FUZZY TOPSIS USING TRIANGULAR FUZZY NUMBER

Fuzzy TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) is an extension of the traditional TOPSIS method that integrates **fuzzy logic** to handle uncertainty and imprecise data. It is often used in **multi-criteria decision-making (MCDM)** problems, where criteria evaluations are subjective and uncertain.

Key Concepts of Fuzzy TOPSIS

1. **Fuzzy Sets:**
   Traditional TOPSIS assumes precise numerical data, but in real-world scenarios, data can be vague or subjective. Fuzzy logic helps represent this uncertainty using fuzzy numbers like **triangular** or **trapezoidal fuzzy numbers**. For example:
   - A rating for "Performance" could be represented as (4, 6, 7), where:
     - **4** is the lowest estimate (pessimistic)
     - **6** is the most likely value
     - **7** is the highest estimate (optimistic)
2. **Ideal and Negative-Ideal Solutions:**
   - **Positive Ideal Solution (PIS):** The best possible value for each criterion.
   - **Negative Ideal Solution (NIS):** The worst possible value for each criterion.
     Fuzzy TOPSIS identifies the alternative that is **closest to the PIS** and **farthest from the NIS**.
3. **Handling Uncertainty in Decision Data:**
   In decision-making, experts may provide their preferences in **linguistic terms** such as "high," "medium," or "low." These are converted into **fuzzy numbers** to account for vagueness.

Steps in Fuzzy Topsis

1. **Define the decision matrix**

- List the alternatives and criteria.
- Use **fuzzy numbers** to express the criteria values for each alternative (e.g., $\tilde{A}=(l,m,u)$, where l = lower bound, m = most likely value, u = upper bound of the fuzzy number).

2. Normalize the fuzzy matrix

- Normalize the fuzzy data to bring all values to a comparable scale.

3. Calculate the weighted normalized fuzzy matrix

- Multiply the normalized matrix by the importance weights (expressed as fuzzy numbers).

4. Determine the fuzzy positive ideal solution (FPIS)

- For each criterion, select the **best value** among all alternatives.
- Similarly, determine the **fuzzy negative ideal solution (FNIS)** by selecting the worst values.

5. **Calculate the distances**

- Measure the **distance** between each alternative and the FPIS and FNIS using a fuzzy distance measure (e.g., Euclidean distance or other metrics adapted for fuzzy numbers).

6. Calculate the closeness coefficient (CC)

- The CC measures how close each alternative is to the FPIS, given by:

$$CC_i = \frac{D_i^-}{D_i^+ + D_i^-}$$

where D_i^- is the distance to the NPIS and D_i^+ is the distance to the FNIS.

7. Rank the alternatives

- Rank the alternatives based on the **closeness coefficient** (higher is better).

1. Tabulated Data

Supplier	Sq	Q	CO2	P	L
1	61	69	18	137	3
2	122	54	24	142	6
3	75	57	30	196	5
4	80	77	55	247	1

Beneficial criteria: Service Quality (Sq), Quality (Q)
Non-beneficial criteria: CO2 emission, Price (P), Lead time (L)

2. Normalization of Fuzzy Matrix

For beneficial criteria (e.g., Service Quality, Quality):

$$. \widetilde{r_{ij}} = \left(\frac{l_{ij}}{u_j^*}, \frac{m_{ij}}{u_j^*}, \frac{u_{ij}}{u_j^*}\right)$$

where u_j^* is the maximum value for the j^{th} criterion.

For non-beneficial criteria (e.g., CO2 Emission, Price, Lead Time):

$$. \widetilde{r_{ij}} = \left(\frac{l_j^*}{u_{ij}}, \frac{l_j^*}{m_{ij}}, \frac{l_j^*}{l_{ij}}\right)$$

where l_j^* is the minimum value for the j^{th} criterion.

3. Weighted Normalized Fuzzy Matrix

Let w_j represent the weight for each criterion. The normalized matrix is multiplied by these weights to create the weighted normalized fuzzy matrix:

$$\widetilde{v_{ij}} = \widetilde{r_{ij}} \times w_j$$

4. Fuzzy Positive Ideal Solution (FPIS) and Fuzzy Negative Ideal Solution (FNIS)

- **FPIS**: For beneficial criteria, the maximum value across alternatives. For non-beneficial criteria, the minimum value.

$$A^+ = \left(max_i \widetilde{v_{ij}}\ for\ beneficial, min_i \widetilde{v_{ij}}\ for\ non-beneficial\right)$$

- **FNIS**: For beneficial criteria, the minimum value across alternatives. For non-beneficial criteria, the maximum value.

$$A^- = \left(min_i \widetilde{v_{ij}}\ for\ beneficial, max_i \widetilde{v_{ij}}\ for\ non-beneficial\right)$$

5. Distance from FPIS and FNIS

The distance between two triangular fuzzy numbers A=(a_1,a_2, a_3) and B=(b_1, b_2 , b_3) is calculated as:

$$d(A,B)=\sqrt{\frac{1}{3}\sum_{i=1}^{3}(a_i - b_i)^2}$$

6. Closeness Coefficient (CC)

The closeness coefficient for each alternative is given by:

$$CC_i = \frac{D_i^-}{D_i^+ + D_i^-}$$

where D_i^+ is the distance to the FPIS and D_i^- is the distance to the FNIS.

7. Ranking the Alternatives

Alternatives are ranked based on their **closeness coefficient (CC)** — the higher the CC, the better the alternative.

: