Using Simulated Annealing Algorithm for Maximizing Social Utility in Dynamic Spectrum Management

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Abstract: **Thousands of equipments of the wireless local access network and the mobile devices are widely used and the demand of dynamic spectrum resources is significantly growing. How to maximize the social utility in modern technique becomes an important issue. In this paper, the simulated annealing based approach which dynamically allocates the power spectrum is proposed to enhance the system efficiency. The objective is to maximize the sum of individual Shannon utilities in a multi-user communication system. Experimental result shows that our approach obtains the acceptable result within an acceptable amount of run time for the larger test cases.**

Key-Words: Simulated annealing, Social utility maximization, Shannon utility, Dynamic spectrum management, Multi-user communication system, Power spectrum.

1 Introduction

Spectrum optimization problem is getting more and more popularly today as the wireless products, such as the wireless local area network, blue tooth goods and the cell phone, are widely used. There are many researches focus on the spectrum management [2][5], including the static and dynamic spectrum management as the wireless products become more and more popular. To accurately manage the spectrum of a multi-user system, herein we discuss the dynamic spectrum management.

From the optimization perspective, the study of dynamic spectrum management can be divided into three kinds of methods, including Nash equilibrium, social utility maximization and competitive economy equilibrium. Chung *et al.* [3] proved that a Nash equilibrium always exists in the noncooperative rate maximization game. Yu *et al.* [8] proposed an iterative water-filling algorithm to efficiently reach the Nash equilibrium for solving multiple power control problem of the digital subscriber loop (DSL for short). Leon Walras *et al.* [4] proposed the competitive equilibrium concept which dynamically balances by the supply and demand to model the market operation. Ye [6] proposed a competitive economy equilibrium solution that may achieve both social economic efficiency and individual optimality in dynamic spectrum management. Lin *et al.* [1] further observed that adjusting budget can meet each user's physical power demand or balance all individual utilities in a competitive spectrum market. It is difficult to optimize the overall performance utility if we only improve the individual performance of individual user. Yu *et al.* [8] proposed method which refined the optimal spectrum management with less complexity and generalized the multi-user power management for orthogonal frequency division multiplex (OFDM for short) systems maximized the overall utility. Some traditional methods can not guarantee to obtain the global optimal solution for the nonconvex problem [7] and the time complexity grows exponentially. Many papers developed the heuristic algorithms to approximate the solution with the acceptable runtime but the solution quality was degenerated. Many methods greedily searched the local solution which reduced the utility for users and channels with the background interference and crosstalk ratio.

Simulated annealing is widely used in many applications [10][11][12][13][14], such as the design and analysis algorithm, the physical design and wireless local network. The simulated annealing based method is an effective technique to obtain the optimal or sub-optimal solutions. The mixed methods, including the simulated annealing and Tabu Search, is utilized to get the global optimal solution [10]. Pan et al proposed the simulated annealing method to handle the multi-mode scheduling [11]. Sadegheih [12] provided the simulated annealing based approach to handle the large circuits. Sadegheih also combined the simulated annealing and evolutionary algorithm during the spanning tree construction [13]. Lin et al. proposed the simulated annealing method to search the feasible floorplan which all obstacles are not overlapping with each other.

The main contributions of the paper are as follows. First, the proposed method effectively maximizes the social utility instead of the individual utility, i.e. the traditional greedy-based approach obtaining the local optimal solution. Second, we propose the simulated annealing method, which searches the solution space globally under the given background interference and crosstalk ratio, maximizes the spectrum utility for *m* users and *n* channels. Third, the proposed method which reduces the redundant searching efficiently obtains the neighbor solution.

The outline of this paper is organized as follows. Section 2 describes the motivation, terminology and problem definition of social utility maximization. Section 3 discusses the simulated annealing based approach to maximize the social utility of the spectrum optimization problem. Experimental result and the conclusion are discussed in Sections 4 and 5, respectively.

2 Preliminary

In this section, we will discuss the terminology, motivation, and problem definition.

2.1 Terminology

In this sub-section, we describe the notations used in the paper in detailed.

x_{ij} : The power allocation for user *i* on channel *j*.

α_{kj}^{i} : The crosstalk rate between users *k* and *i* on channel *j*.

σ_{ij} : The background interference for user *i* on channel *j*.

m : The number of users.

n : The number of channels.

$P_{\max}^{i}$: The allowable maximization power allocation for user *i*.

u_i : The individual utility of the power spectrum under multi-channel for user *i* .

T : The starting temperature for the simulated annealing approach.

