# FINOPTIX
# SUMMER PROJECT

## Portfolio Management, Algorithmic Trading & Risk Optimization Framework


**Project Mentors**

Keshav Bansal

Kushagra Tiwari

Ankit Kumar

Aarush Singh

Team FinOptix

## Executive Summary

This report provides an extensive overview of the FinOptix Summer Project '25, focusing on the development and integration of advanced financial models and trading strategies to optimize portfolio management. We explore **algorithmic trading** principles, comprehensive **risk management** techniques, and key asset pricing models including the **Fama-French Three-Factor Model**, **Markowitz's Mean-Variance Optimization**, and the **Black-Litterman framework**.

Our methodology incorporates detailed backtesting, empirical analysis, and risk mitigation strategies to enhance asset allocation across diverse market conditions. The outcomes reveal significant improvements in risk-adjusted returns and portfolio stability, offering practical tools for financial decision-making. This document serves as a thorough guide for our presentation, ensuring a complete understanding of each project component.

# Contents

# 1 Technical Indicators

Candlestick Plots

Candlestick charts are widely used in technical analysis to represent price movement of an asset within a specified time period. Each candlestick contains information about the opening, closing, high, and low prices.
**Components:**
Each candlestick shows:

- **Open:** Price at the beginning of the time period.
- **High:** Highest price during the time period.
- **Low:** Lowest price during the time period.
- **Close:** Price at the end of the time period.

**Color Interpretation:**

- **Green Candlestick (Bullish):** Closing price > opening price.
- **Red Candlestick (Bearish):** Closing price < opening price.

Candlestick patterns help traders predict potential market movements.

Moving Averages

Moving Averages (MAs) are used to smooth out price fluctuations and identify the direction of trends over time.
**Simple Moving Average (SMA):**
The SMA is the arithmetic mean of closing prices over a period $n$:

$$SMA_t = \frac{1}{n}\sum_{i=0}^{n-1} P_{t-i}$$

**Significance:**
SMA helps in identifying support/resistance and trend directions. Crossovers signal buy/sell opportunities.
**Exponential Moving Average (EMA)** EMA gives more weight to recent prices:

$$EMA_t = \alpha P_t + (1-\alpha)EMA_{t-1} \quad \text{where } \alpha = \frac{2}{n+1}$$

**Significance** EMA reacts faster to recent changes and is useful for short-term trading signals.

### Relative Strength Index

RSI is a momentum oscillator measuring the speed and magnitude of price changes.

$$RS = \frac{\text{Average Gain over } n}{\text{Average Loss over } n}, \quad RSI = 100 - \frac{100}{1 + RS}$$

Typically $n = 14$.
**Significance:**

- **RSI > 70:** Overbought (possible correction).
- **RSI < 30:** Oversold (possible rebound).

RSI helps identify overextended conditions and potential reversals.

### MACD

MACD is a trend-following indicator showing the relationship between two EMAs:

$$\begin{aligned} MACD &= EMA_{12}(P) - EMA_{26}(P) \\ \text{Signal Line} &= EMA_9(MACD) \\ \text{Histogram} &= MACD - \text{Signal Line} \end{aligned}$$

**Significance**

- **MACD > Signal Line:** Bullish signal.
- **MACD < Signal Line:** Bearish signal.
- **MACD crosses zero:** Trend change.

### Bollinger Bands

Bollinger Bands consist of a central SMA and two bands at $\pm k$ standard deviations:

$$\begin{aligned} \text{Middle Band} &= SMA_n(P) \\ \text{Upper Band} &= SMA_n(P) + k \cdot \sigma \\ \text{Lower Band} &= SMA_n(P) - k \cdot \sigma \end{aligned}$$

Where $k = 2$ typically.
**Significance**

- **Price near upper band:** Overbought.
- **Price near lower band:** Oversold.
- **Bands widen:** Increased volatility.
- **Bands narrow:** Possible breakout.

ATR

ATR measures market volatility using the True Range (TR):

$$TR_t = \max(H_t - L_t, |H_t - C_{t-1}|, |L_t - C_{t-1}|)$$

$$ATR_t = \frac{1}{n}\sum_{i=0}^{n-1} TR_{t-i} \quad \text{or} \quad ATR_t = \frac{ATR_{t-1}(n-1) + TR_t}{n}$$

**Significance**

- **High ATR:** High volatility.
- **Low ATR:** Market consolidation.

ATR helps with setting stop-losses and position sizing.

# 2 Risk Management Strategies

Risk management forms the fundamental pillar of our portfolio strategy, focusing on balancing potential returns with acceptable risk levels across various market conditions.

**Static Risk Controls**

**Stop Loss:** Automatically closes positions at predetermined unfavorable prices to cap losses
**Take Profit:** Secures gains at specified favorable price levels

For long trades, Stop Loss is placed below entry price and Take Profit above; for short trades, positions are reversed. While effective for basic risk mitigation, these static levels can be susceptible to sudden market volatility.

