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SS Tutorial - 5

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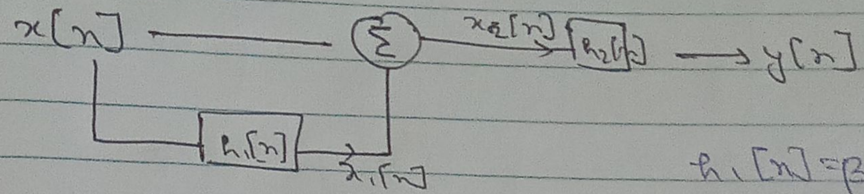
22EC01057

Sol 1 a)



$h_1[n] = \beta\delta[n-1]$

$h_2[n] = \alpha^n u[n]$

For impulse resp $x[n] = \delta[n]$

$x_1[n] = \beta\delta[n-1]$

$x_2[n] = x[n] + x_1[n] = \delta[n] + \beta\delta[n-1]$

$$y[n] = \sum_{-\infty}^{\infty} x_2[k] \cdot h_2[n-k]$$

$$= \sum_{-\infty}^{\infty} (\delta(k) + \beta\delta[k-1])(\alpha^{n-k} u[n-k])$$

$$= \alpha^n u[n] + \beta\alpha^{n-1} u[n-1]$$

$$= \begin{cases} \alpha^n + \beta\alpha^{n-1} & , n \geq 1 \\ \alpha^n & n = 0 \\ 0 & n < 0 \end{cases}$$

b) Yes, it is causal system.
For the system to be causal, $\alpha \leq 1$

Sol 2 $[x(t) * h(t)] * g(t) = y(t) * g(t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$y(t) * g(t) = \int_{-\infty}^{\infty} y(p) \cdot g(t-p)\, dp$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x(\tau)\, h(p-\tau)\, d\tau)\, g(t-p)\, dp$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(\tau)\, h(\sigma)\, g(t-\tau-\sigma)\, d\sigma\, d\tau$$

Here $p - \tau = \sigma$

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Consider

$x(t) * [h(t) * g(t)] = x(t) * y(t)$

$y(t) = \int_{-\infty}^{\infty} h(\tau) g(t-\tau) d\tau = \int_{-\infty}^{\infty} h(\sigma) g(t-\sigma) d\sigma$

$\Rightarrow y(t) * x(t) = \int_{-\infty}^{\infty} x(p) y(t-p) dp$

$= \int_{-\infty}^{\infty} x(\tau) y(t-\tau) d\tau$

$= \iint_{-\infty}^{\infty} x(\tau) h(\sigma) g(t-\tau-\sigma) d\sigma \, d\tau$

Proved

Sol 3

$x[n] \rightarrow \boxed{y_1[n] = x[n] - 0.5x[n-1]} \xrightarrow{y_1[n]} \boxed{h_2[n] = 0.5^n u[n]} \rightarrow y[n]$

$y_1[n] = \delta[n] - 0.5\,\delta[n-1]$

$y[n] = \sum_{k=-\infty}^{\infty} y_1[k] \cdot h_2[n-k]$

$= \sum_{k=-\infty}^{\infty} (\delta[k] - 0.5\delta[k-1])\, 0.5^{n-k} u(n-k)$

$= 0.5^n u[n] - 0.5^n u[n-1]$

$= 0.5^n \delta[n]$

$y[n] = \begin{cases} 1 & n = 0 \\ 0 & \text{, otherwise} \end{cases}$

Sol 4

$h[n] = (n+1)\alpha^n u[n]$

$S[n] = \sum_{k=-\infty}^{\infty} u[k] h[n-k] = \sum_{k=-\infty}^{\infty} u[k] (n-k+1) \alpha^{n-k} u[n-k]$

Put $n-k = p$

$= \sum_{p=0}^{\infty} (p+1) \alpha^p u[n-p]$

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$$s[n] = \sum_{p=0}^{n} (p+1)\alpha^{p} \qquad n \geq 0$$

$$s[n] = 1 + 2\alpha + 3\alpha^2 + \cdots (n+1)\alpha^n$$
$$\alpha s[n] = \alpha + 2\alpha^2 + \cdots n\alpha^n + (n+1)\alpha^{n+1}$$

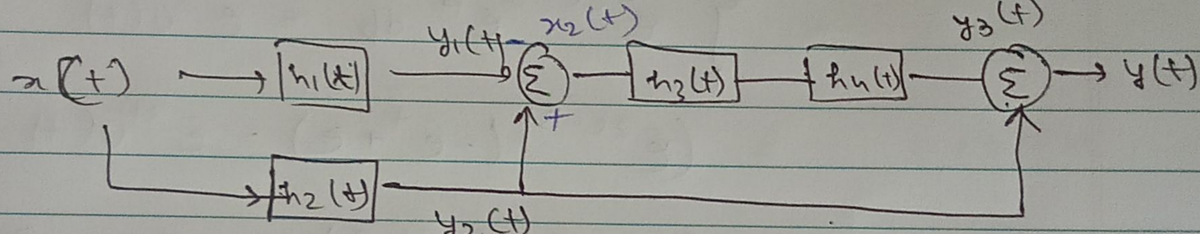
$$(1-\alpha)s[n] = 1 + \alpha + \cdots + \alpha^n - (n+1)\alpha^{n+1}$$