T_c : The current temperature during the simulated annealing approach.

T_e : The terminal temperature for the simulated annealing approach

$u_i(\mathrm{x}_i, \overline{\mathrm{x}}_i)$: The Shannon utility function for user *i*.

2.2 Motivation

In the subsection, we describe the motivation of searching social utility maximization, discuss the Shannon utility and illustrate the social utility maximization with two examples having the different social utility.

First, the motivation of searching the feasible social utility maximization is discussed. According to the above definition, we will discuss the motivation of searching the feasible solution of social utility maximization. Besides, the novel concept is proposed to show the problem of social utility maximization is indeed needed in the modern ear. As is well known, if we maximize the utility for each individual user with multi-channel, it does not guarantee to maximize the overall social utility for the communication system. Since the traditional approach which maximizes the utility for each individual user obtains the local optimal solution by greedy method, we try to apply simulated annealing to solve the spectrum management problem.

Second, the formula of Shannon utility for each individual user is defined in the follows. To explore the social utility maximization, for the three users and two channels, we first explain how to estimate the utility and then discuss the solution with the maximal social utility. In the paper, we evaluate user utility based on the Shannon utility function as follows,

$$u_i(\mathrm{x}_i, \overline{\mathrm{x}}_i) = \sum_{j=1}^{n} \log(1 + \frac{x_{ij}}{\sigma_{ij} + \sum_{k \neq i} \alpha_{kj}^{i} x_{kj}})$$

where x_{ij} is the power allocation for user *i* on channel *j*. α_{kj}^{i} denotes the crosstalk rate between users *k* and *i* on channel *j*. σ_{ij} is the background interference for user *i* on channel *j*.

Third, the first example is used to illustrate the computation of social utility maximization. To simplify, suppose *m*=3, *n*=2 and we have the formula,

$$u_1 = \log(1 + \frac{x_{11}}{\sigma_{11} + \alpha_{21}^{1} x_{21} + \alpha_{31}^{1} x_{31}}) + \log(1 + \frac{x_{12}}{\sigma_{12} + \alpha_{22}^{1} x_{22} + \alpha_{32}^{1} x_{32}})$$

Similarly, we have the formulas for users 2 and 3 are,

$$u_2 = \log(1 + \frac{x_{21}}{\sigma_{21} + \alpha_{11}^{2} x_{11} + \alpha_{31}^{2} x_{31}}) + \log(1 + \frac{x_{22}}{\sigma_{22} + \alpha_{12}^{2} x_{12} + \alpha_{32}^{2} x_{32}})$$

and

$$u_3 = \log(1+\frac{x_{31}}{\sigma_{31}+\alpha_{11}^3 x_{11}+\alpha_{21}^3 x_{21}}) + \log(1+\frac{x_{32}}{\sigma_{32}+\alpha_{12}^3 x_{12}+\alpha_{22}^3 x_{22}})$$

For the parameters, which are randomly assigned and discussed in the experimental results, used in the example, we have the following for u_1,

$$u_1 = \log(1+\frac{x_{11}}{4+0.1x_{21}+0.1x_{31}}) + \log(1+\frac{x_{12}}{7+0.6x_{22}+0.1x_{32}})$$

Similarly, for u_2 and u_3, we have the following,

$$u_2 = \log(1+\frac{x_{21}}{5+0.8x_{11}+0.7x_{31}}) + \log(1+\frac{x_{22}}{6+0.9x_{12}+0.6x_{32}})$$

and

$$u_3 = \log(1+\frac{x_{31}}{3+0.1x_{11}+0.3x_{21}}) + \log(1+\frac{x_{32}}{1+0.7x_{12}+0.5x_{22}}).$$

When we search the assignment solution for user 1, 2, 3 and channel 1, 2, respectively, there are many feasible solutions. The assignment of the first example with three users and two channels is $(x_{11}, x_{12}) = (1,0)$, $(x_{21}, x_{22}) = (0,1)$, $(x_{31}, x_{32}) = (0,1)$. It means that user 1 , 2 and 3 are assigned to channels 1, 2 and 2, respectively. According to the above formulas, the maximization utility of social utility is 0.3800.

In the second example, we try to show the large social utility due to the better assignment of power allocation. Similarly, we remove the detailed formulas in the first example and only give the final assignment of power allocation. When the assignment of the example with three users and two channels is $(x_{11}, x_{12}) = (0,1)$, $(x_{21}, x_{22}) = (0,1)$, $(x_{31}, x_{32}) = (0,1)$. It means that user 1, 2 and 3 are assigned to channels 1, 2 and 2, respectively. According to the above formulas, the maximization utility of social utility is 0.27001. However, from the randomly assignment for maximizing of individual utility, the solutions are not the best results.

