CS345 Assignment 1

You will be informed on Friday about which question(s) are to be submitted.

Q1) Given an $N \times N$ chess board where each cell is indexed by (i, j) where $0 \leq i, j \leq N-1$. Also given constant integers k_0, l_0 in the range $0, \ldots, N-1$. A knight can move from (i, j) to $(i \pm k_0, j \pm l_0)$. Describe an algorithm which takes two cell addressed (a, b) and (c, d) and returns the number of knight moves which can take it from (a, b) to (c, d). If not reachable, then it returns ∞. Give the time and space complexity of your algorithm.

Q2) Consider the following claims.

a. Any undirected graph $G = (V, E)$ with $|E| \leq |V| - 1$ is always cycle-free. Prove or disprove.

b. In any connected undirected graph $G = (V, E)$, any spanning tree of G is a valid DFS tree. Prove or disprove.

c. Given an undirected graph $G = (V, E)$, can it be decided in $O(|V|)$ time that G is cycle free? Justify your claim.

Q3) Using a small example show that Dijkstra's algorithm fails on directed graph with negative weight edges but without negative weight cycles.

Q4) Given a directed edge-weighted graph. Will the weighted shortest path remain shortest after the following operation? Justify your answer.

1. $w_e \to w_e + c$ for all $e \in E$ for a positive constant c.
2. $w_e \to w_e - c$ for all $e \in E$ for a positive constant c.
3. $w_e \to c.w_e$ for all $e \in E$ for a positive constant c.

Q5) Given the road network of a city in which some roads are one-way. Describe an algorithm to find if there is a path to go from a crossing / junction a to crossing / junction b. What is the time complexity?

Q6) In a multi-processing system multiple processors concurrently execute. At various times they require resources (memory, registers, processing units, etc). Each source can be assigned to one process at a time. A process may require one or more resources to complete its current step. It gets stuck until all the required resources are assigned. After all resources are assigned to it, it releases all of them after a fixed amount of time. System is said to be get into a deadlock if at least one process gets stuck forever.

Describe a way to express this situation using graphs and suggest how to detect if the system is in a deadlock.

Q7)

a. Let $G = (V, E, w)$ be an vertex-weighted graph. Describe an algorithm which takes a vertex x as input and computes a shortest path to the maximum weight vertex among all the vertices which are reachable from x.

b. Let N be a sequence of n digits. Let E be a set of some pairs of integers in the set $0, 1, \ldots, n-1$. One *exchange operation* corresponds the swapping of i-th and j-th digit in N where $\{i, j\} \in E$. Design an algorithm to compute the shortest sequence of exchange operations which transforms N into the largest number (as an n-digit number) achievable this way.

Q8) Let G be an edge weighted graph in which non-edge is viewed as an edge with ∞ weight. Let s be a fixed vertex. Let arrays $d[]$ and $parent[]$ store the result of Bellman Ford algorithm. Design an efficient algorithm which takes these arrays as input and a new weight w'_e for edge e. It updates the values of the arrays $d[]$ and $parent[]$ efficiently.