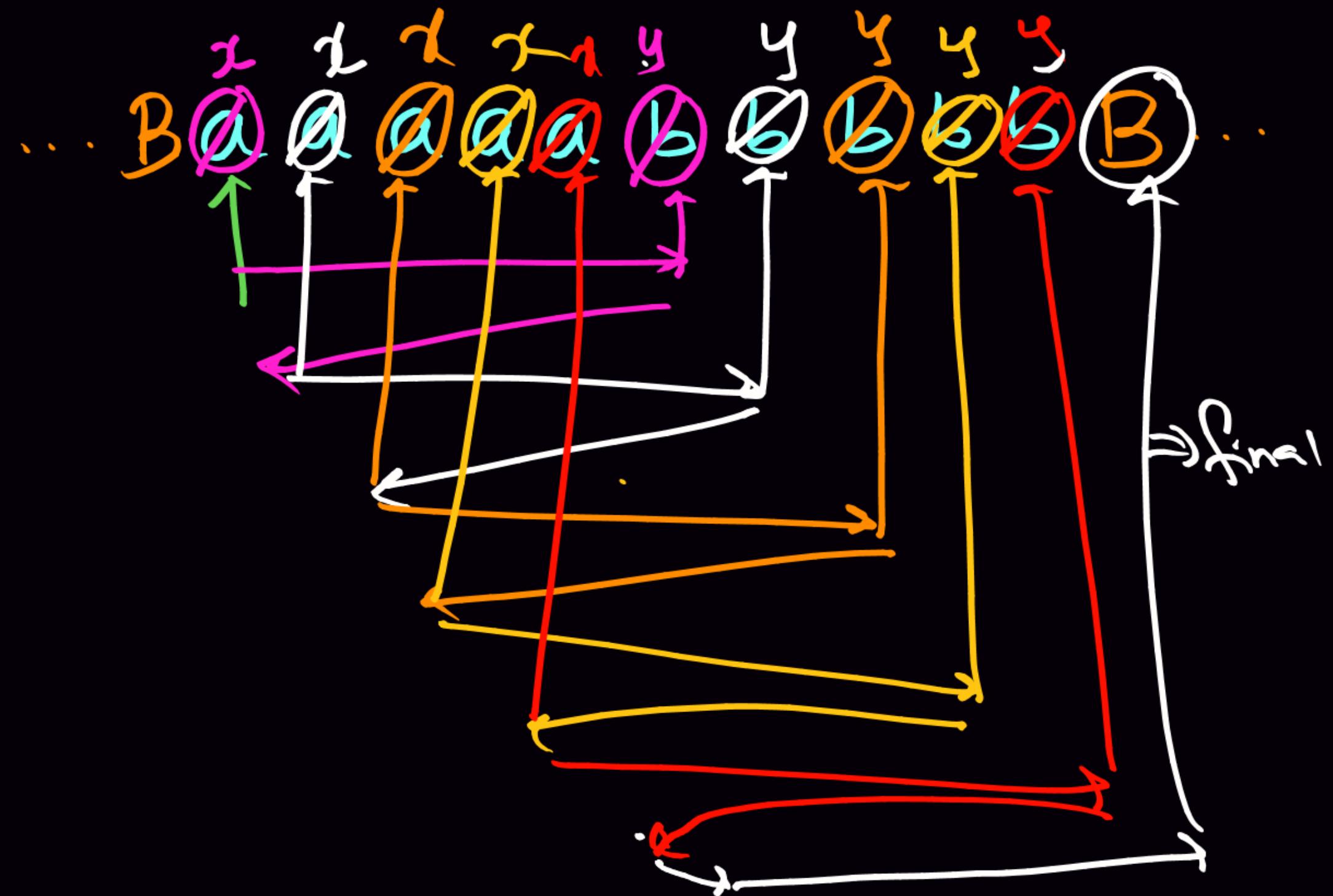
ε ✓
a ✓
aa ✓
aaa ✓
$\vdots$

$L = a^*$

5) $L = \{a^n b^n \mid