1. **a.** .07.

   b. .15 + .10 + .05 = .30.

   c. Let A = the selected individual owns shares in a stock fund. Then $P(A)$ = .18 + .25 = .43. The desired probability, that a selected customer does not shares in a stock fund, equals $P(A') = 1 - P(A) = 1 - .43 = .57$. This could also be calculated by adding the probabilities for all the funds that are not stocks.

2. **a.** $P(A \cup B) = .50 + .40 - .25 = .65$.

   b. $P(\text{neither } A \text{ nor } B) = P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B) = 1 - .65 = .35$.

   c. The event of interest is $A \cap B'$; from a Venn diagram, we see $P(A \cap B') = P(A) - P(A \cap B) = .50 - .25 = .25$.

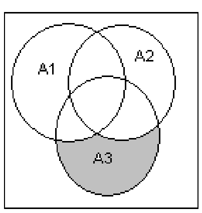
3. **a.** $A_1 \cup A_2$ = "awarded either #1 or #2 (or both)": from the addition rule,
   $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) = .22 + .25 - .11 = .36$.

   b. $A_1' \cap A_2'$ = "awarded neither #1 or #2": using the hint and part (a),
   $P(A_1' \cap A_2') = P((A_1 \cup A_2)') = 1 - P(A_1 \cup A_2) = 1 - .36 = .64$.

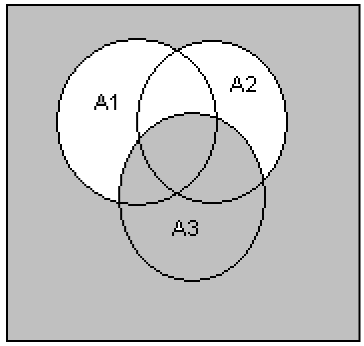
   c. $A_1 \cup A_2 \cup A_3$ = "awarded at least one of these three projects": using the addition rule for 3 events,
   $P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3) =$
   $.22 + .25 + .28 - .11 - .05 - .07 + .01 = .53$.

   d. $A_1' \cap A_2' \cap A_3'$ = "awarded none of the three projects":
   $P(A_1' \cap A_2' \cap A_3') = 1 - P(\text{awarded at least one}) = 1 - .53 = .47$.

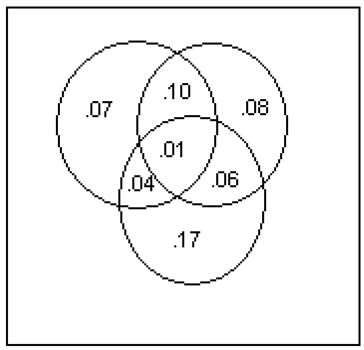
   e. $A_1' \cap A_2' \cap A_3$ = "awarded #3 but neither #1 nor #2": from a Venn diagram,
   $P(A_1' \cap A_2' \cap A_3) = P(A_3) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3) =$
   $.28 - .05 - .07 + .01 = .17$. The last term addresses the "double counting" of the two subtractions.



f. $(A_1' \cap A_2') \cup A_3$ = "awarded neither of #1 and #2, or awarded #3": from a Venn diagram, $P((A_1' \cap A_2') \cup A_3) = P(\text{none awarded}) + P(A_3)$ = .47 (from **d**) + .28 = 75.



Alternatively, answers to **a-f** can be obtained from probabilities on the accompanying Venn diagram:



4. Let A = an adult consumes coffee and B = an adult consumes carbonated soda. We're told that $P(A) = .55$, $P(B) = .45$, and $P(A \cup B) = .70$.

a. The addition rule says $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, so $.70 = .55 + .45 - P(A \cap B)$ or $P(A \cap B) = .55 + .45 - .70 = .30$.

b. There are two ways to read this question. We can read "does not (consume at least one)," which means the adult consumes neither beverage. The probability is then $P(\text{neither } A \text{ nor } B) = P(A' \cap B') = 1 - P(A \cup B) = 1 - .70 = .30$.

The other reading, and this is presumably the intent, is "there is at least one beverage the adult does not consume, i.e. $A' \cup B'$. The probability is $P(A' \cup B') = 1 - P(A \cap B) = 1 - .30$ from **a** = .70. (It's just a coincidence this equals $P(A \cup B)$.)

