

Set - 3

1. Algorithm efficiency refers to how many computation resources an algorithm consumes in relation to its size of input. It is measure of its performance -

- (i) Time efficiency - how fast algorithm runs
- (ii) Space efficiency - how much memory it takes

2. Time complexity

(i) Amount of time an algorithm takes to complete function of input size n

(ii) Example - Linear search
 $O(n)$ Time in worst case, it must check all element

(iii) Merge sort
 $O(n \log n)$ - Divide array $\log n$ time & performs n comparisons at each level

Space complexity

(i) Amount of memory an algorithm takes to complete function of input size n

(ii) Example - Linear search
 $O(1)$
It only uses few extra variable. Memory doesn't grow in array's size.

(iii) Merge sort
 $O(n)$ - requires extra auxillary array of size n to merge sorted halves

3. Let 3D array is $A[D1][D2][D3]$ where

$D1$ = No. of rows

$D2$ = No. of columns

$D3$ = No. of planes

B = Base address ($A[0][0][0]$)

S = Size of one element (4 for int)

- Find address of $A[i][j][k]$
- (i) To find correct depth - $D_1 \times D_2$ elements
 Elements to skip = $k \times (D_1 \times D_2)$
 - (ii) To get column j within slab
 Elements to skip = $j \times D_1$
 - (iii) To get correct row within column
 Element to skip = i
 - (iv) Total elements = $(k \times D_1 \times D_2) + (j \times D_1) + i$
 - (v) Final address
 $A[i][j][k] = B + [(k \times D_1 \times D_2) + (j \times D_1) + i] \times S$

4. Binary search algorithm

BS(arr A, int target)

low = 0

high = length(A) - 1

while low <= high

mid = ~~from~~ (high + low) / 2

// 1. if target is at middle

If $A[mid] == \text{target}$ then

Return mid;

// 2. if target is greater, search right half

Else If $A[mid] < \text{target}$ then

low = mid + 1

// 3. if target is smaller, search left half

else high = mid - 1

END while

Return -1

5. Bubble sort repeatedly steps through the lists, compares adjacent elements & swap them if they are in wrong order

Working - Algorithm uses two nested loops -

(i) Outer loop - This loop runs $n-1$. After each pass, next largest element is guaranteed to be in its final sorted position

(ii) Inner loop - Iterates from beginning of array up to unsorted portion. It compares $A[j]$ to $A[j+1]$ & swaps if $A[j] > A[j+1]$

~~Example - $[3, 1, 2] \rightarrow \text{swap } (3 \ \& \ 1)$
 $[2, 3, 1] \rightarrow \text{swap } (3 \ \& \ 2)$
 $[1, 2, 3]$~~

Example $[3, 2, 1] \rightarrow \text{swap } (3 \ \& \ 2)$

$[2, 3, 1] \rightarrow \text{swap } (3 \ \& \ 1)$

$[2, 1, 3] \rightarrow \text{swap } (2 \ \& \ 1)$

$[1, 2, 3] \rightarrow \text{No swapping as } a[2] < a[3]$

6. `int factorial(int n) {`

`if (n == 0) {`
`return 1; }`

`else {`

`return n * factorial(n-1); }`

(i) Base case - If $n = 0$, stops recursing & returns 1

(ii) Recursive case - If $n > 0$, factorial is defined as $n \times (n-1)!$ where function calls itself with $n-1$ argument

8.

a) Recursion

- (i) It uses call stack. Each time function calls itself, a new stack frame is pushed to frame
- (ii) Easy to write but can be memory intensive. Space complexity is $O(n)$
- (iii) If recursion is too deep, it consumes all available stack memory resulting in a stack overflow error

Iteration

- (i) Uses loop. ~~Heap~~ Memory required is typically just for loop control variables
- (ii) More memory efficient with space complexity $O(1)$
- (iii) Iteration avoids stack overflow due to heap memory which is a much larger memory area

```
6) int fibo (int n) {  
    if (n == 0) {  
        return 0; }  
    else if (n == 1) {  
        return 1; }  
    else {  
        return fibo(n-1) + fibo(n-2); }  
}
```

9. a) Divide

- (i) Algorithm divides unsorted array into two halves.
- (ii) It recursively calls itself on each half.
- (iii) Halves are broken into arrays of size 1. Each array contains one sorted element
- (iv) Division takes $\log n$ ~~times~~ ~~time~~ time at each level

b) Merge

- (i) Merge two sorted arrays into a larger sorted array
- (ii) It is very efficient as it uses two pointers & compares ~~pointers~~ elements at pointers
- (iii) It is repeated until all elements get stored in temporary array
- (iv) Merging process taking linear time, $O(n)$
- (v) Overall efficiency - $O(n \log n)$

Real world applications

- (i) External sorting
- (ii) Stable sorting requirements
- (iii) Parallel & concurrent programming
- (iv) Language standard libraries