

MSO 201A : Homework 5

[1] A machine contains two belts of different lengths. These have times to failure which are exponentially distributed, with means α and 2α . The machine will stop if either belt fails. The failures of the belts are assumed to be independent. What is the probability that the system performs after time α from the start?

[2] Let X be a normal random variable with parameters $\mu = 10$ and $\sigma^2 = 36$. Compute (a) $P(X > 5)$, (b) $P(4 < X < 16)$, (c) $P(X < 8)$.

[3] Let $X \sim N(\mu, \sigma^2)$. If $P(X \leq 0) = 0.5$ and $P(-1.96 \leq X \leq 1.96) = 0.95$, find μ and σ^2 .

[4] It is assumed that the lifetime of computer chips produced by a certain semiconductor manufacturer are normally distributed with parameters $\mu = 1.4 \times 10^6$ and $\sigma^2 = 3 \times 10^5$ hours. What is the approximate probability that a batch of 10 chips will contain at least 2 chips whose lifetime are less than 1.8×10^6 hours?

[5] Let X be a Normal random variable with mean 0 and variance 1, i.e. $N(0,1)$. Prove that

$$P(|X| \geq t) \leq \sqrt{\frac{2}{\pi}} \frac{e^{-t^2/2}}{t}; \forall t > 0.$$

[6] Show that if X is a discrete random variable with values $0, 1, 2, \dots$ then $E(X) = \sum_{k=0}^{\infty} (1 - F(k))$, where $F(x)$ is the distribution function of the random variable X .

[7] The cumulative distribution function of a random variable X defined over $0 \leq x < \infty$ is $F(x) = 1 - e^{-\beta x^2}$, where $\beta > 0$. Find the mean, median and variance of X .

[8] Show that for any $x > 0$, $1 - \Phi(x) \leq \frac{\phi(x)}{x}$, where $\Phi(x)$ is the c.d.f. and $\phi(x)$ is the p.d.f. of standard normal distribution.

[9] A point m_0 is said to be mode of a random variable X , if the p.m.f. or the p.d.f. of X has a maximum at m_0 . For the distribution given in problem [7], if m_0 denotes the mode; μ , the mean and σ^2 , the variance of the corresponding random variable, then

show that $m_0 = \mu\sqrt{2/\pi}$ and $2m_0^2 - \mu^2 = \sigma^2$.

[10] Let X be a Poisson random variable with parameter λ . Find the probability mass function of $Y = X^2 - 5$.

[11] Let X be Binomial random variable with parameters n and p . Find the probability mass function of $Y = n - X$.

[12] Consider the discrete random variable X with the probability mass function

$$P(X = -2) = \frac{1}{5}, \quad P(X = -1) = \frac{1}{6}, \quad P(X = 0) = \frac{1}{5},$$

$$P(X = 1) = \frac{1}{15}, \quad P(X = 2) = \frac{10}{30}, \quad P(X = 3) = \frac{1}{30}.$$

Find the probability mass function of $Y = X^2$.

[13] The probability mass function of the random variable X is given by

$$P(X = x) = \begin{cases} \frac{1}{3} \left(\frac{2}{3} \right)^x & x = 0, 1, 2, \dots \\ 0 & \text{otherwise.} \end{cases}$$

Find the distribution of $Y = X/(X + 1)$.

[14] Let X be a random variable with probability mass function

$$P(X = x) = \begin{cases} e^{-1}, & x = 0 \\ \frac{e^{-1}}{2(|x|)!}, & x \in \{\pm 1, \pm 2, \dots\} \\ 0 & \text{otherwise.} \end{cases}$$

Find the p.m.f. and distribution of the random variable $Y = |X|$.

Useful data

$$\Phi(1/3) = 0.6293, \quad \Phi(5/6) = 0.7967, \quad \Phi(1) = 0.8413, \quad \Phi(4/3) = 0.918$$