Both of these approaches use *deMorgan's laws*, which say that $P(A' \cap B') = 1 - P(A \cup B)$ and $P(A' \cup B') = 1 - P(A \cap B)$.

5. **a.** Let E be the event that at most one purchases an electric dryer. Then E' is the event that at least two purchase electric dryers, and $P(E') = 1 - P(E) = 1 - .428 = .572$.

b. Let A be the event that all five purchase gas, and let B be the event that all five purchase electric. All other possible outcomes are those in which at least one of each type of clothes dryer is purchased. Thus, the desired probability is $1 - [P(A) - P(B)] =$
$1 - [.116 + .005] = .879$.

6. **a.** There are six simple events, corresponding to the outcomes *CDP, CPD, DCP, DPC, PCD*, and *PDC*. Since the same cola is in every glass, these six outcomes are equally likely to occur, and the probability assigned to each is $\frac{1}{6}$.

b. $P(C \text{ ranked first}) = P(\{CPD, CDP\}) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = .333$.

c. $P(C \text{ ranked first and } D \text{ last}) = P(\{CPD\}) = \frac{1}{6}$.

7. **a.** The probabilities do not add to 1 because there are other software packages besides SPSS and SAS for which requests could be made.

b. $P(A') = 1 - P(A) = 1 - .30 = .70$.

c. Since A and B are mutually exclusive events, $P(A \cup B) = P(A) + P(B) = .30 + .50 = .80$.

d. By deMorgan's law, $P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B) = 1 - .80 = .20$.
In this example, deMorgan's law says the event "neither A nor B" is the complement of the event "either A or B." (That's true regardless of whether they're mutually exclusive.)

8. The only reason we'd need at least two selections to find a 75W bulb is if the <u>first</u> selection was <u>not</u> a 75W bulb. There are 6 + 5 = 11 non-75W bulbs out of 6 + 5 + 4 = 15 bulbs in the box, so the probability of this event is simply $\frac{11}{15}$.

9. Let A be that the selected joint was found defective by inspector A, so $P(A) = \frac{724}{10{,}000}$. Let B be analogous for inspector B, so $P(B) = \frac{751}{10{,}000}$. The event "at least one of the inspectors judged a joint to be defective is $A \cup B$, so $P(A \cup B) = \frac{1159}{10{,}000}$.

a. By deMorgan's law, $P(\text{neither } A \text{ nor } B) = P(A' \cap B') = 1 - \mathrm{P}(\mathrm{A} \cup \mathrm{B}) = 1 - \frac{1159}{10{,}000} = \frac{8841}{10{,}000} = .8841$.

b. The desired event is $B \cap A'$. From a Venn diagram, we see that $P(B \cap A') = P(B) - P(A \cap B)$. From the addition rule, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ gives $P(A \cap B) = .0724 + .0751 - .1159 = .0316$.
Finally, $P(B \cap A') = P(B) - P(A \cap B) = .0751 - .0316 = .0435$.

10. **a.** Let S_1, S_2 and S_3 represent day, swing, and night shifts, respectively. Let C_1 and C_2 represent unsafe conditions and unrelated to conditions, respectively. Then the simple events are S_1C_1, S_1C_2, S_2C_1, S_2C_2, S_3C_1, S_3C_2.

b. $P(C_1)$= P($\{S_1C_1, S_2C_1, S_3C_1\}$)= .10 + .08 + .05 = .23.

c. $P(S_1')$ = 1 – P($\{S_1C_1, S_1C_2\}$) = 1 – (.10 + .35) = .55.

11. In what follows, the first letter refers to the auto deductible and the second letter refers to the homeowner's deductible.

a. $P(MH)$ = .10.

b. P(low auto deductible) = $P(\{LN, LL, LM, LH\})$ = .04 + .06 + .05 + .03 = .18. Following a similar pattern, P(low homeowner's deductible) = .06 + .10 + .03 = .19.

c. P(same deductible for both) = $P(\{LL, MM, HH\})$ = .06 + .20 + .15 = .41.

d. P(deductibles are different) = 1 – P(same deductible for both) = 1 – .41 = .59.

e. P(at least one low deductible) = $P(\{LN, LL, LM, LH, ML, HL\})$ = .04 + .06 + .05 + .03 + .10 + .03 = .31.

f. P(neither deductible is low) = 1 – P(at least one low deductible) = 1 – .31 = .69.

