Assignment 2 Comparison of the exponential and running mean for random walk model

PART 1

Group 6:

- Andrei Shumeiko
- Ayush Gupta
- Olga Klyagina
- Simona Nitti


```
clc;
clear;
close all;
```

For Iterations of 3000

Task 1.1

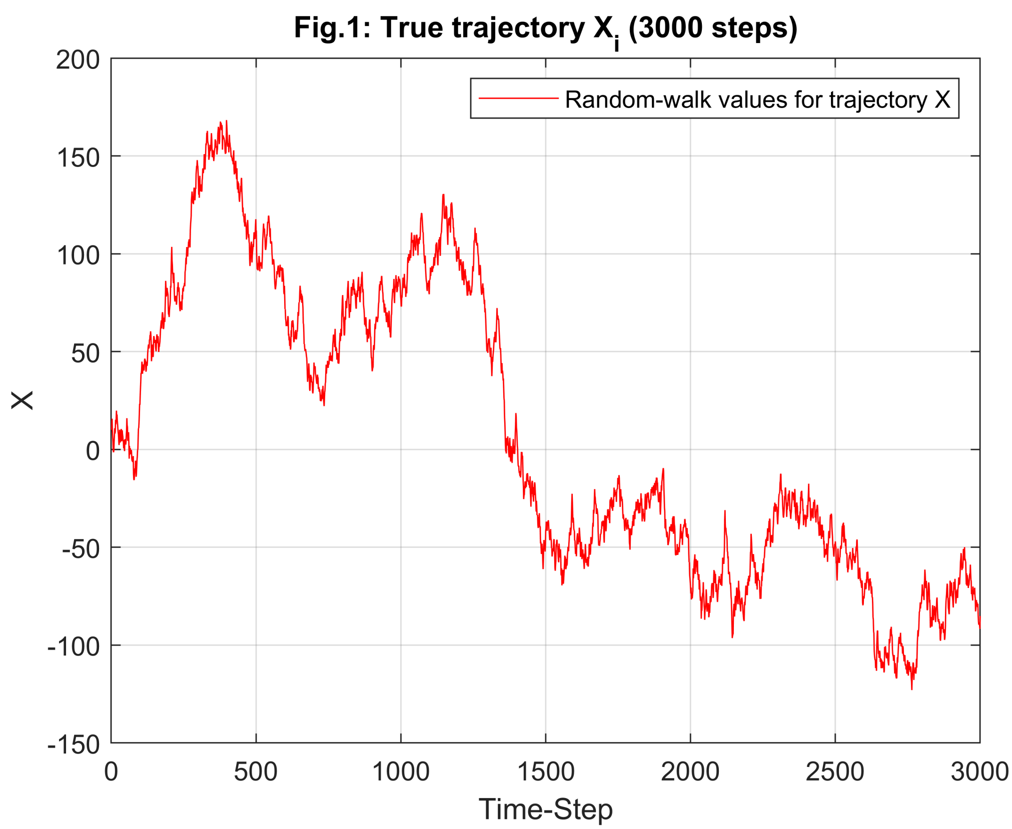
Generation a true trajectory using the random walk model

```
itr = 3000;
X_1 = zeros(itr,1);
X_1(1)=10;
w_variance = 13;

for i=2:itr
    X_1(i) = (sqrt(w_variance) * randn) + X_1(i-1);
end
```

True trajectory graph:

```
figure
plot(X_1,'r')
grid on
xlabel("Time-Step")
ylabel("X")
title('Fig.1: True trajectory X_i (3000 steps)')
legend("Random-walk values for trajectory X")
```



