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Section -N
Roll No. - 28 Date: / / /

Tutorial - 4

Q1 $T(n) = 3T(n/2) + n^2$

$a = 3, \quad b = 2$

$n^{\log_b a}$

$n^{\log_2 3}$

$f(n) = n^2$

$f(n) > n^{\log_2 3}$

$\therefore O(n^2)$

Q2 $T(n) = 4T(n/2) + n^2$

$a = 4 \quad b = 2$

$n^{\log_2 2^2}$

n^2

$f(n) = n^{\log_b a}$

$O(n^2 \cdot \log n)$

Q3 $T(n) = T(n/2) + 2^n$

$a = 1 \quad b = 2$

$n^{\log_2 1}$

~~$f(n)$~~ $\cdot \; n^0 = 1$

$f(n) > n^{\log_b a}$

$O(2^n)$

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Q4 $T(n) = 2^n T(n/2) + n^n$

cannot apply Master theorem because a is not constant.

Q5 $T(n) = 16\,T(n/4) + n$

$a = 16 \quad b = 4$

$n^{\log_4 16}$

$n^{\log_4 4^2}$

n^2

$f(n) = n^{\log_b a}$

$O(n^2)$

Q6 $T(n) = 2\,T(n/2) + n\log n$

$f(n) = n\log n$

$k = 1$

$T(n) = O(n\log^{k+1} n)$

$= O(n\log^2 n)$

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Q7 $T(n) = 2T(n/2) + \frac{n}{\log n}$

Master theorem does not apply because $n/\log n$ is not a polynomial.

Q8 $T(n) = 2T(n/4) + n^{0.51}$

Master theorem does not apply because $n^{0.51}$ is not a polynomial.

Q9 $T(n) = 0.5\,T(n/2) + 1/n$

$a = \frac{1}{2} \qquad b = \frac{1}{2}$

$n^{\log_2 2^{-1}}$

n^{-1}

$f(n) = \log_b 9$

$T(n) = O\left(\frac{1}{n}\log n\right)$

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Q10 $T(n) = 16T(n/4) + n$

$a = 16 \quad b = 4$

$n^{\log_4 4^2}$

$n^2 < f(n)$

$T(n) = O(n)$

Q11 $T(n) = 4T(n/2) + \log n$

$a = 4 \quad b = 2$

$n^{\log_2 2^2}$

n^2

$f(n) < n^{\log_b a}$

$T(n) = O(n^2)$

Q12 $T(n) = \sqrt{n}\, T(n/2) + \log n$

Master theorem does not apply because a is not constant.

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Q.13 $T(n) = 3T(n/2) + n$

$a = 3 \quad b = 2$

$n^{\log_b a} \Rightarrow n^{\log_2 3}$

$f(n) > n^{\log_b a}$

$T(n) = O(n)$

Q.14 $T(n) = 3T(n/3) + \sqrt{n}$

Master theorem is not applicable because $\sqrt{n}$ is not polynomial.

Q.15 $T(n) = 4T(n/2) + cn$

$a = 4 \quad b = 2$

$n^{\log_2 2^2}$

n^2

$= n^{\log_b a} \Rightarrow f(n)$

$T(n) = O(n^2)$

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Q16 $T(n) = 3T(n/4) + n\log n$

$a = 3 \quad b = 4$

$n^{\log_b a} = n^{\log_4 3}$

$f(n) = n\log n$

$k = 1$

$T(n) = \Theta(n\log^{k+1} n)$

$= \Theta(n\log^2 n)$

Q17 $T(n) = 3T(n/3) + n/2$

Master's theorem ~~not~~ applicable

$a = 3, \quad b = 3 \quad f(n) = \frac{n}{2}$

$n^{\log_b a} = n^{\log_3 3} = n$

$T(n) = \Theta\left(\frac{n}{2}\right)$

$f(n) = \log_b a$

$T(n) = \Theta(n\log n)$

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Q18. $T(n) = 6T(n/3) + n^2 \log n$

$a = 6, \quad b = 3$

$f(n) = n^2 \log n$

$n^{\log_b a} = n^{\log_3 6} = n^{1.63}$

$n^{1.63} < n^2 \log n$

$T(n) = O(n^2 \log n)$

Q19 $T(n) = 4T(n/2) + n/\log n$

$a = 4, \quad b = 2, \quad f(n) = n/\log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > n/\log n$

$T(n) = O(n^2)$

Q20 $T(n) = 7T(n/3) + n^2$

$a = 7, \quad b = 3 \quad f(n) = n^2$

$n^{\log_b a} = n^{\log_3 7}$

$n^{\log_3 7} < n^2$

$T(n) = O(n^2)$

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Q21. $T(n) = 64\,T(n/8) - n^2 \log n$

Master's theorem is not applicable

Q22. $T(n) = T(n/2) + n(2 - \cos n)$

Master's theorem is not applicable since $f(n)$ is not a polynomial.