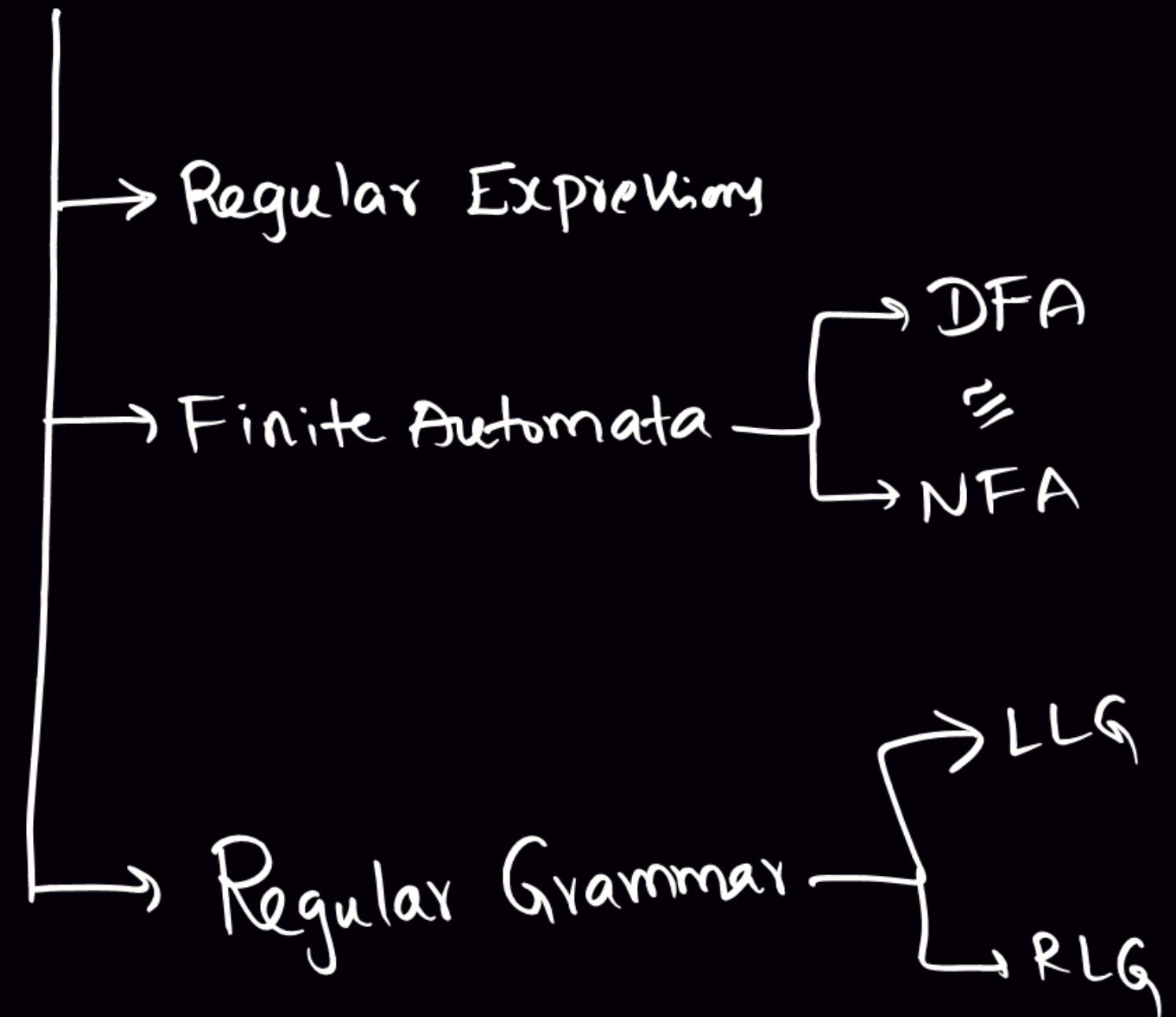
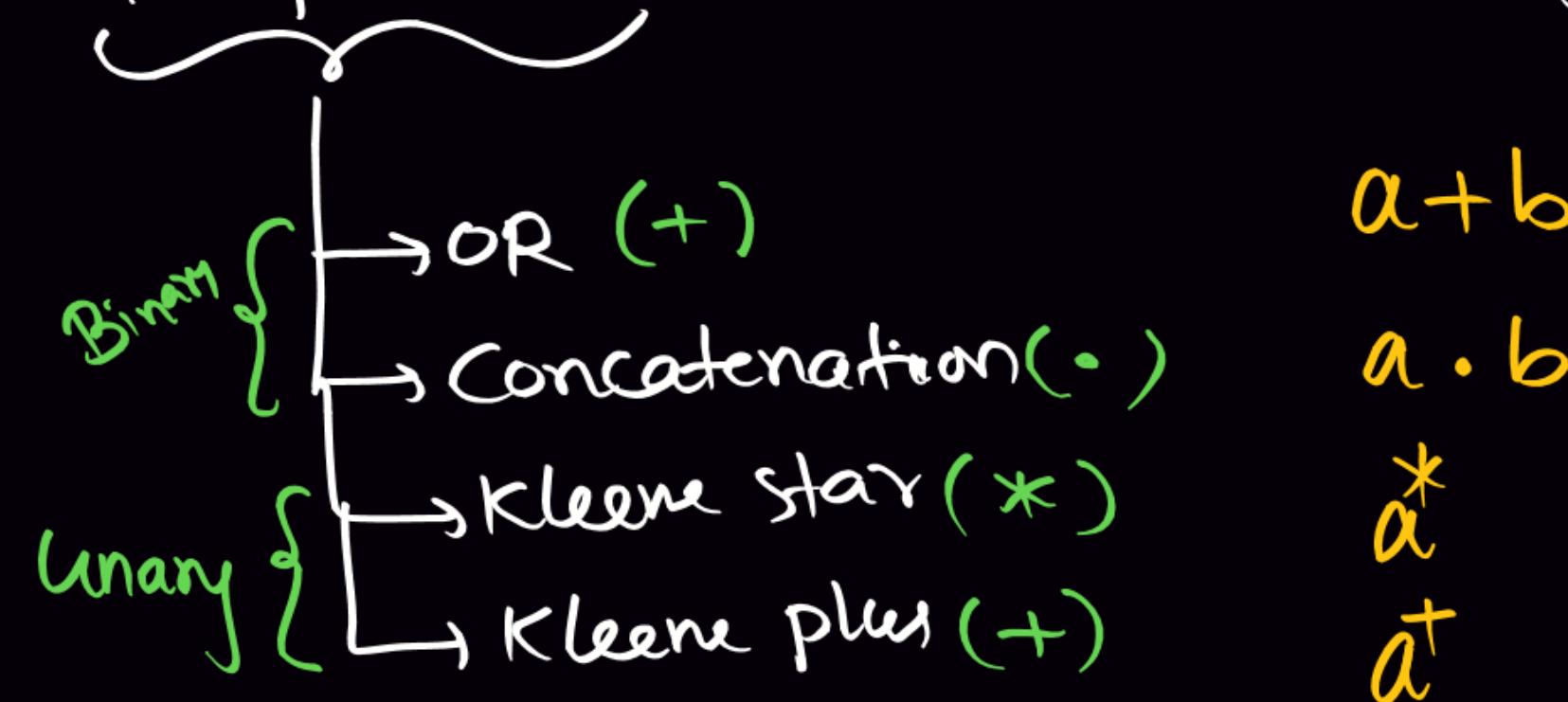