Task 1.2

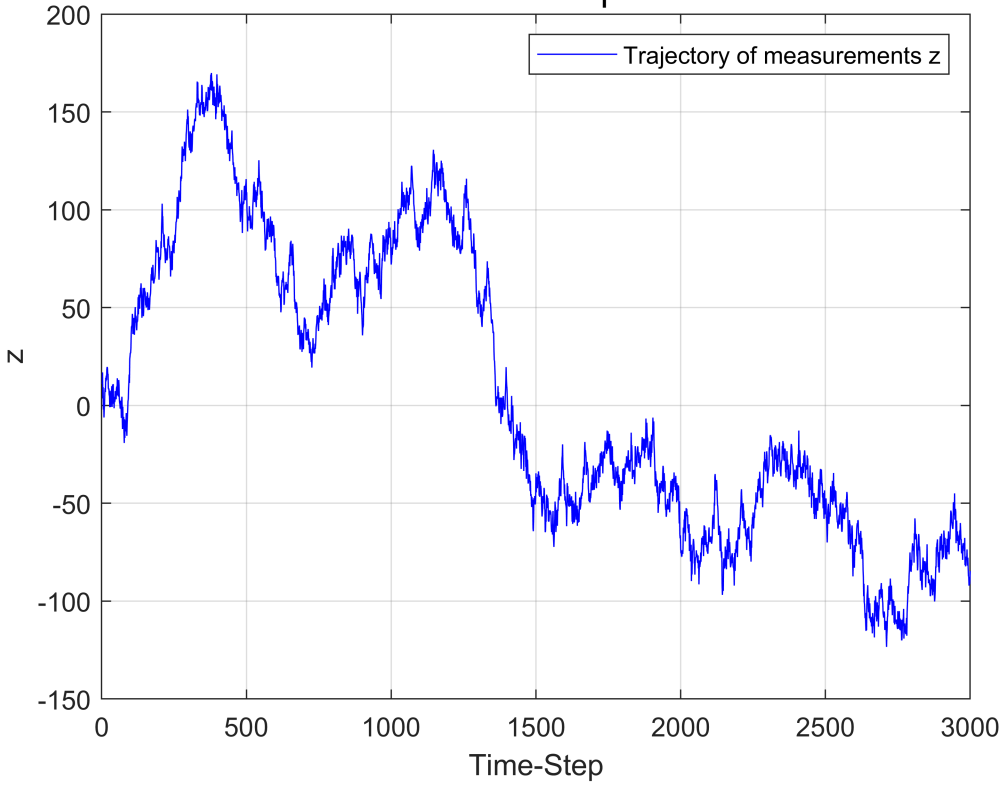
Generating measurements of the process X.

```
n_variance = 8;
n = sqrt(n_variance) * randn(itr,1);
z_1 = X_1 + n;
```

Measurements graph:

```
figure
plot(z_1,'b')
grid on
xlabel("Time-Step")
ylabel("z")
title('Fig.2: Measurements z_i (3000 steps)')
legend("Trajectory of measurements z")
```

Fig.2: Measurements z_i (3000 steps)



Task 2

Identifying 2 and 2 using identification method:

```
for i=2:itr
    v(i-1) = z_1(i)-z_1(i-1);
end

for i=3:itr
    rho(i-2) = z_1(i)-z_1(i-2);
end

new_w_variance_1=v.^2;
Ev=sum(new_w_variance_1)/(itr-1);

new_n_variance_1=rho.^2;
Erho=sum(new_n_variance_1)/(itr-2);

new_w_variance_1 = Erho - Ev
```

```
new_w_variance_1 = 12.4931
```

```
new_n_variance_1 = (Ev - new_w_variance_1)/2
```

```
new_n_variance_1 = 7.5289
```

The calculated variance(w) and variance(n) for iterations of 3000 are much closer to the true value of variance(w) and variance(n).