## 2.1 Advanced Dynamic Strategies

### 2.1.1 Trailing Stop Loss for Long Trades

The **Trailing Stop Loss** dynamically adjusts stop-loss levels as asset prices increase, locking in profits while limiting downside risk.

**Example Implementation**

Initial entry at \$100 with 6% trailing stop:

- Initial stop loss: \$94
- Price rises to \$110 $\rightarrow$ Stop loss moves to \$103.40
- Exit triggered if price drops below updated level

### 2.1.2 Dynamic Exit Condition for Short Trades

Traditional trailing stops prove less effective for short trades. Our **Dynamic Exit Condition** addresses this:

- Initial portfolio value: $x$ dollars

- Exit threshold: $1.06x$
- If value drops to $0.5x$, threshold adjusts to $0.53x$
- Captured 10.89% return in simulated trades

### 2.2 Market Volatility-Based Adjustments

Recognizing external factors' impact on market volatility, particularly in BTC-USDT markets, we introduced multipliers to scale Stop Loss percentages during high-volatility periods when global stock markets are open.

# 3 Backtesting

## 3.1 Introduction to Backtesting

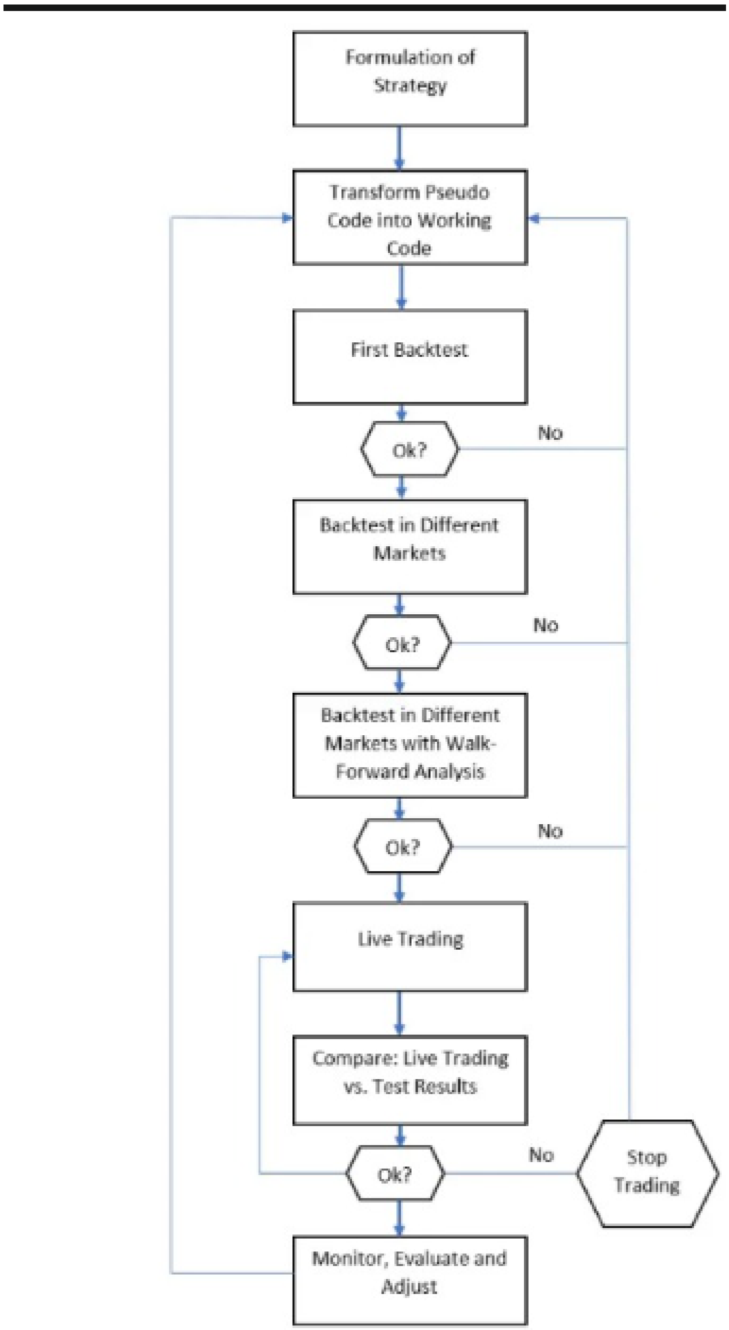



Figure 1: An overview of the backtesting process

**What is backtesting and why do we need it?** Backtesting is a fundamental technique in algorithmic trading that involves testing a trading strategy on historical market data to evaluate its effectiveness and profitability. By simulating how a strategy would have performed in the past, traders can gain insight into its potential performance in the future without risking actual capital.

The goal of backtesting is to assess whether a strategy is robust, consistent, and capable of generating desirable returns while managing risks. It allows traders to identify flaws, optimize parameters, and avoid deploying unprofitable strategies in live markets. A successful backtest does not guarantee future success, but it provides valuable insights into the viability of a strategy. Poor backtest results can indicate that a strategy may be flawed, overly risky, or overly optimized. Backtesting also enables traders to compare multiple strategies, optimize parameters, and avoid impulsive decisions based on real-time experience.

