Project 13
NDOF Earthquake
CEE 536

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Contents

Problem Statement

To show the response of an N-Degree of Freedom system under the effect of earthquake as shown in Figure 1.

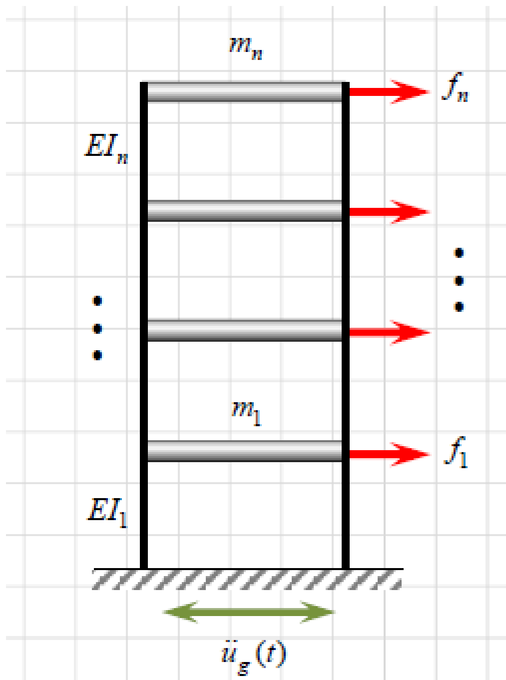

Figure 1

The problem involves using the structure parameters to form equations of motion of the form shown in Figure 2.

$$\begin{aligned} \mathbf{M}\,\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{r}(\mathbf{u}(t)) &= -\ddot{u}_g\mathbf{M1} \\ \mathbf{u}(0) &= \mathbf{u}_o \\ \dot{\mathbf{u}}(0) &= \mathbf{v}_o \end{aligned}$$

Figure 2

The M and r matrix can be assembled using the design stiffness on each element as shown in Figure 3.

$$\mathbf{M} = \begin{bmatrix} m_1 & 0 & 0 & & 0 & 0 \\ 0 & m_2 & 0 & & 0 & 0 \\ 0 & 0 & m_3 & & 0 & 0 \\ & & & \ddots & 0 & 0 \\ 0 & 0 & 0 & 0 & m_{N-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & m_N \end{bmatrix}$$

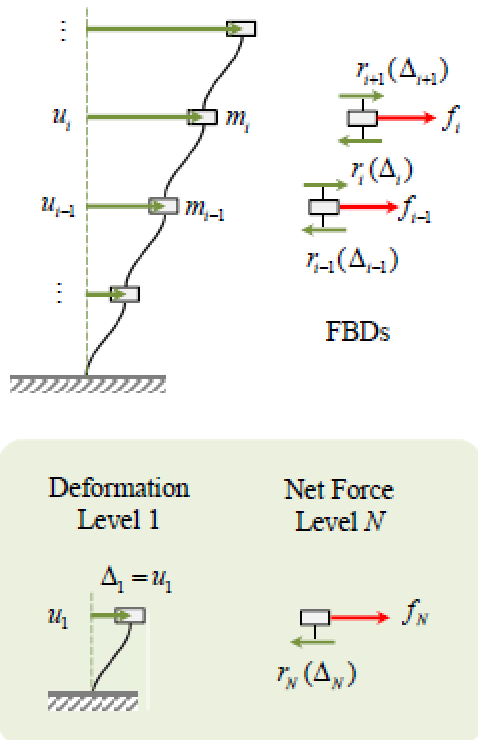

Figure 3

Thus the elements in r matrix can be taken as shown in Figure 4.

$$\mathbf{r} = -\begin{Bmatrix} r_2(\Delta_2) - r_1(\Delta_1) \\ r_3(\Delta_3) - r_2(\Delta_2) \\ \vdots \\ r_{i+1}(\Delta_{i+1}) - r_i(\Delta_i) \\ \vdots \\ -r_N(\Delta_N) \end{Bmatrix}$$

Figure 4

Here the Elastoplastic response is simulated as shown in Figure 5.

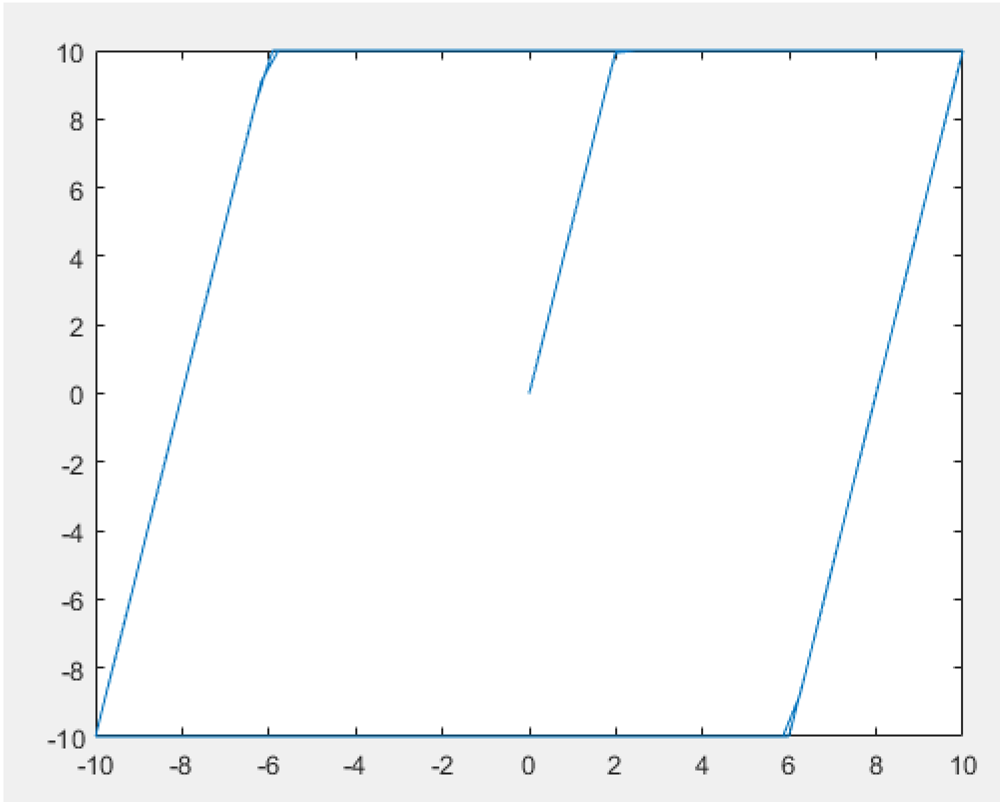

Figure 5

Implementation of Newton's iteration for non-linear response r is shown in Figure 6.

$$x_{i+1} = x_i - \frac{g(x_i)}{g'(x_i)}$$

Figure 6

The u_g matrix is the inertial force under the influence of earthquake as shown in Figure 7.