Task 3

Determine optimal smoothing coefficient in exponential smoothing:

```
x = new_w_variance_1/new_n_variance_1
```

```
x = 1.6593
```

```
alpha = (-x + sqrt(x^2 + 4*x))/2
```

```
alpha = 0.7025
```

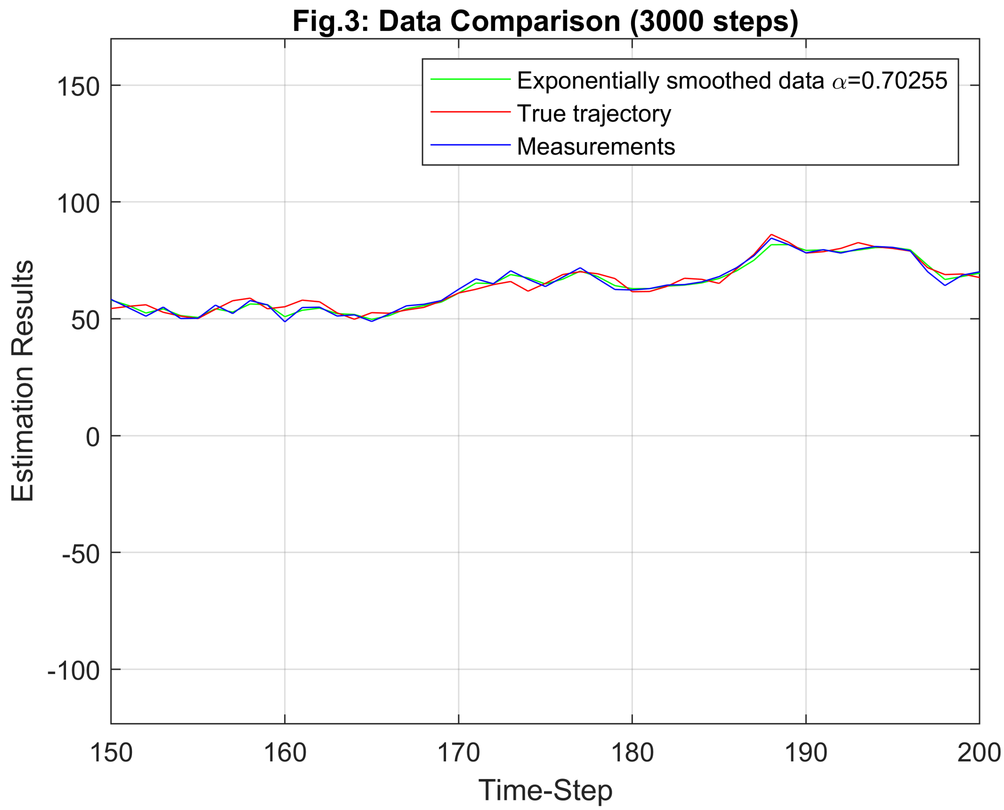
Task 4

Exponential Smoothing:

```
X_smooth = z_1;
for i = 2:itr
    X_smooth(i) = X_smooth(i-1) +  alpha*(z_1(i) - X_smooth(i-1));
end
```

Graph for exponential smoothing with comparison with measurements, true values of process and exponentially smoothed data:

```
figure
plot(X_smooth,'g')
hold on
plot(X_1,'r')
plot(z_1,'b')
grid on
axis([150 200 -inf inf])
xlabel("Time-Step")
ylabel("Estimation Results")
legend(['Exponentially smoothed data \alpha=', num2str(alpha)],'True trajectory','Measurements'
title('Fig.3: Data Comparison (3000 steps)')
xlim([150.0 200.0])
```



For Iterations of 300

Generation a true trajectory using the random walk model

```
clear;
itr = 300;
X_2 = zeros(itr,1);
X_2(1)=10;
w_variance = 13;

for i=2:itr
    X_2(i) = (sqrt(w_variance) * randn) + X_2(i-1);
end
```

True trajectory graph:

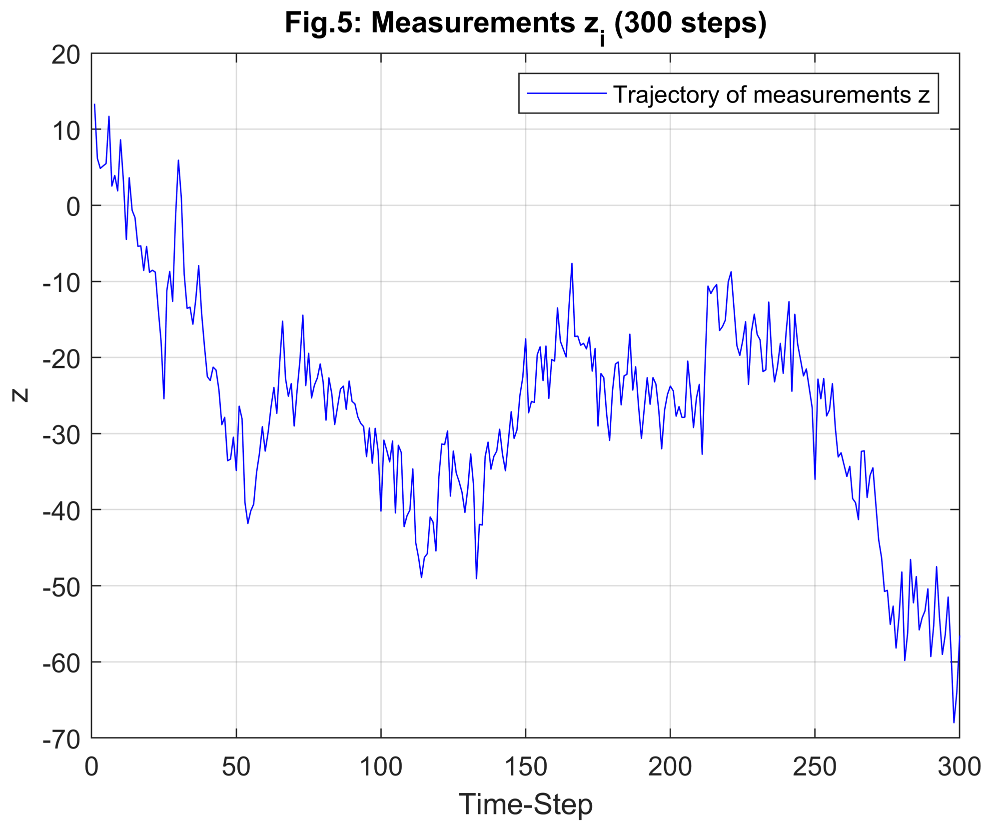
```
figure
plot(X_2,'r')
grid on
xlabel("Time-Step")
ylabel("X")
title('Fig.4: True trajectory X_i (300 steps)')
legend("Random-walk values for trajectory X")
```

Generating measurements of the process X.

```
n_variance = 8;
n = sqrt(n_variance) * randn(itr,1);
z_2 = X_2 + n;
```

Measurements graph:

```
figure
plot(z_2,'b')
grid on
xlabel("Time-Step")
ylabel("z")
title('Fig.5: Measurements z_i (300 steps)')
legend("Trajectory of measurements z")
```



Identifying 2 and 2 using identification method:

```
for i=2:itr
    v(i-1) = z_2(i)-z_2(i-1);
end

for i=3:itr
    rho(i-2) = z_2(i)-z_2(i-2);
end

new_w_variance_2=v.^2;
Ev=sum(new_w_variance_2)/(itr-1);

new_n_variance_2=rho.^2;
Erho=sum(new_n_variance_2)/(itr-2);

new_w_variance_2 = Erho - Ev
```

```
new_w_variance_2 = 12.4747
```

```
new_n_variance_2 = (Ev - new_w_variance_2)/2
```

```
new_n_variance_2 = 5.9860
```

The calculated variance(w) and variance(n) for iterations of 300 are less accurate from that of the true value of variance(w) and variance(n).

