CS203 Assignment 1 Semester I, 2017-18

Due Date: 16th August, 2017 **Maximum Marks: 50**

Instructions

- Submit the assignment at the end of class on or before the due date.
- Yours answers should be precise and clearly written.
- Cheating/plagiarizing in any form will be heavily penalized.
- Late submissions will receive a mark of zero.

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1. (10 points) Suppose that $G = (V, E)$ is a *graph*, where V is the set of *vertices* and E is the set of *edges*. Any two vertices are called *adjacent*, if there is an edge between them. Otherwise, they are called *non-adjacent*. A *bijection* f from V to V is called an *automorphism of* G, if it maps all adjacent pairs of vertices to adjacent pairs of vertices and non-adjacent pair of vertices to non-adjacent pair of vertices. Let $\text{Aut}(G)$ be the set of all automorphisms of G. Show that $\text{Aut}(G)$ forms a group under function composition.

2. (15 points) For a binary string $\mathbf{b} = (b_1, \ldots, b_n) \in \{0,1\}^n$, we can define a function $f_{\mathbf{b}}$ from $\{0,1\}^n$ to $\{+1, -1\}$ as follows: for all $\mathbf{c} = (c_1, \ldots, c_n) \in \{0,1\}^n$,
$$f_{\mathbf{b}}(\mathbf{c}) = (-1)^{\sum_{i=1}^{n} b_i c_i}.$$
Let $F = \left\{ f_{\mathbf{b}} : \mathbf{b} \in \{0,1\}^n \right\}$, the set of all such functions and $*$ be a binary operation defined on F as follows: for any two $\mathbf{a}, \mathbf{b} \in \{0,1\}^n$,
$$f_{\mathbf{a}} * f_{\mathbf{b}}(\mathbf{c}) = f_{\mathbf{a}}(\mathbf{c}) \cdot f_{\mathbf{b}}(\mathbf{c}), \ \forall \mathbf{c} \in \{0,1\}^n.$$
Show that F forms a group under $*$.

3. (10 points) Suppose that G is a finite group. For all $g \in G$, we can define a function f_g from G to G as follows: for all $h \in G$, $f_g(h) = hg$. Let $F_G = \{f_g : g \in G\}$, the set of all such functions. Prove that F_G forms a group under function composition.

4. (15 points) Let G be the set of all 2×2 matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where a, b, c, d are integers modulo p, p is a prime number, such that p does not divide $ad - bc$. Show that G forms a group under matrix multiplication. Also find the size of the group.