Regular Languages:



Regular Expression

→ It uses 4 operators to describe a Regular language.



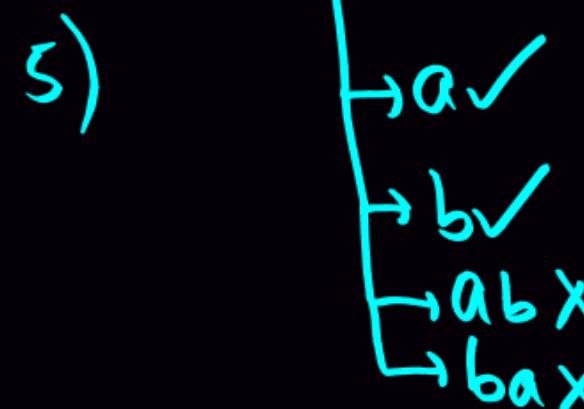
OR

1) $+ , \cup , |$

2) $R_1 + R_2$
 $R_1 \cup R_2$
 $R_1 | R_2$

3) either R_1 or R_2

4) $a+b$



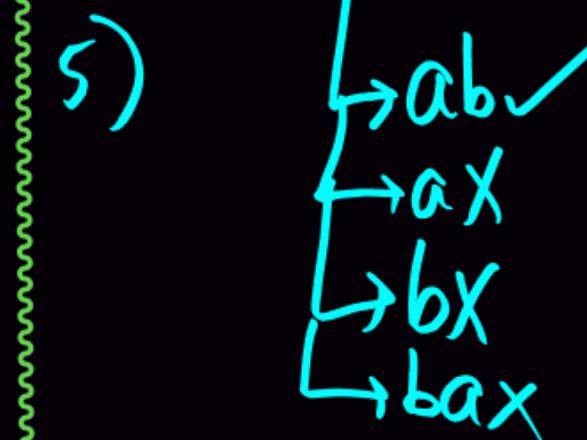
Concatenation

1) \cdot

2) $R_1 \cdot R_2$
 $R_1 R_2$

3) R_1 followed by R_2

4) $\underline{a \cdot b}$



Kleene Star

1) $*$

2) R^*
 $R^* = R^0 + R^1 + R^2 + \dots$

3) zero or more repetitions of R

4) $\underline{a^*} = a^0 + a^1 + a^2 + a^3 + \dots$

$$5) \left\{ \begin{array}{l} a^0 = \epsilon \\ a^1 = a \\ a^2 = aa \\ \vdots \end{array} \right\} a^{\geq 0}$$

