MSO 201 A : Homework 13

1. Suppose that n measurements are to be taken under a treatment condition (X) and another $3n$ measurements are to be taken independently under a control condition (Y). It is thought that the standard deviation of a single observation is about 5 under both conditions. How large n be so that a 95% confidence interval for $\mu_X - \mu_Y$ has a width of 4? Here μ_X and μ_Y are the population means of X and Y, respectively.

2. The manufacturer of bags of cement claims that they fill each bag with 50 pounds of cement. Assume that the amount filled in each bag follows a normal distribution with standard deviation 1.2 pounds. However, there was some complaint that the bags are under-filled with mean filling amount 49 pounds of cement each bag. The decision rule is adopted to shut down the filling machine if the sample mean weight for a sample of 20 bags is below 49.5 lb.

(a) Write the null and alternative hypotheses.

(b) What is the probability of a Type I error?

(c) Calculate the probability of a Type II error.

(d) If you are asked to construct a test of 2% level of significance, what will be your decision rule (i.e., write the rejection region for $\alpha = 0.02$)? State clearly what is your test-statistic.

(e) Suppose a sample of 20 bags has on the average 49.27 lb cements in each bag. What will be your conclusion about the hypotheses at $\alpha = 0.02$?

3. A professor claims that the average score on a recent exam was 83. Assume that the test scores are normally distributed. You ask some people in class how they did, and you record the following scores: 82, 77, 85, 76, 81, 91, 70, and 82. Suppose you want to test whether the professors statement was correct against the alternative that his claim is not correct.

(a) Perform a 10% level of significance test about the professors claim. State clearly the null and alternative hypotheses in terms of appropriate parameter, formula and value of the test statistic, the rejection region and the conclusion in the context of the current problem.

(b) Find the p-value of the test. Interpret its value.

4. The Daytona Beach Tourism Commission recently claimed that the average amount of money a typical college student spends per day during spring break is over 70 USD. Based upon previous research, the population standard deviation is estimated to be 17.32 USD. The Commission surveys 45 students and find that the mean spending is 67.57 USD. Is there evidence that the average amount spent by students is less than 70 USD? Perform a 1% level of significance test stating clearly the null and alternative hypotheses in terms of appropriate parameter, formula and value of the test statistic, the rejection region and the conclusion in the context of the current problem.

5. Let $X_1, X_2, \ldots, X_n$ be a random sample from a $Poisson(\theta)$ population. Suppose that we want to test $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$. Find the rejection region for the most powerful test when $\theta_0 < \theta_1$.

6. Suppose that X is a discrete random variable with $P[X = 0] = \frac{\theta}{4}$, $P[X = 1] = \frac{1-\theta}{2}$ and $P[X = 2] = \frac{2+\theta}{4}$. Consider the following testing of hypothesis problem: $H_0 : \theta = \frac{1}{3}$ against $H_1 : \theta = \frac{2}{3}$. Consider further a Likelihood Ratio Test (LRT) that rejects H_0 if $\Lambda \leq c$, where Λ is a likelihood ratio, and c is a suitable constant.

(a) Find a test statistics Λ if you have a single observation $X = 0$. What are the other values of Λ if the observed value of X is other than 0?

(b) Find the distribution of Λ under H_0.

(c) If $X = 2$ is observed, what is p-value of the test?

7. Show that if $\frac{1}{n}\sum\limits_{i=1}^{n} x_i = 0$, the estimated slope ($\hat{\beta}_1$) and intercept ($\hat{\beta}_0$) are uncorrelated for the linear model : $Y_i = \beta_0 + \beta_1 x_i + e_i$ for $i = 1, \ldots, n$, where e_is are iid normal random variables with mean $= 0$ and variance $= \sigma^2$.

8. Consider a linear model : $Y_i = \beta_0 + \beta_1 x_i + e_i$ for $i = 1, \ldots, n$, where e_is are iid normal random variables with mean $= 0$ and variance $= \sigma^2$.

(a) Find an expression for the least square estimator ($\hat{\beta}$) of β for the model.

(b) Show that $\hat{\beta}$ is unbiased.

(c) Find $Var(\hat{\beta})$.

(d) Suppose that the following data for (x_i, Y_i) are observed: $(1.1, 8.5)$, $(3.5, 9.1)$, $(3.4, 13.4)$, $(5.7, 18.5)$, $(1.7, 6.4)$. Given the data, compute $\hat{\beta}$ and $\hat{\sigma}^2$ and give a 95% confidence interval for β.