## 3.2 How Does Backtesting Work?

Backtesting means testing a trading strategy on past market data to see how it would have performed. We take historical prices and apply our strategy's rules to decide when we would have bought or sold. At each step, we check if a trade should happen, update our virtual money or holdings, and keep track of how the strategy grows or loses money over time. This helps us understand if the strategy is profitable and what kind of risks it carries, without using real money.

**Required Components**

- Historical Data: Typically includes OHLC prices and trading volume.
- Trading Strategy Logic: Rules for entering and exiting trades.
- Performance Tracker: Measures profit/loss and tracks other metrics.

**Walkthrough of a Backtest** A backtest works by looping through historical data and executing trades when conditions are met. Here's a basic flow:

- Start with initial capital (e.g., $10,000).
- At each time step:
  - Check for a buy/sell signal based on strategy logic. (eg. if the MACD line crosses over the signal line, we get a "buy" signal.)
  - Execute a virtual trade if conditions are satisfied.( use all the cash to buy the BTC and calculate the portfolio value and remains based on the type of trade)
  - Update portfolio value. (portfolio value = cash + (holdings * current price ))
- At the end, calculate KPIs like return, Sharpe ratio, and drawdown.(eg. if the sharpe ratio is higher, better the trade ).

Code Snippet

```
initial_capital = 10000
cash = initial_capital
position = 0
portfolio = []

for i in range(1, len(data)):
    price = data['Close'].iloc[i]
    signal = data['signals'].iloc[i]

    # Buy Signal
    if signal == 1 and cash >= price:
        position = cash / price
        cash = 0
    # Sell Signal
    elif signal == -1 and position > 0:
        cash = position * price
        position = 0

    # Portfolio Value = Cash + Value of Held Position
    portfolio_value = cash + position * price
    portfolio.append(portfolio_value)

data['Portfolio_Value'] = portfolio
```

### 3.3 Risk Management in Backtesting

Risk management is a crucial component of any trading strategy and must be included during backtesting to ensure realistic performance analysis. Two widely used techniques are **Static Stop Loss/Take Profit** and **Trailing Stop Loss/Take Profit**. These mechanisms define when a trade should be exited, either to limit losses or lock in profits.

Static Stop Loss and Take Profit

A *static stop loss* and *take profit* are predefined price levels set when a trade is entered. These levels remain fixed throughout the trade's duration.

- **Stop Loss:** Closes the trade if the price moves unfavorably beyond a set threshold to limit losses.
- **Take Profit:** Exits the trade at a predefined profit level to secure gains.

This method is simple, consistent, and well-suited for strategies that follow fixed **risk-reward ratios**.

**Example (Long Position):**

- Entry Price: \$100
- Static Stop Loss: \$95 (i.e., a \$5 risk)
- Static Take Profit: \$110 (i.e., a \$10 profit target)

In this setup, the trade will automatically exit at $95 or $110 depending on which level is hit first, regardless of price movement in between.

**Trailing Stop Loss and Take Profit**

A *trailing stop loss* and *take profit* are dynamic risk-management tools that adjust in real time as the market price moves favorably.

- **Trailing Stop Loss:** Follows the market price at a fixed distance, locking in profits as the price rises.
- **Trailing Take Profit:** (less commonly used) shifts the profit target upward to capture additional gains in a trend.

If the price reverses by the trailing distance, the trade exits automatically. This method is ideal for **trend-following strategies**.

**Example (Long Position with 5% Trailing Stop Loss):**

- Entry Price: $100
- Price rises to $110 → Trailing Stop Loss moves to $104.5 (5% below peak)
- If price drops to $104.5 → Trade exits

This dynamic approach helps protect gains while allowing profitable trades to continue running in strong trends.

## 3.4 Key Performance Indicators (KPIs)

**Total Return Definition:** Total return tells us how much profit or loss the strategy made from the starting point to the end.

**Formula:**

$$\text{Total Return} = \left( \frac{\text{Final Portfolio Value} - \text{Initial Capital}}{\text{Initial Capital}} \right) \times 100$$

**How to Interpret:**

| **Return type** | **Interpretation** |
|---|---|
| Positive | Strategy made a profit |
| Negative | Strategy lost money |
| Higher return | More profitable — but may involve more risk |
| Consistent Return | More stable, often preferred with lower risk |

Table 1: Interpreting Return Types

Code Snippet

**Code Example:**

```
initial_value = 10000
final_value = data['Portfolio_Value'].iloc[-1]
total_return = (final_value - initial_value) / initial_value * 100
print("Total Return:", total_return, "%")
```

**Sharpe Ratio Definition:**The Sharpe Ratio tells us how much extra return a strategy is giving for every unit of risk it is taking.

Imagine two person A and B invest in different strategies:

- Person A makes 12% return per year, but their returns jump up and down a lot.
- Person B makes 10% return, but with very steady performance.