Kleene plus

1) $+$

2) R^+

$$R^+ = R^1 + R^2 + R^3 + \dots$$

3) one or more repetitions of R

4) $\underline{a^+} = a^1 + a^2 + a^3 + \dots$

$$5) \left\{ \begin{array}{l} a^1 = a \\ a^2 = aa \\ \vdots \end{array} \right\} a^{\geq 1}$$

Regular Exp
R

Regular Language
(Regular Set)

$$\epsilon \quad L(\epsilon) = \{\epsilon\}$$

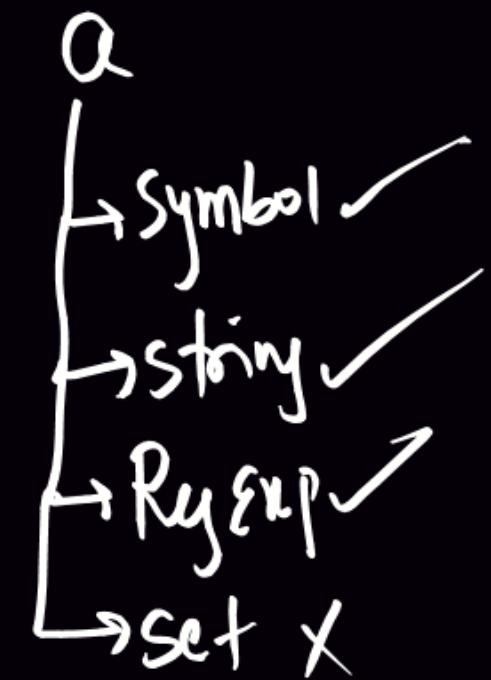
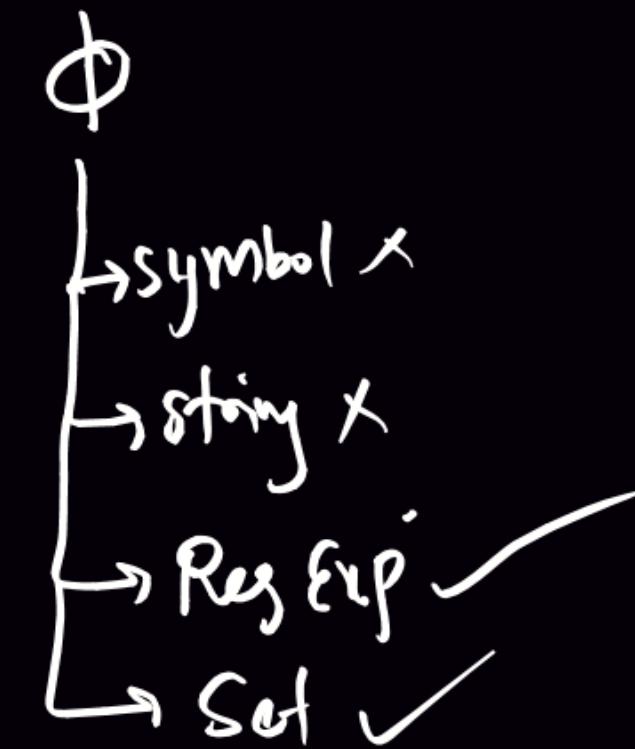
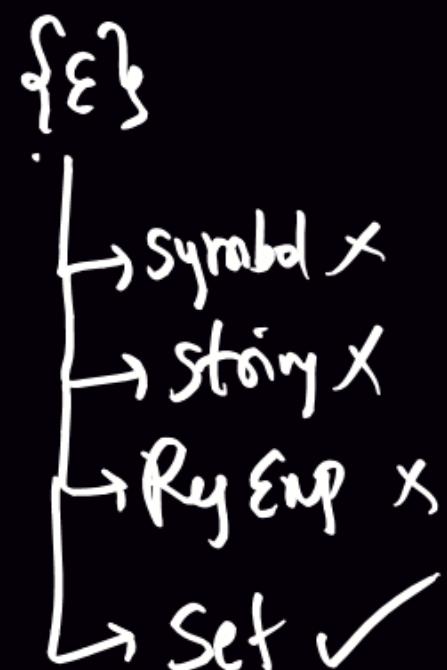
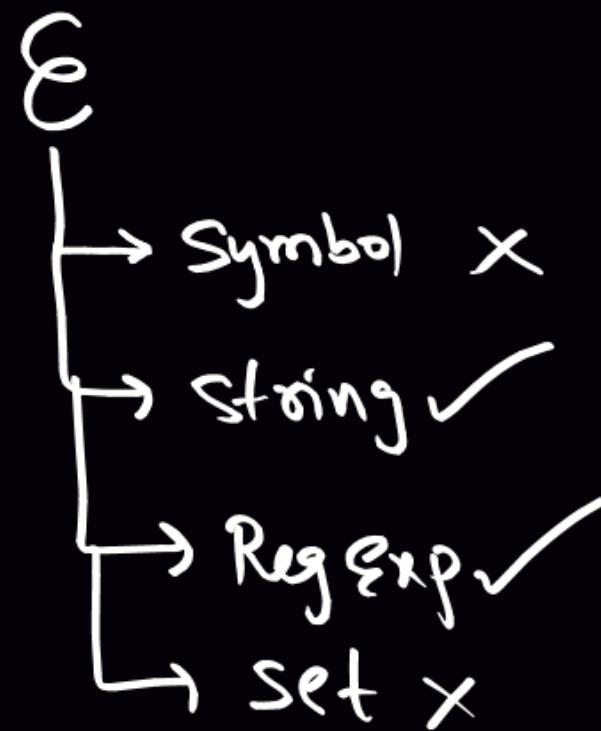
$$\epsilon + a \quad L(\epsilon + a) = \{\epsilon, a\}$$

