Consider $n=2$. $\Rightarrow Y = X_1 + X_2$.

$$P[Y=y] = P[X_1 + X_2 = y], \quad y = 0, \ldots$$

$$= \sum_{x=0}^{y} P[X_1 = x, X_2 = y-x]$$

$$= \sum_{x=0}^{y} P[X_1 = x]\, P[X_2 = y-x] \quad \text{since } X_1 \perp\!\!\!\perp X_2.$$

$$= \sum_{x=0}^{y} \frac{e^{-\lambda_1}\lambda_1^{x}}{x!} \times \frac{e^{-\lambda_2}\lambda_2^{y-x}}{(y-x)!}$$

$$= e^{-(\lambda_1+\lambda_2)} \frac{1}{y!} \sum_{x=0}^{y} \frac{y!}{x!(y-x)!} \lambda_1^{x} \cdot \lambda_2^{y-x}$$

$$= \frac{e^{-(\lambda_1+\lambda_2)}}{y!} \sum_{x=0}^{y} \binom{y}{x} \lambda_1^{x} \lambda_2^{(y-x)}$$

$$= \frac{e^{-(\lambda_1+\lambda_2)} (\lambda_1+\lambda_2)^{y}}{y!} \; *$$

$* \; X_1 + X_2 \sim \text{Poi}(\lambda_1 + \lambda_2)$.