Even though A's return is higher, your friend might have a better Sharpe Ratio — because their strategy is more consistent and less risky.

The Sharpe Ratio helps compare which strategy is truly "better," not just in terms of how much it made, but also how safely it made it

**Formula:**

$$\text{Sharpe Ratio} = \frac{\text{Average Return} - \text{Risk-Free Return}}{\text{Standard Deviation of Return}}$$

**Average Return:** How much profit the strategy makes on average.

**Risk-Free Return:** The return you could earn without taking any risk at all. Think of it like keeping your money in a super-safe place, like a government savings bond or fixed deposit, where you're almost guaranteed to get your money back with a small amount of interest.

**Standard Deviation:** How much the returns fluctuate. More fluctuation means more risk. If the standard deviation is close to the average return, it implies low fluctuation. If it is far from the average, it implies high fluctuation.

**How to Interpret:**

| Sharpe Ratio | Interpretation |
|---|---|
| $< 1$ | Poor – return is not worth the risk taken |
| 1 to 2 | Acceptable – okay for some strategies |
| 2 to 3 | Good – the strategy has solid performance |
| $> 3$ | Excellent – very strong risk-adjusted return |

Table 2: Sharpe Ratio Interpretation Guide

Code Snippet

**Code Example:**

```
import numpy as np
data['Daily_Returns'] = data['Portfolio_Value'].pct_change().dropna()
risk_free_rate = 0.0001
excess_returns = data['Daily_Returns'] - risk_free_rate
sharpe_ratio = excess_returns.mean() / excess_returns.std()
annualized_sharpe = sharpe_ratio * np.sqrt(252)
```

**Maximum Drawdown Definition:** Maximum Drawdown (MDD) tells us the worst fall in our portfolio value from a high point (peak) to a low point (trough) before it recovers. It represents the largest temporary loss during the backtest period. This is important because it shows the risk of deep losses, even if the strategy looks profitable overall. A strategy that makes good returns but also suffers large drops can be dangerous in real-world trading.

**Formulas:**

$$\text{Max Drawdown} = \max\left(\frac{\text{Peak} - \text{Trough}}{\text{Peak}}\right) \times 100$$

**How to Interpret:**

| Maximum Drawdown | Interpretation |
|---|---|
| $< 30\%$ | Very safe and stable |
| 10% to 30% | Moderate risk |
| More than 30% | High risk — big temporary losses |

Table 3: Drawdown Risk Interpretation

**Maximum Dip Definition:** Maximum Dip measures the biggest loss from the entry point of a trade to the lowest point during the trade. It tells you how bad the trade got after entering, regardless of whether it recovered or not. It focuses on entry risk — the amount of pain a trader would have felt after placing a trade.

**Formulas:**

$$\text{Max Dip} = \max\left(\frac{\text{Entry Price} - \text{Lowest Price}}{\text{Entry Price}}\right) \times 100$$

**How to Interpret:**

| Maximum Dip | Interpretations |
|---|---|
| $< 5\%$ | Low dip — safe trade |
| 5%–20% | Moderate dip — normal fluctuations |
| $> 20\%$ | High dip — risky or badly timed entry |

Table 4: Interpretation of Maximum Dip

**Code Example for Max Dip and Max Drawdown:**

```
a = 0
max___drawdown = []
max___dip = []

for i in range(0, ((int(len(trades) / 2)) * 2), 2):
    drawdown = []
    index1 = trades[i]
    index2 = trades[i + 1]
    stocks = num_stocks[a]
    remain = remains[a]

    max1 = dd1['Portfolio_Value'].iloc[index1]
    min1 = dd1['Portfolio_Value'].iloc[index1]

    for j in range(index1, index2 + 1):
        portfolio = dd1['Portfolio_Value'].iloc[j]
        max1 = max(max1, portfolio)
        min1 = min(min1, portfolio)
        drawdown_percent = ((max1 - portfolio) / portfolio) * 100
        drawdown.append(drawdown_percent)

    max___drawdown.append(max(drawdown))
    dip_from_entry = ((dd1['Portfolio_Value'].iloc[index1] - min1) /
                      dd1['Portfolio_Value'].iloc[index1]) * 100
    max___dip.append(dip_from_entry)
    a += 1
```

```
twd1['Max Drawdown for Trade'] = max___drawdown
twd1['Max Dip for Trade'] = max___dip
```

### 3.5 Benefits and Limitations of Backtesting

**Benefits**

- **Risk-free testing:** Allows you to test strategies without using real money.
- **Strategy refinement:** Helps tweak entry/exit rules and parameters to improve performance.
- **Early problem detection:** Identifies weaknesses or risky behaviors in your strategy.
- **Performance comparison:** Lets you objectively compare different strategies using metrics.

**Limitations**

- **Overfitting risk:** Strategy may be too closely tailored to past data and fail in the future.
- **No guarantee of future success:** Past performance does not always reflect future market behavior.
- **Assumption bias:** Unrealistic assumptions (like perfect execution, zero fees) can mislead results.
- **Bad or incomplete data:** Poor-quality historical data can give inaccurate outcomes.

