H.W.

21) $S \to aS \mid bS \mid A$

$A \to aB \Rightarrow A = a(a+b)^*$

$B \to aB \mid bB \mid \varepsilon \Rightarrow B = (a+b)^*$

$\boxed{L = (a+b)^* a (a+b)^*}$

22) $S \to Sa \mid Sb \mid A$

$A \to Ba \Rightarrow A = (a+b)^* a$

$B \to Ba \mid Bb \mid \varepsilon \Rightarrow B = (a+b)^*$

$\boxed{L = (a+b)^* a (a+b)^*}$

23) $S \to \underbrace{aS \mid bS}_{(a+b)^* S} \mid a$ $\quad \boxed{L = \Sigma^* a}$

24) $S \to Sa \mid Sb \mid a$ $\quad \boxed{L = a\Sigma^*}$

25) $S \to aA \mid bB$

$A \to aA \mid \varepsilon \Rightarrow A = a^*$

$B \to bB \mid \varepsilon \Rightarrow B = b^*$

$L = a^+ + b^+$

26) $S \to Aa \mid Bb$

$A \to Aa \mid \varepsilon \Rightarrow A = a^*$

$B \to Bb \mid \varepsilon \Rightarrow B = b^*$

$L = a^+ + b^+$

27) $S \to abS \mid \varepsilon$ $\quad \boxed{L = (ab)^*}$

28) $S \to aA \mid bA$

$A \to a \mid b$

$L = (a+b)A$

$= (a+b)(a+b) = (a+b)^2$

Language L (Set)

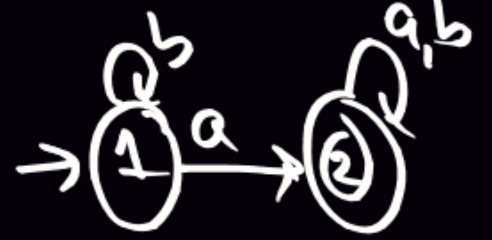
Regular Set

Def1: L has FA

Def2: L has Reg Exp

Def3: L has Reg Grammar

Def4: L has finite no. of equivalence classes



min DFA

$[1]: b^*$

$[2]: b^* a (a+b)^*$

$[1] \cup [2] = (a+b)^*$

Non-Regular Set

Def1: no FA exist for L

Def2: no Reg Exp

Def3: no RG

Def4: inf equivalence classes

L

- Finite Lang
  - ⇓ Regular
  - $\{a^n b^n \mid n < 100\}$
- Inf Set
  - L over 1 symbol
    - Forms A.P.
      - ⇓ Regular
      - $\{a^{2n+5}\}$
    - Not Forms A.P.
      - ⇓ Not Regular
      - $\{a^{2^n}\}$
  - L over >1 symbol
    - Regular
      - $a^* b^*$
      - $(ab)^*$
    - Not Regular
      - $a^n b^n$
      - $a^* b^{2^n}$
      - $a^n b^n c^{2^k}$

1) $\{a^m b^n \mid m,n \geq 0\} = a^* b^*$

2) $\{w \mid w \in a^* b^*\} = a^* b^*$

3) $\{a^m b^n\} = a^* b^*$

$\Rightarrow$ Inf, Reg

4) $\{a^m b^n \mid m < n < 100\} \Rightarrow$ Fin, Reg

5) $\{a^m b^n \mid \underbrace{m > n} > 100\} \Rightarrow$ Inf, Not Reg

6) $\{a^m b^n \mid \underbrace{m \neq n}_{m<n \text{ OR } m>n}\} \Rightarrow$ Inf, Not Reg

7) $\{a^m b^n \mid m = n\} \Rightarrow$ Inf, Not Reg

8) $\{w \mid w \in (a+b)^*\} = (a+b)^*$ Inf, Reg

9) $\{w \mid w \in (a+b)^*, \#_a(w) = \text{even}, \#_b(w) = \text{odd}\}$ Inf, Reg

10) $\{w \mid w \in (a+b)^*, \#_a(w) = \#_b(w)\}$ Inf, Not Reg

*** 11) $\{ww \mid w \in a^*\} = \{\varepsilon, a^2, a^4, a^6, a^8, \dots\} = (aa)^*$ Inf, Reg

12) $\{\underbrace{w\#w}_{\text{String}} \mid w \in a^*\} = \{w\#w \mid w \in \{\varepsilon, a, aa, aaa, \dots\}\} = \{\#, a\#a, a^2\#a^2, \dots\} = a^n\#a^n$

