Experiment–5: Continuous Time Fourier Series

Signals and Systems Lab(EC2P002)
School of Electrical Sciences, IIT Bhubaneswar
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Agenda of the Experiment

In this lab session, we will learn about:

1. Symbolic expressions in MATLAB and their usage
2. Continuous-time Fourier series
3. Gibbs phenomenon

Symbolic Expressions and Numerical Integration in MATLAB

1. Type *help syms* to learn about constructing symbolic expressions.
2. Execute the following expressions
   syms t;
   *func = sin(2*pi*t);*
3. How is *func* stored in MATLAB workspace? (That is, what do you see if you type *func* in the command prompt and press enter?)
4. By defining *syms t* prior to *func*, we have created a symbolic expression in terms of the variable t. Type *help subs* to see how to numerically evaluate the symbolic expression for a given value of t.
5. Use *subs* to evaluate *func* for $t = -1 : 0.01 : 1$.
6. Type *help int* to learn about numerical integration. Use *int()* to evaluate the integral of *func* and *func*^{2} over one period. Verify the correctness of the results using pen-and-paper calculation.

In class we learned the Fourier series *synthesis* and *analysis* equations to be

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \mathrm{e}^{jk\omega_0 t}$$

and

$$a_k = \frac{1}{T_0} \int_{T_0} x(t) \mathrm{e}^{-jk\omega_0 t} \, \mathrm{d}t,$$

respectively, where $x(t)$ is a continuous-time periodic signal with fundamental period T_0 and fundamental frequency $\omega_0 = 2\pi / T_0$.

1. Write a function named *FourierAnalysis()* that takes three inputs: the symbolic definition of a periodic time-domain function $x(t)$, the fundamental period T_0 of $x(t)$, and a vector K with integer elements. The outputs of *FourierAnalysis()* are the Fourier coefficients a_ks for each element k in K, and the corresponding frequencies $\omega_k = k\omega_0 = k2\pi / T_0$.

2. Use the *FourierAnalysis()* function to determine the Fourier series coefficients a_ks for $x_1(t) = \cos(100\pi t)$ and $x_2(t) = \sin(100\pi t)$. Keep in mind that $x_1(t)$ and $x_2(t)$ must be symbolic expressions.
3. Define $x(t)$ to be a square wave of fundamental period 1 s, whose one period over $-0.5 \leq t \leq 0.5$ is given by

$$x(t) = \begin{cases} 1, & |t| \leq 0.25 \\ 0, & 0.25 \leq |t| \leq 0.5. \end{cases}$$

   (One way to generate such a signal is to consider the application of the signum function to a sinusoidal signal.)
4. Use the *FourierAnalysis()* function to determine the coefficients a_k for $k = -10 : 10$. Are the coefficients real or complex? Why?
5. Give a stem plot of $|a_k|$ vs w_k.
6. Determine a_k via pen-and-paper calculations and verify whether your function is working correctly.

Fourier Series Synthesis Equation and Gibbs Phenomenon

1. Use the a_ks computed in the previous exercise to find an approximation of $x(t)$ using the truncated Fourier series expression

$$x_N(t) = \sum_{k=-N}^{N} a_k \mathrm{e}^{j\omega_k t}.$$

2. Plot $x_N(t)$ against $t = -2 : 0.01 : 2$ for $N = 1, 5, 10, 25,$ and 50. For each N, record the following:
   a The maximum percentage overshoot of $x_N(t)$ relative to $x(t)$. Since the largest value of $x(t)$ is 1, the percentage maximum overshoot is the maximum value of $x_N(t)$ minus 1.
   b The amplitude corresponding to the points at which the rising edge of $x(t)$ intersects $x_N(t)$.
   c The time difference between a rising edge of $x(t)$ and the nearest peak of $x_N(t)$.

3. Relate the the above observations to the *Gibbs phenomenon*. (Review Section 3.4 of the textbook.)