$$a^* \quad L(a^*) = \{a^n \mid n \geq 0\} = \{\epsilon, a, a^2, a^3, \dots\}$$

$$\emptyset_{\text{empty exp}} \quad L(\emptyset) = \{\} = \emptyset_{\text{empty set}}$$

$$a+b \quad L(a+b) = \{a, b\}$$

$$\Sigma = \{a, b\}$$



$$L(\epsilon) = \{\epsilon\}$$

$$L(\phi) = \phi = \{\}\}$$

$$1) \phi + \phi = \phi$$

$$2) \varepsilon + \varepsilon = \varepsilon$$

$$3) a + a = a$$

$$4) \boxed{R + R = R}$$

$$5) \phi + \varepsilon = \varepsilon$$

$$6) \varepsilon + \phi = \varepsilon$$

$$7) \phi + a = a$$

$$8) a + \phi = a$$

$$\text{same} \quad 9) a + \varepsilon = \text{same}$$

$$10) \varepsilon + a = \text{same}$$

$a \neq a + \varepsilon$

$$11) \phi \cdot \phi = \phi$$

$$12) \varepsilon \cdot \varepsilon = \varepsilon$$

$$13) a \cdot a = a a = a^2$$

$$14) R \cdot R = R R = R^2$$

$$15) \phi \cdot \varepsilon = \phi$$

$$16) \varepsilon \cdot \phi = \phi$$

$$17) \phi \cdot a = \phi$$

$$18) a \cdot \phi = \phi$$

$$19) \varepsilon \cdot a = a$$

$$20) a \cdot \varepsilon = a$$

$$\text{I}) R + \phi = R$$

$$\text{II}) R \cdot \phi = \phi$$

$$\text{III}) R \cdot \varepsilon = R$$

$$21) \quad \phi^* = \phi^0 + \phi^1 + \phi^2 + \dots = \varepsilon + \underbrace{\phi_0 + \phi_1 + \phi_2 + \dots}_{\phi} = \varepsilon + \phi = \boxed{\varepsilon}$$

