Approach 1:- Distribution f^n based approach:-

Let F_U be the dist^n f^n of $U(X_1, \cdots, X_n)$.

$$F_U(y) = P[U(X_1, \cdots, X_n) \leq y]$$

$$= \int\int \cdots \int_{U(x_1, x_2, \cdots, x_n) \leq y} f_{X_1, \cdots, X_n}(x_1, \cdots, x_n)\, dx_1 \cdots dx_n .$$

Hence, $f_Y(y) = \frac{d}{dy} F_U(y)$.

Example 1:- $X_1, \cdots, X_n$ indep Poisson (λi), $i = 1, \cdots, n$.

Want to know the dist^n / p.m.f. of $Y = \sum_{i=1}^{n} X_i$.