MSO 201 : Home work 1

1. A mutual fund company offers its customers a variety of funds: a money-market fund, three different bond funds (short, intermediate, and long-term), two stock funds (moderate and high-risk), and a balanced fund. Among customers who own shares in just one fund, the percentages of customers in the different funds are as follows:

Money-market	20%	High-risk stock	18%
Short bond	15%	Moderate-risk stock	25%
Intermediate bond	10%	Balanced	7%
Long bond	5%		

A customer who owns shares in just one fund is randomly selected.

a. What is the probability that the selected individual owns shares in the balanced fund?

b. What is the probability that the individual owns shares in a bond fund?

c. What is the probability that the selected individual does not own shares in a stock fund?

2. Consider randomly selecting a student at a certain university, and let A denote the event that the selected individual has a Visa credit card and B be the analogous event for a MasterCard. Suppose that $P(A) = .5$, $P(B) = .4$, and $P(A \cap B) = .25$.

a. Compute the probability that the selected individual has at least one of the two types of cards (i.e., the probability of the event $A \cup B$).

b. What is the probability that the selected individual has neither type of card?

c. Describe, in terms of A and B, the event that the selected student has a Visa card but not a MasterCard, and then calculate the probability of this event.

3. A computer consulting firm presently has bids out on three projects. Let $A_i = \{\text{awarded project } i\}$, for $i = 1, 2, 3$, and suppose that $P(A_1) = .22$, $P(A_2) = .25$, $P(A_3) = .28$, $P(A_1 \cap A_2) = .11$, $P(A_1 \cap A_3) = .05$, $P(A_2 \cap A_3) = .07$, $P(A_1 \cap A_2 \cap A_3) = .01$. Express in words each of the following events, and compute the probability of each event:

a. $A_1 \cup A_2$

b. $A_1' \cap A_2'$ [*Hint:* $(A_1 \cup A_2)' = A_1' \cap A_2'$]

c. $A_1 \cup A_2 \cup A_3$ **d.** $A_1' \cap A_2' \cap A_3'$

e. $A_1' \cap A_2' \cap A_3$ **f.** $(A_1' \cap A_2') \cup A_3$

4. Suppose that 55% of all adults regularly consume coffee, 45% regularly consume carbonated soda, and 70% regularly consume at least one of these two products.

a. What is the probability that a randomly selected adult regularly consumes both coffee and soda?

b. What is the probability that a randomly selected adult doesn't regularly consume at least one of these two products?

5. Consider the type of clothes dryer (gas or electric) purchased by each of five different customers at a certain store.
   a. If the probability that at most one of these purchases an electric dryer is .428, what is the probability that at least two purchase an electric dryer?
   b. If P(all five purchase gas) = .116 and P(all five purchase electric) = .005, what is the probability that at least one of each type is purchased?

6. An individual is presented with three different glasses of cola, labeled C, D, and P. He is asked to taste all three and then list them in order of preference. Suppose the same cola has actually been put into all three glasses.
   a. What are the simple events in this ranking experiment, and what probability would you assign to each one?
   b. What is the probability that C is ranked first?
   c. What is the probability that C is ranked first and D is ranked last?

7. Let A denote the event that the next request for assistance from a statistical software consultant relates to the SPSS package, and let B be the event that the next request is for help with SAS. Suppose that $P(A) = .30$ and $P(B) = .50$.
   a. Why is it not the case that $P(A) + P(B) = 1$?
   b. Calculate $P(A')$.
   c. Calculate $P(A \cup B)$.
   d. Calculate $P(A' \cap B')$.

8. A box contains six 40-W bulbs, five 60-W bulbs, and four 75-W bulbs. If bulbs are selected one by one in random order, what is the probability that at least two bulbs must be selected to obtain one that is rated 75 W?

9. Human visual inspection of solder joints on printed circuit boards can be very subjective. Part of the problem stems from the numerous types of solder defects (e.g., pad nonwetting, knee visibility, voids) and even the degree to which a joint possesses one or more of these defects. Consequently, even highly trained inspectors can disagree on the disposition of a particular joint. In one batch of 10,000 joints, inspector A found 724 that were judged defective, inspector B found 751 such joints, and 1159 of the joints were judged defective by at least one of the inspectors. Suppose that one of the 10,000 joints is randomly selected.
   a. What is the probability that the selected joint was judged to be defective by neither of the two inspectors?
   b. What is the probability that the selected joint was judged to be defective by inspector B but not by inspector A?

10. A certain factory operates three different shifts. Over the last year, 200 accidents have occurred at the factory. Some of these can be attributed at least in part to unsafe working conditions, whereas the others are unrelated to working conditions. The accompanying table gives the percentage of accidents falling in each type of accident–shift category.

