CS 580

FINAL REVIEW SESSION.

(2). Problem P is NP-hard.

→ 3 SAT

(1) Convert instances of 3 SAT to problem P.

(2) Convert instances of P to 3SAT.

def 3SAT (ϕ) :

(1) Preprocessing → x (instance of P)

(2) Solve $P(x)$

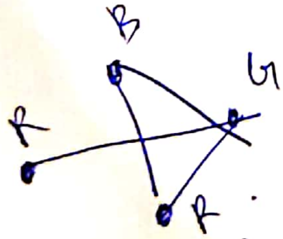
(3) Return output.

⇒ Prove that 4- color is NP-Hard.

3-color. $\longrightarrow$ 4 color.

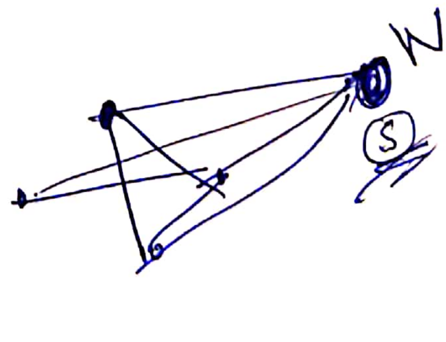
G, {R, B, G} R, B, G, W

undirec.



G.

⇒



G'

Clm G has a 3-coloring iff G' has a 4-coloring.

Proof.

Direction 1 $\Rightarrow$

If G has valid 3-coloring then G' has a 4-coloring

Direction 2 $\Leftarrow$

If G' has a 4-coloring, then G has a 3-coloring.

(5) To show 4-SAT is NP-Hard.

3-SAT $\Longrightarrow$ 4-SAT.

ϕ $\qquad$ ϕ'

C_1 and C_2 and ... C_m

$a_1 \vee b_1 \vee c_1$

$\Rightarrow$ $(a_1 \vee b_1 \vee c_1 \vee x_1)$ and $(a_2 \vee b_2 \vee c_2 \vee x_2)$ and ... $(a_m \vee b_m \vee c_m \vee x_m)$.

false

$(a_1 \vee b_1 \vee c_1) \Rightarrow$ ① $(a_1 \vee b_1 \vee c_1 \vee x_1) \to T$

and

② $(a_1 \vee b_1 \vee c_1 \vee \overline{x_1}) \to F$

Claim

ϕ is satisfiable iff ϕ' is satisfiable

$\Rightarrow$ If ϕ is satisfiable then ϕ' is satisfiable.

$\Rightarrow$ If ϕ' is satisfiable then ϕ is satisfiable.

α'

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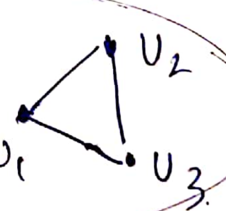
each vertex $u \longrightarrow$



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⑦. Subset of size 1 $\to$ poly time

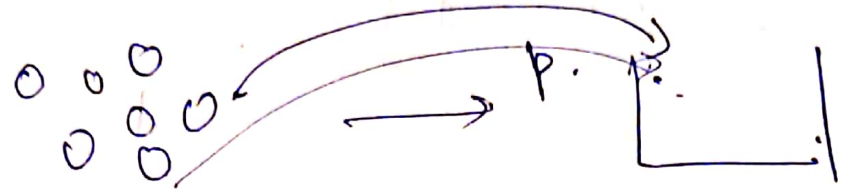
\+ size 2 $\to$ $^nC_2 \times$ polytime

\+ ⋮

\+ size 20 $\to$ $^nC_{20} \times$ polytime

$O(n+m)$

(8) m balls and 1 bin



For each ball, define

$x_i = 1$ if ball i ends up in the bin.

$\quad\;\; 0$ otherwise.