From the viewpoint of social utility, we can further improve the social utility under the same parameter. According to the above parameters, we can maximize the social utility with, $(x_{11}, x_{12}) = (1,0)$, $(x_{21}, x_{22}) = (1,0)$, $(x_{31}, x_{32}) = (0,1)$ and the maximal social utility is 0.465. By searching the feasible solutions as many as possible, our simulated annealing method usually obtains the solution which maximizes the social utility for multi-user and multi-channel with background interference and crosstalk ratio. By the observation, it motivates us to develop the simulated annealing method to maximize the social utility for spectrum management problem with multi-user and multi-channel.

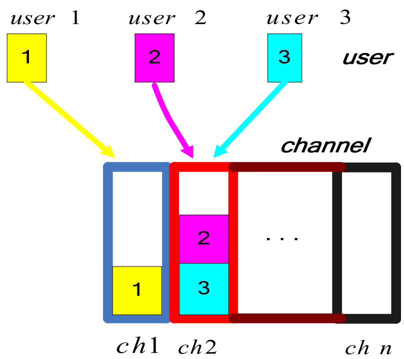

Figure 1. A first solution with feasible social utility for spectrum management problem

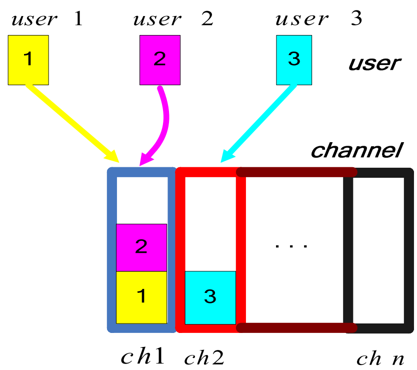

Figure 2. The second solution with maximal social utility for spectrum management problem

Obviously, the different assignments of all power allocations lead to the different social utility. It means that searching the feasible assignment of power allocation among all feasible solution is very effective and efficient to increase the social utility maximization For the social utility maximization problem in the paper, how to solve the problem by using the simulated annealing based approach motivate us to study the paper.

2.3 Problem Definition

The problem solved in this paper is to maximize the sum of individual Shannon utility in a multi-user communication system with background interference and crosstalk.

The problem of social utility maximization is defined as follow,

$$\max \sum_{i=1}^{m} \mathrm{u}_i$$

$$s.t. \quad \sum_{j=1}^{n} x_{ij} \le P_{\max}^{i}, i = 1,2,\ldots,m,$$

$$x_{ij} \ge 0.$$

$$\mathrm{u}_i(\mathrm{x}_i, \overline{\mathrm{x}}_i) = \sum_{j=1}^{n} \log(1 + \frac{x_{ij}}{\sigma_{ij} + \sum_{k \ne i} \alpha_{kj}^{i} x_{kj}})$$

where m denotes the number of users, n denotes the number of channels, u_i is the Shannon utility function for user i, x_{ij} represents the power allocated on channel j by user i, $P_{\max}^{i}$ is the power constraint for user i, σ_{ij} is the crosstalk ratio for user i on channel j and α_{kj}^{i} is the interference ratio for user i from user k on channel j.

3 Our Simulated Annealing Approach

In this section, a simulated annealing approach will be proposed to solve the social utility maximization problem. The pseudo code of the basic simulated annealing algorithm is shown in Figure 3. There are some issues that we must consider in our approach, for example, initial temperature, cost function, ending temperature, cooling rate, neighbor perturbations and so on. The design flow of the proposed method is shown in Figure 4. Finally, the example of five users and five channels is used to illustrate the solutions with different social utilities. The following are the description of our simulated annealing approach in detail.

1. Temperature: During the simulated annealing procedure, the starting and ending temperatures are given as T and T_e, respectively. T is assigned as the cost of the initial solution and T_e is assigned as $T/10$.

2. Cost function : For each user i, we have the utility as follows,

$$u_i(\mathrm{x}_i, \overline{\mathrm{x}}_i) = \sum_{j=1}^{n} \log(1 + \frac{x_{ij}}{\sigma_{ij} + \sum_{k \ne i} \alpha_{kj}^{i} x_{kj}})$$

where x_{ij} is the power allocation for user i on channel j. α_{kj}^{i} denotes the crosstalk rate between users k and i on channel j. σ_{ij} is the background interference for user i on channel j. To maximize the social utility, the cost function is defined as follows,

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \log(1 + \frac{x_{ij}}{\sigma_{ij} + \sum_{k \ne i} \alpha_{kj}^{i} x_{kj}})$$

3. Initial solution: The solution quality is estimated by the Shannon utility function. The user power allocation of the initial solution is assigned with the consideration of the background interference.