n \geq 1\} = \{ab, aabb, aaabbb, a^4 b^4, \ldots\}$
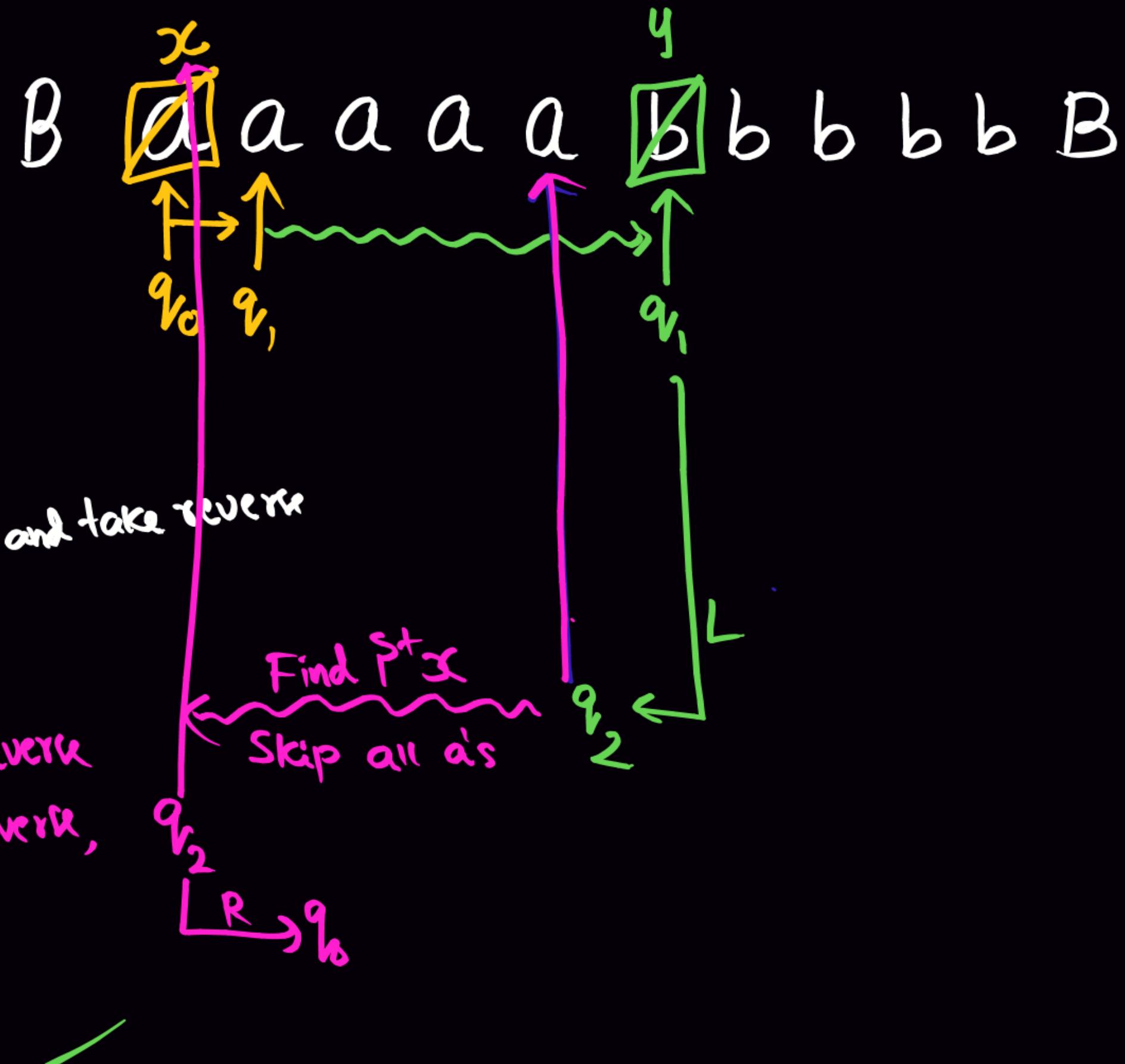
1st Scan:

q_0 : Replace a with x
move to q_1

q_1 : Skip all a's,
Replace b with y and take reverse
move to q_2

q_2 : Skip all a's in Reverse
to Find 1st x in reverse,
move to q_0

B a a a a a a b b b b b B

x

y

q_0 q_1

q_1

Find 1st x
Skip all a's

q_2

L

q_2

$\lfloor R \rightarrow q_0$
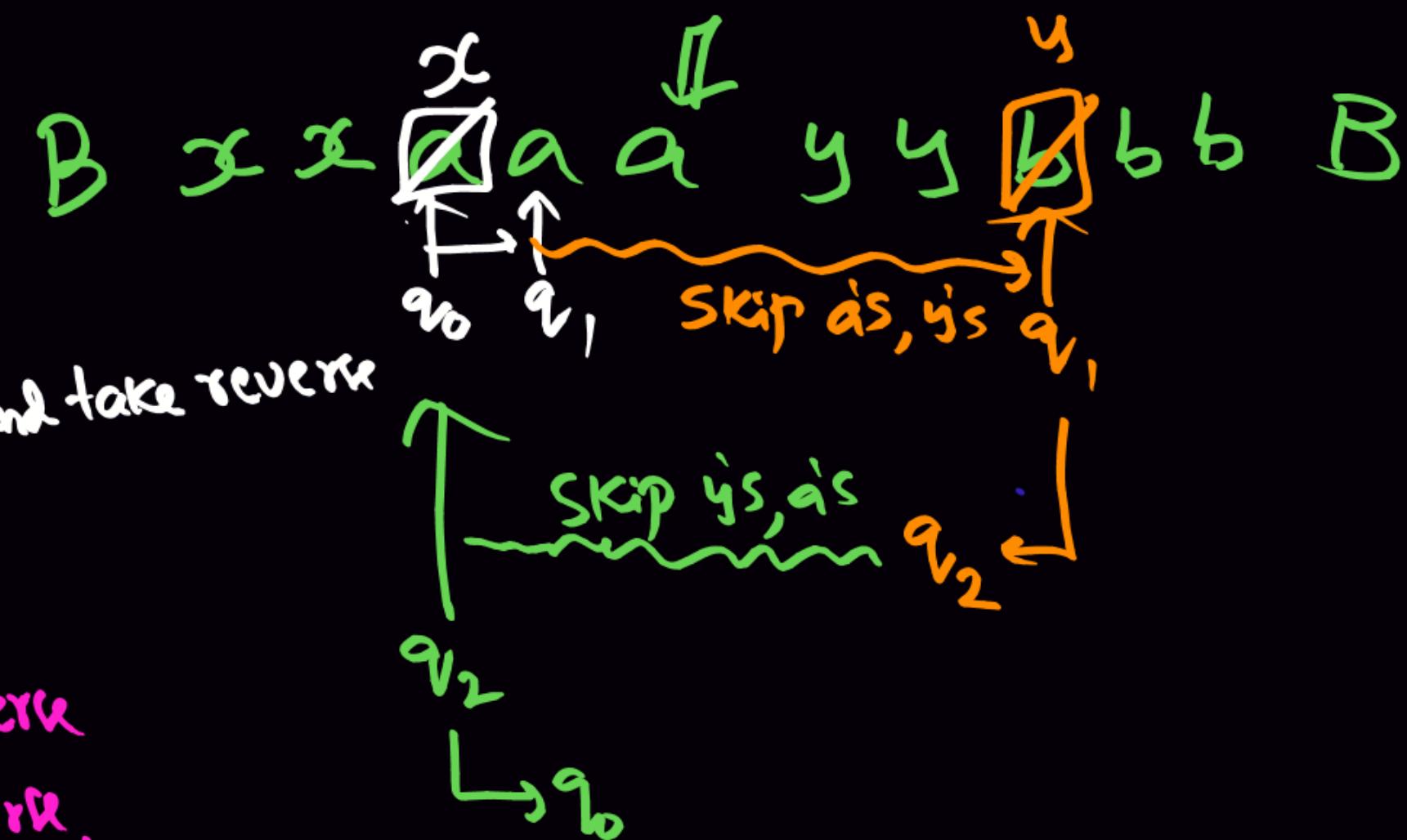
3rd scan:

B a a a a a b b b b b B
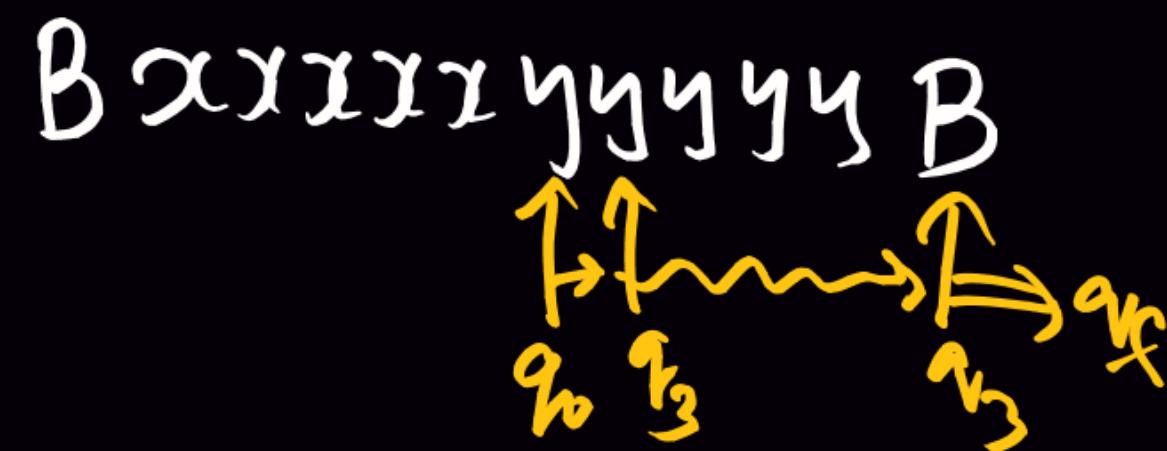
q_0 : Replace a with x
    move to q_1

q_1 : Skip all a's, y's

Replace b with y and take reverse
    move to q_2

q_2 : Skip all a's, y's in Reverse
    to Find 1st x in reverse,
    move to q_0

x   ⇓   y
B x x ⌀ a a   y y ⌀ b b B
         ↑   ↑
       q_0 q_1 skip a's, y's q_1

         skip y's, a's q_2

q_2

 q_0

Last Scan:

$$B\ x\ x\ x\ x\ y\ y\ y\ y\ y\ B$$
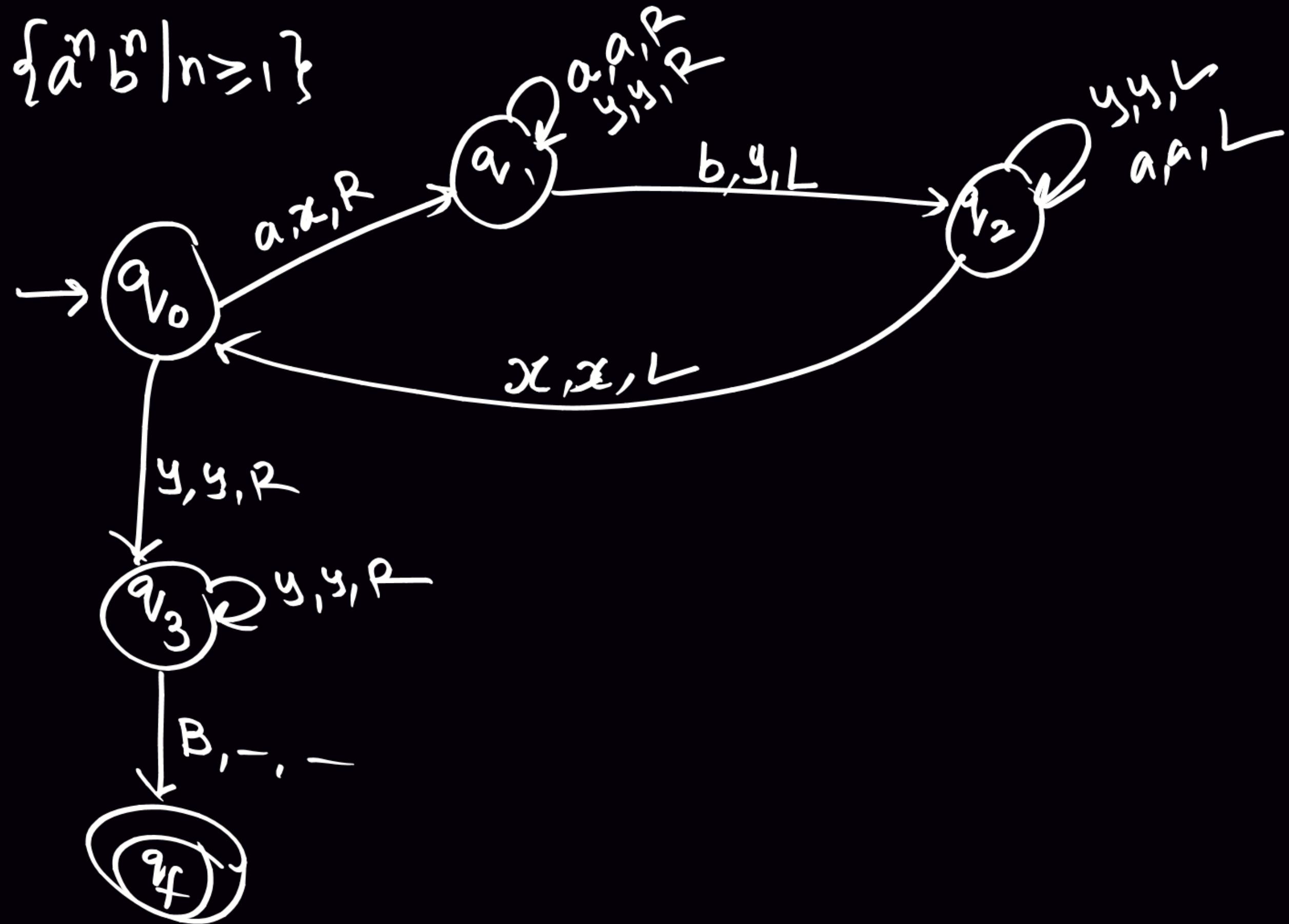