Data Structures

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Binary Tree Data Structures

Definition of a Node

A non-divisible unit of information of a large data structure such as a linked list, a tree, or a graph. A node would also contain links (pointers) to other nodes. The linked nodes are related in some sort of relations.

Trees

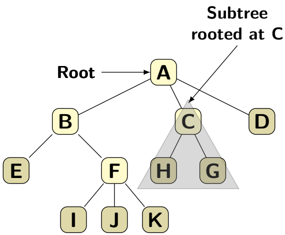
Definition of a Tree

A tree T can be empty (null tree having no node), or may consist of

1. A special node, designated as r, called the root.
2. A set of trees k trees $T_1, T_2, \ldots, T_k$ (some could possibly empty) with roots $r_1, r_2, \ldots, r_k$ respectively.

T is constructed by making $r_1, r_2, \ldots, r_k$ as children of r. Tree $T_1, T_2, \ldots, T_k$ are called subtrees of T.

Tree

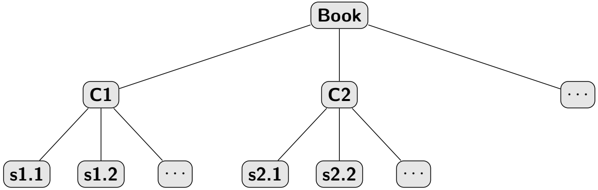


Farthest leaf nodes: I, J, K
Leaf node: D, E, G, H, I, J, K
Depth of I, J, K: 3
Height of tree: 3

Terminology

- Type of nodes: **internal** or **leaf**.
- A is an **ancestor** of all nodes including itself.
- All nodes including A are **descendant**s of A.
- **Ordered tree**: **siblings** are ordered from left to right.
- Degree of a tree is k: if no node has more than k children.

Tree



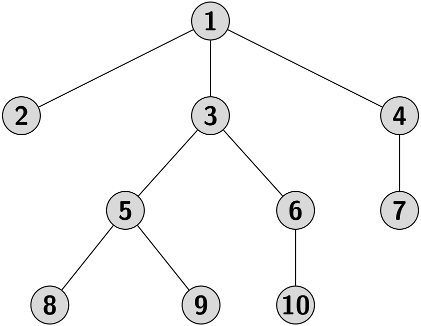
Tree Path

- Above figure depicts is an example of a tree.
- Parent-child relations shown by lines.
- A path: $n_1, n_2, \ldots, n_k$, such that $n_i = parent(n_{i-1})$.
- Length of a path is 1 less than number of nodes.

Ordered Trees

- Ordered trees have special importance.
- Ordered trees are presented by drawing on a plane.
- Child nodes are listed using some specific order (counter clockwise).
- If children of each node is placed at a given distance down.
- Root is placed on the top.

Example of an Order Tree



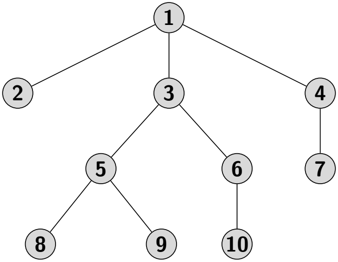
- Children of node 1: 2-first child, 3-second child, 4-third child.
- Sometimes ordering is defined by left-to-right.

Tree Traversal

- We can systematically order nodes of a tree in many ways.
- Three most important ordering are: Preorder, Postorder and Inorder.
- Recursive definition of these orderings are as follows:
  - If a tree T is empty then empty list is the preorder, postorder and inorder listing of T.
  - If T consist of only one node, then the node by itself is the listing in all three orderings.
- Otherwise, let T be a tree with root r and k subtrees $T_1, T_2, \ldots, T_k$.

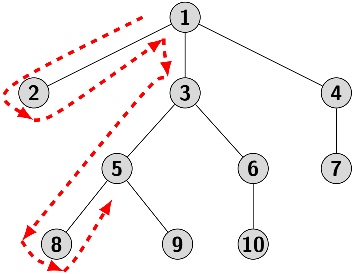
Tree Traversal

- Preorder listing of T: list root r of T, preorder list of all subtree $T_1, T_2, \ldots, T_k$ in left to right order.
- Postorder listing of T: postorder list of all subtree $T_1, T_2, \ldots, T_k$ in left to right order followed by root r of T.
- Inorder listing of T: inorder listing of T_1 followed by root r and then inorder listing of each group of nodes $T_2, T_3, \ldots, T_k$ in inorder.