Backtesting is a crucial first step in developing and evaluating a trading strategy. It allows traders to simulate trades on historical data and measure performance using objective metrics like return, drawdown, and Sharpe Ratio.

It helps in refining strategies, identifying weaknesses, and avoiding risky or unprofitable setups before risking real capital. However, backtesting is not enough on its own. Market conditions change, and strategies that perform well in the past may fail in the future.

## 4 Fama-French Three-Factor Model

The **Fama-French Three-Factor Model** enhances traditional asset pricing by incorporating additional risk factors beyond market risk, providing a comprehensive framework for analyzing stock returns within our project. Developed by Eugene Fama and Kenneth French, this model extends the Capital Asset Pricing Model (CAPM) by addressing empirical anomalies in stock performance, making it a pivotal tool in our portfolio management strategies.

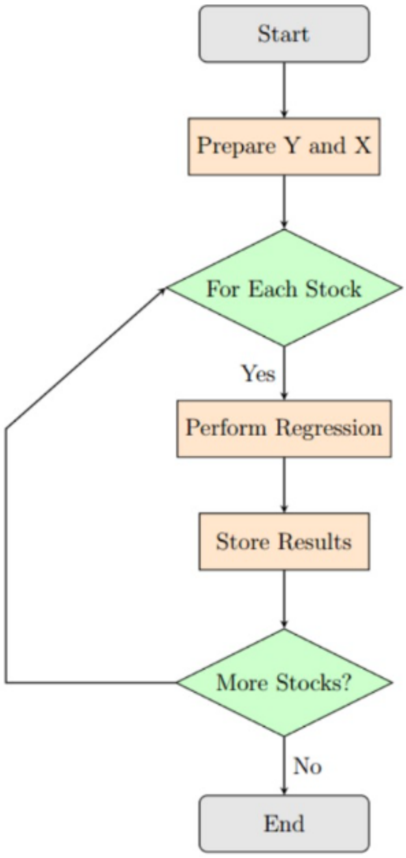



Figure 2: An overview of the Fama French Model

### 4.1 Enhancement Over CAPM

The CAPM focuses solely on market risk to predict expected returns, using the formula:

$$E(R_i) = R_f + \beta_i \cdot (E(R_m) - R_f)$$

Where:

- $E(R_i)$: Expected return on the asset
- $R_f$: Risk-free rate
- $\beta_i$: Beta coefficient of market sensitivity
- $E(R_m) - R_f$: Market risk premium

However, CAPM often underestimates returns for small-cap and value stocks. The Fama-French model addresses this limitation by adding two factors—size and value—to explain variations in returns more effectively:

$$R_p - R_f = \alpha + \beta_m(R_m - R_f) + \beta_{smb} \cdot SMB + \beta_{hml} \cdot HML$$

Where:

- $R_p$: Return of the portfolio or asset

- $R_f$: Risk-free rate
- $R_m$: Market return
- $SMB$: Small Minus Big, capturing size effect
- $HML$: High Minus Low, capturing value effect
- $\alpha$: Abnormal return not explained by factors
- $\beta_m, \beta_{smb}, \beta_{hml}$: Sensitivities to respective factors

### 4.2 Key Factors and Their Construction

**Core Components of the Model**

The Fama-French Three-Factor Model incorporates three distinct factors to describe stock returns:

- **Market Risk Premium ($R_m - R_f$)**: Represents the excess return of the market over the risk-free rate, akin to CAPM, reflecting general market exposure.
- **Size Factor (SMB - Small Minus Big)**: Captures the historical outperformance of small-cap companies over large-cap companies, calculated as the difference between average returns of small-cap and large-cap portfolios.
- **Value Factor (HML - High Minus Low)**: Reflects the tendency of value stocks (high book-to-market ratio) to outperform growth stocks (low book-to-market ratio), computed as the difference between average returns of high and low book-to-market portfolios.

### 4.3 Empirical Evidence and Theoretical Insights

**Historical Performance Insights**

Empirical data analyzed during our project confirms key observations:

- Small-cap stocks consistently yield higher returns accompanied by increased risk compared to large-cap stocks.
- Value stocks with high book-to-market ratios outperform growth stocks over extended periods, indicating a value premium.
- The model explains over 90% of diversified portfolio returns, a significant improvement over CAPM's 70% within sample data.

The model's ability to account for cross-sectional variations in stock returns stems from its recognition of size and value as systematic risk factors, beyond mere market risk. This aligns with findings that higher returns are associated with higher risk in these dimensions, supporting the efficient market hypothesis to an extent, though anomalies remain.