4. Cooling rate: In our simulated annealing approach, the temperature reduction is set to be 0.90. It means that the next temperature will be $0.9 \times T_c$, where T_c is the current temperature.

5. Stopping constraints: Our algorithm terminates when the current temperature is reached to the stopping temperature or the solution can not be improved anymore.

6. The neighbor perturbations: For the initial solution, we provide the movements to perturb the power spectrum of each user. If the quality of current solution is near the best solution so far, we then perturb the power with minor change. Otherwise, the power of each user has a significant change if the solution quality is far from the best solution. To enhance the algorithm performance, a coefficient, *min_step*, is applied to be the maximum displacement for all power allocations. The coefficient will be reduced with the temperature reduction simultaneously. Therefore, the maximum displacement for all power allocations will be decreasing as time goes on.

The above description is for the procedure of the simulated annealing method. Figure 4 shows the whole design flow of the proposed method. The key points will be described briefly as follows.

First, the initial solution is obtained with consideration of the background interference. Then, the algorithm searches the neighbor solution by perturbation the power allocation for multi-users and multi-channels. During the searching procedure, we accept and reject the solution by the proposed cost function which is discussed in Section 3. Finally, the algorithm stops if the temperature reaches the stopping temperature or the solution of social utility will not improve anymore.

Finally, we use an example with 5 users and 5 channels to illustrate the solutions with different social utilities. In Figure 5(a), the ILP formulations are shown. Figure 5(b), (c) and (d) are the results of the different power allocations. In Figure 6, the corresponding results are shown.

```
int accept (struct feasible_solution f, g)
{
    float Δc;
    Δc ← c(g) – c(f);
    if (Δc ≤ 0)
        return 1;
    else return (e^(-Δc / T) > random(1));
}

simulated_annealing()
{
    struct feasible_solution f, g;
    float T;

    f ← initial_solution();
    do{
        do{
            g ← "some element of N(f)";
            if (accept (f, g))
                f ← g;
        }while(!thermal_equilibrium());
        T ← new_temperature(T);
    }while(!stop());
    "report f";
}
```

Figure 3. Simulated Annealing algorithm

Table 1. The coefficient of our experiment

coefficient	parameter	value
The upper bound of power allocations	$P^i_{\max}$	1
The maximum displacement for all power allocations	*min_step*	0.5
Cooling rate	k	0.9
Initial temperature	T	Cost (Init solution)
Terminal temperature	T_e	T/ 10

4 Experimental Result

All experiments are implemented using C++ language on the Intel CPU 3GHz machine with the 2GB memory. The objective of this work is to maximize the social utility under the multi-user and multi-channel with the background interference and crosstalk ratio. The background interference *(crosstalk ratio)* is randomly assigned in the ranges from 1 to 9 *(from 0.1 to 0.9)*. Besides, the relative parameters which we used in the paper are listed in Table 1. Our simulated annealing algorithm, which reduces the redundant searching, efficiently obtains the solution to maximize social utility.

We investigate the efficiency of our proposed simulated annealing approach. The experiment result is shown in Table 2. For the small test case with few users and few channels, our simulated annealing method efficiently solves the problem with acceptable runtime. Even though for the largest test case, including 50 users and 50 channels, our method still maximizes the social utility within 1 hour.

To show the superiority of convergence of our proposed method by using two examples, including u2030_1 and u3010_1, Figures 7 and 8 are used to illustrate the procedure of the simulated annealing based method searching the feasible solutions of maximization social utility. In Figure 7, the *x*- and *y*-axis denote the numbers of the feasible solutions during the searching procedure and the corresponding value of social utility for each solution. Besides, the minimization and maximization of social utilities are 3.61226 and 5.26686, respectively.

5 Conclusion

In the paper, the dynamic spectrum optimization problem is solved by the proposed simulated annealing based approach. According to the literature [2], even in the two user case, the problem is NP-hard. Due to the problem complexity, there are few researches to explore the efficient and effective methods to obtain the better solution of social utility maximization. Therefore, we try to provide a simulated annealing approach which searches the feasible solution of social utility maximization as many as possible to solve the problem efficiently. Experimental result shows that our method get the acceptable social utility with the acceptable runtime of the simulated annealing based approach even though for the largest test case.