		Unsafe Conditions	Unrelated to Conditions
Shift	*Day*	10%	35%
	Swing	8%	20%
	Night	5%	22%

Suppose one of the 200 accident reports is randomly selected from a file of reports, and the shift and type of accident are determined.
   a. What are the simple events?
   b. What is the probability that the selected accident was attributed to unsafe conditions?
   c. What is the probability that the selected accident did not occur on the day shift?

11. An insurance company offers four different deductible levels—none, low, medium, and high—for its homeowner's policyholders and three different levels—low, medium, and high—for its automobile policyholders. The accompanying table gives proportions for the various categories of policyholders who have both types of insurance. For example, the proportion of individuals with both low homeowner's deductible and low auto deductible is .06 (6% of all such individuals).

	Homeowner's			
Auto	**N**	**L**	**M**	**H**
L	.04	.06	.05	.03
M	.07	.10	.20	.10
H	.02	.03	.15	.15

Suppose an individual having both types of policies is randomly selected.
   a. What is the probability that the individual has a medium auto deductible and a high homeowner's deductible?
   b. What is the probability that the individual has a low auto deductible? A low homeowner's deductible?
   c. What is the probability that the individual is in the same category for both auto and homeowner's deductibles?
   d. Based on your answer in part (c), what is the probability that the two categories are different?
   e. What is the probability that the individual has at least one low deductible level?
   f. Using the answer in part (e), what is the probability that neither deductible level is low?

12. The route used by a certain motorist in commuting to work contains two intersections with traffic signals. The probability that he must stop at the first signal is .4, the analogous probability for the second signal is .5, and the probability that he must stop at at least one of the two signals is .6. What is the probability that he must stop
   a. At both signals?
   b. At the first signal but not at the second one?
   c. At exactly one signal?

13. The computers of six faculty members in a certain department are to be replaced. Two of the faculty members have selected laptop machines and the other four have chosen desktop machines. Suppose that only two of the setups can be done on a particular day, and the two computers to be set up are randomly selected from the six (implying 15 equally likely outcomes; if the computers are numbered $1, 2, \ldots, 6$, then one outcome consists of computers 1 and 2, another consists of computers 1 and 3, and so on).
   a. What is the probability that both selected setups are for laptop computers?
   b. What is the probability that both selected setups are desktop machines?
   c. What is the probability that at least one selected setup is for a desktop computer?
   d. What is the probability that at least one computer of each type is chosen for setup?

14. Show that if one event A is contained in another event B (i.e., A is a subset of B), then $P(A) \le P(B)$. [*Hint:* For such A and B, A and $B \cap A'$ are disjoint and $B = A \cup (B \cap A')$, as can be seen from a Venn diagram.] For general A and B, what does this imply about the relationship among $P(A \cap B)$, $P(A)$ and $P(A \cup B)$?

15. The three most popular options on a certain type of new car are a built-in GPS (A), a sunroof (B), and an automatic transmission (C). If 40% of all purchasers request A, 55% request B, 70% request C, 63% request A or B, 77% request A or C, 80% request B or C, and 85% request A or B or C, determine the probabilities of the following events. [*Hint:* "A or B" is the event that at least one of the two options is requested; try drawing a Venn diagram and labeling all regions.]
   a. The next purchaser will request at least one of the three options.
   b. The next purchaser will select none of the three options.
   c. The next purchaser will request only an automatic transmission and not either of the other two options.
   d. The next purchaser will select exactly one of these three options.

16. A certain system can experience three different types of defects. Let $A_i (i = 1,2,3)$ denote the event that the system has a defect of type i. Suppose that

$$P(A_1) = .12 \quad P(A_2) = .07 \quad P(A_3) = .05$$
$$P(A_1 \cup A_2) = .13 \quad P(A_1 \cup A_3) = .14$$
$$P(A_2 \cup A_3) = .10 \quad P(A_1 \cap A_2 \cap A_3) = .01$$

   a. What is the probability that the system does not have a type 1 defect?
   b. What is the probability that the system has both type 1 and type 2 defects?
   c. What is the probability that the system has both type 1 and type 2 defects but not a type 3 defect?
   d. What is the probability that the system has at most two of these defects?

17. An academic department with five faculty members—Anderson, Box, Cox, Cramer, and Fisher—must select two of its members to serve on a personnel review committee. Because the work will be time-consuming, no one is anxious to serve, so it is decided that the representative will be selected by putting the names on identical pieces of paper and then randomly selecting two.
   a. What is the probability that both Anderson and Box will be selected? [*Hint:* List the equally likely outcomes.]
   b. What is the probability that at least one of the two members whose name begins with C is selected?
   c. If the five faculty members have taught for 3, 6, 7, 10, and 14 years, respectively, at the university, what is the probability that the two chosen representatives have a total of at least 15 years' teaching experience there?

18. In Exercise 5, suppose that any incoming individual is equally likely to be assigned to any of the three stations irrespective of where other individuals have been assigned. What is the probability that
   a. All three family members are assigned to the same station?
   b. At most two family members are assigned to the same station?
   c. Every family member is assigned to a different station?