### 4.4 Limitations and Critiques

Despite its strengths, the Fama-French Three-Factor Model has notable limitations:

- **Missing Anomalies:** It does not capture other documented effects such as momentum, where past winners continue to outperform, leading to extensions like the Carhart Four-Factor Model.
- **Data Mining Concerns:** Critics argue that size and value factors may result from overfitting historical data, questioning their persistence in future or different market contexts.
- **Theoretical Ambiguity:** While market risk has a clear basis in CAPM, the theoretical underpinnings of size and value premiums are less defined, with explanations ranging from financial distress to behavioral biases.
- **Country-Specific Factors:** The factors are often country-specific, performing better with local rather than global data, as observed in markets like the U.S., Canada, Japan, and the U.K.

### 4.5 Application in Portfolio Management

In our project, the Fama-French Three-Factor Model was applied to enhance portfolio construction and risk assessment. Using regression analysis on historical data for selected stocks, we estimated factor sensitivities ($\beta_m, \beta_{smb}, \beta_{hml}$) to understand how market, size, and value risks impact expected returns. This approach guided asset selection by tilting portfolios towards small-cap or value stocks when favorable premiums were identified, aligning with our goal of optimizing risk-adjusted returns.

**Practical Implementation**

Key applications in our project included:

- **Risk Assessment:** Determining portfolio sensitivity to size and value factors alongside market risk for comprehensive risk profiling.
- **Performance Attribution:** Decomposing portfolio returns to identify contributions from each factor, aiding in understanding outperformance or underperformance sources.
- **Strategic Allocation:** Adjusting portfolio weights to exploit historical premiums in small-cap and value stocks, based on empirical factor loadings.

### 4.6 Conclusion

The Fama-French Three-Factor Model served as a critical framework in our project, offering a more robust explanation of stock returns compared to CAPM by incorporating size and value factors. Its empirical success in explaining over 90% of portfolio return variations provided valuable insights for portfolio optimization. However, limitations such as the exclusion of momentum and theoretical ambiguities were acknowledged, prompting integration with other models like Black-Litterman for enhanced applicability. This model remains a cornerstone of our asset pricing analysis, balancing empirical rigor with practical utility in financial decision-making.

## 5 Markowitz Mean Variance Optimization

Markowitz's MVO underpins modern portfolio theory, guiding our approach to constructing optimal portfolios by balancing risk and return effectively.It provides a quantitative framework for constructing portfolios that maximize expected return for a given level of risk (variance), or equivalently, minimize risk for a given level of return.

### 5.1 Key Components

**Key Components**

**Expected Returns:** Weighted average of asset returns Given a portfolio of $n$ assets with weights $\mathbf{w} = [w_1, w_2, \ldots, w_n]^T$, and expected returns $\boldsymbol{\mu} = [\mu_1, \mu_2, \ldots, \mu_n]^T$, the expected return of the portfolio is:

$$\mathbb{E}[R_p] = \mathbf{w}^T \boldsymbol{\mu}$$

**Risk:** Measured as portfolio variance or standard deviation Let $\Sigma$ be the $n \times n$ covariance matrix of asset returns. The portfolio variance is:

$$\sigma_p^2 = \mathbf{w}^T \Sigma \mathbf{w}$$

**Covariance:** In Markowitz Mean-Variance Optimization, the covariance matrix $\Sigma$ captures the pairwise covariances between asset returns in a portfolio. It is defined as:

$$\Sigma = \begin{bmatrix} \mathrm{Var}(R_1) & \mathrm{Cov}(R_1, R_2) & \cdots & \mathrm{Cov}(R_1, R_n) \\ \mathrm{Cov}(R_2, R_1) & \mathrm{Var}(R_2) & \cdots & \mathrm{Cov}(R_2, R_n) \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{Cov}(R_n, R_1) & \mathrm{Cov}(R_n, R_2) & \cdots & \mathrm{Var}(R_n) \end{bmatrix}$$

where:

$$\mathrm{Cov}(R_i, R_j) = \mathbb{E}\left[(R_i - \mu_i)(R_j - \mu_j)\right]$$

Let $\mathbf{R} = [R_1, R_2, \ldots, R_n]^T$ denote the random return vector and $\boldsymbol{\mu} = \mathbb{E}[\mathbf{R}]$ the expected return vector, then the covariance matrix can be compactly expressed as:

$$\Sigma = \mathbb{E}[(\mathbf{R} - \boldsymbol{\mu})(\mathbf{R} - \boldsymbol{\mu})^T]$$

**Efficient Frontier:** Optimal risk-return combinations which we get on solving the optimization problem of MVO.

### 5.2 Objective

The standard form of the Markowitz optimization problem is to optimize:

$$\min_{\mathbf{w}} \quad \mathbf{w}^T \Sigma \mathbf{w}$$

subject to:

$$\mathbf{w}^T \boldsymbol{\mu} = \mu_p \quad \text{(target return)}$$

$$\sum_{i=1}^{n} w_i = 1 \quad \text{(fully invested portfolio)}$$

In the case of short selling we introduce another constraint as:

$$w_i \geq 0 \quad \forall i$$

### 5.3 Efficient Frontier

The set of optimal portfolios forms the **efficient frontier** — the boundary of the minimum variance for a given return.

**Efficient Frontier**

Portfolios on the frontier are optimal. Any portfolio below the frontier is inefficient (higher risk for the same return).