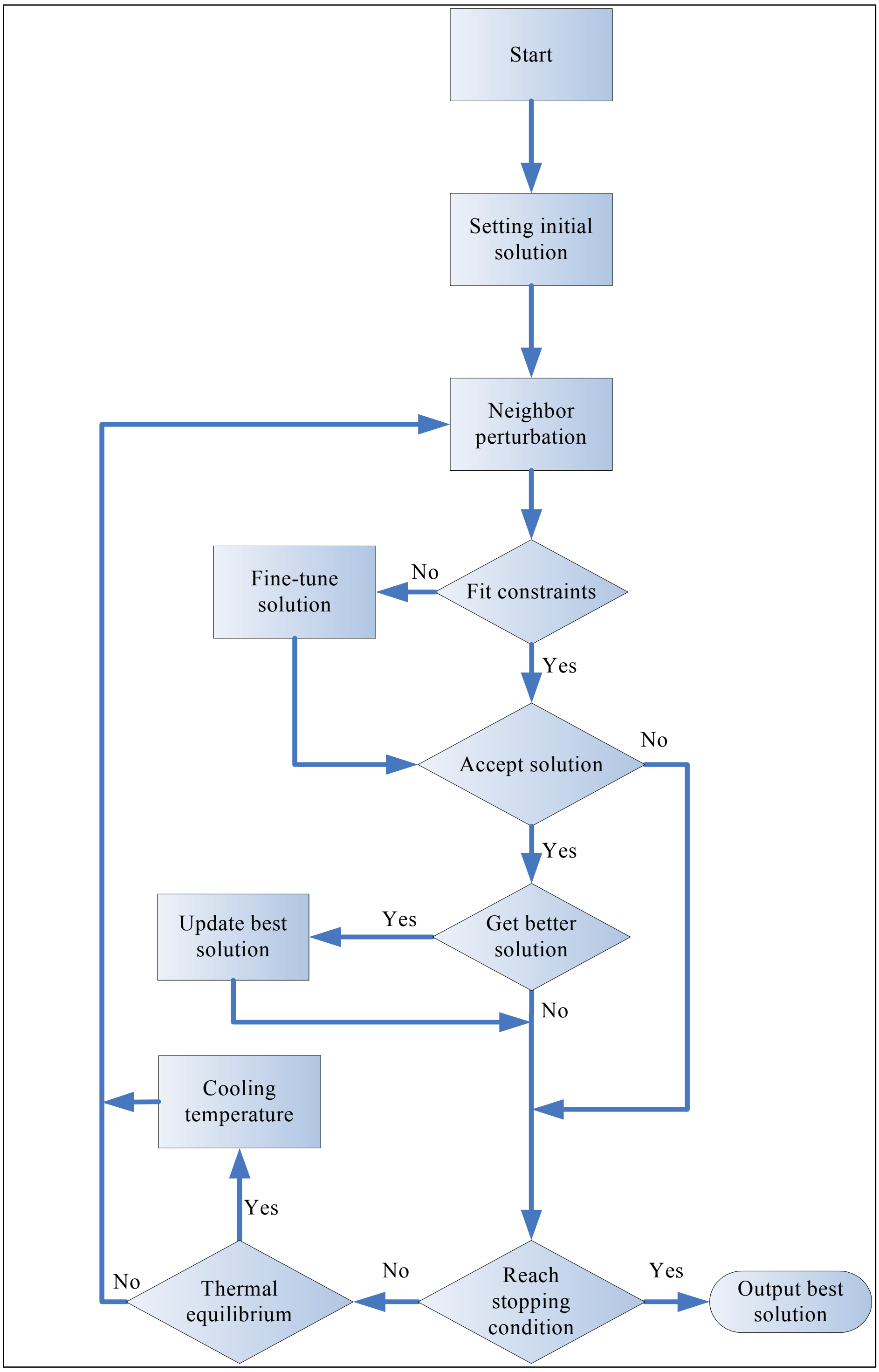

Figure 4. The design flow of the proposed approach

$$\max \sum_{i=1}^{m} u_i$$

$$u_1 = \log(1+\frac{x_{11}}{9+0.4x_{21}+0.5x_{31}+0.9x_{41}+0.2x_{51}}) + \log(1+\frac{x_{12}}{9+0.8x_{22}+0.4x_{32}+0.8x_{42}+0.8x_{52}}) + \log(1+\frac{x_{13}}{2+0.4x_{23}+0.6x_{33}+0.8x_{43}+0.7x_{53}})$$
$$+\log(1+\frac{x_{14}}{5+0.3x_{24}+0.7x_{34}+0.7x_{44}+0.6x_{54}}) + \log(1+\frac{x_{15}}{9+0.8x_{25}+0.3x_{35}+0.3x_{45}+0.4x_{55}})$$