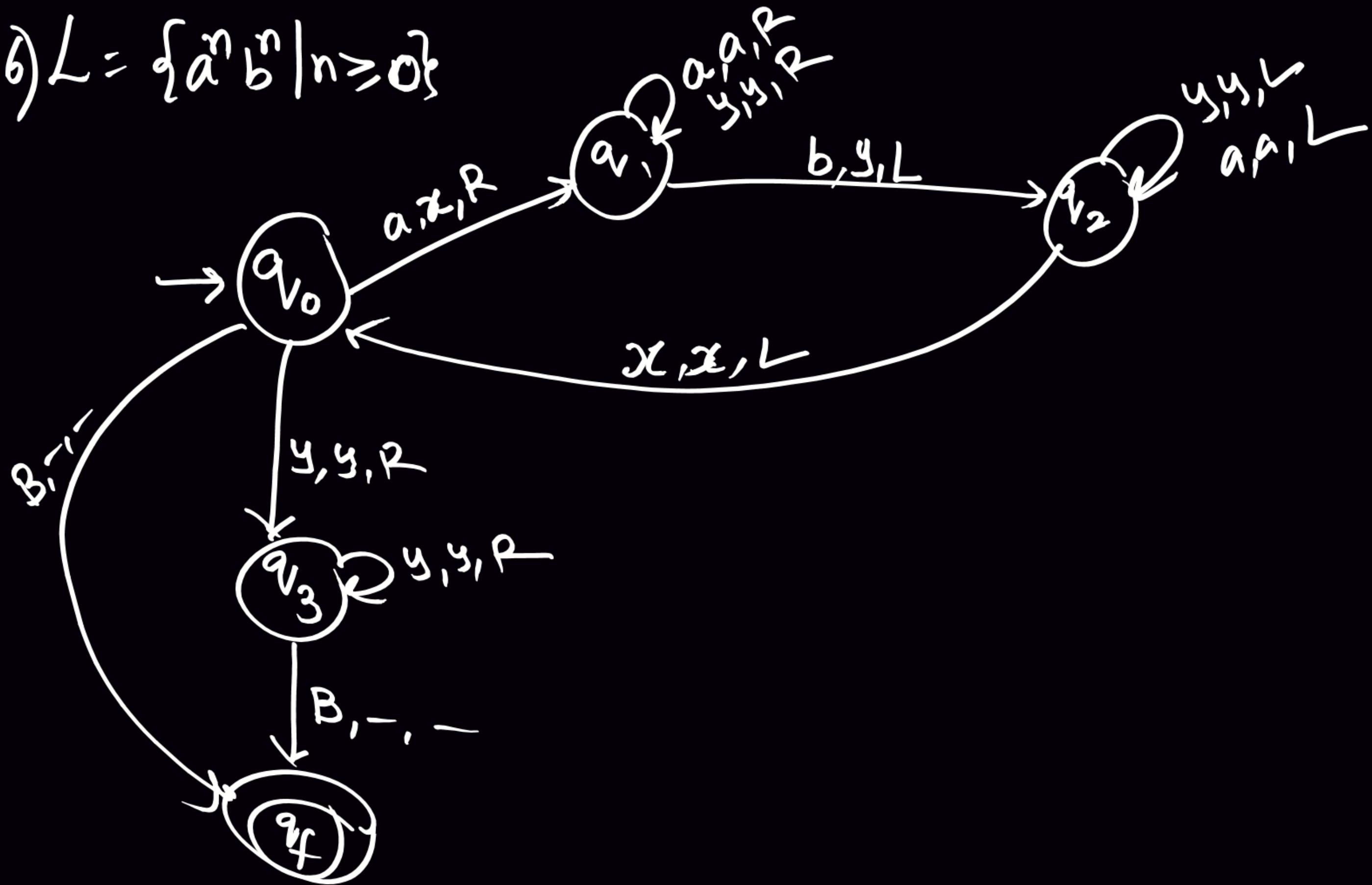
q_0: If it sees y, move to q_3

