ESO207A: Data Structures and Algorithms
Theoretical Assignment 4

Deadline: April 18, 2018

April 7, 2018

Instructions

- **You are not allowed (explicitly) to use the internet for this assignment** unless otherwise specified.
- All submissions must be in the pdf format.
- Please write precise answers.
- Marks of subparts of questions may not be evenly distributed.
- This assignment may take time and hence you are advised to start early.

Problem 1. (40 marks) The DFS that you learnt in class is not just a tool to traverse a graph but a very powerful tool. You have seen its application in computing strongly connected components of a directed Graph. We will see one more application of DFS in this assignment. Note that we will be dealing only with **directed** graphs unless otherwise specified.
Consider a directed Graph G = (V,E). Let $|V| = m$ and $|E| = n$. A graph G is said to be a **Unique Path Graph** iff for every $u \in V$, if $v \in V$ is reachable from u then there exists a unique path from u to v.

Given a graph $G = (V, E)$ the task is to determine whether G is a unique path graph.

1 [20 marks] Consider the DFS tree T_v of a vertex v. If G is a unique path graph can there exist (1)forward edges, (2)cross edges and (3) backward edges in T_v? Extend your argument to all vertices of G to give a *necessary and sufficient condition* (iff) for G to be a unique path graph. Prove it.

2 [20 marks] Convert and maybe modify the above into an $\mathcal{O}(m^2)$ time algorithm and give a pseudo-code for your algorithm. (A C code will not fetch any credit. An $\mathcal{O}(mn)$ time algorithm will fetch 25% credit).

Problem 2. (60 marks) Consider a set of N Boolean variables $X = \{x_1, \ldots, x_N\}$. Consider an expression E in the Conjunctive Normal Form (CNF) with K conjunctions such that $E = a_1 \wedge a_2 \wedge \ldots a_K$ where each a_k has the form $a_k = y_i \vee y_j$ where y_i, y_j are either variables in X or their negations. For example, E can be the expression $E = (x_1 \vee x_3) \wedge (\neg x_1 \vee x_4) \wedge (x_2 \vee x_3) \wedge (x_2 \vee \neg x_4)$.

Your task is to determine whether a given expression E is satisfiable, that is, does there exist an assignment of truth values of variables in X such that E becomes true. If yes, then you must report the corresponding assignment of truth values.

The following sub-parts will guide you towards the solution

1 [15 marks] Observe that $a \vee b \equiv \neg a \implies b$. Convert E into this new form and use it to construct a graph. Clearly specify your Vertex set and Edge set.

2 [25 Marks] Relate the problem to a problem covered in class to determine whether E is satisfiable. The following observations may help

   (a) a and $\neg a$ cannot have the same truth value

   (b) If $a \implies b \implies c \implies a$, then a, b and c must have the same truth value.

3 [20 Marks] Finally if E is satisfiable, give a method to assign truth values to the variables in X.