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Section - H
Roll No. - 27

DAA

Tutorial - 6

Ans 1) Minimum spanning tree - A MST is a subset of the edges of a connected, edge-weighted undirected graph that connects all the vertices together, without any cycles and with the minimum possible total edge weight.

Applications -

i) Consider n station are to be linked using a communication network and lying of communication link b/w any two station involved a cost. The ideal solution could be ~~a~~ to exact a subgraph termed as minimum cost spanning tree.

ii) Suppose you meant to construct highways or railroads spanning several cities then we can use the concept of minimum spanning tree.

iii) Design LAN

iv) Laying pipelines connecting offshore drilling sities, refineries and consume markets.

Ans 2 Prim's algo - $O((V+E)\log)$

Space complexity $O(V)$

Kruskal's algo

Time complexity - $O(E(\log V))$

Space complexity - $O(|V|)$.

Dijkstra algo

Time Complexity - $O(V^2)$

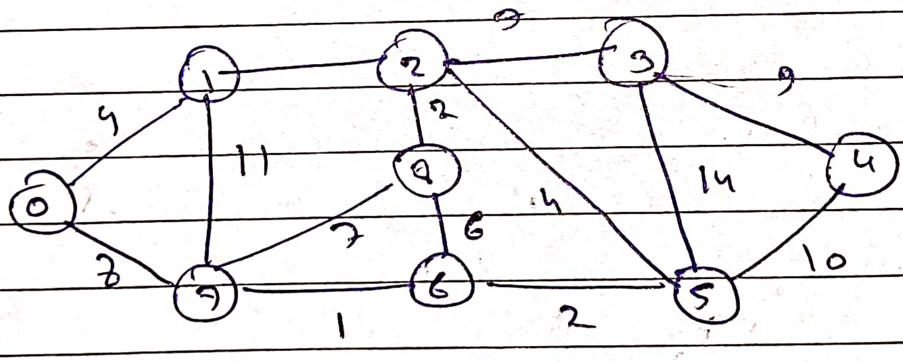
Space Complexity - $O(V^2)$

Bellman ford

Time Complexity - $O(VE)$