12. Let A = motorist must stop at first signal and B = motorist must stop at second signal. We're told that $P(A)$ = .4, $P(B)$ = .5, and $P(A \cup B)$ = .6.

a. From the addition rule, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, so .6 = .4 + .5 – $P(A \cap B)$, from which $P(A \cap B)$ = .4 + .5 – .6 = .3.

b. From a Venn diagram, $P(A \cap B') = P(A) - P(A \cap B)$ = .4 – .3 = .1.

c. From a Venn diagram, P(stop at exactly one signal) = $P(A \cup B) - P(A \cap B)$ = .6 – .3 = .3. Or, P(stop at exactly one signal) = $P([A \cap B'] \cup [A' \cap B]) = P(A \cap B') + P(A' \cap B) = [P(A) - P(A \cap B)] + [P(B) - P(A \cap B)]$ = [.4 – .3] + [.5 – .3] = .1 + .2 = .3.

13. Assume that the computers are numbered 1-6 as described and that computers 1 and 2 are the two laptops. There are 15 possible outcomes: (1,2) (1,3) (1,4) (1,5) (1,6) (2,3) (2,4) (2,5) (2,6) (3,4) (3,5) (3,6) (4,5) (4,6) and (5,6).

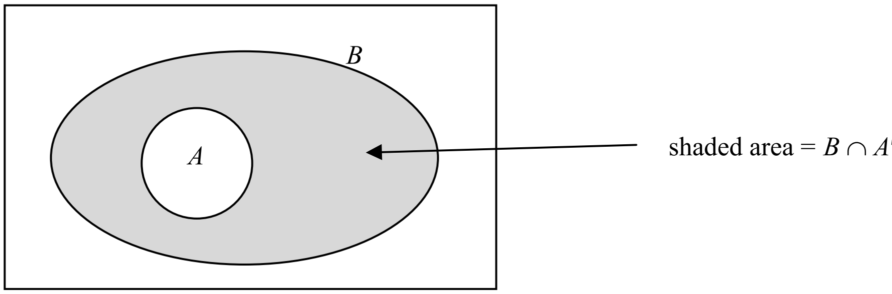
a. P(both are laptops) = $P(\{(1,2)\})$ = $\frac{1}{15}$ =.067.

b. P(both are desktops) = $P(\{(3,4)\ (3,5)\ (3,6)\ (4,5)\ (4,6)\ (5,6)\})$ = $\frac{6}{15}$ = .40.

c. P(at least one desktop) = 1 – P(no desktops) = 1 – P(both are laptops) = 1 – .067 = .933.

d. P(at least one of each type) = 1 – P(both are the same) = 1 – [P(both are laptops) + P(both are desktops)] = 1 – [.067 + .40] = .533.

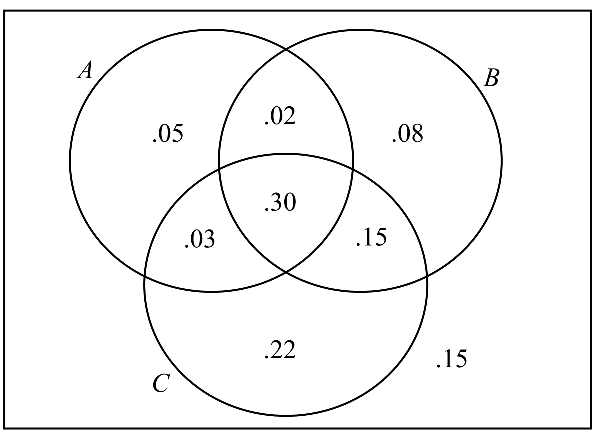
14. Since A is contained in B, we may write $B = A \cup (B \cap A')$, the union of two mutually exclusive events. (See diagram for these two events.) Apply the axioms:
$P(B) = P(A \cup (B \cap A')) = P(A) + P(B \cap A')$ by Axiom 3. Then, since $P(B \cap A') \geq 0$ by Axiom 1, $P(B) = P(A) + P(B \cap A') \geq P(A) + 0 = P(A)$. This proves the statement.



