Regional Geoid Modelling

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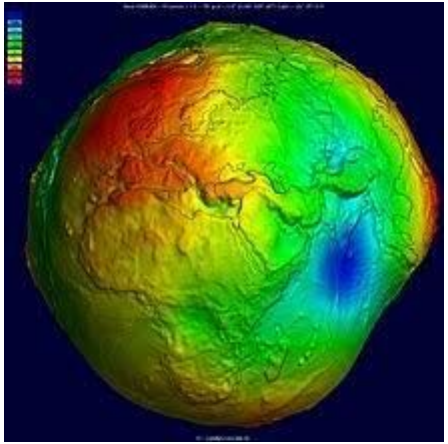
Introduction



Geoid represents the true shape of the Earth, it is an equipotential surface that approximately coincides with the global mean sea level.

Geoid is the vertical reference system used for height measurement on surface of Earth, hence it’s precise determination is important.

In this project we make use of airborne gravity data, measured over New Mexico, United States to estimate the regional best fitting geoid.

We make use of the remove-restore method for geoid computation. The ellipsoid used for calculation is WGS84

Literature Review

- In traditional **remove-restore method**, the basic idea is to shift topographic masses and produce a downward shift in data from the Earth's surface to the geoid.

$$\Delta g_0^H = \Delta g_P - A_P + A_{P_0}^C = \Delta g_P + \delta A$$

- Here, Δg_P is the free-air gravity anomaly which is obtained after removing both free-air correction and the corrections due to atmosphere.
- Then, the geoidal heights are restored using the reduced gravity anomaly obtained from the above equation, combined with long-wavelength undulations and the contribution due to indirect effects of terrain.

$$N = N_{GM} + N_{\Delta g} + N_h$$

- The geoid undulations due to reduced gravity anomalies are computed using Strokes integral, as shown in equation below.

$$N = \frac{R}{4\pi\gamma} \iint_\sigma (\Delta g + C + \delta\Delta g - \Delta g_{GGM})\, S(\psi) d\sigma$$

Literature Review (Cont)

- The unmodified strokes integration kernel S(ψ) which is the function of spherical distance between the computational point P and the data point Q, and it is given as shown in the equation below.

$$S(\psi) = \frac{1}{\sin(\psi/2)} - 6\sin(\psi/2) + 1 - 5\cos\psi - 3\cos\psi \ln[\sin(\psi/2) + \sin^2(\psi/2)]$$

- However, since strokes integral requires a continuous gravity data over the whole of the Earth's surface, and since its an impractical requirement **modified strokes integration kernels** are used as suggested by Featherstone et al. 1998.

$$S^{ME}(\psi) = S(\psi) - S(\psi_\circ) \text{ for } (0 < \psi < \psi_\circ)$$

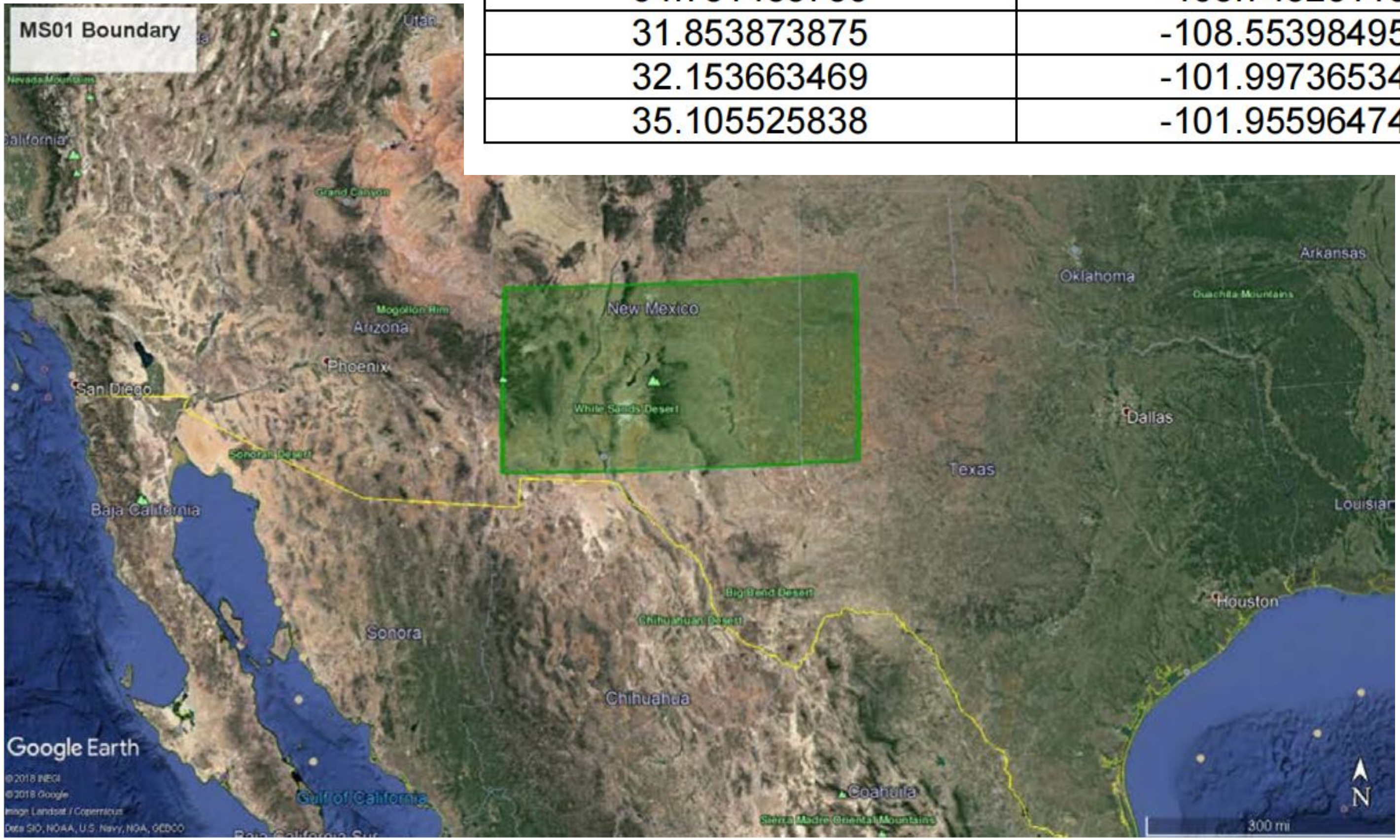
where, ψ_0 is the optimal cap size, and is empirically determined through the comparison of the levelling derived geoid.

- The idea of modifying Stokes' kernel is to reduce the truncation error when limiting the area of integration to a cap around the computation point

Study Area

Table 1: Latitude and Longitude Coordinates of Corner Points Defining Block MS01

Latitude (decimal degrees)	Longitude (decimal degrees)
34.781488780	-108.748231133
31.853873875	-108.553984956
32.153663469	-101.997365340
35.105525838	-101.955964747



Study Area

Data Format

Block + Line Number

Latitude

Ellipsoidal Height

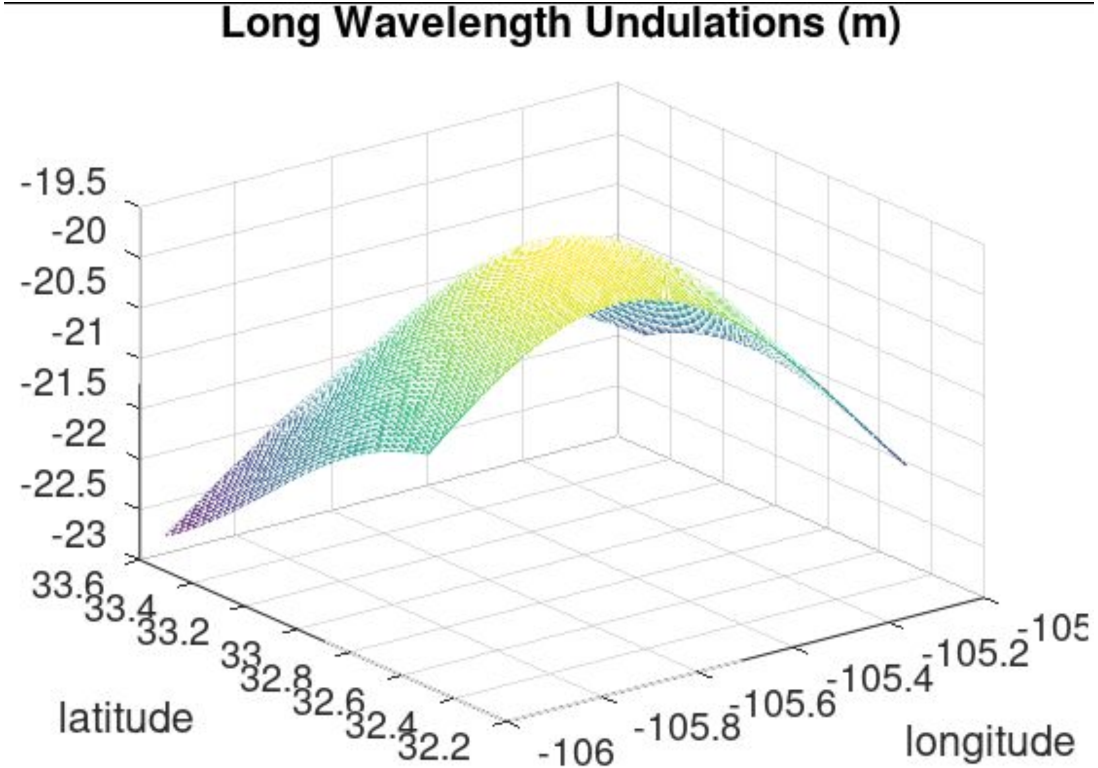
PN02131	20160913055217000	45.49191038	-124.60713684	6277.983	978683.65
PN02131	20160913055218000	45.49307147	-124.60735348	6278.475	978684.01
PN02131	20160913055219000	45.49423231	-124.60757023	6278.902	978684.36
PN02131	20160913055220000	45.49539295	-124.60778694	6279.221	978684.68
PN02131	20160913055221000	45.49655349	-124.60800350	6279.407	978684.99
PN02131	20160913055222000	45.49771397	-124.60822002	6279.476	978685.29
PN02131	20160913055223000	45.49887443	-124.60843652	6279.458	978685.56
PN02131	20160913055224000	45.50003492	-124.60865285	6279.381	978685.83
PN02131	20160913055225000	45.50119546	-124.60886897	6279.270	978686.08
PN02131	20160913055226000	45.50235603	-124.60908504	6279.142	978686.32
PN02131	20160913055227000	45.50351668	-124.60930113	6278.985	978686.54

Time

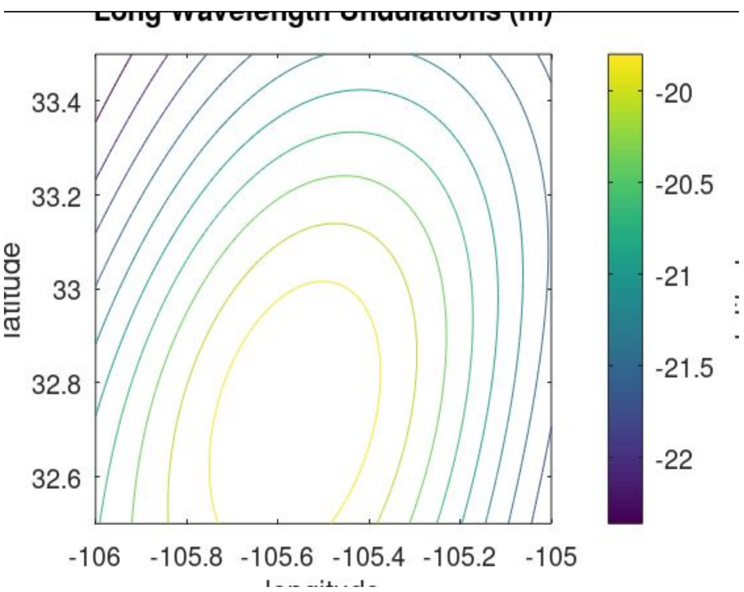
Longitude

Observed Gravity

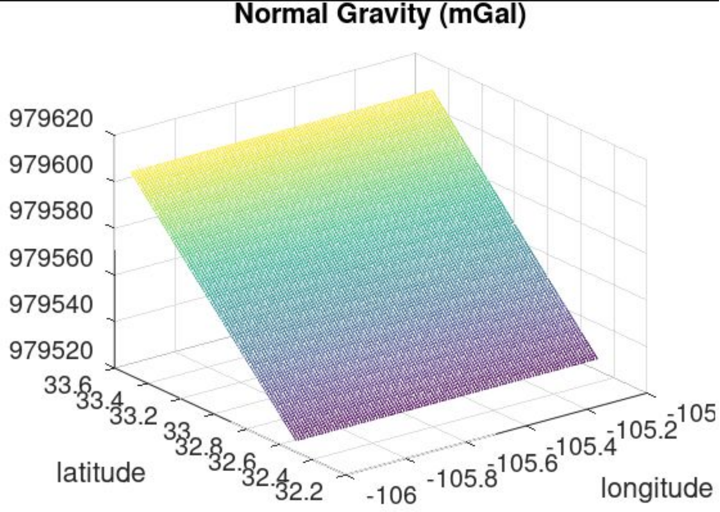
Long Wavelength Undulations (N_{CGM})

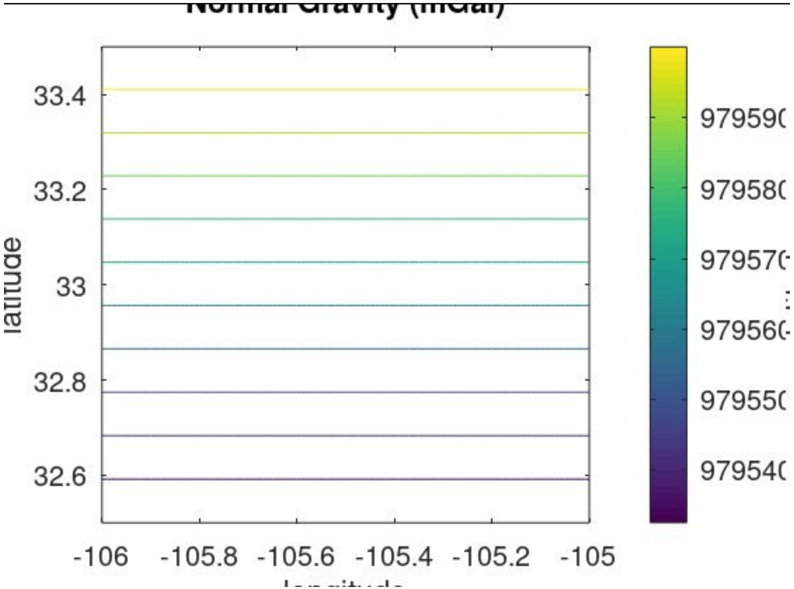


Orthometric height = elevation + N_{GGM}



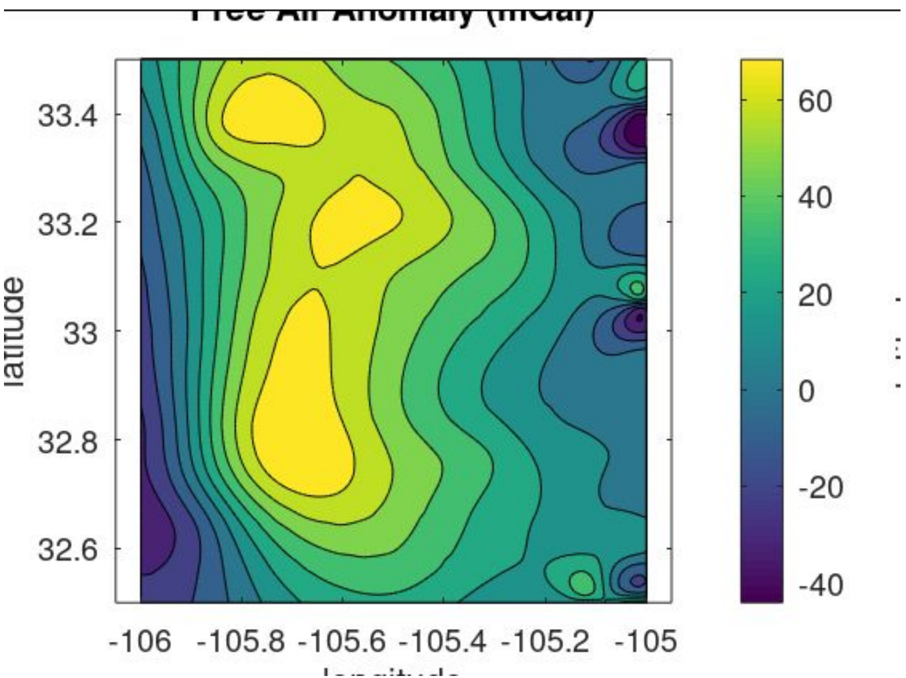
Normal Gravity





We compute Normal gravity at the observed gravity data points values using the Somigliana-Pizzetti formula. We need to remove normal gravity from observed gravity data to calculate the free air gravity anomaly

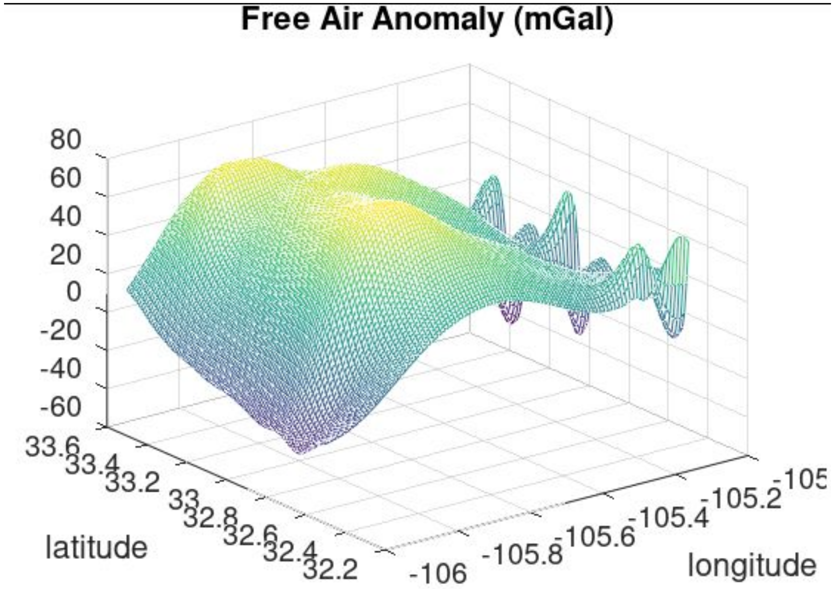
Free Air Gravity Anomaly



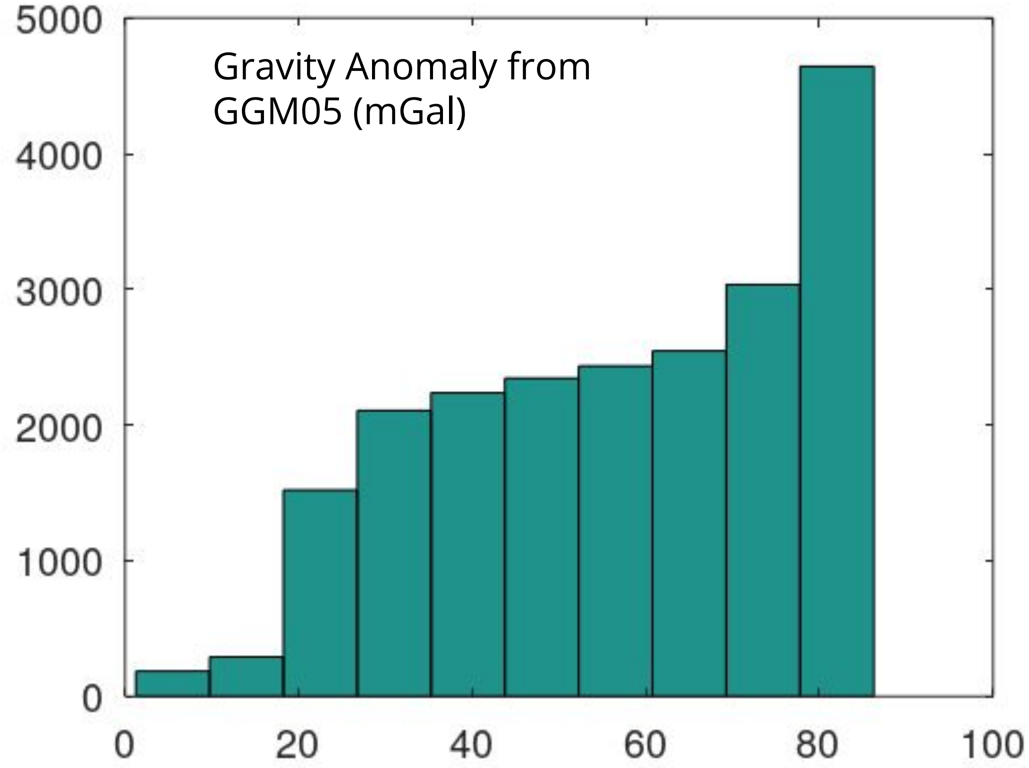
Free Air Anomaly is calculated by the formula

$\Delta g = (g_{observed} + \delta g_{Free\ air}) - \gamma$

Where γ is the normal gravity, $\delta g_{Free\ air}$ is the free air gravity disturbance, $g_{observed}$ is the observed value of gravity.

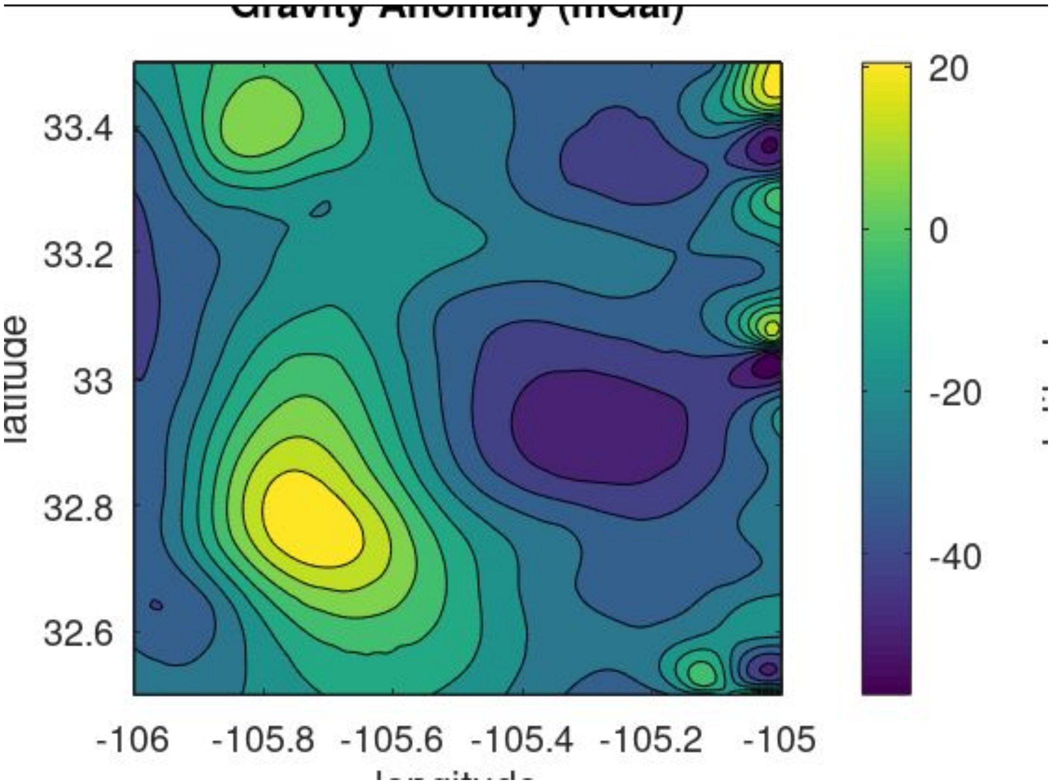


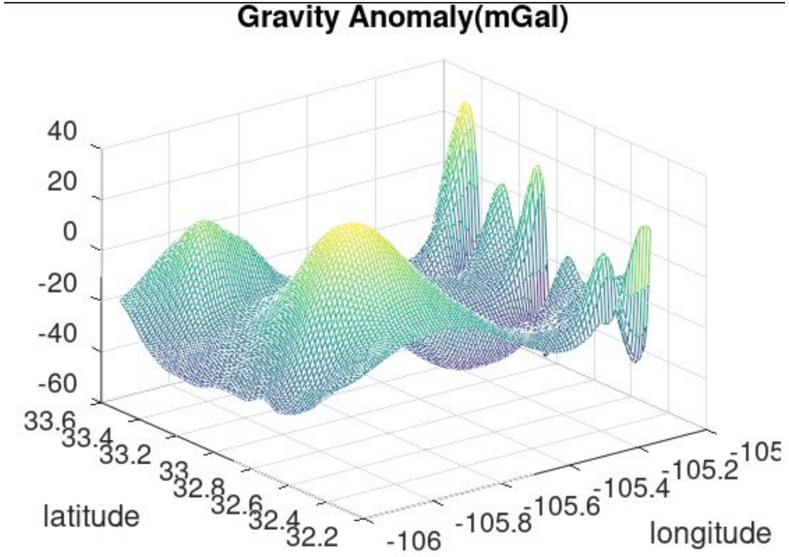
Long Wavelength Gravity Anomaly



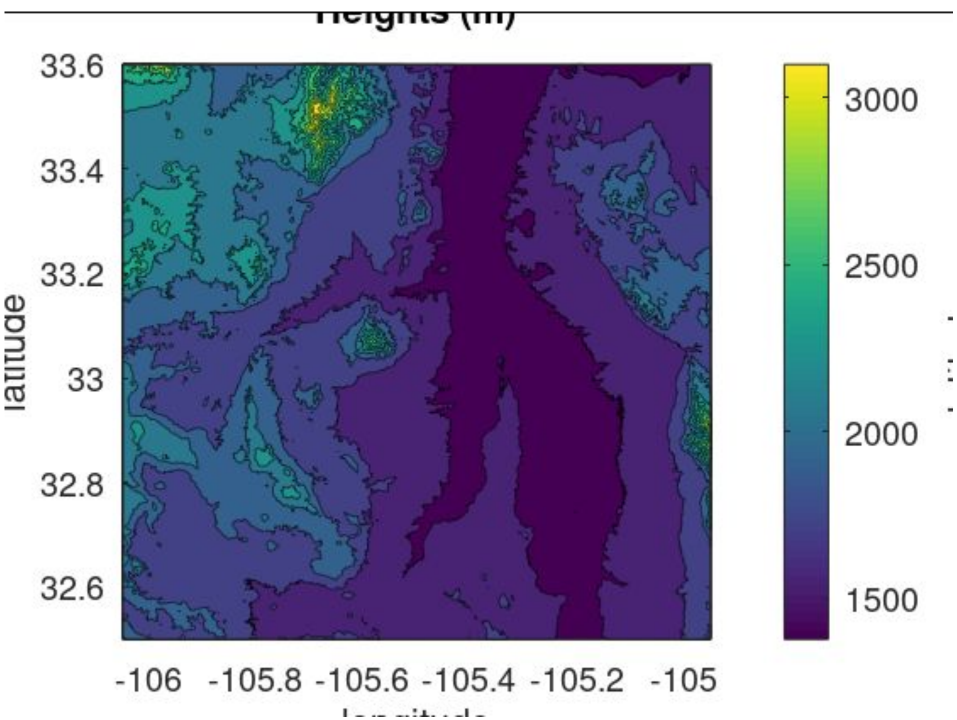
Distribution of Long Wavelength Gravity Anomaly, as present in the GGM05

Gravity Anomaly After Removal of Long Wavelength Gravity Anomaly and Attraction due to atmosphere

