Consider the following:

$10^a < f_{2n} < 10^{a+1}$ where $n \ge 1$

$f_{2n} = \frac{1}{\sqrt{5}}\left(\phi^{2n} - \frac{1}{\phi^{2n}}\right) = \frac{\phi^{4n}-1}{\sqrt{5}\,\phi^{2n}}$ where $\phi = \frac{1+\sqrt{5}}{2}$

i.e. $10^a < \frac{\phi^{4n}-1}{\sqrt{5}\,\phi^{2n}} < \frac{\phi^{4n}}{\sqrt{5}\,\phi^{2n}} = \frac{\phi^{2n}}{\sqrt{5}}$

$a < 2n\log\phi - \log\sqrt{5}$

$n > \frac{a+\log\sqrt{5}}{2\log\phi}$

and

$f_{2n-1} < f_{2n} < 10^{a+1}$ where $n \ge 1$

$\frac{\phi^{4n-2}+1}{\sqrt{5}\,\phi^{2n-1}} < 10^{a+1}$

$\frac{1}{\sqrt{5}}\phi^{2n-1} < 10^{a+1}$

$(2n-1)\log\phi - \log\sqrt{5} < a+1$

$n < \frac{a+1+\log\sqrt{5}}{2\log\phi} + \frac{1}{2}$

So, we have $\frac{a+\log\sqrt{5}}{\log\phi} < 2n < \frac{a+1+\log\sqrt{5}}{\log\phi} + 1$ where $n \ge 1$

Similarly, $\frac{a+\log\sqrt{5}}{\log\phi} - 1 < 2n+1 < \frac{a+1+\log\sqrt{5}}{\log\phi}$ where $n \ge 0$

Making use of the fact that $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^n = \begin{pmatrix} f_{n+1} & f_n \\ f_n & f_{n-1} \end{pmatrix}$ and $f_{n+5} > 10 f_n$,

we can obtain the solution proivded by me in Ruby.