$$u_2 = \log(1+\frac{x_{21}}{6+0.8x_{11}+0.3x_{31}+0.8x_{41}+0.7x_{51}}) + \log(1+\frac{x_{22}}{9+0.1x_{12}+0.3x_{32}+0.7x_{42}+0.9x_{52}}) + \log(1+\frac{x_{23}}{5+0.6x_{13}+0.4x_{33}+0.9x_{43}+0.3x_{53}})$$
$$+\log(1+\frac{x_{24}}{4+0.2x_{14}+0.1x_{34}+0.3x_{44}+0.6x_{54}}) + \log(1+\frac{x_{25}}{5+0.3x_{15}+0.6x_{35}+0.5x_{45}+0.7x_{55}})$$

$$u_3 = \log(1+\frac{x_{31}}{1+0.7x_{11}+0.7x_{21}+0.4x_{41}+0.5x_{51}}) + \log(1+\frac{x_{32}}{2+0.1x_{12}+0.9x_{22}+0.2x_{42}+0.4x_{52}}) + \log(1+\frac{x_{33}}{3+0.7x_{13}+0.9x_{23}+0.2x_{43}+0.9x_{53}})$$
$$+\log(1+\frac{x_{34}}{3+0.8x_{14}+0.7x_{24}+0.2x_{44}+0.6x_{54}}) + \log(1+\frac{x_{35}}{1+0.1x_{15}+0.3x_{25}+0.8x_{45}+0.2x_{55}})$$

$$u_4 = \log(1+\frac{x_{41}}{9+0.3x_{11}+0.9x_{21}+0.1x_{31}+0.9x_{51}}) + \log(1+\frac{x_{42}}{3+0.5x_{12}+0.3x_{22}+0.2x_{32}+0.1x_{52}}) + \log(1+\frac{x_{43}}{6+0.2x_{13}+0.4x_{23}+0.3x_{33}+0.5x_{53}})$$
$$+\log(1+\frac{x_{44}}{6+0.1x_{14}+0.2x_{24}+0.7x_{34}+0.7x_{54}}) + \log(1+\frac{x_{45}}{5+0.4x_{15}+0.2x_{25}+0.9x_{35}+0.6x_{55}})$$

$$u_5 = \log(1+\frac{x_{51}}{1+0.4x_{11}+0.4x_{21}+0.7x_{31}+0.6x_{41}}) + \log(1+\frac{x_{52}}{3+0.9x_{12}+0.6x_{22}+0.9x_{32}+0.2x_{42}}) + \log(1+\frac{x_{53}}{5+0.7x_{13}+0.7x_{23}+0.8x_{33}+0.8x_{43}})$$
$$+\log(1+\frac{x_{54}}{9+0.6x_{14}+0.2x_{24}+0.3x_{34}+0.3x_{44}}) + \log(1+\frac{x_{55}}{9+0.7x_{15}+0.2x_{25}+0.5x_{35}+0.1x_{45}})$$

(a) ILP formulations

x_{ij}	j=1	j=2	j=3	j=4	j=5
i=1	1	0	0	0	0
i=2	0	1	0	0	0
i=3	0	0	1	0	0
i=4	0	0	0	1	0
i=5	0	0	0	0	1
utility = 0.3291580					

(b) Power assignment and social utility

x_{ij}	j=1	j=2	j=3	j=4	j=5
i=1	1	0	0	0	0
i=2	1	0	0	0	0
i=3	1	0	0	0	0
i=4	1	0	0	0	0
i=5	1	0	0	0	0
utility = 0.3590798					

(c) Power assignment and social utility

x_{ij}	j=1	j=2	j=3	j=4	j=5
i=1	0	0	1	0	0
i=2	0	0	0	1	0
i=3	0	0	0	0	1
i=4	0	1	0	0	0
i=5	1	0	0	0	0
utility = 1.000000					