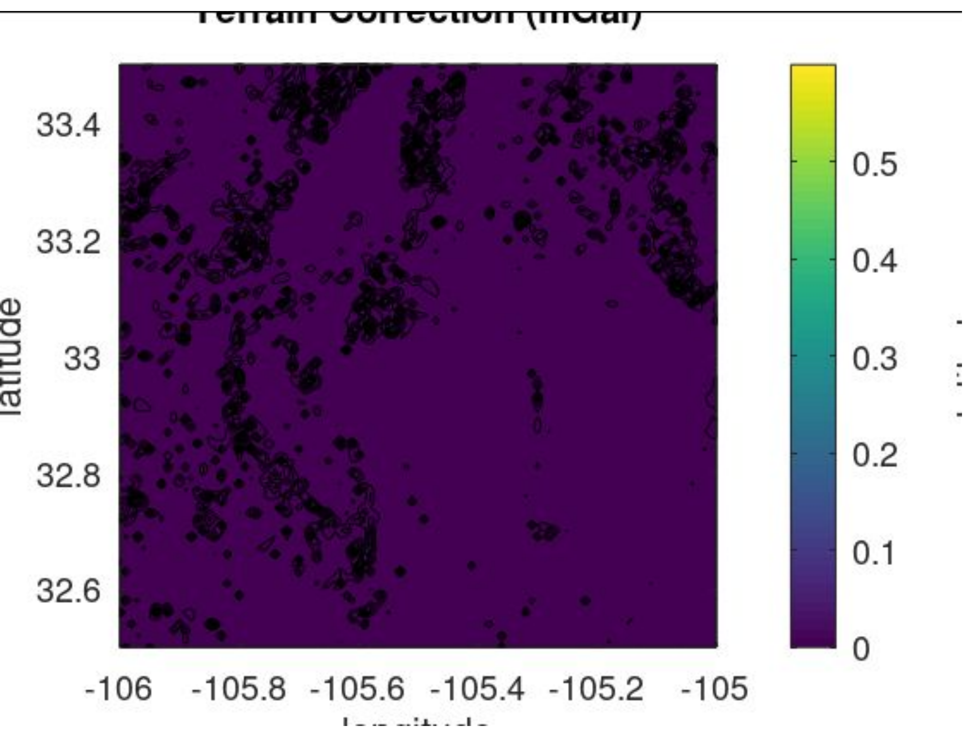


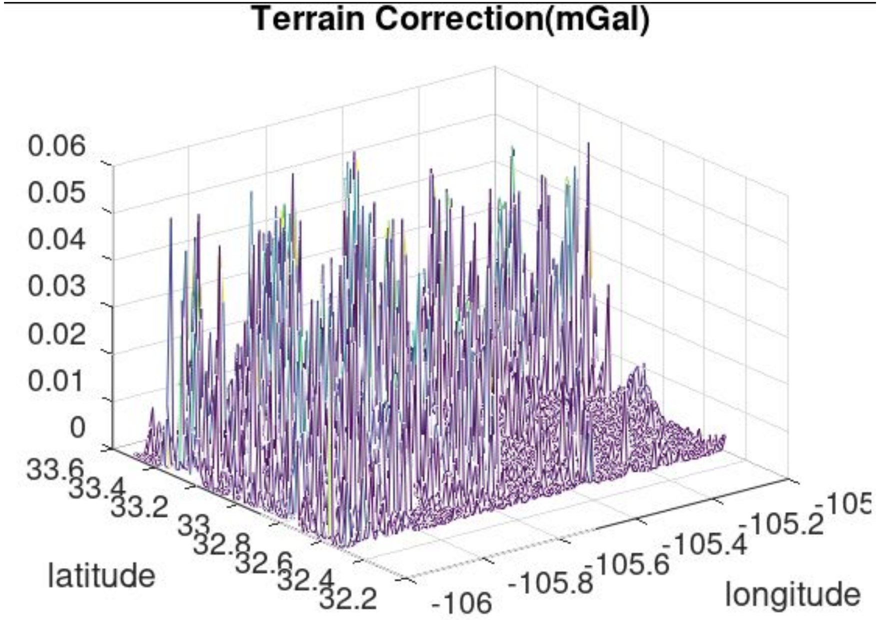


Heights from DEM



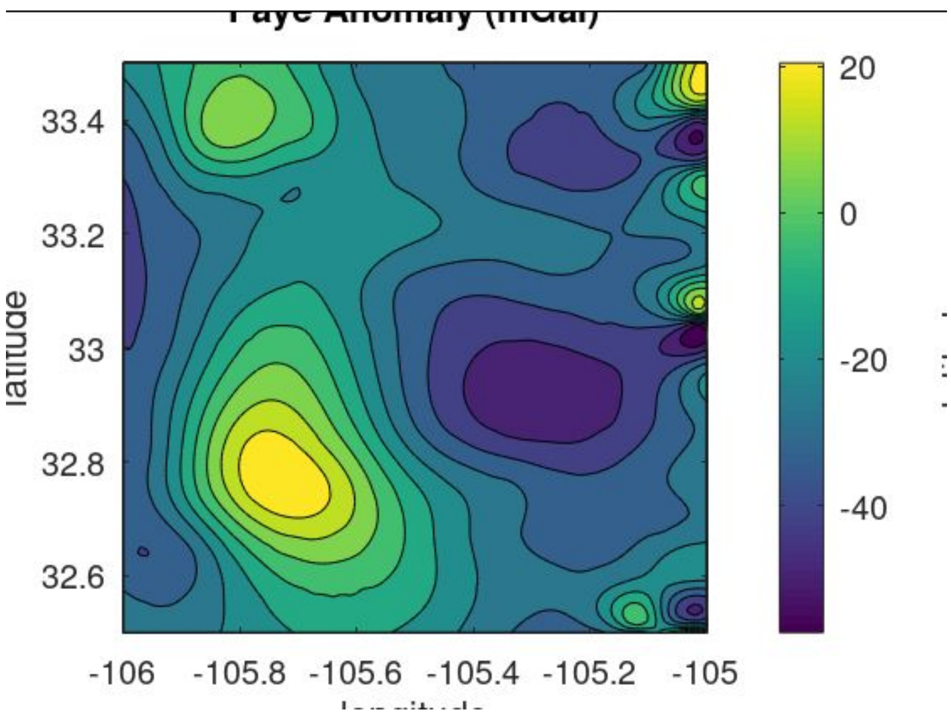
Terrain Correction

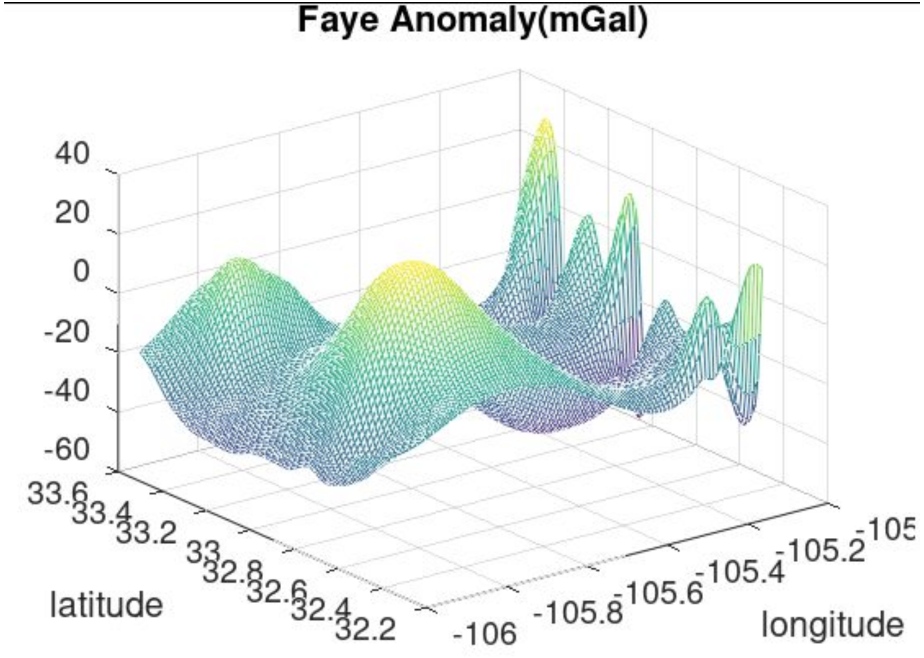




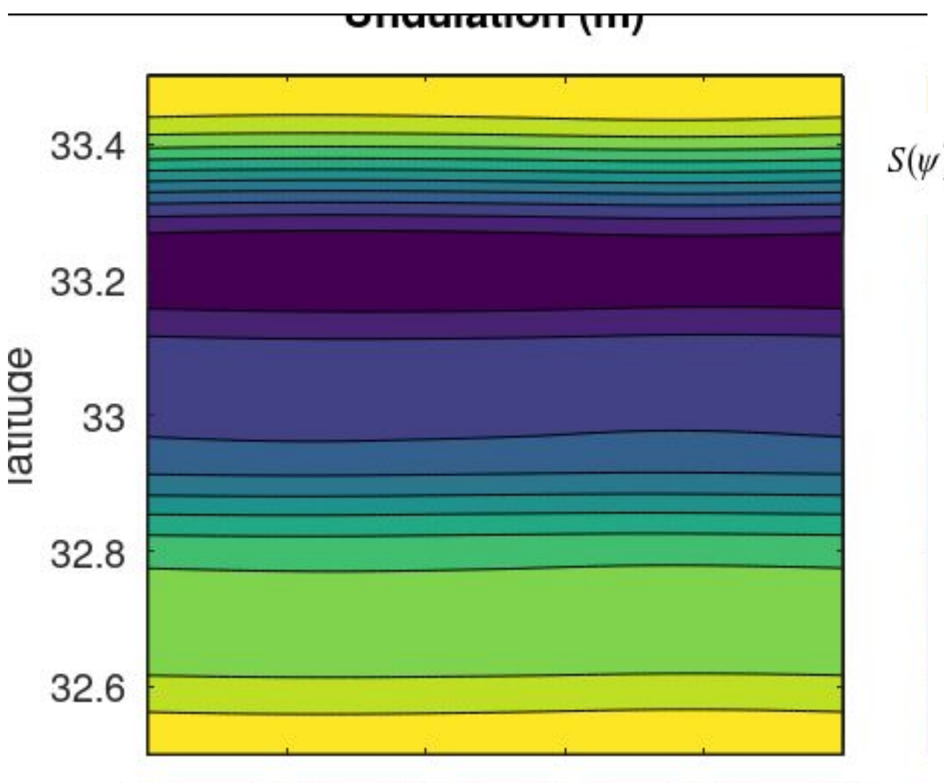
Faye Anomaly

Faye Anomaly is calculated by removing the Terrain Correction from Gravity anomaly after removal of long wavelength anomaly and atmospheric atmosphere.



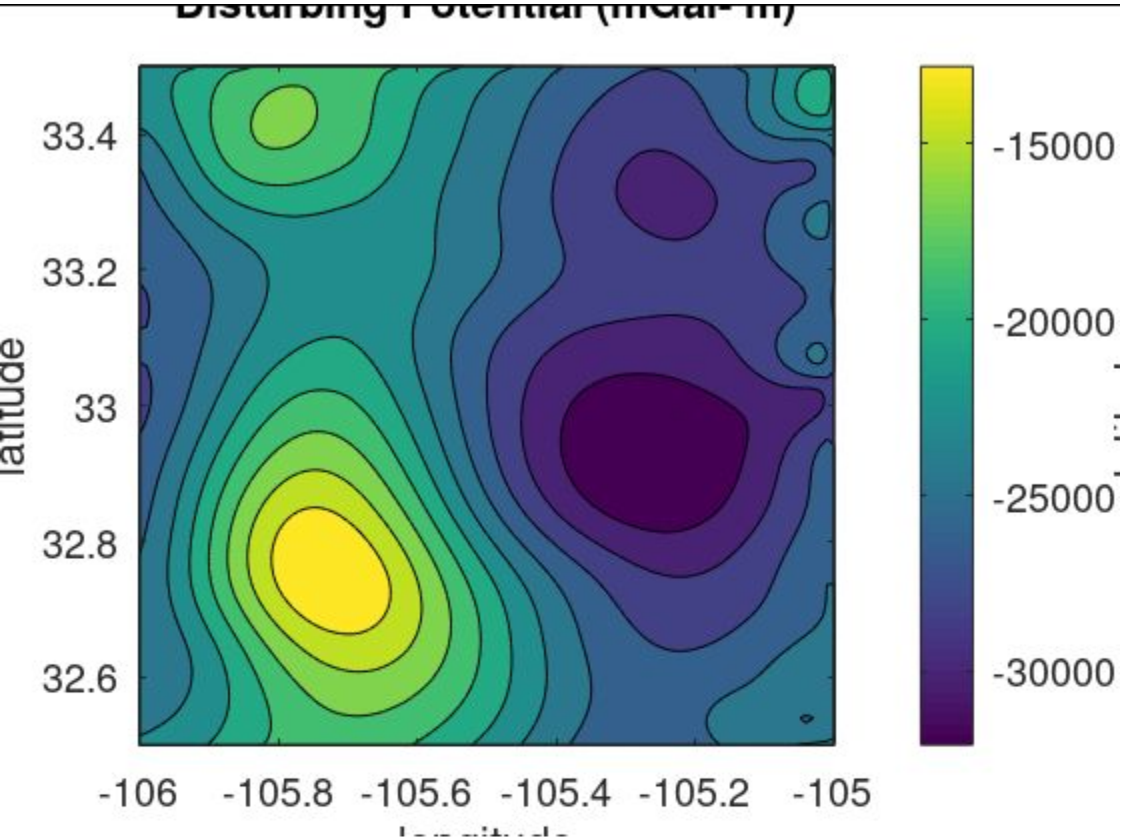


Disturbing Potential : Standard Stokes Kernel



$$S(\psi) = \frac{1}{\sin(\psi/2)} - 6\sin(\psi/2) + 1 - 5\cos\psi - 3\cos\psi \ln[\sin(\psi/2) + \sin^2(\psi/2)]$$

Disturbing Potential: Modified Kernel

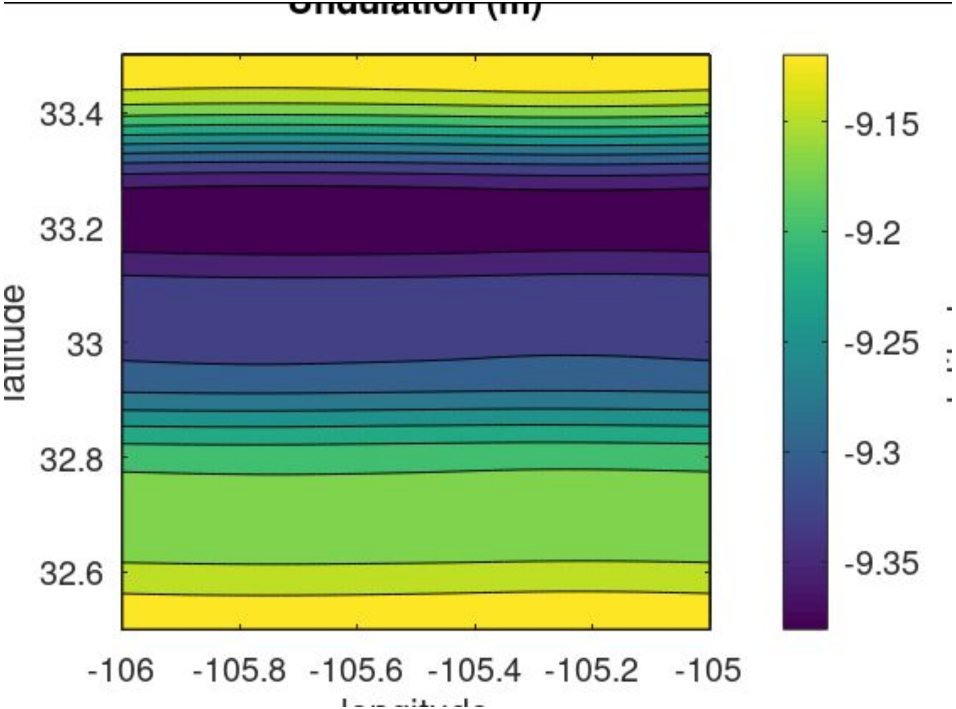


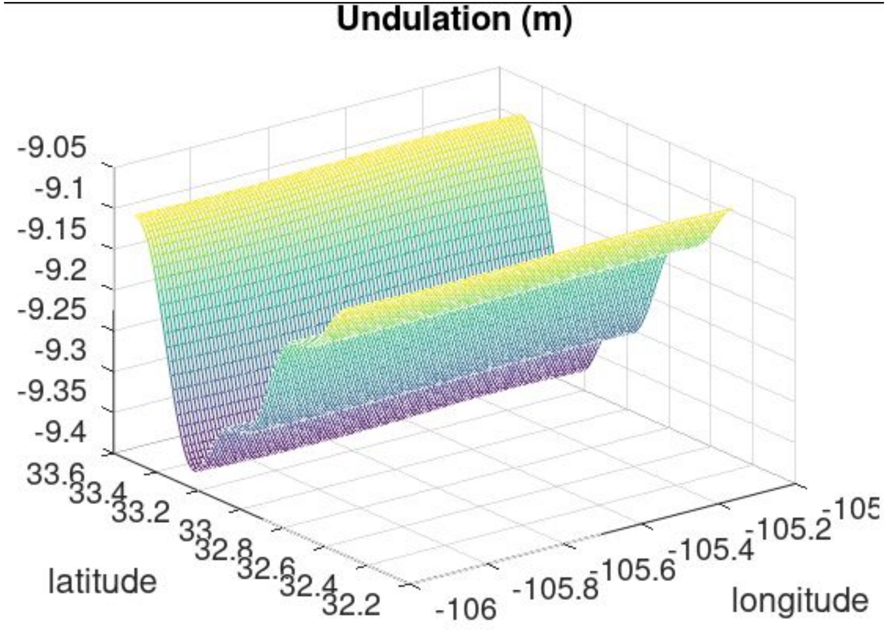
$$x = (\lambda_Q - \lambda_P)\cos\varphi_Q,$$

$$y = \varphi_Q - \varphi_P.$$

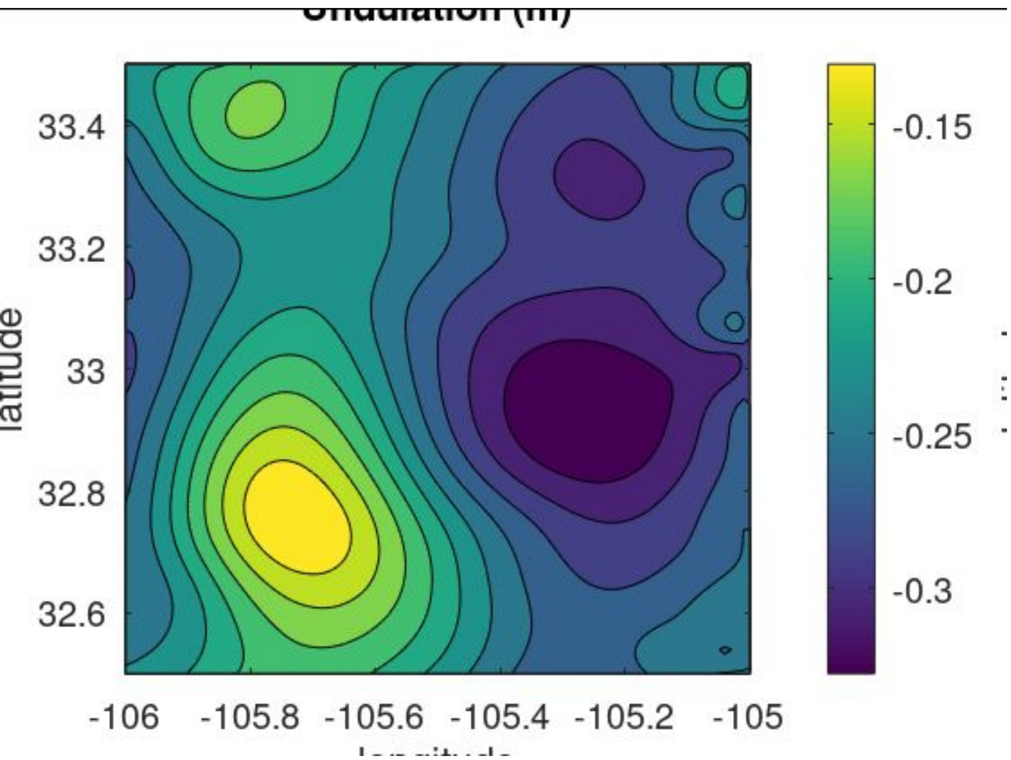
$$K(x, y) = \frac{2}{\sqrt{x^2 + y^2}}.$$

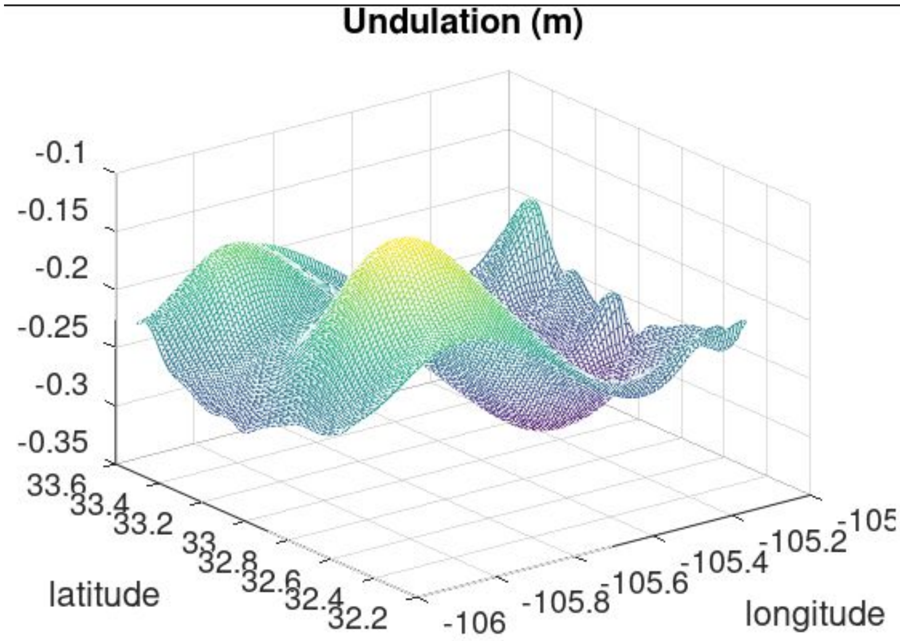
Undulation


33.4
33.2
33
32.8
32.6
-106
-105.8
-105.6
-105.4
-105.2
-105
-9.15
-9.2
-9.25
-9.3
-9.35

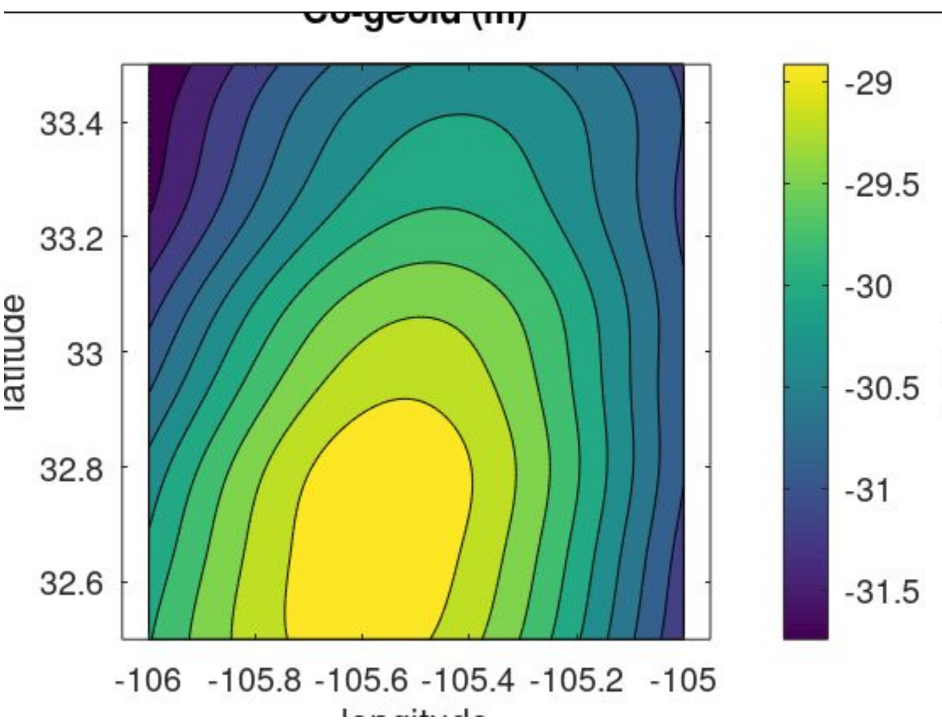

Undulation (m)
-9.05
-9.1
-9.15
-9.2
-9.25
-9.3
-9.35
-9.4
33.6
33.4
33.2
33
32.8
32.6
32.4
32.2
latitude
-106
-105.8
-105.6
-105.4
-105.2
longitude

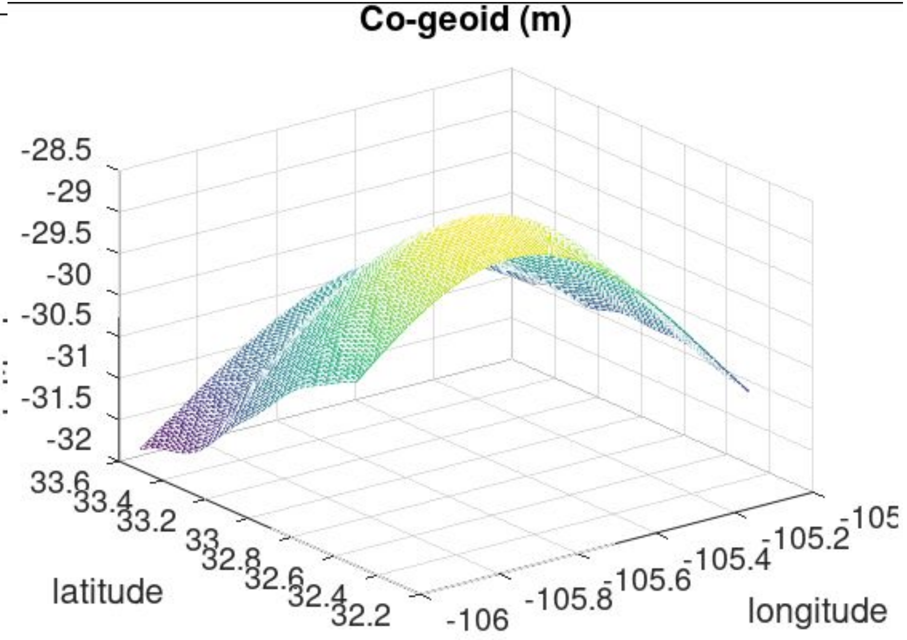
Undulation: Modified Kernel



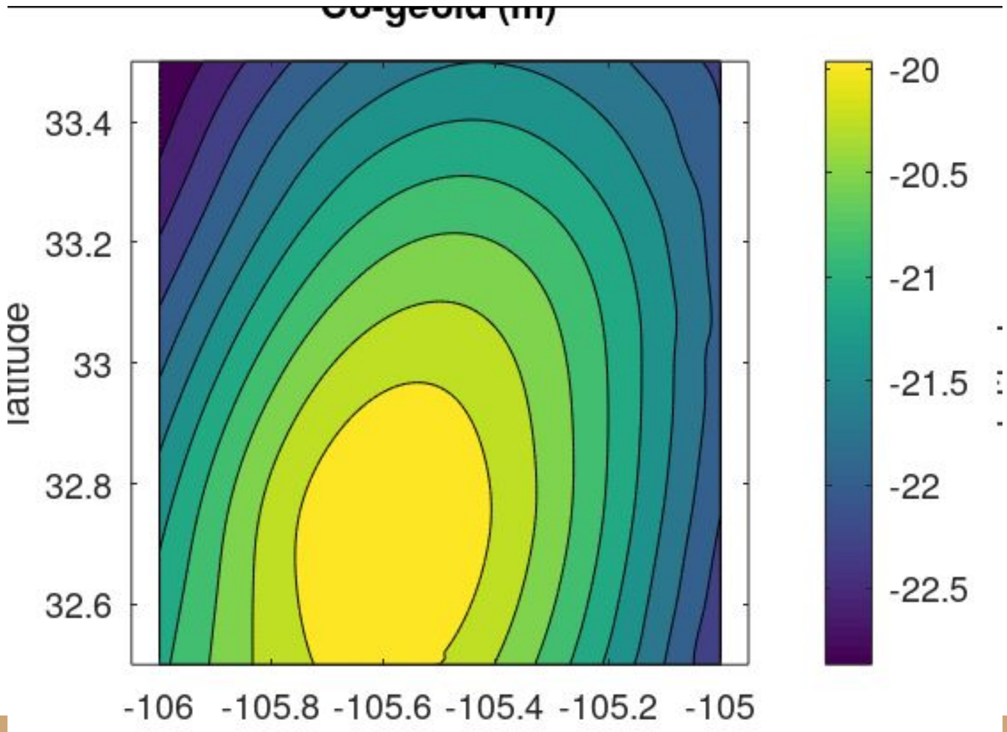


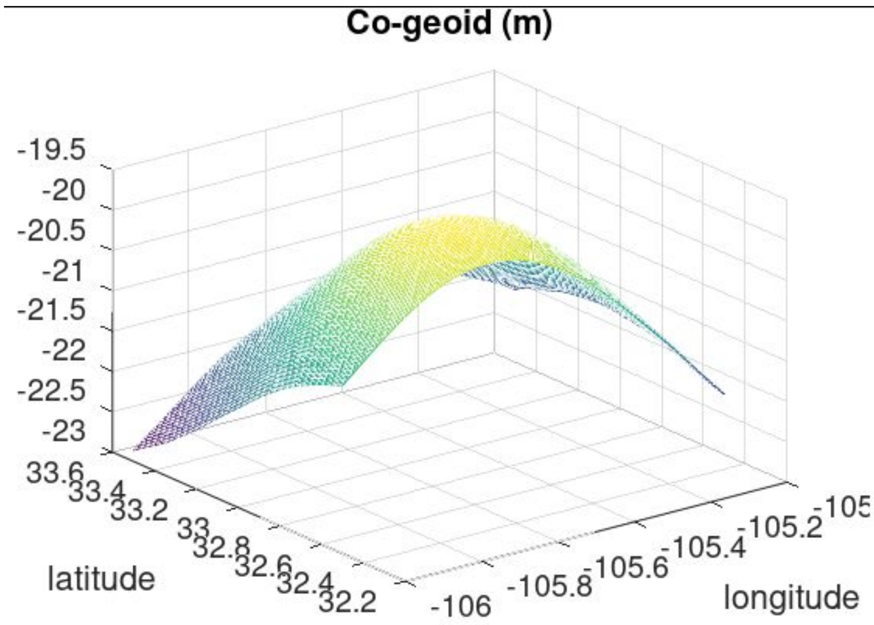
Co-Geoid


Co-geoid (m)
33.4
33.2
33
32.8
32.6
latitude
-106
-105.8
-105.6
-105.4
-105.2
-105
-29
-29.5
-30
-30.5
-31
-31.5

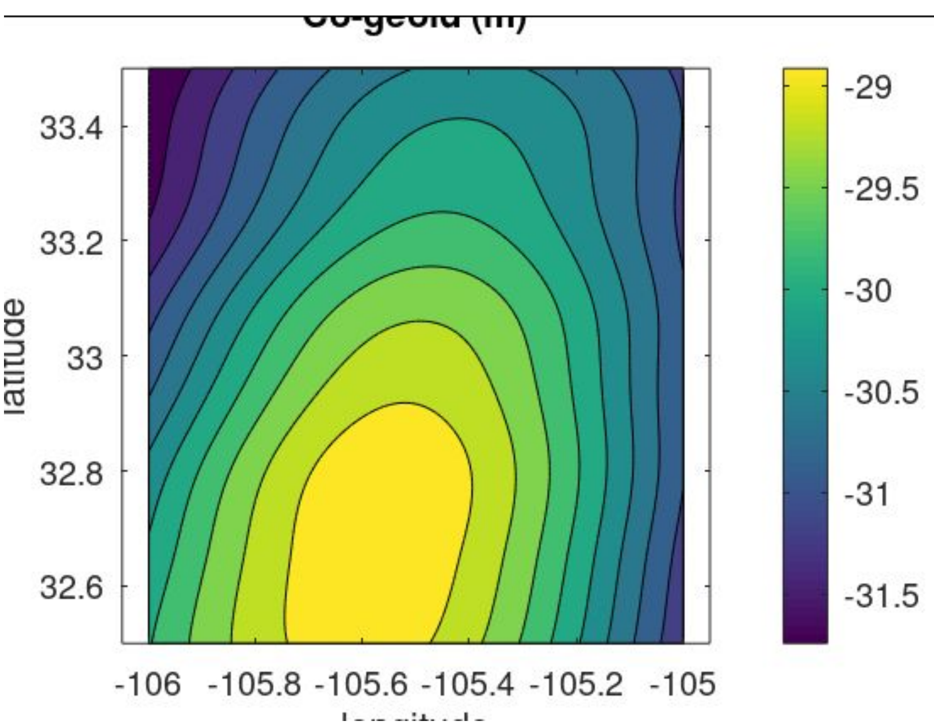

Co-geoid (m)
-28.5
-29
-29.5
-30
-30.5
-31
-31.5
-32
33.6
33.4
33.2
33
32.8
32.6
32.4
32.2
latitude
-106
-105.8
-105.6
-105.4
-105.2
-105
longitude

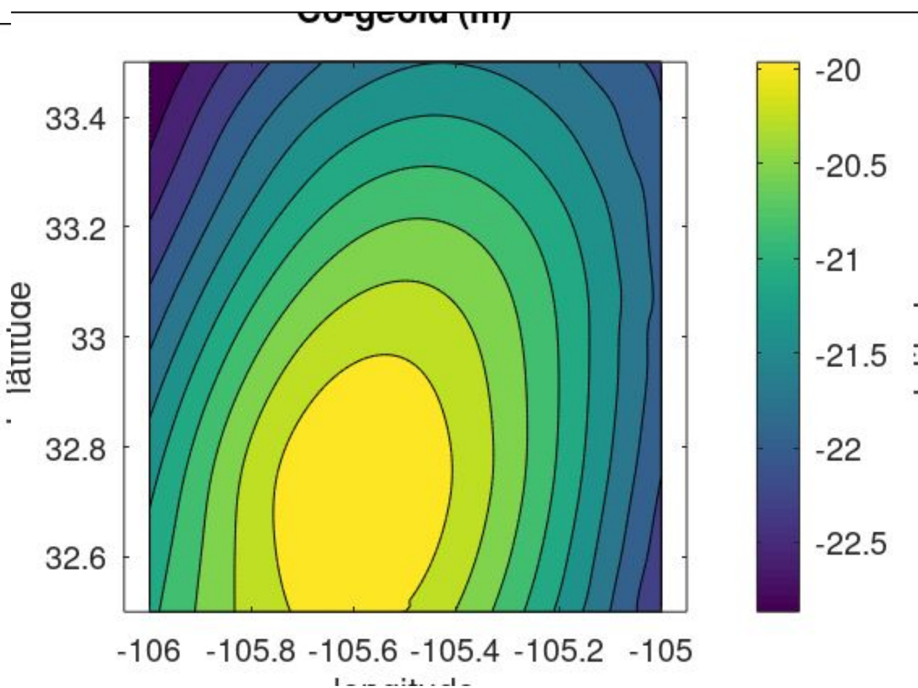
Co-Geoid: Modified Kernel



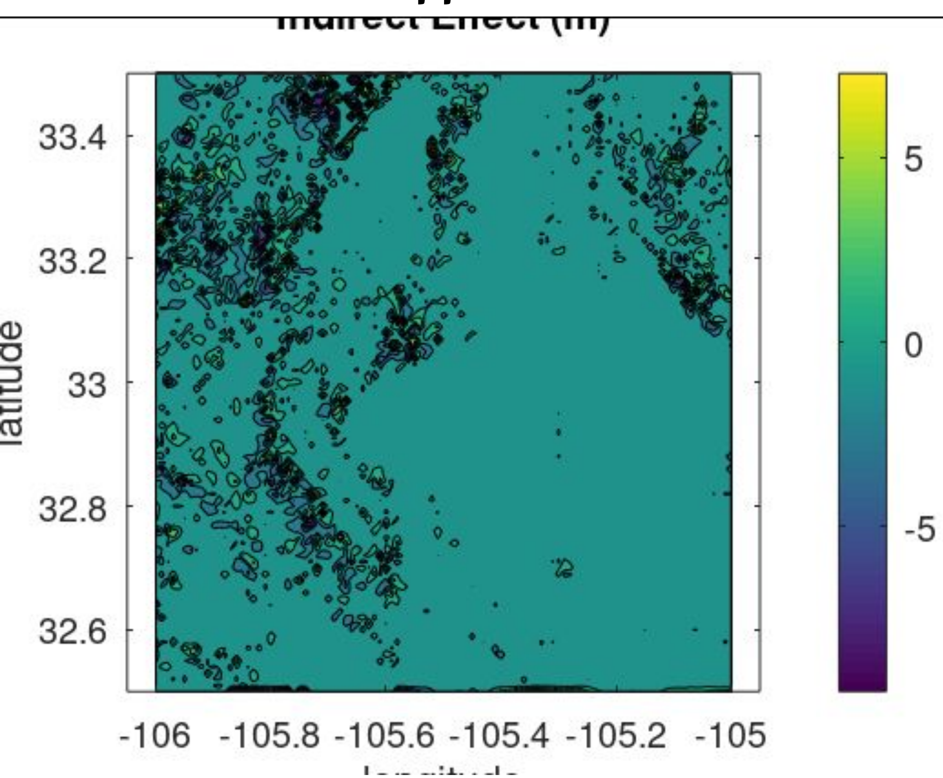


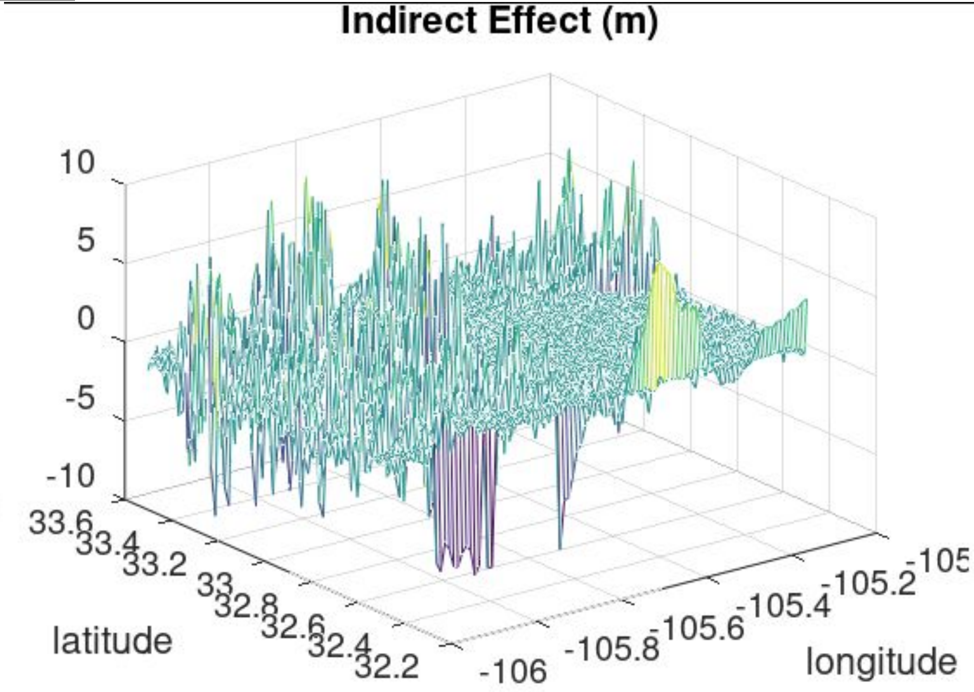
Cogeoid : Standard Kernel vs. Modified Kernel



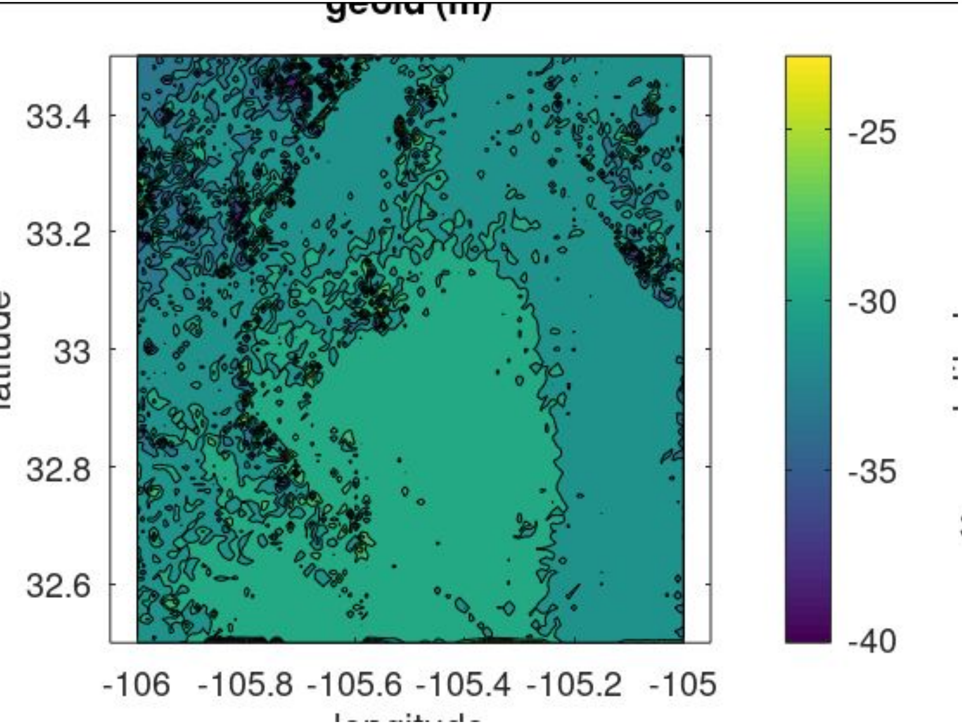


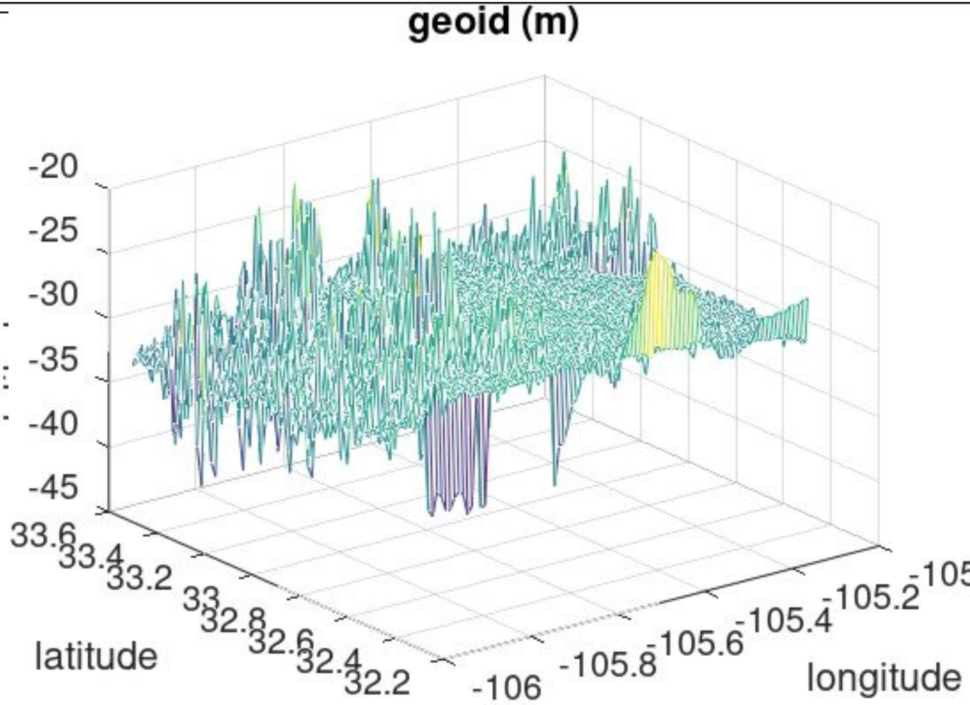
Indirect Effect



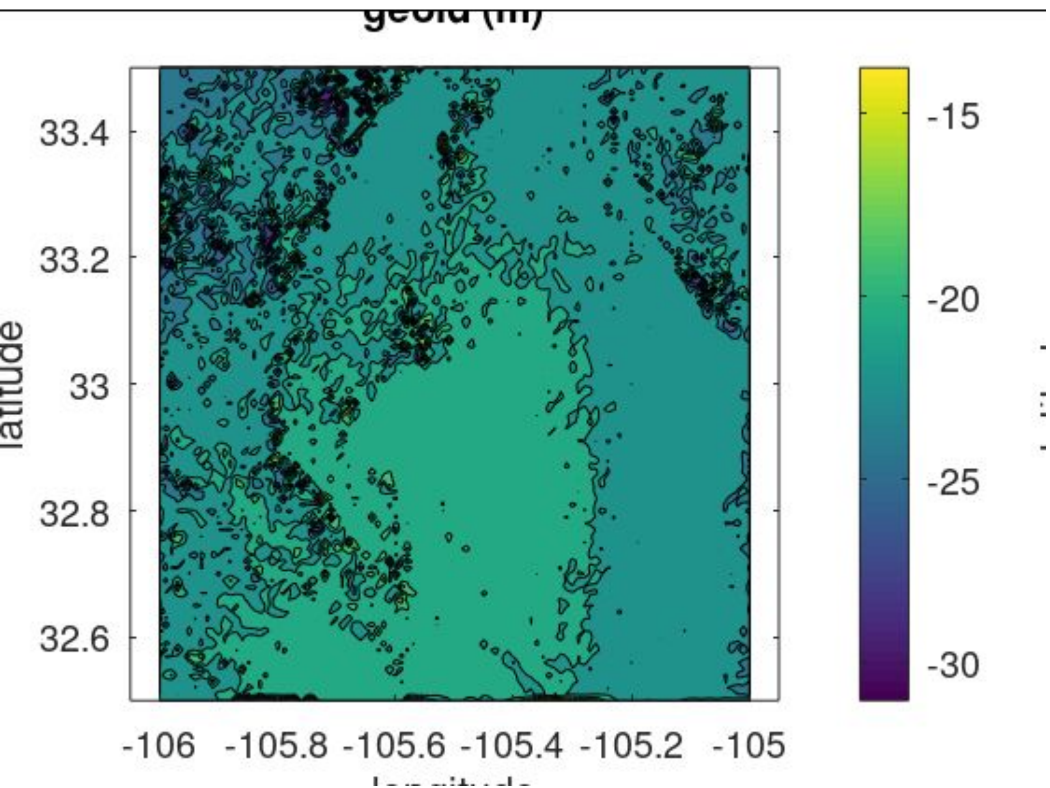


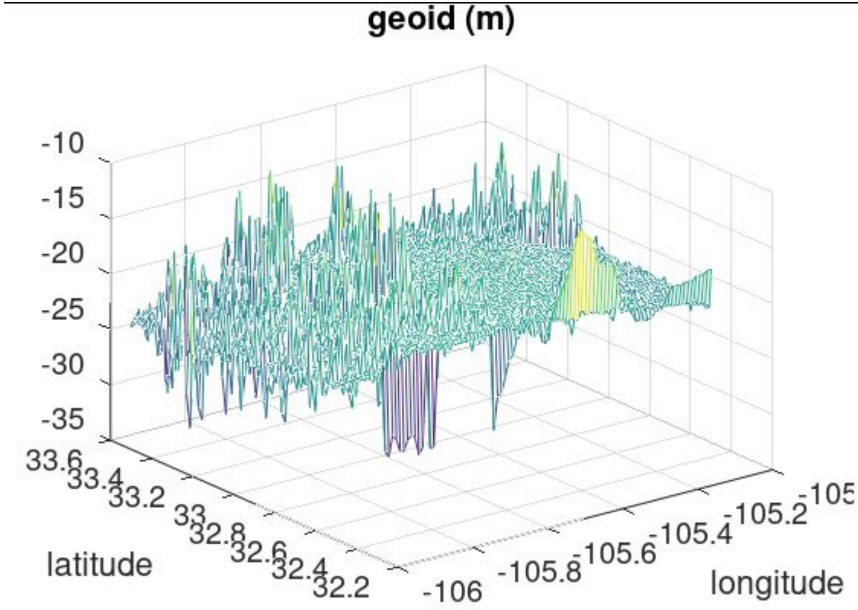
Geoid


geoid (m)
33.4
33.2
33
32.8
32.6
-25
-30
-35
-40
-106
-105.8
-105.6
-105.4
-105.2
-105

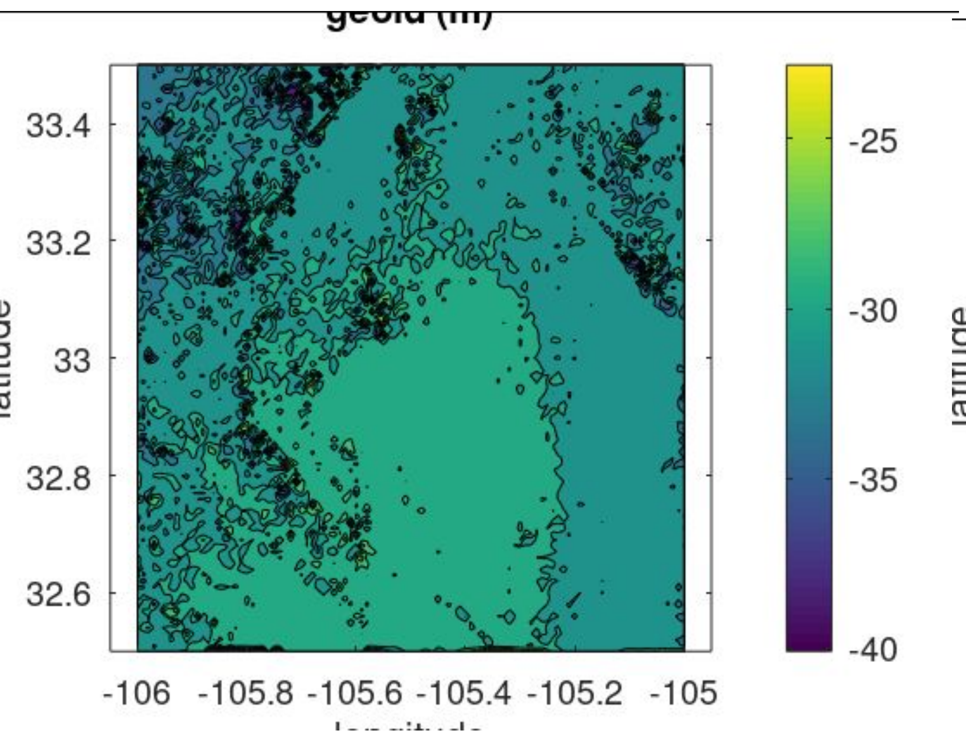

geoid (m)
-20
-25
-30
-35
-40
-45
33.6
33.4
33.2
33
32.8
32.6
32.4
32.2
latitude
-106
-105.8
-105.6
-105.4
-105.2
longitude

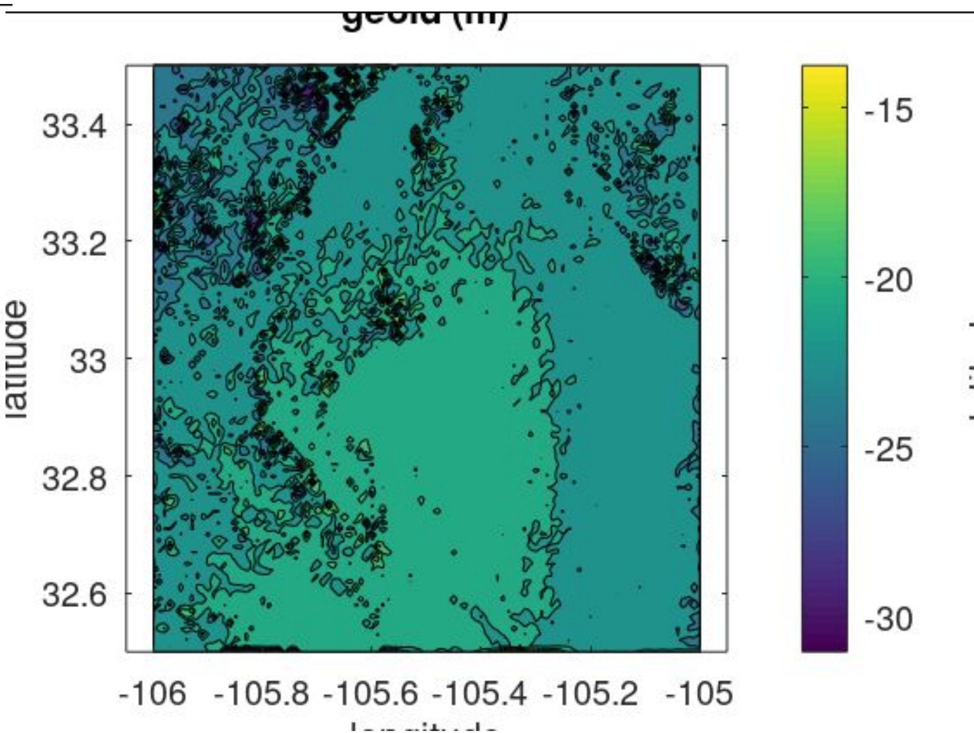
Geoid: Modified Kernel





Geoid : Standard Kernel vs. Modified Kernel





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- Ellmann, Artu. (2004). The geoid for the Baltic countries determined by the least squares modification of Stokes' formula.
- Dongmei Guo, Huiyou He, Peng Sang, Precise geoid computation using Stokes-Helmert's scheme and strict integrals of topographic effects, Geodesy and Geodynamics
- Varga, M., Pitoňák, M., Novák, P. *et al.* Contribution of GRAV-D airborne gravity to improvement of regional gravimetric geoid modelling in Colorado, USA. *J Geod* 95, 53 (2021)
- McCubbine, J.C., Amos, M.J., Tontini, F.C. *et al.* The New Zealand gravimetric quasigeoid model 2017 that incorporates nationwide airborne gravimetry