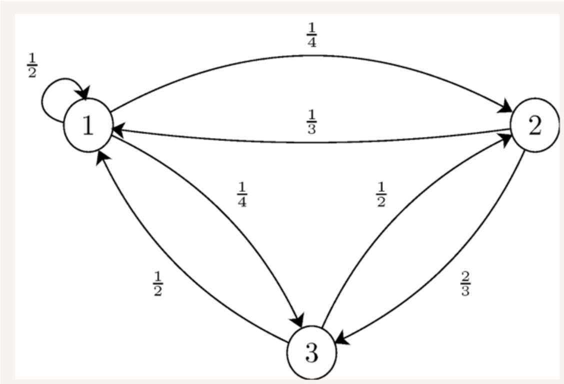
Practice Midterm Solution

Markov Chain 1.a:



b:

First, we obtain

$$\begin{aligned} P(X_1 = 3) &= 1 - P(X_1 = 1) - P(X_1 = 2) \\ &= 1 - \frac{1}{4} - \frac{1}{4} \\ &= \frac{1}{2}. \end{aligned}$$

We can now write

$$\begin{aligned} P(X_1 = 3, X_2 = 2, X_3 = 1) &= P(X_1 = 3) \cdot p_{32} \cdot p_{21} \\ &= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{3} \\ &= \frac{1}{12}. \end{aligned}$$

Markov Chain 2

$$T = \begin{array}{c c} & \begin{array}{cc} M & E \end{array} \\ \begin{array}{c} M \\ E \end{array} & \begin{pmatrix} 0.8 & 0.2 \\ 0.7 & 0.3 \end{pmatrix} \end{array}$$

After one study period,

$$\begin{array}{c} \begin{array}{cc} M & E \end{array} \\ \begin{pmatrix} 60 & 40 \end{pmatrix} \end{array} \begin{array}{c c} & \begin{array}{cc} M & E \end{array} \\ \begin{array}{c} M \\ E \end{array} & \begin{pmatrix} 0.8 & 0.2 \\ 0.7 & 0.3 \end{pmatrix} \end{array} = \begin{array}{c} \begin{array}{cc} M & E \end{array} \\ \begin{pmatrix} 76 & 24 \end{pmatrix} \end{array}$$

So in the very next study period, there will be 76 students do maths work and 24 students do the English work.

After two study periods,

$$\begin{array}{c} \begin{array}{cc} M & E \end{array} \\ \begin{pmatrix} 76 & 24 \end{pmatrix} \end{array} \begin{array}{c c} & \begin{array}{cc} M & E \end{array} \\ \begin{array}{c} M \\ E \end{array} & \begin{pmatrix} 0.8 & 0.2 \\ 0.7 & 0.3 \end{pmatrix} \end{array} = \begin{pmatrix} 60.8+16.8 & 15.2+7.2 \end{pmatrix}$$

$$= \begin{pmatrix} 77.6 & 22.4 \end{pmatrix}$$

After two study periods there will be 78 (approx) students do maths work and 22 (approx) students do English work.

Markov Chain 3

	Professional	Skilled	Unskilled
Professional	.8	.1	.1
Skilled	.2	.6	.2
Unskilled	.25	.25	.5

so that the transition matrix for this chain is

$$\mathsf{P} = \begin{pmatrix} .8 & .1 & .1 \\ .2 & .6 & .2 \\ .25 & .25 & .5 \end{pmatrix}$$

with

$$\mathsf{P}^2 = \begin{pmatrix} 0.6850 & 0.1650 & 0.1500 \\ 0.3300 & 0.4300 & 0.2400 \\ 0.3750 & 0.3000 & 0.3250 \end{pmatrix},$$

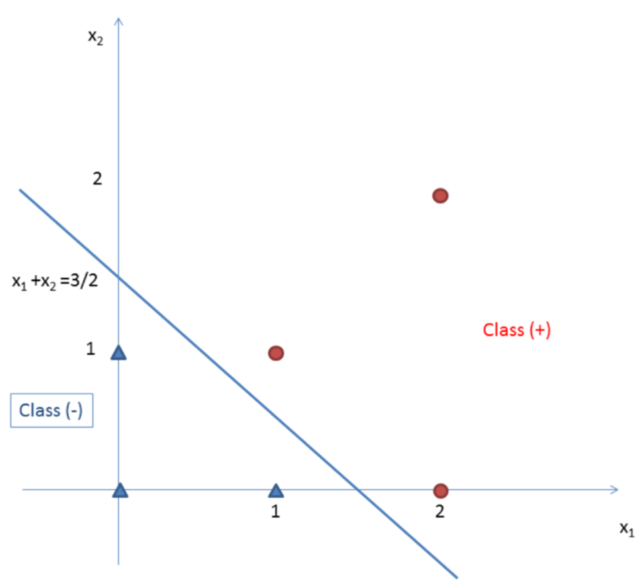
and thus the probability that a randomly chosen grandson of an unskilled labourer is a professional man is 0.375.

SVM 1:

SVMs find a linear function of the form $f(\mathbf{x}) = \langle \mathbf{w} \cdot \mathbf{x} \rangle + b$ where $\mathbf{x}$ is the input vector, $\mathbf{w}$ is a vector of weights, and b is the *bias*. An input vector $\mathbf{x_i}$ is assigned to the positive (1) or negative class (-1) as follows

$$y_i = \begin{cases} 1 & if \langle \mathbf{w} \cdot \mathbf{x}_i \rangle + b \geq 0 \\ -1 & if \langle \mathbf{w} \cdot \mathbf{x}_i \rangle + b < 0 \end{cases}$$

SVM 2a:Yes, see picture below: (know how to express the separability using equations)



b. See the line in above graph. (Be sure to know how to find it using R)

SVM 3a:

$$\begin{aligned} -b &\geq 1 \\ -(2w_1 + 2w_2 + b) &\geq 1 \\ 2w_1 + b &\geq 1 \\ 3w_1 + b &\geq 1 \end{aligned}$$

b: Be sure to know how to use svm() in R to find the parameters

$$(b^* = -1, w_1^* = 1, w_2^* = -1)$$