$\Pr(X_i = 1) = p$

$\Pr(X_i = 0) = 1 - p.$

$E[X] = ?$

$X = \sum_{i=1}^{m} x_i.$

$E[X] = E\left(\sum_{i=1}^{m} x_i\right) \overset{\text{linearity of expectation}}{=} \sum_{i=1}^{m} E[x_i] = mp$

$E[X_i] = 1 \cdot \Pr(X_i = 1) + 0 \Pr(X_i = 0)$

$= p.$

$$\Pr[X \geq 2E[X]] \leq 100$$

$$\leq 1$$

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(1) Markov's Inequality.

$$\Pr[X \geq a] \leq \frac{E[X]}{a}.$$

$$a = 2E[X]$$

$$\Pr[X \geq 2E[X]] \leq 1/2$$

(2) Chernoff Bounds.

$$\Pr[X \geq (1+\delta)\mu] \leq e^{-\mu \delta^2/2+\delta}.$$

$\delta = 1$

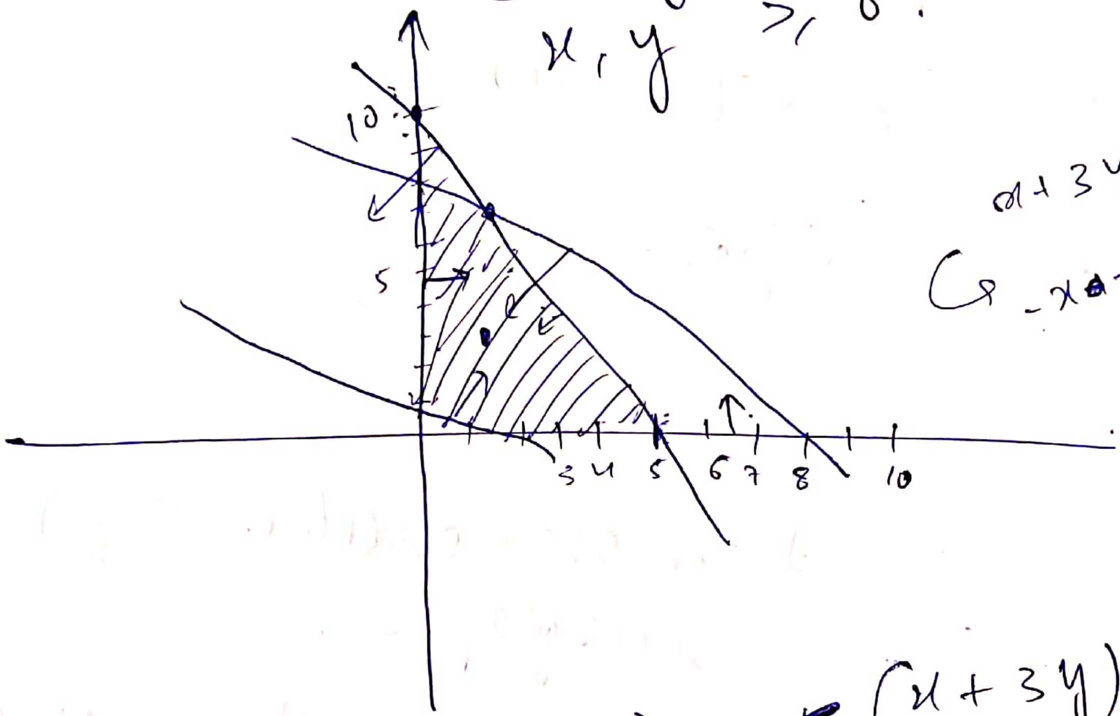
9) max $2x + 3y$

$2x + y \le 10 \rightarrow y_1$

$x + y \le 8 \rightarrow y_2$

$-(x + 3y) \le -2 \rightarrow y_3$

$x, y \geq 0$.



$x + 3y \geq 2$.

$\hookrightarrow -x - 3y \le -2$.

$(2x+y)y_1 + (x+y)y_2 - (x+3y)y_3 \le 10y_1 + 8y_2 - 2y_3$

$\hookrightarrow$

$2x+3y \le x(2y_1 + y_2 - y_3)$

$+ y(y_1 + y_2 - 3y_3)$

$\le 10y_1 + 8y_2 - 2y_3$

$\underline{X}$ is a r.v. continuous.

→ Show that X can take values atleast

Case (1) $E[X] = 1$

Case (2) $E[X] = 10$

Case 3 $E[X] = 5$

(6) X = # monochromatic size r subgraphs.

$X_i = 1$ if subgraph i is monochromatic

$$Pr(X_i = 1) = \frac{1}{2^{{}^rC_2}} + \frac{1}{2^{{}^rC_2}} = \frac{2}{2^{{}^rC_2}}$$

$$E[X] = E\left(\sum X_i\right) \overset{\text{Linearity}}{=} \sum E[X_i]$$

$$= {}^nC_r \frac{2}{2^{{}^rC_2}}$$

for certain values of n & r, < 1