Space Complexity - $O(E)$

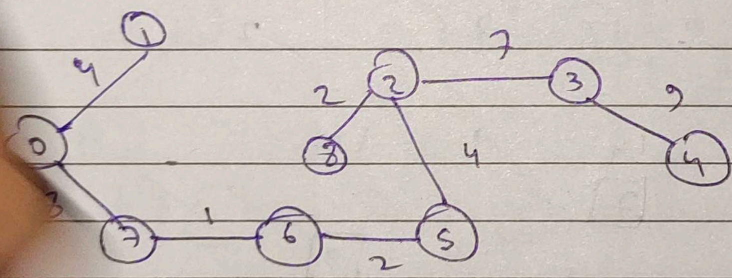
Ans 3



Kruskal's algo

O	V	W	
6	7	1	✓
5	6	2	✓
2	8	2	✓
0	1	4	✓
2	5	4	✓
6	8	6	✗
2	3	7	✓
7	8	7	✗
0	7	8	✓
1	2	8	✗
4	3	9	✓
4	5	10	✗

O	V	W	
1	7	11	✗
3	5	14	✗

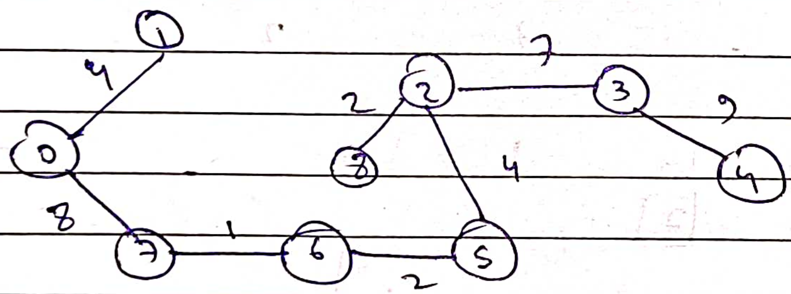


weight = 1+2+2+2+4+4+7+8+9 = 37

Kruskal's algo

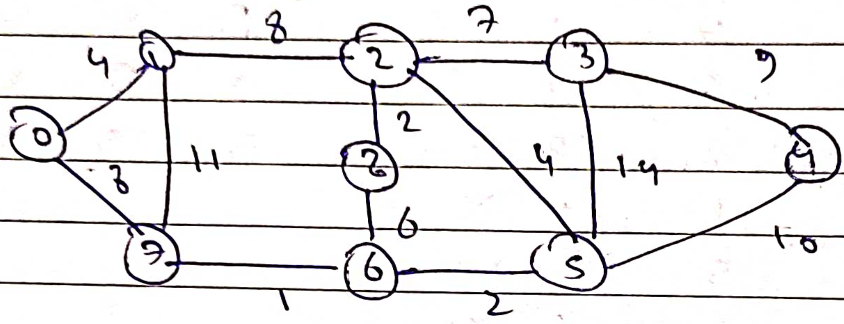
O	V	W	
6	7	1	✓
5	6	2	✓
2	8	2	✓
0	1	4	✓
2	5	4	✓
6	8	6	✗
2	3	7	✓
7	8	7	✗
0	7	8	✓
1	2	8	✗
4	3	9	✓
4	5	10	✗

O	V	W	
1	7	11	✗
3	5	14	✗



weight = 1+2+2+2+4+4+7+8+9 = 37

Prim's algo



weight

0	1	2	3	4	5	6	7	8
$\boxed{0}$	∞	∞	∞	∞	∞	∞	∞	∞
	$\boxed{4}$						8 $\boxed{8}$	
	8	8				$\boxed{1}$		7
	11		7		4	1		$\boxed{2}$
			7		2			6
	4	14	1	10				
		$\boxed{7}$						
				$\boxed{9}$				

Key

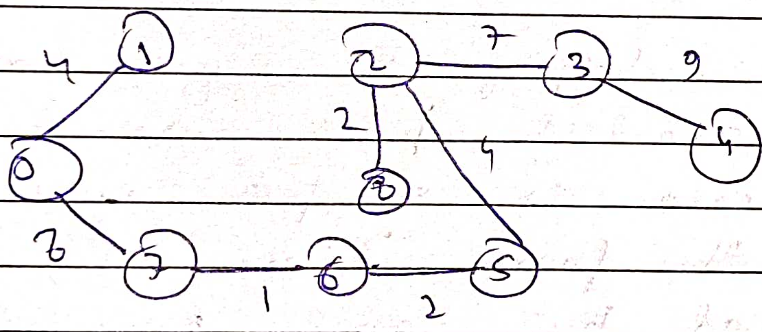
		4	7	9				
	4	~~8~~	~~14~~	~~16~~	2	1	8	~~6~~ 2
0	~~∞~~	~~∞~~	~~∞~~	~~∞~~	~~∞~~	~~∞~~	~~∞~~	~~∞~~
0	1	2	3	4	5	6	7	8

MST

	T	T	T	T	T	T	T	T
T	~~F~~	~~F~~	~~F~~	~~F~~	~~F~~	~~F~~	~~F~~	~~F~~
0	1	2	3	4	5	6	7	8

Parent

		~~8~~ 2		3				
	0	~~1~~	5	8	6	7	0	~~1~~ 2
-1	~~-1~~	~~-1~~	~~-1~~	~~-1~~	~~-1~~	~~-1~~	~~-1~~	~~-1~~
0	1	2	3	4	5	6	7	8