Inf, Not Reg

13) $\{\underbrace{ww}_{\text{String}} \mid w \in (a+b)^*\} \Rightarrow$ Inf, Not Reg

abaa abaa

14) $\{\underbrace{w\#w}_{\text{String}} \mid w \in (a+b)^*\} \Rightarrow$ Inf, Not Reg

15) $\{w_1 \# w_2 \mid w_1, w_2 \in a^*\} = a^* \# a^*$ Inf, Reg

16) $\{ww^R \mid w \in a^*\} = ww = (aa)^*$ Inf, Reg

17) $\{w \# w^R \mid w \in a^*\} = w \# w = a^n \# a^n$ Inf, Not Reg

18) $\{ww^R \mid w \in (a+b)^*\} \Rightarrow$ Inf, Not Reg

a baa aaba

19) $\{w \# w^R \mid w \in (a+b)^*\} \Rightarrow$ Inf, Not Reg

20) $\{\underbrace{w}_{\varepsilon, a}\underbrace{w}_{\varepsilon, a}\underbrace{w}_{\varepsilon, a} \mid w \in a^*\} = a^{3n} = (aaa)^*$ $\rightarrow \varepsilon$, $\rightarrow a^3$

21) $\{w_1 w_2 w_3 \mid w_1, w_2, w_3 \in a^*\} = a^* a^* a^* = a^*$

Reg, Inf

22) $\{www \mid w \in (a+b)^*\} \Rightarrow$ Inf, Not Reg

23) $\{w_1 w_2 w_3 \mid w_1, w_2, w_3 \in (a+b)^*\} =$

Inf, Reg

$\{www \mid w \in (a+b)^*\}$

$\varepsilon\ \varepsilon\ \varepsilon \longrightarrow \varepsilon$

$a\ a\ a \longrightarrow a^3$

$b\ b\ b \longrightarrow b^3$

aa aaaa

ababab

$(a+b)^*(a+b)^*(a+b)^* = (a+b)^*$

$\varepsilon \quad \varepsilon$

$w \in (a+b)^*$ $\quad w_1, w_2, w_3 \in (a+b)^*$

$www \neq \underbrace{w_1 w_2 w_3}$

$= (a+b)^*(a+b)^*(a+b)^*$

$= (a+b)^*$

$w^3 \mid w \in (a+b)^*$

$\{w^3 \mid w = (a+b)^*\}$ Reg

$\{www \mid w \in (a+b)^*\}$ Not Reg

$w \in \{\varepsilon, a, b, aa, ab, ba, \ldots\}$

24) $\{wwx \mid w, x \in (a+b)^*\}$

25) $\{wxw \mid$ " $\}$

26) $\{xww \mid$ " $\}$

27) $\{ww^Rx \mid$ " $\}$

28) $\{wxw^R \mid$ " $\}$

29) $\{xww^R \mid$ " $\}$

$\Rightarrow$ To prove put $w = \varepsilon$

$\Rightarrow x = (a+b)^*$ Regular

$w = ab, x = aaa$

$wwx = \underbrace{ababaaa}_{w = \varepsilon,\ x = ababaaa}$

30) $\{ww x \mid w, x \in (a+b)^+\}$

31) $\{w x w \mid w, x \in (a+b)^+\}$

32) $\{x w w \mid w, x \in (a+b)^+\}$

Not Reg

33) $\{w w^R x \mid w, x \in (a+b)^+\}$ Not Reg

34) $\{w x w^R \mid w, x \in (a+b)^+\} = w(a+b)^+ w^R$

*** Reg

$= [a(a+b)^+ a] + [b(a+b)^+ b]$

35) $\{x w w^R \mid w, x \in (a+b)^+\}$

Not Reg

31) $w x w \mid w, x \in (a+b)^+$

$w(a+b)^+ w$

$w = a \Rightarrow a(a+b)^+ a$

$w = b \Rightarrow b(a+b)^+ b$

$w = aa \Rightarrow aa(a+b)^+ aa$ ✓