$$22) \quad \varepsilon^* = \varepsilon$$

$$23) \quad \alpha^* = \alpha^0 + \alpha^1 + \alpha^2 + \dots$$

$$24) \quad R^* = R^0 + R^1 + R^2 + R^3 + \dots$$

$$\boxed{R^0 = \varepsilon}$$

$$25) \quad \phi^+ = \phi$$

$$\varepsilon^0 = \varepsilon$$

$$26) \quad \varepsilon^+ = \varepsilon$$

$$\alpha^0 = \varepsilon$$

$$27) \quad \alpha^+ = \alpha + \alpha\alpha + \alpha\alpha\alpha + \dots$$

$$\phi^0 = \varepsilon$$

$$28) \quad R^+ = R^1 + R^2 + R^3 + \dots$$

$$b^0 = \varepsilon$$

$$(ab)^0 = \varepsilon$$

$$(a+b)^0 = \varepsilon$$

$$29) \quad a \cdot \overset{*}{a} = a \cdot (\underset{\substack{\uparrow \\ \text{Diagram: } \square}}{\varepsilon + a + aa + \dots}) = a\varepsilon + a \cdot a + a \cdot aa + \dots$$

$$= a + \overset{*}{a^2} + \overset{*}{a^3} + \dots = \boxed{\overset{*}{a}}$$

$$30) \quad \overset{*}{a} \cdot a = \overset{*}{a}$$

$$31) \quad a + \overset{*}{a} = a + [\varepsilon + a + aa + \dots] = \underline{a} + \varepsilon + \underline{a^2} + \underline{a^3} + \dots = \overset{*}{a}$$

$$32) \quad \overset{*}{a} + a = \overset{*}{a}$$

$$33) \quad \vec{a} + \vec{a} = \vec{a}$$

$$34) \quad \vec{a} \cdot \vec{a} = (\underbrace{\vec{a}^1 + \vec{a}^2 + \vec{a}^3 + \dots}_{\text{f}}) \cdot (\underbrace{\vec{a}^1 + \vec{a}^2 + \vec{a}^3 + \dots}_{\text{f}})$$
$$= (\vec{a})^2$$

$$\begin{aligned} &= \vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{a}^2 + \vec{a} \cdot \vec{a}^3 + \dots + \underbrace{\vec{a} \cdot \vec{a}}_{\vec{a}^2}, \underbrace{\vec{a} \cdot \vec{a}}_{\vec{a}^2}, \underbrace{\vec{a} \cdot \vec{a}}_{\vec{a}^2}, \dots \\ &= \vec{a}^2 + \vec{a}^3 + \vec{a}^4 + \dots \\ &= \vec{a}^2 \vec{a}^* = \underbrace{\vec{a} \vec{a}}_{*} = \vec{a} \vec{a} = \vec{a} \vec{a} \end{aligned}$$

$$35) (\overset{*}{a})^{100} = \overset{*}{a}\overset{*}{a}\overset{*}{a}\dots \text{100 times} = \overset{*}{a}$$

$$36) (\overset{100}{a^*})^* = \text{same}$$

$$37) (\overset{*}{a})^* = (\overset{*}{a})^0 + (\overset{*}{a})^1 + (\overset{*}{a})^2 + \dots = \overset{*}{a}$$

$$38) (\overset{*}{a})^+ = (\overset{*}{a})^1 + (\overset{*}{a})^2 + \dots = \overset{*}{a}$$

$$39) (\overset{+}{a})^* = \underbrace{(\overset{+}{a})^0}_{\text{1}} + (\overset{+}{a})^1 + (\overset{+}{a})^2 + \dots = \overset{*}{a}$$

$$40) (\overset{+}{a})^+ = \overset{+}{a}$$

$$\text{I) } (\overset{*}{a})^2 = \overset{*}{a} \cdot \overset{*}{a} = \overset{*}{a}$$

$$\text{II) } (\overset{*}{a}^2)^* = (aa)^* = \epsilon + aa + a^4 + a^6 + \dots$$

$$(\overset{*}{a}^2)^* \neq (\overset{*}{a}^2)^*$$

Note:

$$\text{I) } \tilde{a}^* = \varepsilon + \tilde{a}^+ = (\tilde{a} + \varepsilon)^* = (\tilde{a} + \varepsilon)^+ = (\tilde{a}^*)^2 = (\tilde{a}^*)^* = (\tilde{a}^*)^+ = (\tilde{a}^+)^*$$

$$\text{II) } (a+b)^* = \varepsilon + (a+b)^+ = (\tilde{a} b)^* \tilde{a}^* = (a+b)^* \tilde{a}$$