(d) Power assignment and social utility

Figure 5. The example with five users and five channels

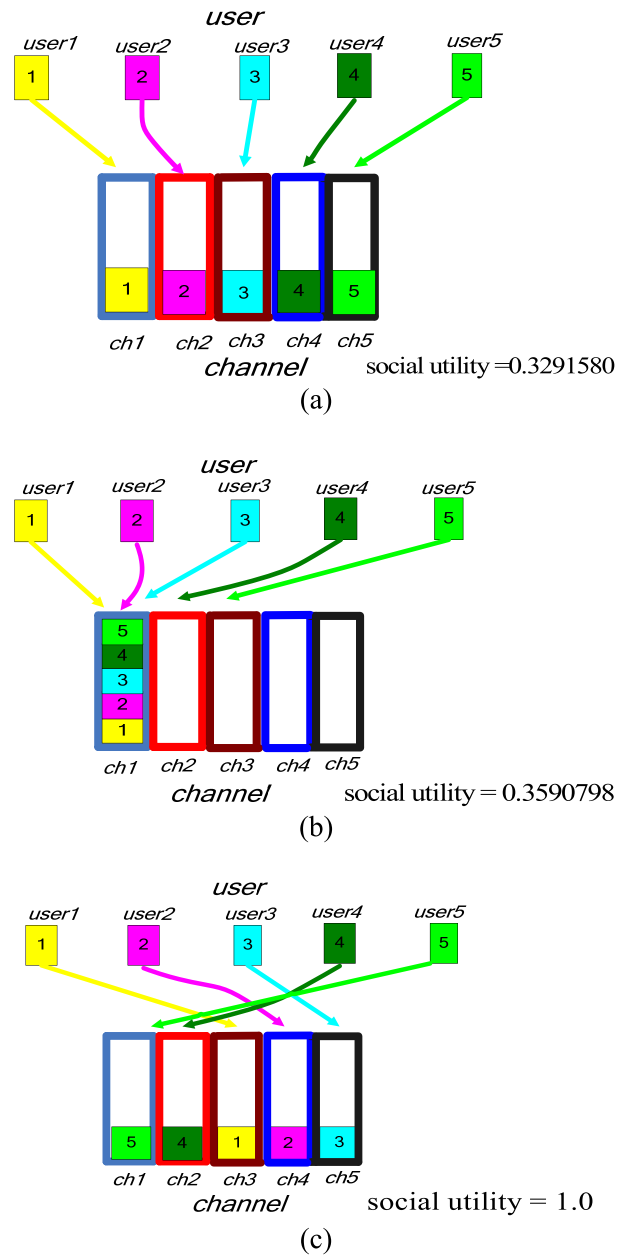

Figure 6. Power assignment in Figure 5(b),(c) and(d).

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Table 2. Experimental result of spectrum management

Test case	User number	Channel number	CPU(sec)	Utility
u10c10_1	10	10	<1s	2.18163
u10C20_1	10	20	6s	2.74449
u10c30_1	10	30	14s	3.0728
u10c40_1	10	40	24s	3.24
u10c50_1	10	50	41s	3.26243
u20c10_1	20	10	7s	3.74007
u20c20_1	20	20	28s	5.06596
u20c30_1	20	30	70s	5.26686
u20c40_1	20	40	243s	5.73952
u20c50_1	20	50	557s	5.85862
u30c10_1	30	10	60s	4.43878
u30c20_1	30	20	221s	6.77981
u30c30_1	30	30	532s	7.13375
u30c40_1	30	40	773s	8.06903
u30c50_1	30	50	1231s	8.44176
u40c10_1	40	10	40s	5.14411
u40c20_1	40	20	358s	7.93071
u40c30_1	40	30	972s	9.6839
u40c40_1	40	40	1094s	9.25846
u40c50_1	40	50	3571s	9.50717
u50c10_1	50	10	126s	5.26709
u50c20_1	50	20	751s	8.87619
u50c30_1	50	30	1823s	10.7217
u50c40_1	50	40	2876s	11.1269
u50c50_1	50	50	2709s	12.3902