q_3: skip all y's to reach B, If it reaches B, ken go to final

$$L = \{a^n b^n \mid n \geq 1\}$$

State diagram of a Turing machine:

- Start arrow into q_0
- $q_0 \xrightarrow{a, x, R} q_1$
- q_1 self-loop: a, a, R and y, y, R
- $q_1 \xrightarrow{b, y, L} q_2$
- q_2 self-loop: y, y, L and a, a, L
- $q_2 \xrightarrow{x, x, L} q_0$
- $q_0 \xrightarrow{y, y, R} q_3$
- q_3 self-loop: y, y, R
- $q_3 \xrightarrow{B, -, -} q_f$

6) $L = \{a^n b^n \mid n \geq 0\}$

State diagram transitions:

- $q_0 \xrightarrow{a, x, R} q_1$
- q_1 self-loop: a, a, R and y, y, R
- $q_1 \xrightarrow{b, y, L} q_2$
- q_2 self-loop: y, y, L and a, a, L
- $q_2 \xrightarrow{x, x, L} q_0$
- $q_0 \xrightarrow{y, y, R} q_3$
- q_3 self-loop: y, y, R
- $q_3 \xrightarrow{B, -, -} q_f$
- $q_0 \xrightarrow{B, -, -} q_f$

7) $L = \{a^n b^n c^n \mid n \geq 1\}$

H.W.

x y z

B x x x $\cancel{a}$ a y y y $\cancel{b}$ b z z $\cancel{c}$ c B

q_0 q_1 q_1 q_2 q_3

L

q_3

q_3

$\rightarrow q_0$

8) 1's Complement of Binary i/p.