### 5.4 Advantages and Limitations

**Advantages**

- Provides a quantitative, theoretically grounded approach to diversification
- Forms the basis of modern portfolio theory and CAPM

**Limitations**

- Assumes normally distributed returns and stable covariances
- Sensitive to input estimation errors
- No guarantee of outperformance out-of-sample

We address these through integrated models and sensitivity analyses.

## 6 Black- Litterman Model

The **Black-Litterman Model** provides a sophisticated framework for portfolio optimization, blending market equilibrium with subjective investor views.

The key innovation of the Black–Litterman model lies in its ability to systematically blend market consensus with individual investor insights, creating more stable and realistic portfolio allocations.

### 6.1 Investor Views

The key innovation of the Black–Litterman model is its ability to incorporate subjective views of the investor about future returns. These views are usually related to specific assets or asset classes and may involve an expected return different from the market equilibrium return.

These views can be categorized as:

1. **Absolute views**: An investor believes a particular asset will have a return of 10%
2. **Relative views**: An investor believes Asset A will outperform Asset B by 5%

### 6.2 Bayesian Framework

The model applies Bayesian inference to combine the CAPM equilibrium returns (prior) with the investor's views (new information) to generate a posterior distribution of expected returns.

This results in adjusted returns that account for both the market equilibrium and the investor's subjective expectations, weighted by the investor's confidence in their views.

The Bayesian update can be expressed as:

$$\text{Posterior} \propto \text{Prior} \times \text{Likelihood} \tag{1}$$

### 6.3 Reverse Optimization

Instead of starting with expected returns (as in traditional mean-variance optimization), the Black–Litterman model starts by inferring the expected returns implied by the market (market equilibrium returns) using the market capitalization weights.

This process is called *reverse optimization*, where the equilibrium returns are derived by assuming that the market portfolio is mean-variance efficient.

The implied equilibrium returns are calculated as:

$$\boldsymbol{\mu} = \delta \boldsymbol{\Sigma} \boldsymbol{w}_{mkt} \tag{2}$$

where:

- $\boldsymbol{\mu}$ = Vector of implied equilibrium returns
- $\delta$ = Risk aversion coefficient
- $\boldsymbol{\Sigma}$ = Covariance matrix of asset returns
- $\boldsymbol{w}_{mkt}$ = Market capitalization weights

The key innovation of the Black–Litterman model is its ability to incorporate subjective views of the investor about future returns. These views are usually related to specific assets or asset classes and may involve an expected return different from the market equilibrium return.

These views can be categorized as:

1. **Absolute views**: An investor believes a particular asset will have a return of 10%
2. **Relative views**: An investor believes Asset A will outperform Asset B by 5%

### 6.4 Mathematical Foundation

**Equilibrium Returns:**

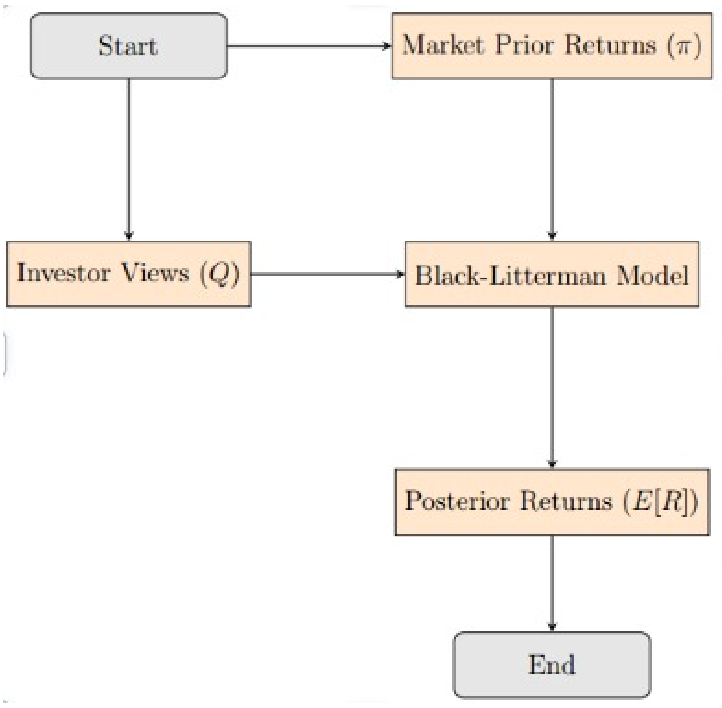
$$\Pi = \lambda \Sigma W_m$$

**Posterior Returns:**

$$\mu_{BL} = \left[(\tau\Sigma)^{-1} + P^T \Omega^{-1} P\right]^{-1} \left[(\tau\Sigma)^{-1}\Pi + P^T \Omega^{-1} Q\right]$$

Where:

$$\Pi = \text{Equilibrium excess returns} \tag{3}$$
$$\lambda = \text{Risk aversion coefficient} \tag{4}$$
$$\Sigma = \text{Covariance matrix} \tag{5}$$
$$W_m = \text{Market capitalization weights} \tag{6}$$


Start
Market Prior Returns ($\pi$)
Investor Views ($Q$)
Black-Litterman Model
Posterior Returns ($E[R]$)
End

Steps of the Black-Litterman Model

## 6.5 Step 1: Market Equilibrium

Use the global market portfolio (e.g., stocks, bonds) and the covariance matrix of returns to compute the implied excess equilibrium returns. These are based on market capitalization weights and can be thought of as the market's expected returns if no investor had any special information or views.