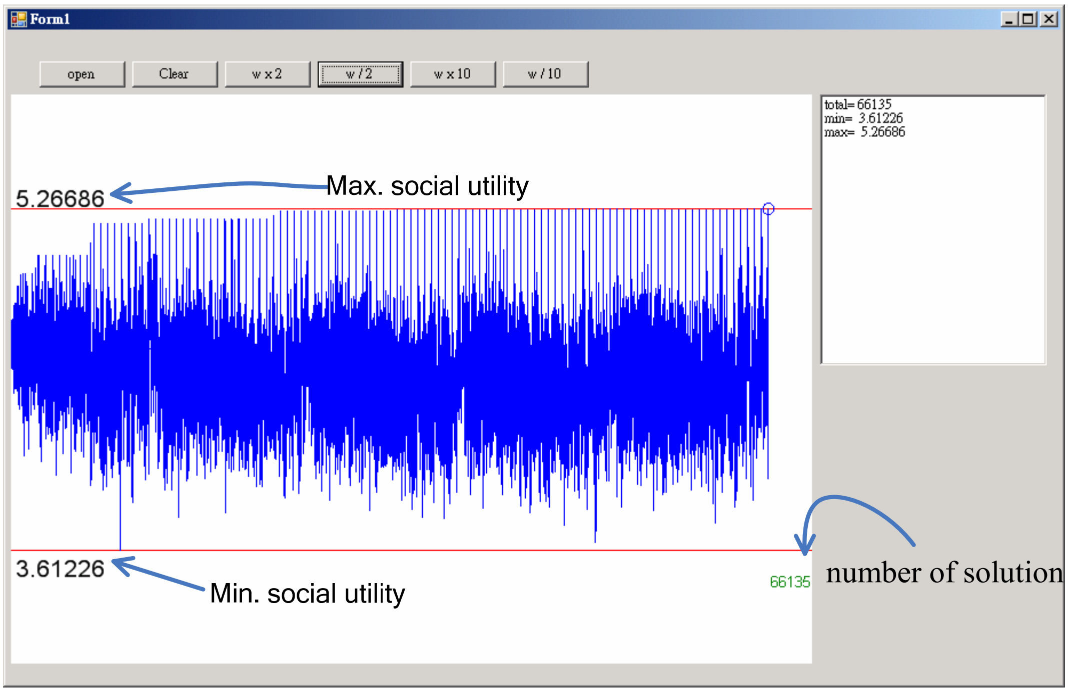

Figure 7. The tour of solution finding for u2030_1

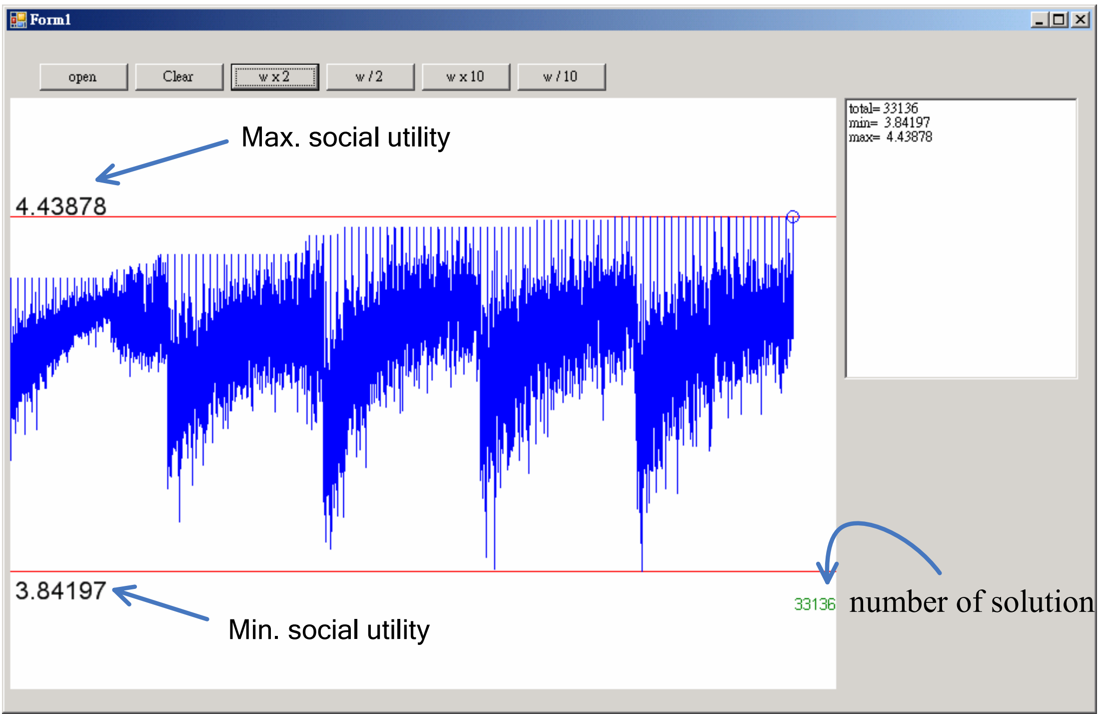

Figure 8. The tour of solution finding for u3010_1