10111 → [ ] → 01000

0, 1, R
1, 0, R

B, B, _
Halt

01000

B 1 0 1 1 1 B

Halt
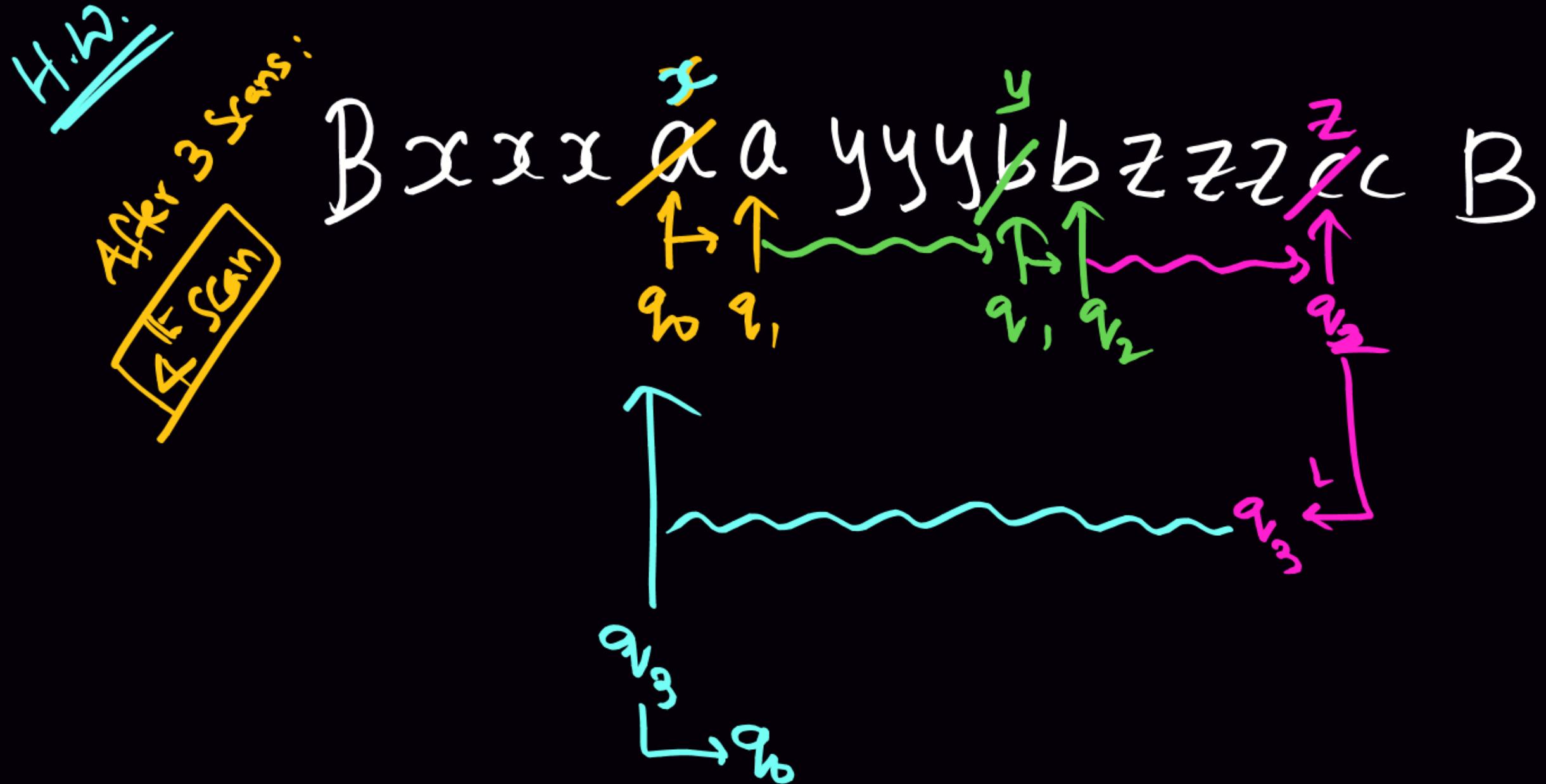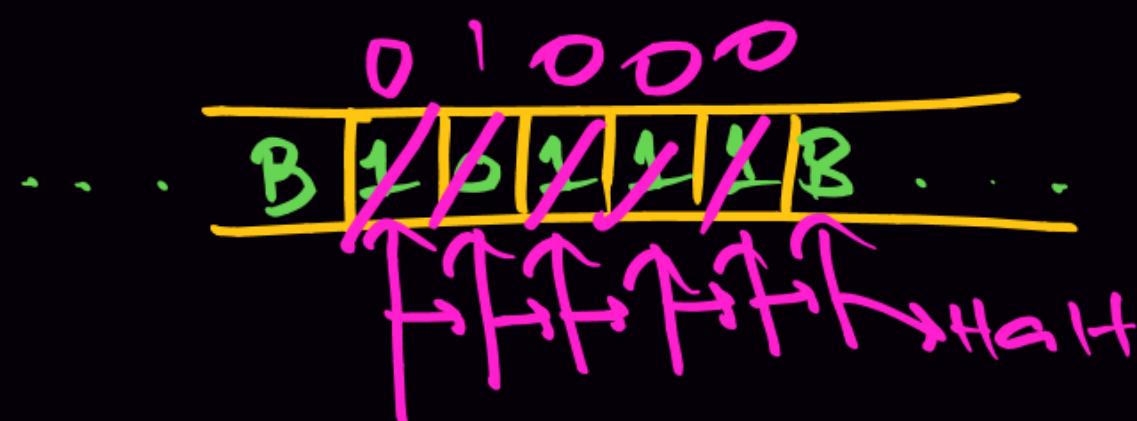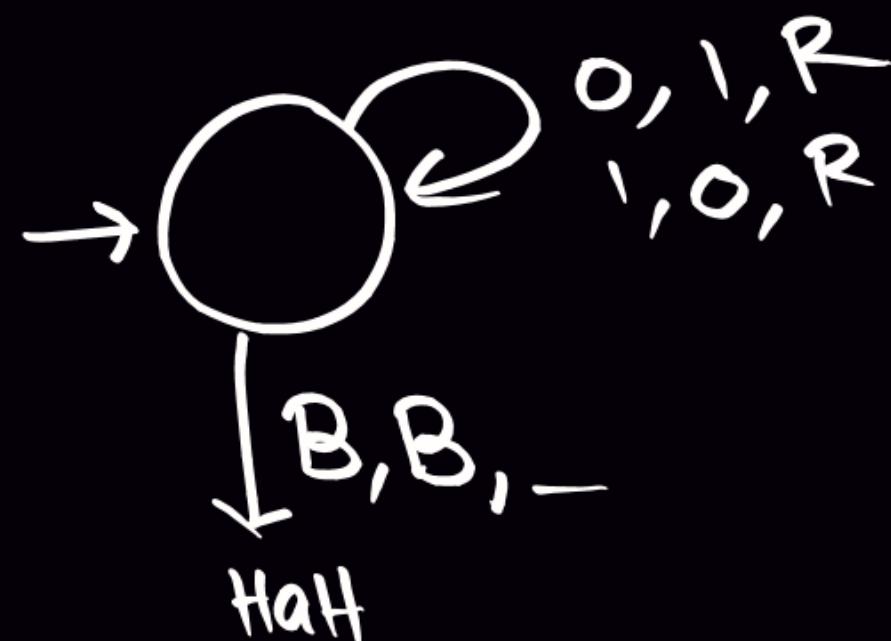
2's complement

Addition of Binary

Subtraction "

multiplication "

$\{ ww^R | w \in (a+b)^* \}$

$\{ w \# w | $ "