$$s[n] = \left(\frac{1}{1-\alpha}\right)\left[\frac{1-\alpha^{n+1}}{1-\alpha} - (n+1)\alpha^{n+1}\right]$$

$$s[n] = \begin{cases} \left[\dfrac{1}{(\alpha-1)^2} - \dfrac{\alpha\,\alpha^n}{(\alpha-1)^2} + \dfrac{\alpha}{\alpha-1}(n+1)\alpha^n\right] & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$s[n] = \left[\frac{1}{(\alpha-1)^2} - \frac{\alpha\,\alpha^n}{(\alpha-1)^2} + \left(\frac{\alpha}{\alpha-1}\right)(n+1)\alpha^n\right]u[n]$$

Sol 5



$$h_1(t) = \delta(t-1), \qquad h_2(t) = e^{-2t}u(t)$$
$$h_3(t) = \delta(t-1) \qquad h_4(t) = e^{-3(t+2)}u(t+2)$$

$$y_1(t) = \delta(t) * \delta(t-1)$$
$$= \delta(t-1)$$

$$y_2(t) = \delta(t) * h_2(t) = \cancel{\delta(t) *}\ h_2(t)$$
$$= e^{-2t}u(t)$$

Now $h_5(t) = h_3(t) * h_4(t)$

$$= \delta(t-1) * h_4(t) = h_4(t-1)$$
$$= e^{-3(t+1)}u(t+1)$$

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$x_2(t) = y_2(t) - y_1(t)$

$= e^{-2t} u(t) - \delta(t-1)$

$y_3(t) = x_2(t) * h_5(t)$

$$= \int_{-\infty}^{\infty} e^{-2\tau} u(\tau)\, h(t-\tau)\, d\tau - \int_{-\infty}^{\infty} \delta(\tau-1)\, h_5(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} e^{-3t+\tau-3} u(\tau) u(t-\tau+1)\, d\tau - e^{-3t} u(t)$$

$$= \int_{0}^{\infty} e^{-3t+\tau-3} u(t-\tau+1)\, d\tau - e^{-3t} u(t)$$

Let $t-\tau = p$

$$= \int_{t}^{-\infty} e^{-2t-p-3} u(p+1)\,(-dp) - e^{-3t} u(t)$$

$$= \int_{-\infty}^{t} e^{-2t-p-3} u(p+1)\, dp - e^{-3t} u(t)$$

$$= \int_{-1}^{t} e^{-2t-3} e^{-p}\, dp - e^{-3t} u(t)$$

$$= (e^{-2t-3})(-e^{-t}+e) - e^{-3t} u(t)$$

$$= (e - e^{-t})(e^{-2t-3})\, u(t+1) - e^{-3t} u(t)$$

Now $h(t) = y_3(t) + y_2(t)$

$$= (e - e^{-t})(e^{-2t-3})\, u(t+1) - e^{-3t} u(t) + e^{-2t} u(t)$$

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(6) a) $\phi_{xy}(t) = \int_{-\infty}^{\infty} x(t+\tau)\, y(\tau)\, d\tau$

$\phi_{yx}(t) = \int_{-\infty}^{\infty} y(t+\tau)\, x(\tau)\, d\tau$

Let $t+\tau = u$, $\tau = u-t$

$= \int_{-\infty}^{\infty} y(u)\, x(u-t)\, du$

$= \phi_{yx} = \int_{-\infty}^{\infty} x(\tau - t)\, y(\tau)\, d\tau$

We see that

$\phi_{yx}(t) = \phi_{xy}(-t)$

(b) odd part $= \dfrac{\phi_{xx}(t) - \phi_{xx}(-t)}{2}$

$\phi_{xx}(-t) = \phi_{xx}(t)$

odd part $= 0$

c) $\phi_{xy}(t) = \int_{-\infty}^{\infty} x(t+\tau)\, y(\tau)\, d\tau$

$= \int_{-\infty}^{\infty} x(t+\tau) \cdot x(t+\tau)\, d\tau$ $\qquad \tau + t = u$

$= \int_{-\infty}^{\infty} x(t+u-T)\, x(u)\, du$

$\phi_{xy}(t) = \phi_{xx}(t-T)$

For ϕ_{yy}

$\phi_{yy} = \int_{-\infty}^{\infty} y(t+\tau)\, y(\tau)\, d\tau$

$= \int_{-\infty}^{\infty} y(t+\tau+T)\, x(\tau+T)\, d\tau$

$= \int_{-\infty}^{\infty} x(t+u)\, x(u)\, du$

$\Rightarrow \boxed{\phi_{yy} = \phi_{xx}}$