## 6.6 Step 2: Formulate Views

The investor introduces their views on one or more assets or asset classes, either in absolute terms (e.g., "I expect Asset X to return 7%") or in relative terms (e.g., "Asset A will outperform Asset B by 3%").
Each view is accompanied by a confidence level. This confidence determines how much weight the model should give to the investor's view versus the market equilibrium.
Views are typically expressed in matrix form:

$$\boldsymbol{P}\boldsymbol{\mu} = \boldsymbol{Q} + \boldsymbol{\epsilon} \tag{7}$$

where:

- $\boldsymbol{P}$ = Picking matrix (identifies which assets the views relate to)
- $\boldsymbol{Q}$ = Vector of view values
- $\boldsymbol{\epsilon}$ = Vector of view errors with covariance matrix $\boldsymbol{\Omega}$

## 6.7 Step 3: Blend Views and Equilibrium Returns

The investor's views are combined with the market equilibrium returns using Bayesian techniques. The model adjusts the equilibrium returns by incorporating the investor's views while taking into account the confidence levels. If the investor is highly confident, the views will have a larger influence. If the investor is less confident, the market equilibrium returns will dominate.

## 6.8 Step 4: Posterior Returns

The output is a new set of expected returns for each asset in the portfolio, called *posterior returns*. These are a blend of the equilibrium market returns and the investor's views.
The posterior expected returns are calculated as:

$$\boldsymbol{\mu}_{BL} = \boldsymbol{\mu} + \boldsymbol{\Sigma}\boldsymbol{P}^T(\boldsymbol{P}\boldsymbol{\Sigma}\boldsymbol{P}^T + \boldsymbol{\Omega})^{-1}(\boldsymbol{Q} - \boldsymbol{P}\boldsymbol{\mu}) \tag{8}$$

The posterior covariance matrix is:

$$\boldsymbol{\Sigma}_{BL} = \boldsymbol{\Sigma} - \boldsymbol{\Sigma}\boldsymbol{P}^T(\boldsymbol{P}\boldsymbol{\Sigma}\boldsymbol{P}^T + \boldsymbol{\Omega})^{-1}\boldsymbol{P}\boldsymbol{\Sigma} \tag{9}$$

## 6.9 Step 5: Optimize Portfolio

Once the posterior returns are obtained, they are used in the traditional mean-variance optimization framework to construct the optimal portfolio.
The resulting portfolio is adjusted for the investor's views but still remains close to the efficient frontier due to the initial reliance on market equilibrium.
The optimal portfolio weights are:

$$\boldsymbol{w}^* = \frac{1}{\delta}\boldsymbol{\Sigma}_{BL}^{-1}\boldsymbol{\mu}_{BL} \tag{10}$$

### 6.10 Advantages of the Black–Litterman Model

**Stability:**
Traditional mean-variance optimization is highly sensitive to small changes in expected returns, often leading to extreme and unrealistic portfolio weights. The Black–Litterman model smooths out this sensitivity by anchoring on market equilibrium returns.
**Incorporates Investor Views:**
It allows investors to systematically incorporate their views into the portfolio optimization process, giving them more control over the outcome without completely abandoning the market information.
**Reduced Estimation Error:**
Traditional models rely on historical returns to estimate expected returns, which can be error-prone. The Black–Litterman model reduces estimation errors by incorporating market equilibrium information, which is generally considered more reliable.
**Balanced Portfolio:**
The model avoids extreme portfolios, providing more reasonable asset allocations by maintaining a balance between market consensus and individual views.

The Black–Litterman model is a powerful tool for portfolio construction, blending market equilibrium with individual investor views in a systematic and coherent way. It improves the stability of portfolio optimization by relying on a solid foundation of market data, while still allowing flexibility for subjective opinions on expected returns.

## 7 Conclusion

**Project Achievements**

The FinOptix Summer Project '25 successfully demonstrates a comprehensive framework for portfolio management through:

- Integration of algorithmic trading principles with advanced risk management
- Application of sophisticated financial models (Fama-French, MVO, Black-Litterman)
- Notable improvements in risk-adjusted returns and portfolio stability
- Empirical validation through extensive backtesting across diverse market conditions

### 7.1 Key Performance Metrics

Our integrated approach achieved:

- **Mean Returns:** 0.10 to 0.18
- **Standard Deviations:** 0.02 to 0.18
- **Superior Sharpe Ratios:** Outperforming benchmark portfolios
- **Enhanced Stability:** Across varying market conditions