- Preorder listing: 1, 2, 3, 5, 8, 9, 6, 10, 4,7.
- Postorder listing: 2, 8, 9, 5, 10, 6, 3, 7, 4, 1.
- Inorder listing: 2, 1, 8, 5, 9, 3, 10, 6, 7, 4.

Walk Around the Tree



Euler Tour

- A walk around the tree treating edges as walls.
- Start the walk outside the tree, starting at the root,
- Stay as close to the tree as possible.
- Move anti clockwise till reaching back to the root.

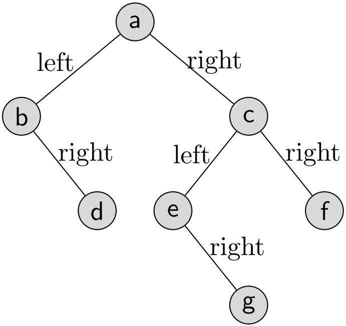
Tree Traversal Terminology

- Every node v is visited three times in the walk around a tree.
  - First time on the left before walking around left subtree of v.
  - Second time from below after having traversed around all nodes of left subtree of v.
  - Third time on the right after having traversed the right subtree of v.
- For distinguishing the three different visits to a node, use following terminology:
  - First time (before the Euler tour of v' left subtree)
  - Second time (between the Euler tours of v' two subtrees)
  - Third time (after the Euler tour of v' right subtree)

Tree Traversal Terminology

- For preorder traversal: list the node when it is visited for the "first time" during Euler tour.
  - Listing will be: 1, 2, 3, 5, 8, 9, 6, 10, 4, 7.
- For inorder traversal: is not defined for general tree but only for binary tree. However, it may sometimes defined as visiting a node "second time" during Euler traversal.
  - Listing will be: 2, 1, 8, 5, 9, 3, 10, 6, 7, 4.
-
- For postorder traversal: list the node when it is visited "third time" during Euler traversal.
  - Listing will be: 2, 8, 9, 5, 10, 6, 3, 7, 4, 1.

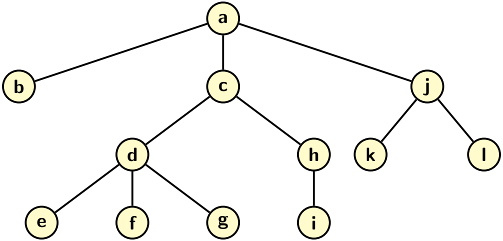
Binary Trees



Binary Trees

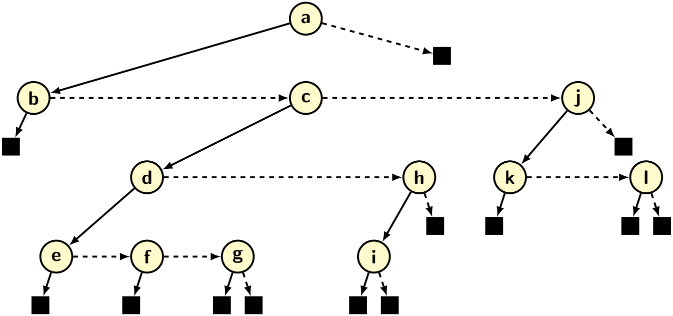
- If arity $k = 2$, then we have a binary tree.
- We distinguish between two children as left and right.
- Pictorial convention is to draw left child extended to the left and right child to right.

General Tree as Binary Tree



- Leftmost child is considered as the left child.
- A non empty right sibling of a node becomes as its right child.

Equivalent Extended Binary Tree

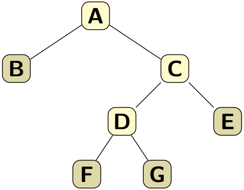


- Only internal nodes have information.
- All leaf nodes are external (null) nodes indicated by black squares.

Traversal of Equivalent Binary Tree

- When a general tree is represented as a binary tree, algorithms binary trees can be used process the general tree.
- But, inorder traversal of the equivalent binary tree does not make any sense.
- In a general tree a node may have more than two children.
- So, inserting a visit to parent node between children is difficult specially when there are odd number of children.

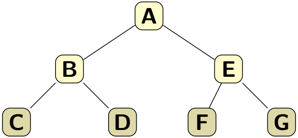
Important Binary Trees



A strictly Binary tree

Strictly Binary Tree

In a strictly binary tree, every internal node has two children.

