The tradeoff between observation accuracy and performance in control systems

Project proposal document
for Visiting Undergraduate Research Program
by

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1 Introduction

Over the past few decades the fields of communication and control have been developed extensively to support the emerging needs of the society such as higher speed internet, better telephone communication networks, wireless access to devices over Internet of Things (IoT), reliable long distance satellite communication, home automation and many other related fields. Communication theory is mainly concerned with the transmission of information from one point to another reliably without putting much concern over what is done with that information once it is transmitted. ~~Whereas,~~ control theory mainly deals with the feedback of information in-order to achieve better performing systems, which are not prone to disturbances, noises and other process variations. In the past few decades, ~~the concept of networked control systems have gained prominence as we have aimed to~~ control systems without physical inter-connection between ~~its~~ subsystems. Signals between sensors, controllers and the plant ~~being controlled~~ are transmitted over communication channels. Networked control finds wide applications but also poses newer challenges which control theory deosn't deal with. There are various additional constraints that the designer has to keep in mind while desigining the control system such as noise in the communication channels, quantization effects for digital communication devices, time delays, packet losses etc. ~~Often, to achieve desired performance out of a system we aim to design optimal controllers.~~

Optimal control design deals with optimization (often minimization) of a performance index subject to system state equation constraints to obtain an optimal control law. When the performance index is a quadratic function of states and control input (for a deterministic system), this optimal control design is commonly referred to as the Linear Quadratic Regulator (LQR) control problem. ~~Though,~~ the LQR control gives good robustness features~~, but~~ a big disadvantage with LQR design is that it requires measurement of all the states. Usually in physical systems, this is not possible. Even when it is possible to measure all the states, it is usually very expensive. To overcome this problem, the states are computationally estimated and then the estimated states are used to design the control. The estimation based control poses its own challenges. The estimation of states for physical systems is seldom free of noise. This inaccuracy in estimation results in deterioration of the performance. This project aims to deal with ~~this~~ trade-off between estimation (observation) accuracy and performance in networked control systems.

It is intuitively understood that the better the observation accuracy, the better the system performance would be. In this document, we will first cover the literature in brief and some well known theoretical concepts ~~related to this project would be covered in some detail~~. We will end with proposing the major objectives for the project~~ and a rough plan to achieve the same will be presented~~.

2 Background and Literature Review

To study the trade-off between observation accuracy and minimum achievable performance index for a system, various concepts need to be reviewed. We are ~~specifically~~ interested in ~~this trade-off~~ for a networked control system facing constraints such as quantized estimation (limited data rate of communication) and multiplicative uncertainity in control signal or system state(s). These form the background of the work that would be done in this project. Also, the control scheme that we would be dealing with the most in this project is the Linear Quadratic Guassian (LQG) control. We start by going over the well-known concepts of partially observed LQG problem. ~~Also, it is important to note here that the discussions in this document is limited to an introductory level only and mathematical proofs of various statements made have been skipped. Proper references are given and the reader is advised to refer to the same for further details.~~

2.1 Classical Partially Observed LQG Problem

For a stochastic system with a quadratic performance index, the LQG control scheme ~~involved estimating~~ the states from the available outputs and then ~~using~~ these states for control so that the performance index is minimized. Consider the stochastic system, the state equation can be written as

$$x_{t+1} = Ax_t + Bu_t + \Gamma w(t) \tag{1}$$

The output equation is

$$y_t = Cx_t + Lv_t \tag{2}$$

where w_t is the noise in the plant which is assumed to be zero mean guassian and v_t is the measurement noise, which is also assumed to be zero mean gussian noise. We also assume that the initial value of the state x, x_0 is known and that w, v and x_0 are all independent of each other~~. It is also assumed~~ that the system is stationary which ensures that the mean and covariances will not change with time. The following block diagram representation gives a clearer picture of LQG control.

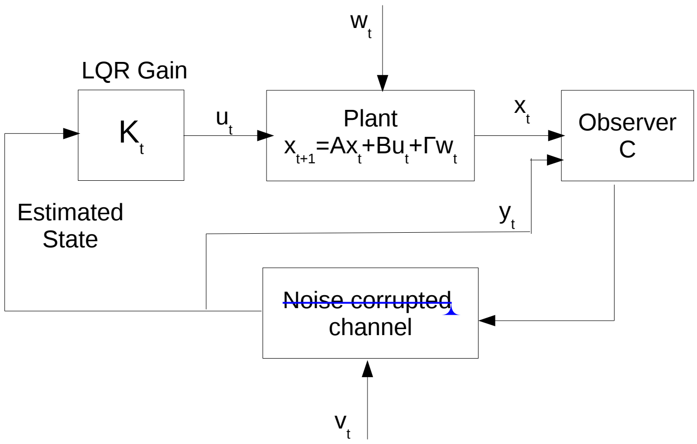

Figure 1: Representative Block Diagram of Classical Partially Observed LQG Control

Our aim is to find a control law u_t so that the following performance index is minimized

$$J(\cdot) = E\left(\sum_{t=0}^{N-1}\left(x_t^T Q x_t + u_t^T R u_t\right) + x_N^T Q x_N\right) \tag{3}$$

The control law is given by

$$u_t = K_t E[x_t|y_t] \tag{4}$$

where K_t is the LQR gain and $E[x_t|y_t]$ is the minimum mean square error estimate of x_t given the measurements y_t. ~~This can be found out by~~ Kalman Filter ~~design.~~ As we can see in Eq.(4), the estimate of the state is used for control and not the original states. It can be proved that this control law ~~leads to an optimal control design.~~ ~~Also,~~ the calculation of the LQR gain K_t and the MMSE estimate of the state can be performed independent of each other. ~~This is the well known Separation Principle. This has been covered in detail in~~ [1].