Optimal smoothing coefficent

```
x = new_w_variance_2/new_n_variance_2
```

```
x = 2.0840
```

```
alpha = (-x + sqrt(x^2 + 4*x))/2
```

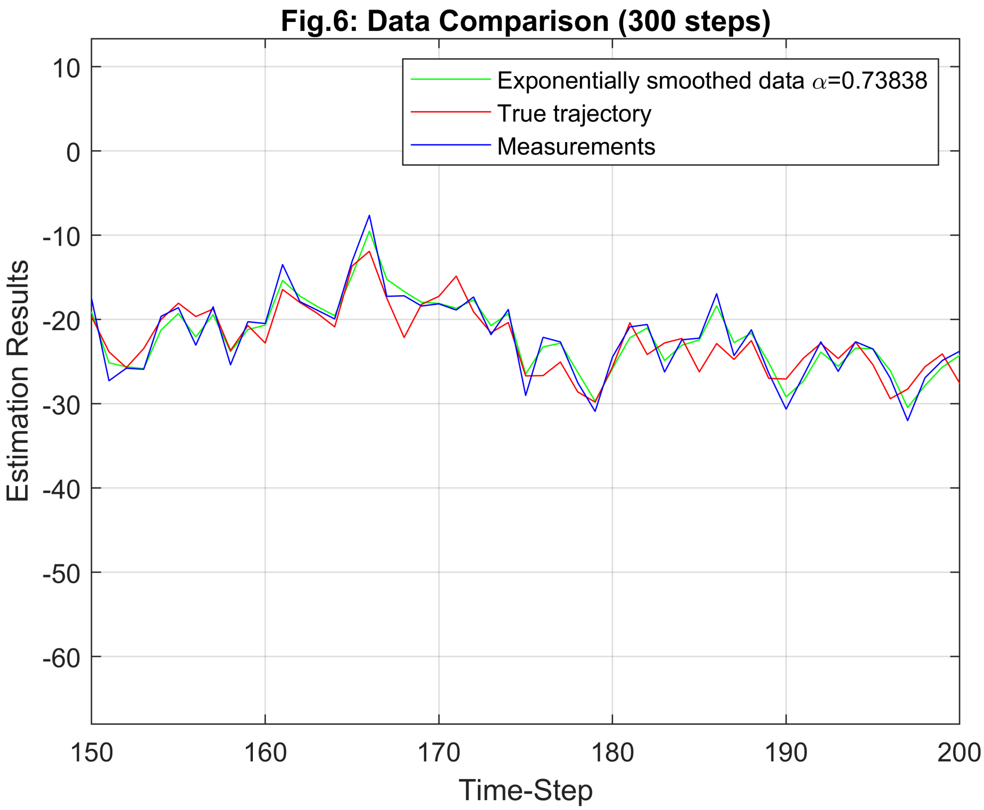
```
alpha = 0.7384
```

Exponential Smoothing:

```
X_smooth = z_2;
for i = 2:itr
    X_smooth(i) = X_smooth(i-1) +  alpha*(z_2(i) - X_smooth(i-1));
end
```

Graph for exponential smoothing with comparison with measurements, true values of process and exponentially smoothed data:

```
figure
plot(X_smooth,'g')
hold on
plot(X_2,'r')
plot(z_2,'b')
grid on
axis([150 200 -inf inf])
xlabel("Time-Step")
ylabel("Estimation Results")
legend(['Exponentially smoothed data \alpha=', num2str(alpha)],'True trajectory','Measurements'
title('Fig.6: Data Comparison (300 steps)')
```



Conclusions:

- The true tracjectory was constructed with random noise at the given variance of 13 and initial condition. Measurements were created with the true trajectory and given variance of 8. The true trajectory and measurement are a set of random values which depends on the previous iterate.
- The variance(w) and variance(n) were calculated using the identification method for both the case of 3000 and 300 iterations. The values of variance(w) and variance(n) for 3000 iterations were much closer and precise with the true values of the variance(w) and variance(n) than in case of 300 iterations. This lets us conclude that more precise accuracy can be obtained for more number of iterations in a given model.
- The optimal smoothing coefficient (alpha) for exponential smoothing was calculated. It lies in the range from 0 to 1. The value of alpha changes with change in the noise (random values). For this problem optimal alpha value is about 0.68 in both cases (300 and 3000 points).
- A comparitive graph is plotted between the true values, measurements and exponentially smoothen data at that particular value of alpha for both 3000 and 300 iteration cases. The exponentially smoothen data follows the measurement data with very small disturbances and reduced noise, thus providing good results.