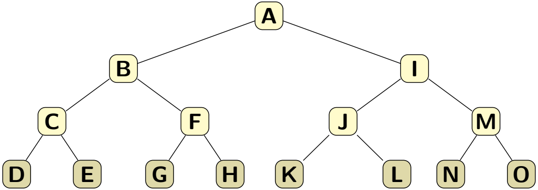


A Full Binary tree

A Full Binary Tree

A full binary tree is a strictly binary tree in which all leaves appear at the bottom most level.

Array Representation

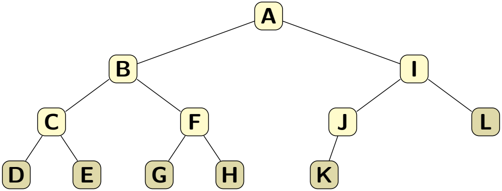


tree[]	-1	A	B	I	C	F	J	M	D	E	G	H	K	L	N	O
	0							7								15

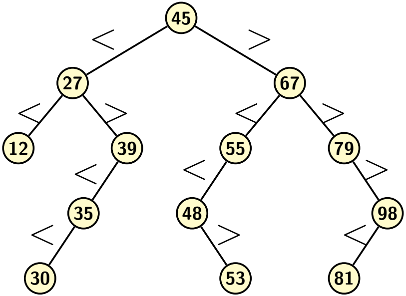
- Children of node i are at $2i$ and $2i + 1$.
- All nodes except node 1 has a parent.
- Array representation also works for complete binary tree.
  - Let child of i is at $2i$ unless $2i > n$.
  - If $2i > n$ then node i has no left child.

A Complete Binary Tree

A complete binary of height h consists of a full binary tree of height $h-1$, in which all internal nodes except at most one at height $h-1$ have two children and appear as far left as possible.



Important Binary Trees



Binary Search Tree

- All key values stored at any node belonging to left subtree is less than the key stored at the root.
- All key values stored at any node belonging to right subtree is greater than the key stored at the root.
- The above two properties hold at any internal node.

Properties of Binary Trees

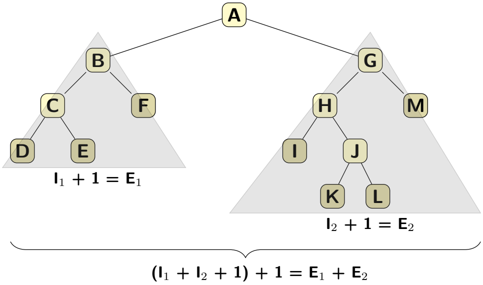
- It is assumed height an empty tree is undefined.
- The height of a tree with one node is 0.
- A tree branch contributes 1 to height.
- A non empty binary tree with height $h \geq 0$, has at least $h + 1$ nodes and at most $2^{h+1} - 1$ nodes.
- The height of a binary tree with n nodes is at most $n - 1$ and at least $\lceil \log_2(n + 1) \rceil$ - 1.
  - Let h be the height, then $h + 1 \leq n \leq 2^{h+1} - 1$.
  - Implies that $\log(n + 1) \leq h + 1$.

Note: You may assume either the number of nodes or the number edges not both.

Leaf nodes and Internal Nodes

Number of leaf nodes in a strictly binary tree is one more than the number internal nodes.

- Every internal node has 2 children.
- A tree of height 2 has $I = 1$, $E = 2$.
- A height 3 has $I = 2, E = 3$ or $I = 3, E = 4$.

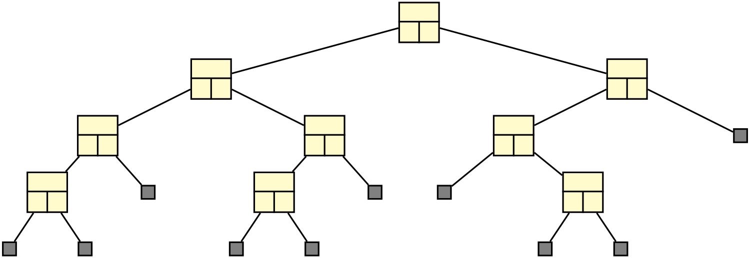


Common Operations on ADT Tree

- **newTree()**: Create a new empty tree.
- **findRoot()**: Given a tree returns its root.
- **isRoot()**: Returns true if the given node is the root.
- **findParent()**: Given a node returns its parent.
- **children():** Given a node returns the list of its children.
- **isInternal()**: Returns true if the given node is an internal node.
- **isLeaf()**: Returns true if the given node is a leaf node.
- **exhangeNode()**: Given two nodes swaps the information held by them.
- **replaceValue()**: Given a node and a value replaces old value by the new value.