$$= (\varepsilon + a+b)^+ = (\tilde{b} a)^* \tilde{b}^* = \tilde{a}^* (a+b)^*$$

$$= (\varepsilon + a+b)^* = \tilde{a}^* (b \tilde{a}^*)^*$$

$$= (\tilde{a}^* \tilde{b}^*)^* = \tilde{b}^* (a \tilde{b}^*)^*$$

$$= (\tilde{b}^* \tilde{a}^*)^*$$

$$= (\tilde{a}^* + \tilde{b}^*)^*$$

$$= (\tilde{a}^* + \tilde{b}^+)^+ = (\tilde{a}^* + b)^* - (\tilde{a}^* + b)^+$$

$(\bar{a}^* \bar{b}^*)^*$ \Rightarrow It will generate every string

$$\epsilon = (\bar{a}^0 \bar{b}^0) = (\bar{a}^0 \bar{b}^0)^1 = (\bar{a}^0 \bar{b}^0)^2 = \dots$$

$$a = (\bar{a}^1 \bar{b}^0) = (a\epsilon) = a$$

$$b = (\bar{a}^0 \bar{b}^1)$$

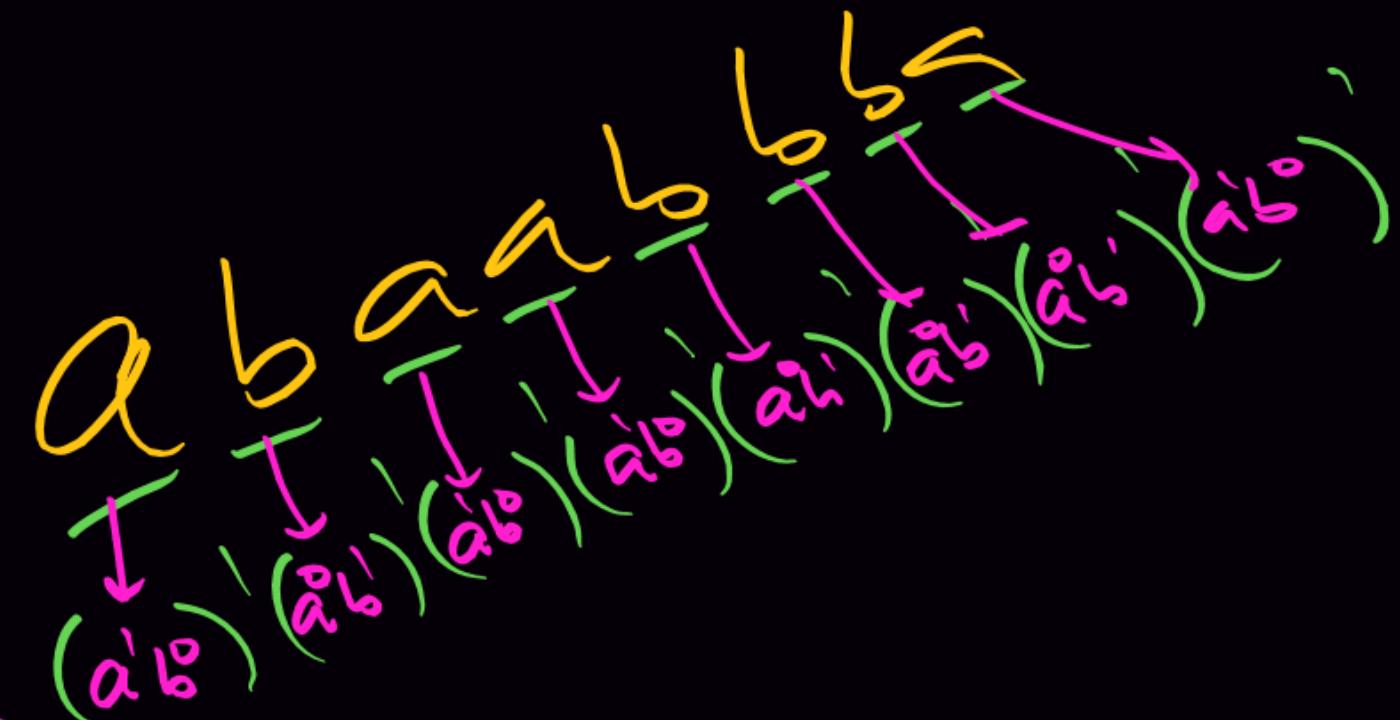
$$aa = (\bar{a}^2 \bar{b}^0)$$

$$ab = (\bar{a}^1 \bar{b}^1)$$

$$ba = (\bar{a}^0 \bar{b}^1)(\bar{a}^1 \bar{b}^0)$$

$$bb = (\bar{a}^0 \bar{b}^2)$$

$$\dots$$



$(\bar{a}^* \bar{b}^*)^* \neq \bar{a}^* \bar{b}^*$

$$(a+\varepsilon)^+ = (a+\varepsilon)^1 + (a+\varepsilon)^2 + (a+\varepsilon)^3 + \dots$$

