CS203 Assignment 2 Semester I, 2017-18

Due Date: 25th August, 2017 **Maximum Marks: 50**

Instructions

- Submit the assignment at the end of class on or before the due date.
- Yours answers should be precise and clearly written.
- Cheating/plagiarizing in any form will be heavily penalized.
- Late submissions will receive a mark of zero.
- For the definition of cyclic group, you are allowed to refer any material.
- Any doubts regarding the assignment can be raised in the discussion forum on moodle.

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1. (10 points) Use the Orbit-counting (Burnsides) Lemma to find a formula for the number of ways of coloring the faces of a cube with k colors. Assume that two colored cubes which differ by a rotation are identical. Repeat for coloring's of the edges, and of the vertices.

2. (10 points) Prove the following statement

   1. Every cyclic group of order n is isomorphic to $\mathbb{Z}_n$.
   2. Every subgroup of a cyclic group is cyclic.
   3. For any cyclic group of order n; prove that given any divisor d of n, there exists a unique subgroup of order d.
   4. In a cyclic group of order n, how many generators are there ?
   5. Prove the following equality:
   $$\sum_{d|n} \phi(d) = n$$
   where ϕ is euler's totient function.

3. (10 points) Suppose you manufacture an identity card by punching two holes in an 3×3 grid. How many distinct cards can you produce? Use Orbit-counting (Burnsides) Lemma.

4. (20 points)
   1. Let G be a finite abelian group such that it does not have any non-trivial subgroup. By non-trivial subgroup we mean that subgroup other than identity and complete group. Then prove that G is a cyclic group whose order is a prime number. (**3 points**)
   2. Let G and H be two finite abelian groups and θ be a homomorphism from G to H. Prove that for every element $a \in G$, the order of $\theta(a)$ divides order of a.(**2 points**)
   3. Let G be a finite abelian group and p be a prime divisor of $o(G)$. Let H be a subgroup of G such that p does not divide $o(H)$ but G/H has an element of order p. Then prove that G also has an element of order p. $o(G)$ represents the order of group G. (**7 points**)
   4. Now using induction on the order of group, prove that for every finite abelian group G and every prime divisor p of $o(G)$, there exists an element in G of order p.(**8 points**)