Linked List Implementation

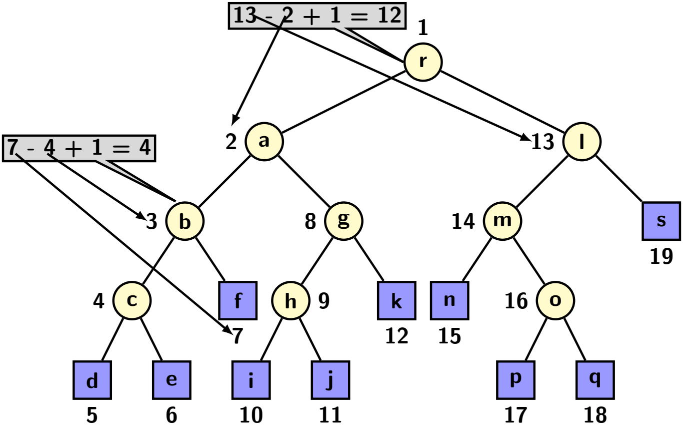
```
typedef struct treenode NODE;
    struct treenode{
        int info;
        treenode *left;
        treenode *right;
} TreeNode;
```



Tree Traversal

- The walk around the tree or ”Euler tour” besides generating three traversals, can also be used other types of traverals.
  - Initialize a counter to 0 to start Euler tour.
  - Increment counter first time a node is visited.
  - In $C_{left}(v)$ store the counter value before traversing v's left subtree.
  - In $C_{right}(v)$ store the counter value after traversing right subtree v.
  - Find the difference $C_{right}(v) - C_{left}(v)$ and add 1.
- The above value gives the number of descendants of v.

Descendant Computation



Preorder Traversal

```c
void preorder(NODE *t) {
    if (t!=NULL) {
        printf("%d\t",t->data); // visit the root
        preorder(t->left);      // left subtree
        preorder(t->right);     // right subtree
    }
}
```

- Postorder and inorder traversals can be also be performed in similar way.

Creating Tree

```
NODE *create() {
    NODE *p;
    int x;
    printf("Enter data(-1 for no data):");
    scanf("%d",&x);
    if(x==-1)
        return NULL;
    p=(NODE *)malloc(sizeof(NODE));
    p->data=x;
    printf("Enter left child of %d:\n",x);
    p->left=create();
    printf("Enter right child of %d:\n",x);
    p->right=create();
    return p;
}
```

Membership Search

```
NODE * search(NODE *t, int x) {
    NODE *p;
    if ((t == NULL) || (t->data == x))
        return t;
    p = search(t->left,x);
    if (p !=NULL)
      return p;
    p = search(t->right, x);
    if (p != NULL)
        return p;
    else
        return NULL;
}
```

Number of Descendants

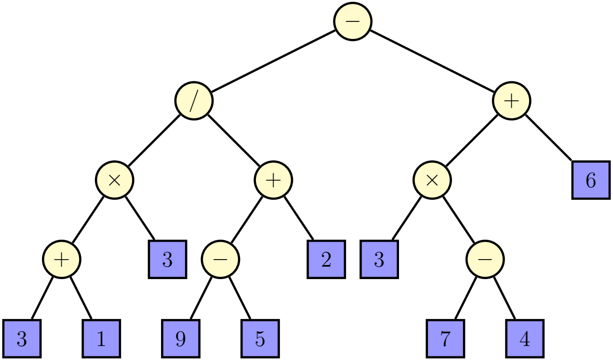
- First find the pointer to element in the tree.
- Then use Euler tour to compute number of descendants.

```
int EulerTour(NODE *t, int x) {
    int no_descendants = 0; // Initialization
    if (t==NULL)
        return no_descendants;
    else {
        counter++;
        if (isInternal(t)) {
            no_descendants += EulerTour(t->left, x);
            no_descendants += EulerTour(t->right, x)
                ;
        }
        return no_descendants;
    }
}
```

Printing Expression

```
Algorithm PrintExpression(T,v) {
    if (T.isExternal(v))
        print "value" stored in v;
    else {
        print "(";
        PrintExpression(T,T.leftChild(v));
        print "operator" stored in v
        PrintExpression(T,T.rightChild(v));
        print ")";
    }
}
```

Example of Printing Expression



- **Infix**: (((((3+1) * 3))/((9-5)+2))+((3 * (7-4)) + 6))
- **Prefix**: - / * + 3 1 3 + - 9 5 2 + * 3 - 7 4 6
- **Postfix**: 3 1 + 3 * 9 5 - 2 + / 3 7 4 - * 6 + -