2.2 Quantization Constraints in Networked Control Systems

As mentioned in the previous section, guassian noises in the plant and measurement might not be the only ~~constraints in LQG control design to estimate the states correctly~~. If in Fig.(1) the observer data is transmitted via a communication channel to the controller. Other than the additive guassian noise v_t, the control performance would also be dependent on the data rate at which the communication is done. The limitation on this feedback data rate affects the system performance ~~and there~~ exists a trade-off between communication rate and optimal performance.

For a noiseless scalar plant with parameter $|a| > 1$, [2] shows that the minimum data rate required to keep the plant bounded is $\log_2 |a|$ bits per sample. This is known as the Data Rate Theorem. Similar results have been dervied for linear state-space systems which we are concerned with in this project. The reader is referred to [3] and references therein for further details.

2.3 Multiplicative Uncertainty in Networked Control Systems

Among various constraints in a networked control system such as time delays, data rate constraints, packet loss etc. is the parameter uncertainty constraint. The robust control techniques aim to deal with uncertain plant parameters and it is a very well researched topic ~~in that sense~~ ([4], [5]). Often, this parametric uncertainty takes a form of multiplicative uncertainty in either the control signal or the states.

From the control perspective, various techniques have been proposed to design controllers for systems with multiplicative uncertainty. Convex optimization based techniques have also been proposed using linear matrix inequality (LMI) techniques for control design [6]. An uncertainty threshold principle has been proposed in [7] which provides an explicit threshold on the uncertainties that can be tolerated in the systems with multiplicative uncertainties. An example of multiplicative uncertainty in a control system is explained using the following block diagram. The system shown below has multiplicative uncertainty in its states. Designing an observer based controller for such a system ~~would pose additional constraint because of~~ this uncertainty.

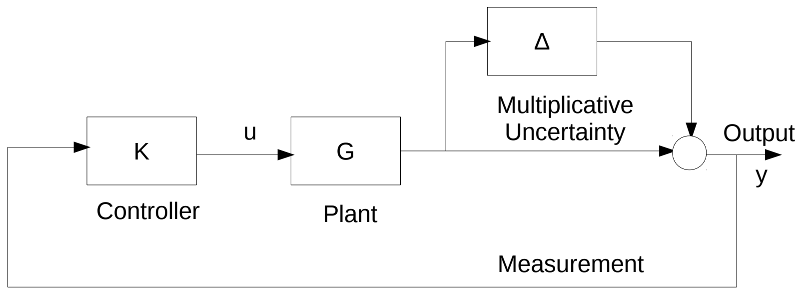

Figure 2: Multiplicative Uncertainty

The uncertain parameter is Δ which changes the plant transfer function effectively by a factor of $(1+\Delta)$. From Fig.(2) we have, the uncertain plant transfer function, $G_u = G(1 + \Delta)$, which is a multiplicative uncertainty in the states of the process.
The presence of such multiplicative uncertainties also affects the system performance. The trade-off between the magnitude of such uncertainties present in the system and the observation accuracy that can be achieved would be of interest to us in this project.

3 Objectives

For a given stochastic scalar system,

$$X_{t+1} = A_t X_t + BU_t \tag{5}$$
$$Y_t = CX_t + W_t \tag{6}$$

where X_{t+1} is the state being estimated from measurements Y_t. For this stochastic system, A_t is a random variable with known distribution and W is the noise in measurement.

The project objectives with respect to this scalar system can be expressed in brief as follows:

1. As described in Section(1), the controller observes Y_t and this is used to estimate the states. Under this observation, we would aim to find out the minimum $\lim_{t\to\infty} E[X_t^2]$ achievable in the limit.

2. To improve the estimation accuracy (and hence the performance) we may be interested to estimate the state X_{t+1} by making n measurements $Y_{t_1}, Y_{t_2}, ...Y_{t_n}$ of the output. From Eq.(6), we then have,

$$Y_{t_i} = CX_t + W_{t_i} \tag{7}$$

for $i = 1, 2, ..., n$ and where W_{t_i} are i.i.d. noise in the measurement. Clearly, the more observations we make the better the controller's knowledge of the system state is. However, the measurements can be expensive. So, this project aims ~~at dealing with this~~ tradeoff between the number of observations n and the minimum attainable mean square estimate $\lim_{t\to\infty} E[X_t^2]$.

4 Approach and Time-line

Initially, some more time would be spent on the literature related to the work in this project. The knowledge of linear quadratic control theory (LQR and LQG control), Riccati equations and their solutions in MATLAB would be very helpful in carrying out the research work on this project successfully and in the given period of time. For the first few weeks, after the literature has been reviewed completely, we would aim to draw out extensions from the existing theory on the trade-off between the MMSE estimate of the states and the observation accuracy for a simple scalar stochastic system. Once this is done, the case where n measurements are made would be studied and its effect on improvement in performance would be demonstrated. Proficiency with MATLAB programming would be an added advantage in completing this project in the given time. Some of my previous codes written using MATLAB's Robust Control Toolbox, Convex Optimization Toolbox, Control Systems Toolobx and Digital Signal Processing Toolbox are available at [8] and [9] respectively. Also, a repository of all future codes for this project is available at [10]. It would also have codes for progress reports and project proposal, including all images and diagrams.

References

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