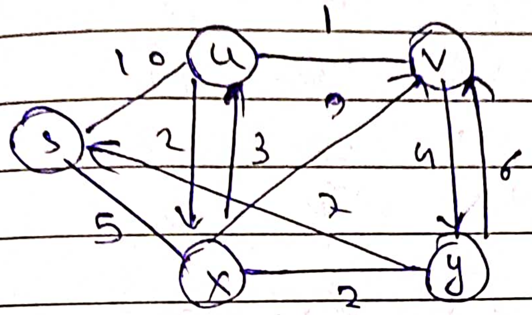
weight = 4 + 8 + 1 + 2 + 4 + 2 + 7 + 9

= 37

Ans 4 i) The shortest path may change. The reason is there may be different no. of edges in different paths from 's' to 't'. eg Let shortest path be of weight 15 & has edge 5. Let there be another path with 2 edges and total weight 25. The weight of the shortest path is increased ~~by~~ (5*10) and becomes 15 + 50. Weight of other path is increased to (2*10) and becomes 25 + 20 so the shortest path changes to the other path with weight as 45.

ii) If we multiply all edges weight by , the shortest path don't change. The reason is simple, weight of all path from 's' to 't' get multiplied by same amount. The no. of edges on a path don't matter. It's like changing limits of weight.

Ans 5 Dijkstra algo.



A	B	C	D	E
0	∞	∞	∞	∞
	10	3	∞	∞
	7		11	5

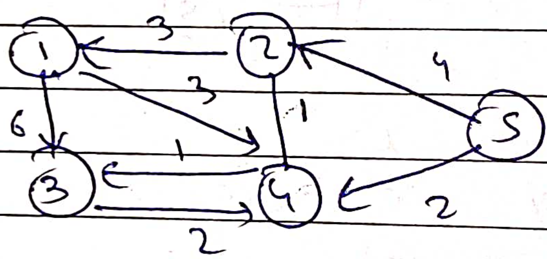
s	u	v	x	y
0	∞	∞	∞	∞
	10		5	
	8	14		7
		13		

node	shortest distance
u	8
x	5
v	13
y	7

Bellman ford algo

	s	u	v	x	y
1st	0	10	∞	5	∞
2nd	0	10	11	5	∞
3rd	0	8	9	5	7
4th	0	8	9	5	7

Ans 6



$$G_1 = \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & \infty & \infty & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ \infty & 4 & \infty & 2 & 0 \end{bmatrix}$$

$$G_2 = \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ 4 & 1 & 1 & 0 & \infty \\ 7 & 4 & 13 & 2 & 0 \end{bmatrix}$$

$$G_3 = \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ 4 & 1 & 1 & 0 & \infty \\ 7 & 4 & 13 & 2 & 0 \end{bmatrix}$$

$$G_4 = \begin{bmatrix} 0 & 4 & 4 & 3 & \infty \\ 3 & 0 & 7 & 6 & \infty \\ 6 & 3 & 0 & 2 & \infty \\ 4 & 1 & 1 & 0 & \infty \\ 6 & 3 & 3 & 2 & 0 \end{bmatrix}$$

$$G_5 = \begin{bmatrix} 0 & 4 & 4 & 3 & \infty \\ 3 & 0 & 7 & 6 & \infty \\ 6 & 3 & 0 & 2 & \infty \\ 4 & 1 & 1 & 0 & \infty \\ 6 & 3 & 2 & 2 & 6 \end{bmatrix}$$

$$G_3 = \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ 4 & 1 & 1 & 0 & \infty \\ 7 & 4 & 13 & 2 & 0 \end{bmatrix}$$

$$G_4 = \begin{bmatrix} 0 & 4 & 4 & 3 & \infty \\ 3 & 0 & 7 & 6 & \infty \\ 6 & 3 & 0 & 2 & \infty \\ 4 & 1 & 1 & 0 & \infty \\ 6 & 3 & 3 & 2 & 0 \end{bmatrix}$$

$$G_5 = \begin{bmatrix} 0 & 4 & 4 & 3 & \infty \\ 3 & 0 & 7 & 6 & \infty \\ 6 & 3 & 0 & 2 & \infty \\ 4 & 1 & 1 & 0 & \infty \\ 6 & 3 & 2 & 2 & 6 \end{bmatrix}$$