$$= \underbrace{a+\varepsilon}_{\text{---}} + \underbrace{\varepsilon+a\varepsilon} + \underbrace{\varepsilon+a+a^2\varepsilon+}_{\text{---}} \dots$$

$$= \varepsilon + a + a^2 + a^3 + \dots$$

$$= a^*$$

$R^+ = R^1 + R^2 + R^3 + \dots$

$a+\varepsilon$ - $\begin{cases} \varepsilon \\ a \\ a^* \end{cases}$

$$(a+\varepsilon)^2 = \overline{(a+\varepsilon)(a+\varepsilon)}$$

$\begin{matrix} \rightarrow \varepsilon \\ \rightarrow a \\ \rightarrow a^* \end{matrix}$

Note:

I) $a \cdot b \neq b \cdot a$ [\cdot is not commutative]

II) $a+b = b+a$ [$+$ is commutative]

III) $R+\phi = \phi+R = R$ [ϕ is Identity exp for $+$]

IV) $R \cdot \epsilon = \epsilon \cdot R = R$ [ϵ is Identity exp for \cdot]

V) $R \cdot \phi = \phi \cdot R = \phi$ [ϕ is Dominator for \cdot
(annihilator)]

VI) $R + \Sigma^* = \Sigma^* + R = \Sigma^*$ [Σ^* is dominator for $+$]

$$\left. \begin{array}{l} \text{VII)} \quad a.(b+c) = ab + ac \\ \text{VIII)} \quad (b+c).a = ba + ca \end{array} \right\} \cdot \text{ can be distributed over +}$$

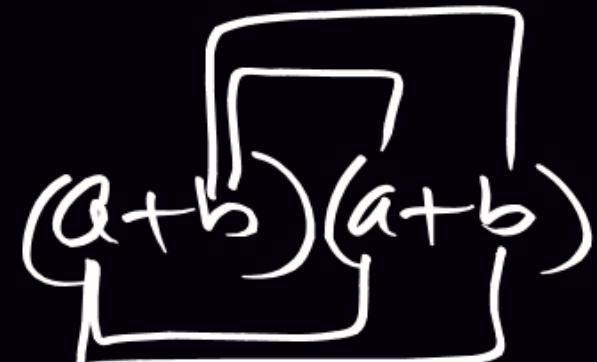
$$\text{IX)} \quad a+(b+c) \neq (a+b).(a+c)$$

$$\Sigma^* = \Sigma^0 + \Sigma^1 + \Sigma^2 + \Sigma^3 + \dots$$

$\Sigma = \{a\}$

$$\begin{aligned} \Sigma^* &= a^0 + a^1 + a^2 + a^3 + \dots \\ &= \epsilon + a + aa + aaa + \dots \end{aligned}$$

generates all strings over $\Sigma = \{a\}$



aa+ab+ba+bb

$\Sigma = \{a, b\}$

$$\begin{aligned} \Sigma^* = \{a, b\}^* &= (a+b)^* = \underbrace{(a+b)}_0 + \underbrace{(a+b)}_1 + \underbrace{(a+b)}_2 + \dots \\ &= \epsilon + a + b + aa + ab + ba + bb \end{aligned}$$

Write Regular Expression for following Regular Languages.

$$1) L = \{\} \text{ over } \Sigma = \{a\}$$

$$R = \phi$$

$$= \phi \cdot a$$

$$= \phi \cdot a^*$$

$$= \phi \cdot aa$$

⋮

$$2) L = \{\} \text{ over } \Sigma = \{a, b\}$$

$$R = \phi$$

$$= \phi \cdot a$$

$$= b \cdot \phi$$

$$= a \cdot \phi \cdot b$$

$$= a^* \cdot \phi$$

$$= b^* \cdot \phi$$

⋮

$$3) L = \{\} \text{ over } \Sigma = \{a, b, c\}$$

$$R = \phi$$

$$4) L = \{\epsilon\} \text{ over } \Sigma = \{a, b\} \implies R = \epsilon^* = \phi^* = a^0 = (ab)^0$$

$$5) L = \{w \mid w \in (a+b)^*, |w| \geq 1\} \implies (a+b)^+$$

$$6) L = \{w \mid w \in (a+b)^*, |w| \geq 0\} \implies (a+b)^*$$

$$7) L = \{w \mid w \in (a+b)^*, |w|=2\} \implies (a+b)^2 = aa + ab + ba + bb$$

$$8) L = \{w \mid w \in (a+b)^*, |w| \leq 2\} \implies \epsilon + (a+b) + (a+b)^2 = (\epsilon + a + b)^2$$

$$9) L = \{w \mid w \in (a+b)^*, |w| \geq 2\} \implies (a+b)^2 (a+b)^*$$

$$10) L = \{w \mid w \in (a+b)^*, |w| \leq 100\} \implies (\epsilon + a + b)^{100}$$

