MLDS-413 Introduction to Databases and Information Retrieval

Homework 1: Number representations

Name 1: _____

NetID 1: _____

Name 2: _____

NetID 2: _____

Instructions

You should submit this homework assignment via Canvas. Acceptable formats are word files, text files, and pdf files. Paper submissions are not allowed and they will receive an automatic zero.

As explained during lecture and in the syllabus, assignments are done in groups. The groups have been created and assigned. Each group needs to submit only one assignment (i.e., there is no need for both partners to submit individually the same homework assignment).

Each group can submit solutions multiple times (for example, you may discover an error in your earlier submission and choose to submit a new solution set). We will grade only the last submission and ignore earlier ones.

Make sure you submit your solutions before the deadline. The policies governing academic integrity, tardiness and penalties are detailed in the syllabus.

Question 1. Unsigned Integer Representation (10 points – 1 point per row)

Please fill out the blank parts of the table below to (i) express the following numbers in binary, (ii) calculate their hexadecimal representation (as shown in class), and (iii) calculate their decimal value as a sum of powers of two.

For example: $52_{10} = 0011\ 0100_2 = 0x34 = 1*2^5 + 1*2^4 + 0*2^3 + 1*2^2 + 0*2^1 + 0*2^0 = 32 + 16 + 4$

	DECIMAL NUMBER	BINARY	HEXADECIMAL	SUM OF POWERS OF TWO
EXAMPLE	52	**0011 0100**	**0x34**	**32 + 16 + 4**
1A)	6			
1B)	19			
1C)	22			
1D)	38			
1E)	42			
1F)	155			
1G)	612			
1H)	1819			
1I)	2293			
1J)	3176			

Question 2. Signed Integer Representation (10 points – 1 point per row)

Please fill out the blank parts of the table below as needed to calculate the 8-bit two's complement binary representation of the following signed decimal numbers. |x| denotes the absolute value of x. If particular number(s) below cannot be represented as 8-bit signed binary integer(s), please indicate which one(s) and explain why. The explanation will then carry the points of the corresponding row(s). Please remember that:

- You may need to add zeros to the left of the number to make it an 8-bit binary number.
- In two's complement signed integer representation, for every (positive or negative) integer x, it holds: -x = ~x + 1, where ~x is the complement of x (calculated by flipping the bits of x).
- If you don't need a column to calculate the 8-bit signed integer binary representation of a number, you can leave that column blank (as in example 1).

For example: 82_{10} = 0101 0010_2. Note that we added a zero at the front to make it an 8-bit binary number. The sign bit (most significant bit of its 8-bit binary representation) is 0, indicating a positive integer, so the 8-bit signed integer binary representation of 82_{10} is 0101 0010_2.

However, to calculate the 8-bit signed integer representation of -82 we need to calculate its two's complement. We have $|-82_{10}| = 82_{10}$ = 0101 0010_2. Flipping the bits gives 1010 1101_2. Finally, adding one gives 1010 1110_2. Thus, the 8-bit signed binary representation of -82_{10} is 1010 1110_2.

	VALUE OF INTEGER X	|X| IN 8-BIT BINARY	~|X| IF NEEDED	8-BIT SIGNED INTEGER REPRESENTATION
EXAMPLE 1	82	**0101 0010 = 82_{10}**		**0101 0010 = 82_{10}**
EXAMPLE 2	-82	**0101 0010 = 82_{10}**	**1010 1101**	**1010 1110 = -82_{10}**
2A)	-1			
2B)	-19			
2C)	22			
2D)	-38			
2E)	-42			
2F)	68			
2G)	-100			
2H)	127			
2I)	128			
2J)	-129			

If you found any numbers that you cannot represent as 8-bit signed integers, indicate which ones and why they cannot be represented in the space below.

Question 3. Binary Pattern Representation (10 points)

BIT PATTERN 1	BIT PATTERN 2
1101 1110 1010 1101 1011 1110 1110 1111	0100 1100 0100 1111 0100 1100 0010 0001

What value does bit pattern 1 represent when interpreted as a

3a) **(2 points)** 32-bit signed integer in two's complement arithmetic?

3b) **(2 points)** 32-bit unsigned integer?

3c) **(1 point)** write the value in question 3b as a hexadecimal number

What value does bit pattern 2 represent when interpreted as a

3d) **(2 points)** 32-bit signed integer in two's complement arithmetic?

3e) **(2 points)** 32-bit unsigned integer?

3f) **(1 point)** UTF-8 text?

Question 4. Accuracy of Integer and Floating Point Representations (20 points – 1 per row)

In the table below, answer "***exactly***," "***approximately***," or "***no way***" to indicate how the following (base ten) numbers can be represented as 32-bit signed integers, 64-bit signed integers, single precision (32-bit) floats, and double precision (64-bit) floats. If the number can be rounded to a value that is representable and the relative error is less than 10^{-3}, then the best answer is "approximately." For example, I would say that 2.5 is "no way" 3, but 2.9999999 is "approximately" 3. The relative error definition is in the reading material and was explained in the lab session.

For example, consider the number $1.0000000005 \times 10^{30}$. It cannot be represented as 32-bit signed integer nor as a 64-bit signed integer (because it is too large). However, it can be represented approximately as a single precision float (with rounding – there are not enough bits in the mantissa of the 32-bit float to represent the 0.0000000004 fractional part of the number, but the relative error is $<10^{-3}$). Moreover, it can be represented exactly as a double precision float.

You can use online floating-point conversion tools (e.g., http://www.binaryconvert.com/convert_float.html).

	BASE TEN NUMBER	32-BIT *SIGNED* INTEGER	64-BIT *SIGNED* INTEGER	32-BIT FLOATING POINT	64-BIT FLOATING POINT
EXAMPLE	1.0×10^{20}	**no way**	**no way**	**approximately**	**exactly**
4A	-0.5				
4B	1/3 (one third)				
4C	0.1				
4D	1/16 (one sixteenth)				
4E	17.5				
4F	4,000,000,000				
4G	-4,000,000,001				
4H	$\sqrt{2}$				
4I	2,000,000,000				
4J	2,000,000,001				
4K	20,000,000				
4L	20,000,000.25				
4M	33,554,432.25				
4N	33,554,432.9				
4O	9,123,000,000,000,000,000				
4P	-9,123,123,123,000,000,000				
4Q	9,123,123,123,123,123,123				
4R	pi				
4S	infinity				
4T	zero				