For general events A and B (i.e., not necessarily those in the diagram), it's always the case that $A \cap B$ is contained in A as well as in B, while A and B are both contained in $A \cup B$. Therefore, $P(A \cap B) \leq P(A) \leq P(A \cup B)$ and $P(A \cap B) \leq P(B) \leq P(A \cup B)$.

15. By rearranging the addition rule, $P(A \cap B) = P(A) + P(B) - P(A \cup B) = .40 + .55 - .63 = .32$. By the same method, $P(A \cap C) = .40 + .70 - .77 = .33$ and $P(B \cap C) = .55 + .70 - .80 = .45$. Finally, rearranging the addition rule for 3 events gives
$P(A \cap B \cap C) = P(A \cup B \cup C) - P(A) - P(B) - P(C) + P(A \cap B) + P(A \cap C) + P(B \cap C) = .85 - .40 - .55 - .70 + .32 + .33 + .45 = .30$.

These probabilities are reflected in the Venn diagram below.



a. $P(A \cup B \cup C) = .85$, as given.

b. $P(\text{none selected}) = 1 - P(\text{at least one selected}) = 1 - P(A \cup B \cup C) = 1 - .85 = .15$.

c. From the Venn diagram, $P(\text{only automatic transmission selected}) = .22$.

d. From the Venn diagram, $P(\text{exactly one of the three}) = .05 + .08 + .22 = .35$.

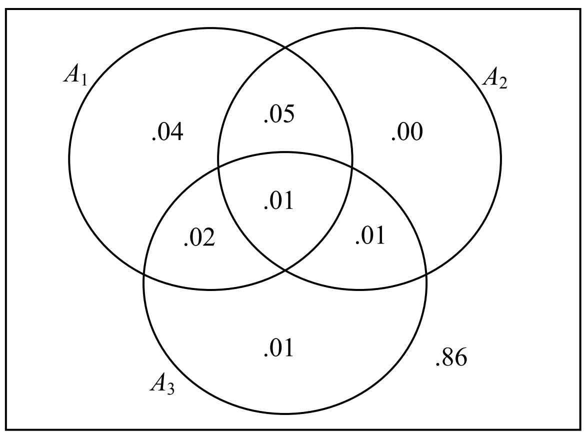
16. These questions can be solved algebraically, or with the Venn diagram below.

a. $P(A_1') = 1 - P(A_1) = 1 - .12 = .88$.

b. The addition rule says $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Solving for the intersection ("and") probability, you get $P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2) = .12 + .07 - .13 = .06$.

c. A Venn diagram shows that $P(A \cap B') = P(A) - P(A \cap B)$. Applying that here with $A = A_1 \cap A_2$ and $B = A_3$, you get $P([A_1 \cap A_2] \cap A_3') = P(A_1 \cap A_2) - P(A_1 \cap A_2 \cap A_3) = .06 - .01 = .05$.

d. The event "at most two defects" is the complement of "all three defects," so the answer is just $1 - P(A_1 \cap A_2 \cap A_3) = 1 - .01 = .99$.



17. There are 10 equally likely outcomes: {A, B} {A, Co} {A, Cr} {A,F} {B, Co} {B, Cr} {B, F} {Co, Cr} {Co, F} and {Cr, F}.

a. P({A, B}) = $\frac{1}{10}$ = .1.

b. P(at least one C) = P({A, Co} or {A, Cr} or {B, Co} or {B, Cr} or {Co, Cr} or {Co, F} or {Cr, F}) = $\frac{7}{10}$ = .7.

c. Replacing each person with his/her years of experience, P(at least 15 years) = P({3, 14} or {6, 10} or {6, 14} or {7, 10} or {7, 14} or {10, 14}) = $\frac{6}{10}$ = .6.

18. Recall there are 27 equally likely outcomes.

a. P(all the same station) = P((1,1,1) or (2,2,2) or (3,3,3)) = $\frac{3}{27} = \frac{1}{9}$.

b. P(at most 2 are assigned to the same station) = 1 – P(all 3 are the same) = $1 - \frac{1}{9} = \frac{8}{9}$.

c. P(all different stations) = P((1,2,3) or (1,3,2) or (2,1,3) or (2,3,1) or (3,1,2) or (3,2,1)) $= \frac{6}{27} = \frac{2}{9}$.