Practice Midterm:

Markov Chain Q1:
Consider the Markov chain with three states, S={1,2,3}, that has the following transition matrix

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}.$$

a. Draw the transition diagram for the Markov chain.
b. If we know $P(X_1=1) = P(X_1 = 2) = 0.25$, find $P(X_1 = 3, X_2=2, X_3=1)$.

Markov Chain Q2:
80% of students who do maths work during one study period, will do the maths work at the next study period. 30% of students who do english work during one study period, will do the english work at the next study period.Initially there were 60 students do maths work and 40 students do english work. Calculate,

(a) The transition probability matrix.
(b) The number of students who do maths work, english work for the next subsequent 2 study periods in R.

Markov Chain Q3:
Assume that a man's profession can be classified as professional, skilled labourer, or unskilled labourer. Assume that, of the sons of professional men, 80 percent are professional, 10 percent are skilled labourers, and 10 percent are unskilled labourers. In the case of sons of skilled labourers, 60 percent are skilled labourers, 20 percent are professional, and 20 percent are unskilled. Finally, in the case of unskilled labourers, 50 percent of the sons are unskilled labourers, and 25 percent each are in the other two categories. Assume that every man has at least one son, and form a Markov chain by following the profession of a randomly chosen son of a given family through several generations.

(a) Write the transition probability matrix.
(b) Find the probability that a randomly chosen grandson of an unskilled labourer is a professional man.

SVM Q1:
Consider the linear SVM model, write the linear function used by the SVM, and explain how an input vector x is assigned to the classes using the linear function.

SVM Q2:
Consider the following training data:

| class | $x_1$ | $x_2$ |
|---|---|---|
| $+$ | 1 | 1 |
| $+$ | 2 | 2 |
| $+$ | 2 | 0 |
| $-$ | 0 | 0 |
| $-$ | 1 | 0 |
| $-$ | 0 | 1 |

(a) Are the two classes linearly separable?
(b) Find the maximum margin linear classifier (You can use R).

SVM Q3:
Consider the following linearly separable 4 samples,

$$\mathrm{X} = \begin{bmatrix} 0 & 0 \\ 2 & 2 \\ 2 & 0 \\ 3 & 0 \end{bmatrix} \quad \mathbf{y} = \begin{bmatrix} -1 \\ -1 \\ +1 \\ +1 \end{bmatrix}$$

(a) Write out the SVM optimization problem. i.e. min a, subject to a + 1 $\geq$ 0.
(b) Solve the optimization problem. (You can use R)