CS 580 Algorithm Design, Analysis, And Implementation
Fall 2021 Vassilis Zikas HW 4

Due November Nov. 5 at 11:59 p.m.

1. (30 points) Sara is playing a game called *Button Mash* with her boyfriend Charles and her sister Elizabeth. In Button Mash, all players are trying to cooperatively achieve the highest total score after n rounds. Each round, one of the players (or no players possibly), can choose to push a button to earn points. Sara, Elizabeth, and Charles are each given an array (J, E, and C respectively) of length n containing integers (positive or negative) where the ith element denotes the amount of points they would earn if that person pushed the button on the ith round. For example, $J[5]$ would be the amount of points that the team would earn on the 5th round if Sara pushed the button. There are two restrictions to the button pushing:

   - At most one player can push the button each round.
   - No player can push the button in two consecutive rounds.

   Design and analyze an algorithm that returns the maximum amount of points that the team can earn along with an array of length n denoting who pushed the button each round to obtain that score.

2. (30 points) Let $G : (V, E)$ be an undirected graph, let $s, t, v \in V$ be three distinct vertices in the graph. Design and analyze an efficient algorithm to find a *path* from s to t that goes though v. Recall that a path cannot repeat vertices.

3. (40 = 20 + 20 points) There may be multiple LP formulations of a problem, for example max flow. Consider the following questions:

   (a) In lecture, we formulated the *max flow problem* as a linear program. Write another way of modeling the max flow as a linear program by defining variables for each s-t path. What is an interpretation of this LP – explain in sentences, as was done in class for the Chicago vs. Detroit Pizza example? In particular, explain what each variable, the objective and each constraint means.

   (b) What is the dual of the above LP? What is an interpretation of the LP – explain in sentences, as was done in class for the Chicago vs. Detroit Pizza example? In particular, explain what each variable, the objective and each constraint means.

4. (2 points) Have you assigned pages on Gradescope?

5. **(*) 2nd Bonus Problem.** (30 points) Jane has been employed to repair a 2-lane road. Specifically, the 2-lane road is modeled as a 2 dimensional array of length n and width 2. Jane is given the location of the patches that are damaged (see the example below). Jane's objective is to repair the road in minimum cost. She has to work with the following constraints:

   - She can only cover rectangular patches i.e. she cannot have L shaped patches.
   - She needs to cover all the bad parts.
   - She can only use P rectangular patches in total.

Her cost is directly proportional to the area she covers. Give an efficient algorithm for Jane to repair the road. You need to give the optimal area and the optimum repair patches.

In the example below, the optimal way of repairing the road using P = 2 patches (green-filled area) is given as under. The optimal area is 8 units.

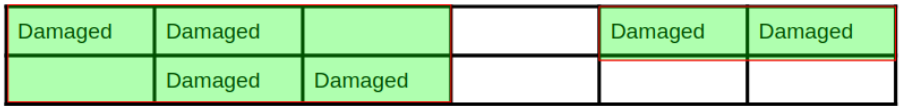

Figure 1: Optimal road patch for P = 2