MSO 201A : Homework 10

1. Let $X_1, X_2, \ldots, X_n$ be a random sample from an exponential distribution with p.d.f.

$$f_X(x) = \frac{1}{\beta} e^{-\frac{x}{\beta}}, \quad x > 0.$$

Show that $\overline{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$ is an unbiased estimator of β.

2. Let $X_1, X_2, \ldots, X_n$ be a random sample from Uniform $(0, \theta)$, where $\theta > 0$. Show that $\frac{n+1}{n} X_{(n)}$ and $2\overline{X}_n$ are both unbiased estimator of θ. Here $X_{(n)} = \max\{X_1, \ldots, X_n\}$, and $\overline{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$.

3. Let $X_1, X_2, \ldots, X_n$ be a random sample from Poisson distribution with the parameter θ; $\theta > 0$. Find an unbiassed estimator of $\theta e^{-2\theta}$.

4. Let $X_1, X_2, \ldots, X_n$ be a random sample from Binomial $(1, \theta)$, where $0 \leq \theta \leq 1$. Find an unbiased estimator of $\theta^2(1-\theta)$.

5. Let X_1 and X_2 be independent random samples with densities $f_1(x_1) = \theta e^{-\theta x_1}$ and $f_2(x_2) = 2\theta e^{-2\theta x_2}$, ~~as the~~ respectively, where $\theta > 0$ is an unknown parameter, and $0 < x_1, x_2 < \infty$. Find a sufficient statistic for θ.

6. Let $X_1, X_2, \ldots, X_n$ be a random sample from a Beta (α, β) distribution $(\alpha > 0, \beta > 0)$ with p.d.f.

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma\alpha\,\Gamma\beta}\, x^{\alpha-1}(1-x)^{\beta-1}, \quad x \in (0,1)$$

$$= 0 \quad \text{otherwise}$$

Show that

(a) $\prod_{i=1}^{n} X_i$ is sufficient for α if β is ~~unknown~~ known

(b) $\prod_{i=1}^{n} (1-X_i)$ is sufficient for β if α is known.

(c) $\left(\prod_{i=1}^{n} X_i, \prod_{i=1}^{n} (1-X_i)\right)$ is jointly sufficient for (α, β) if both α and β are unknown.

7. Let $X_1, X_2, \ldots, X_n$ be a random sample from Uniform $\left(\theta - \frac{1}{2}, \theta + \frac{1}{2}\right)$, where $\theta \in \mathbb{R}$. Find a sufficient statistic for θ.