$w = ab \Rightarrow ab(a+b)^+ ab$ ✗

$w = ba \Rightarrow ba(a+b)^+ ba$ ✗

$w = bb \Rightarrow bb(a+b)^+ bb$ ✓

34) $w(a+b)^+ w^R \mid w \in (a+b)^+$

$\in \{a, b, aa, ab, ba, bb, \dots\}$

$w = a$: $a(a+b)^+ a$

$w = b$: $b(a+b)^+ b$

$w = aa \Rightarrow aa(a+b)^+ aa$ ✓

$w = ab \Rightarrow ab(a+b)^+ ba$ ✓

$w = ba \Rightarrow ba(a+b)^+ ab$ ✓

$w = bb$ ✓

36) $\{wxwy \mid w,x,y \in (a+b)^+\} = a\Sigma^+a\Sigma^+ + b\Sigma^+b\Sigma^+$ Reg

$\boxed{axay + bxby}$

37) $\{xwyw \mid w,x,y \in (a+b)^+\} = \Sigma^+a\Sigma^+a + \Sigma^+b\Sigma^+b$ Reg

38) $\{xww^Ry \mid w,x,y \in (a+b)^+\} = \Sigma^+aa\Sigma^+ + \Sigma^+bb\Sigma^+$ Reg

Reg 39) $\{a^{2n+52}\} = (aa)^* a^{52}$

Reg 40) $\{a^{100n}\} = (a^{100})^*$

Not Reg 41) $\{a^{n^2}\} = \{a^{0^2}, a^{1^2}, a^{2^2}, a^{3^2}, \ldots\}$

Not Reg 42) $\{a^{n^{100}}\} = \{a^{0}, a^{1}, a^{2^{100}}, a^{3^{100}}, \ldots\}$

Not Reg 43) $\{a^{2^n}\}$

Not Reg 44) $\{a^{57^n}\}$

45) $\{a^{\text{Prime}}\}$ Not Reg

46) $\{a^{\text{Prime}^2}\}$ Not Reg

47) $\{a^{n!}\}$ Not Reg

48) $\{a^{n^n}\}$ Not Reg $= \{a^{1^1}, a^{2^2}, a^{3^3}, a^{4^4}, \ldots\}$
$= \{a, a^4, a^{27}, \ldots\}$

49) $\{a^{m^n}\} = a^*$ Reg

Put n=1

50) $\{a^{2n} b^{3K}\} = (aa)^* (bbb)^*$ Reg

Regular

51) $\{a^{2n} \mid n \geq 0\}^* = a^{2n} = (aa)^*$

52) $\{a^{prime}\}^* = (a^2 + a^3 + a^5 + a^7 + a^{11} + \cdots)^* = \{\varepsilon, \cancel{a}, a^2, a^3, a^4, a^5, \ldots\} = \text{Reg}$

$\boxed{\varepsilon + aaa^*}$

53) $\{a^{n^2} \mid n \geq 0\}^* = \underset{n=1}{a^*} \cup \cdots = a^*$

54) $\{a^{2^n} \mid n \geq 0\}^* = \underset{n=0}{a^*} \cup \cdots = a^*$

55) $\{a^{n^n} \mid n \geq 0\}^* = \underset{n=1}{a^*} \cup \cdots = a^*$

56) $\{a^{m^n} \mid m, n \geq 0\}^* = \underset{\substack{m=1 \\ n=1}}{a^*} \cup \cdots = a^*$

57) $\{w \mid w \in (0+1)^*, n_0(w) = n_1(w)\}$

*** 58) $\{w \mid w \in (0+1)^*, n_{01}(w) = n_{10}(w)\}$

59) $\{w \mid w \in (0+1)^*, n_{00}(w) = n_{11}(w)\}$

H.W.

60) $\{w \mid w \in (0+1)^*, n_{000}(w) = n_{111}(w)\}$

*** 61) $\{w \mid w \in (0+1)^*, n_{001}(w) = n_{100}(w)\}$

62) $\{w \mid w \in (0+1)^*, \text{Decimal}(w) \text{ is divisible by } 1024\}$

Next:

↳ closure properties (operations)

$\cup$

$\cap$

Difference

Complement

$\cdot$

Reversal