

Assignment 2 Comparison of the exponential and running mean for random walk model

PART 2

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```
clc;  
clear;  
close all;
```

Task 1: Generate True trajectory

Using the random walk model,

```
n = 300;  
X = zeros(n,1);  
X(1) = 10;  
  
v_w = 28^2; % variance  
sigma_w = sqrt(v_w); % standard deviation  
  
for i = 2:n  
    w = sigma_w*randn; % normally distributed random noise  
    X(i) = w + X(i-1);  
end
```

Task 2: Generate Measurements

```
v_eta = 97^2; % variance  
sigma_eta = sqrt(v_eta); % standard deviation  
eta = sigma_eta*randn(n,1); % normally distributed random noise  
  
z = eta + X; % measurements
```

Task 3: Determine optimal smoothing coefficient in exponential smoothing

```
chi = v_w/v_eta
```

```
chi = 0.0833
```

```
alfa = (-chi + sqrt(chi^2 + 4*chi))/2
```

```
alfa = 0.2500
```

Task 4: Measurement error and window size M

```
M = floor((2-alfa)/alfa)
```

```
M = 7
```

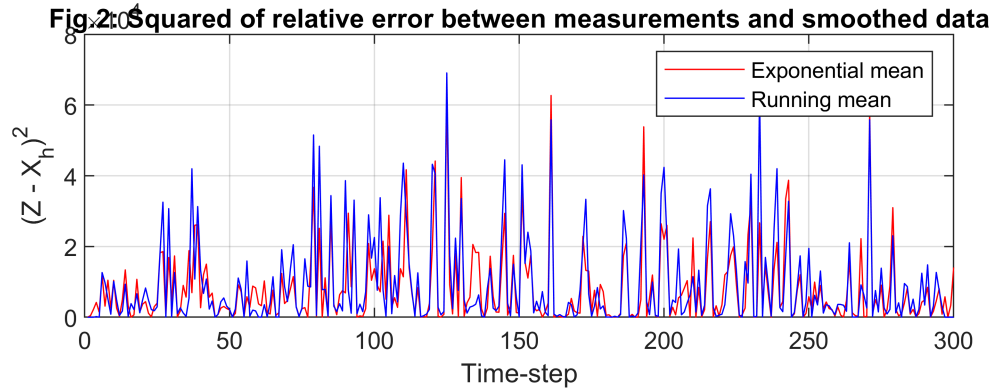
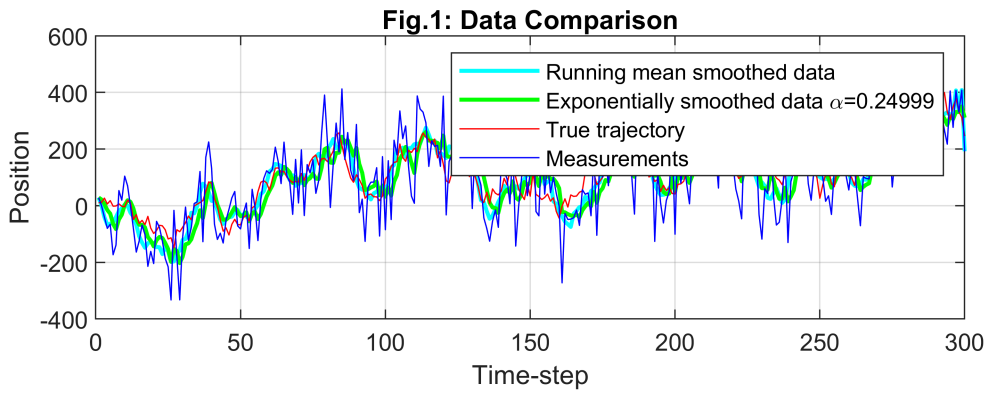
```
v_rm = v_eta/M;  
v_es = v_eta*alfa/(2-alfa);
```

Task 5: Running mean and Exponential mean

```
X_rm = z;  
for i= (M+1)/2:n-(M+1)/2  
    X_rm(i) = 0;  
    for k = i-(M-1)/2:i+(M-1)/2  
        X_rm(i) = X_rm(i) + z(k);  
    end  
    X_rm(i) = X_rm(i)/M;  
end  
  
X_es = z;  
for i = 2:n  
    X_es(i) = X_es(i-1)+alfa*(z(i)-X_es(i-1));  
end
```

Comparitive Plots

```
figure  
subplot(2,1,1)  
plot(X_rm, 'c', "LineWidth",1.5)  
hold on  
plot(X_es, 'g', "LineWidth",1.5)  
plot(X, 'r', "LineWidth",0.5)  
plot(z, 'b', "LineWidth",0.5)  
title('Fig.1: Data Comparison')  
legend('Running mean smoothed data', ['Exponentially smoothed data \alpha=', num2str(alfa)], 'True data')  
xlabel('Time-step')  
ylabel('Position')  
grid on  
  
subplot(2,1,2)  
plot((z-X_es).^2, "r")  
hold on  
plot((z-X_rm).^2, "b")  
grid on  
xlabel('Time-step')  
ylabel('(Z - X_h)^2')  
legend('Exponential mean', 'Running mean')  
title('Fig.2: Squared of relative error between measurements and smoothed data')
```



Conclusions

- The true trajectory X was created for 300 steps using the random walk model with initial and variance conditions given.
- Measurements z were generated from the random walk model of X with the given variance.
- The optimal smoothing coefficient α comes to be 0.25 showing the best possible smoothing conditions.
- Using the round values the window size M comes to be 7 which is optimal for smoothing conditions.
- For the determined window size M and smoothing coefficient α plot between Running mean, Exponential mean, true trajectory and measurements is analysed. This shows as the parameters are for optimal conditions this curve is the best possible smoothen trajectory for the calculated measurements. Even though both methods are in conditions of equal errors conditioned by measurement errors, exponential smoothing with optimal α tends to cause more delay than the running mean.
- A plot of the relative error and smoothen data for both the exponential and running mean is studied. The error for running mean has higher peaks but it gradually lowers as the iterations are increased compared to the exponential mean.