Definition of a Graph

- A graph G consists of a pair of sets V, E denoted by $G = (V, E)$.
- V: vertex set.
  - Each vertex $v \in V$ may represent some records, objects or a piece of information.
- E: edge set.
  - Each edge $e \in E$ links (relates) one pair of distinct vertices $u \neq v \in V$.
  - There is at most one edge which relates two distinct vertices.

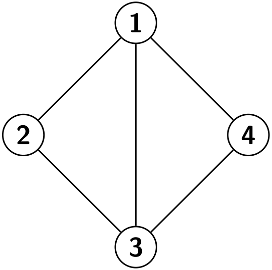
Definition of a Graph

- G is undirected if each edge represents an unordered pair, i.e., $e = (u, v) = (v, u)$.
- In an undirected graph, E may define upto $\binom{|V|}{2}$ relations among vertices.
- If $(u, v) \neq (v, u)$, then the edges are said to be directed:
  - The edge (u, v) is oriented from u to v.
  - The edge (v, u) is oriented from v to u.
- When edges in a graph G are directed, G is known as directed.
- A directed graph may have upto $|V|(|V| - 1)$ edges.

Graph Terminology

- A graph $H = (V_H, E_H)$ is a subgraph of $G = (V_G, E_G)$, if $V_H \subseteq V_G$ and $E_H \subseteq E_G$.
- A simple path in a graph is a sequence of distinct vertices $v_1, v_2, \ldots, v_k$ where $(v_i, v_i + 1) \in E$, for $1 \leq i \leq k - 1$.
- A cycle is a simple path in which the start and end vertices are same, i.e., $v_1 = v_k$.
- G is connected if there is a path between any two pair of distinct vertices in G.
- A connected component of a graph G is a maximally connected subgraph of G
- A graph which does not have any cycle is called acyclic.
- An acyclic undirected graph is a tree.

Adjacency Matrix Representation

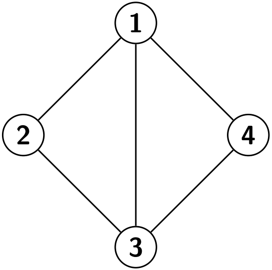


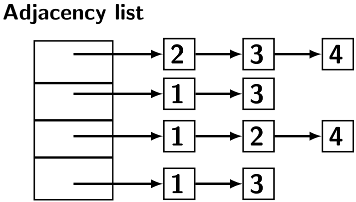
	1	2	3	4
1	0	1	1	1
2	1	0	1	0
3	1	1	0	1
4	1	0	1	0

- Adjancy matrix is a $|V| \times |V|$ matrix in which each row and each column represents a vertex. For a undirected graph $A[i,j] = A[j,i]$.

$$A[i,j] = \begin{cases} 1, \text{ if } (i,j) \in E \\ 0, \text{ otherwise} \end{cases}$$

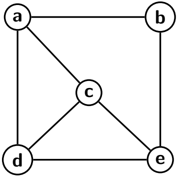
Adjacency List Representation





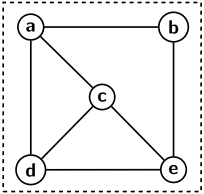
- Each list represents adjacency relations corresponding to a vertex.

Examples of Graphs

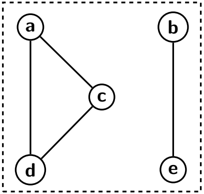


- $V = \{a, b, c, d, e\}$
- $E = \{(a, b), (a, b), (a, d), (b, e),$
  $(c, d), (c, e), (d, e)\}$
- Degree of a vertex v: # edges incident on v.
- $deg(a) = 3$, $deg(b) = 2$, $deg(c) = 3$, $deg(d) = 3$, $deg(e) = 3$,
- # of odd degree vertices is even.

Examples of Graphs



Connected graph



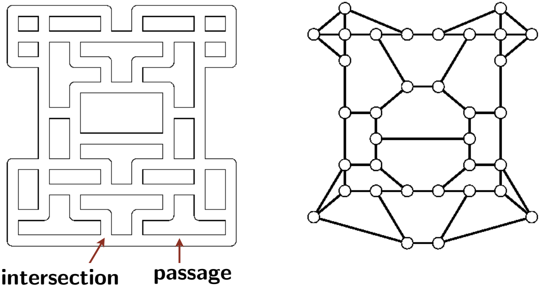
Disconnected graph

- Connected graphs: $\exists$ a path between any two vertices.
- Disconnected graphs: Having more than one connected subgraphs.

Applications of Graph

- Tremaux was obsessed with problem of finding path out of a maze.
- He came up with technique as follows:
  - Unroll a ball of thread to trace of path that is already traversed.
  - Mark each intersection by putting a mark (color).
  - Retrace back to recent most intersection when no new visit options are present.

Maze to Graph



From Chapter 4 of Robert Sedgewick and Kevin Wayne's Algorithm book.