$\epsilon \rightarrow$ zero length

$a+b \rightarrow$ 1 len

$\epsilon+a+b \rightarrow$ 0 or 1 length
at most 1 len

$$(\epsilon+a+b)^2 = (\epsilon+a+b)(\epsilon+a+b)$$

= $\epsilon+a+b+aa+ab+ba+bb$
at most 2 length

$(\epsilon+a+b)^{100}$ = All strings upto 100 length

$$11) L = \{w \mid w \in (a+b)^*, w \text{ starts with } 'a'\} \Rightarrow a(a+b)^* = a\Sigma^*$$

$$12) L = \{w \mid w \in (a+b)^*, w \text{ starts with } aa \text{ or } bb\} = (aa+bb)\Sigma^*$$

$$13) L = \{w \mid w \in (a+b)^*, w \text{ ends with } 'a'\} = \Sigma^*a = (a+b)^*a$$

$$14) L = \{w \mid w \in (a+b)^*, w \text{ ends with } aa \text{ or } bb\} = \Sigma^*(aa+bb)$$

$$15) L = \{w \mid w \in (a+b)^*, w \text{ contains } 'a'\} = \Sigma^*a\Sigma^* = (a+b)^*a(a+b)^*$$

$$16) L = \{w \mid w \in (a+b)^*, w \text{ contains } aa \text{ or } bb\} = \Sigma^*(aa+bb)\Sigma^*$$

$$17) \{a^m b^n \mid m, n \geq 0\} \Rightarrow \overset{*}{ab}^*$$

$$18) \{a^m b^n \mid m \geq 0, n \geq 1\} \Rightarrow \overset{*}{a^m} b^+$$

$$19) \{a^m b^n \mid m \geq 1, n \geq 0\} \Rightarrow \overset{+}{a^m} b^*$$

$$20) \{a^m b^n \mid m \geq 1, n \geq 1\} \Rightarrow \overset{+}{a^m} b^+$$

$$21) \{a^m \mid m = \text{even}\} \Rightarrow (aa)^*$$

$$22) \{a^m \mid m = \text{odd}\} \Rightarrow a(aa)^* = (aa)^* a$$

$$23) \{a^m b^n \mid m = \text{even}, n = \text{odd}\} \Rightarrow (aa)^* b(bb)^*$$

$$24) \{a^m b^n \mid m+n = \text{even}\} \subseteq a^{\text{even}} b^{\text{even}} + a^{\text{odd}} b^{\text{odd}} \Rightarrow (aa)^*(bb)^* + a(aa)^* b(bb)^*$$

$$25) \{a^m b^n \mid m+n = \text{odd}\} \Rightarrow a^{\text{odd}} b^{\text{even}} + a^{\text{even}} b^{\text{odd}} = a(aa)^* (bb)^* + (aa)^* b(bb)^*$$

$m+n = \text{even}$

↓

Both m & n are even

or

Both m & n are odd

$$26) \{w \mid w \in (a+b)^*, \#_a(w)=2\} \Rightarrow b^* a b^* a b^*$$

$$27) \{w \mid w \in (a+b)^*, \#_a(w) \leq 2\} \Rightarrow b^* + b^* a b^* + b^* a b^* a b^* \\ \hookrightarrow b^* (\epsilon+a) b^* (\epsilon+a) b^*$$

$$28) \{w \mid w \in (a+b)^*, \#_a(w) \geq 2\} \Rightarrow \Sigma^* a \Sigma^* a \Sigma^*$$

$$29) \{w \mid w \in (a+b)^*, |w| = \text{even}\} \stackrel{\text{Length of } w \text{ is even}}{\Rightarrow} [(a+b)^2]^*$$

$$30) \{w \mid w \in (a+b)^*, |w| = \text{odd}\} \Rightarrow [(a+b)^2]^* (a+b)$$

no. of a's in w
 $\#_a(w)$
 $n_a(w)$

H.W.

$$\left\{ \begin{array}{l} 31) \{w \mid w \in (a+b)^*, \#_a(w) = \text{even}\} \\ 32) \{w \mid w \in (a+b)^*, \#_a(w) = \text{odd}\} \end{array} \right.$$