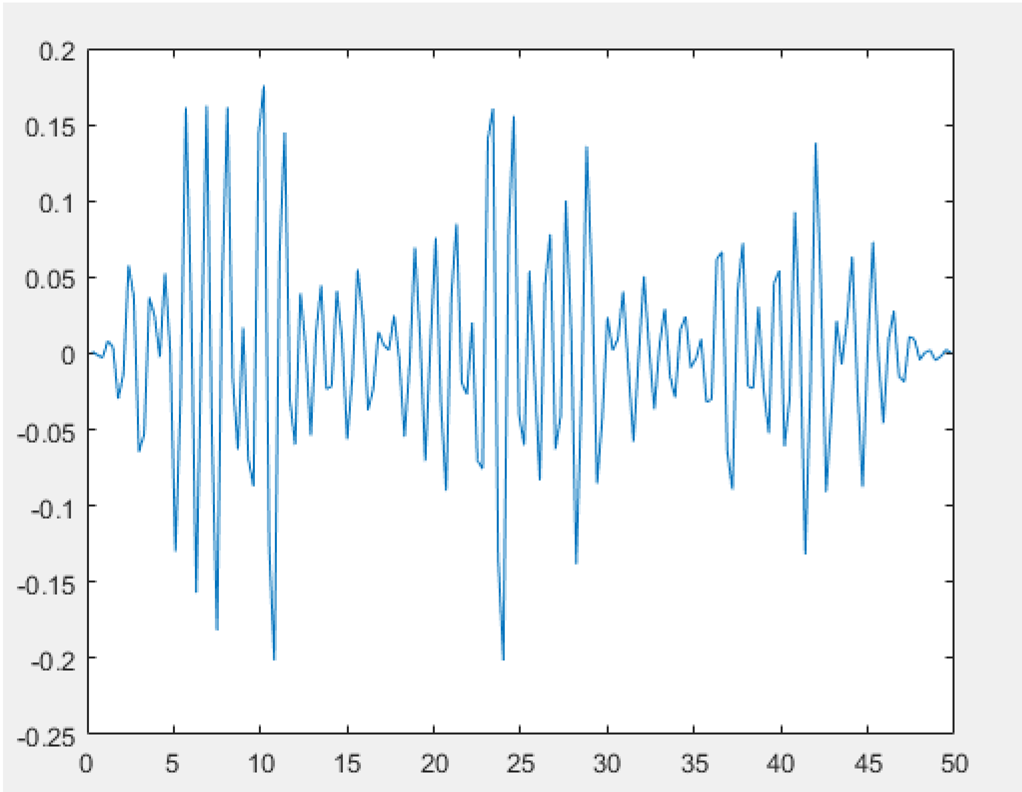

Figure 7

The parameters used for the earthquake are as shown in Figure 8.

```
%   Earthquake Parameters
Shp = [2 3 3 1]';
Amp = [2 3 5 1 3 4]';
Frq = [1 5 8 30 4 6]';
Phs = [0 0 0 0 0 0]';
Peak = .2;
N = 3;
EQtype = 2;
```

Figure 8

Solution

Newmark's Method is used for iterations in the given problem with the following equations as shown in Figure 7.

$$\mathbf{v}_{n+1} = \mathbf{v}_n + h\left(\gamma\,\mathbf{a}_n + (1-\gamma)\,\mathbf{a}_{n+1}\right)$$
$$\mathbf{u}_{n+1} = \mathbf{u}_n + h\mathbf{v}_n + h^2\left(\beta\,\mathbf{a}_n + (\tfrac{1}{2}-\beta)\,\mathbf{a}_{n+1}\right)$$

Figure 9

$a_i = (F_{i-1} – b_n*K - (v_{i-1} + h*Y*a_{i-1})*C)/(K_{Eff})$ -Eq1

Here $K_{eff} = M + h^2(1/2 - \beta)xK + hx(1-\gamma)xC$

Using the above equations a response plot was plotted. A movie was created to visualize the effect of forced vibration on an NDOF system.

Resonance modes are shown below:

Modal Frequencies

0.0156

0.0610

0.1325

0.2245

0.3294

0.4386

0.5435

0.6355

0.7070

0.7524

Conclusion

- NDOF can closely approximate the structural response due to forced vibration.
- When external load frequency matches the modal frequency of the structure it results in large deformations.
- The damping effect is extremely sensitive to the damping coefficient value.
- Damping and yielding reduce the effect of resonance.
- **The stories without damping yield before the damped ones.**
- **The resonance modes when in sync with the earthquake frequency lead to large deformations.**
- **The initial conditions do not have a significant effect on the response.**
- **Additional force has insignificant effect compared to Earthquake.**