Information - Performance Tradeoffs in Control

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Overview

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- ★ Control and Communication
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  - ○ Control over AWGN
  - ○ Rate-limited Control
  - ○ Multiplicative Uncertainties
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Control : An Overview

Interesting control examples

Control Techniques

Commonly used

- Hit and Trial : Experimental techniques
- Open loop control
- Frequency domain based methods
- Optimal control
- Robust control
- Intelligent control
- Nonlinear and adaptive control

In this project

Stochastic optimal control - LQG Control

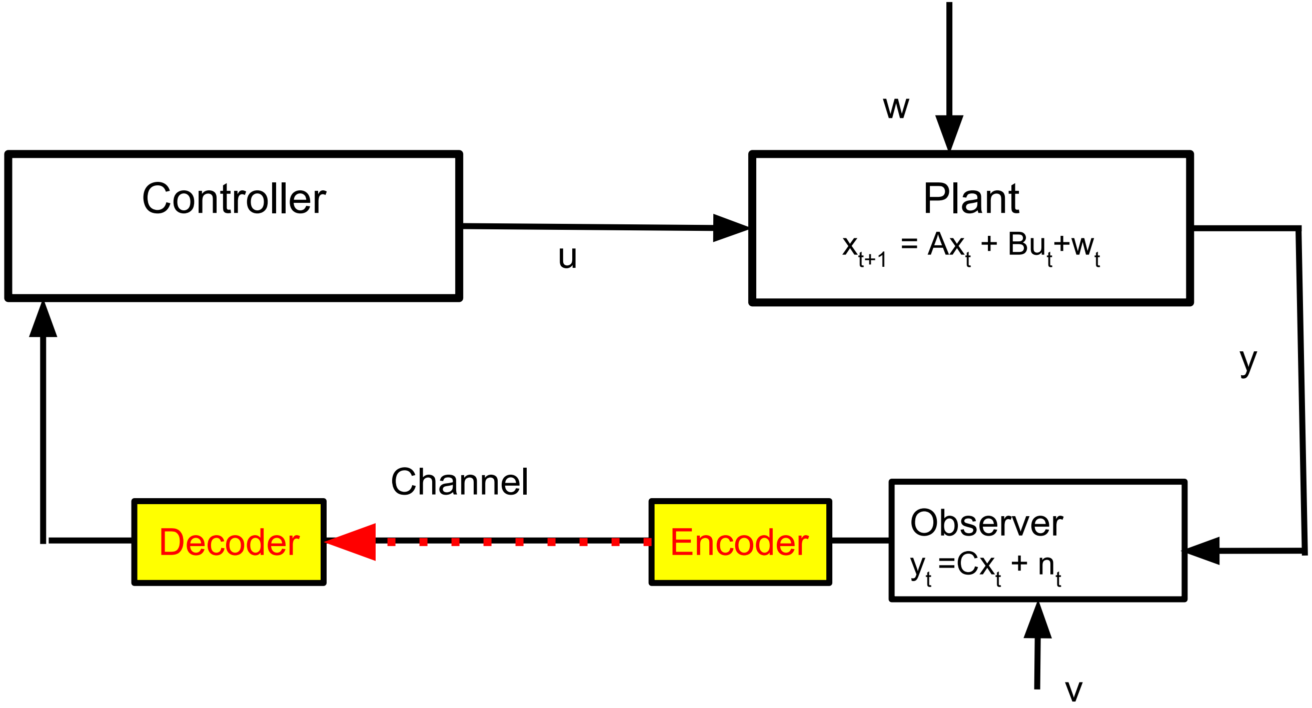
$$x_{t+1} = A_t x_t + B_t u_t + w_t$$
$$y_t = C_t x_t + v_t$$

Performance Cost for LQG:

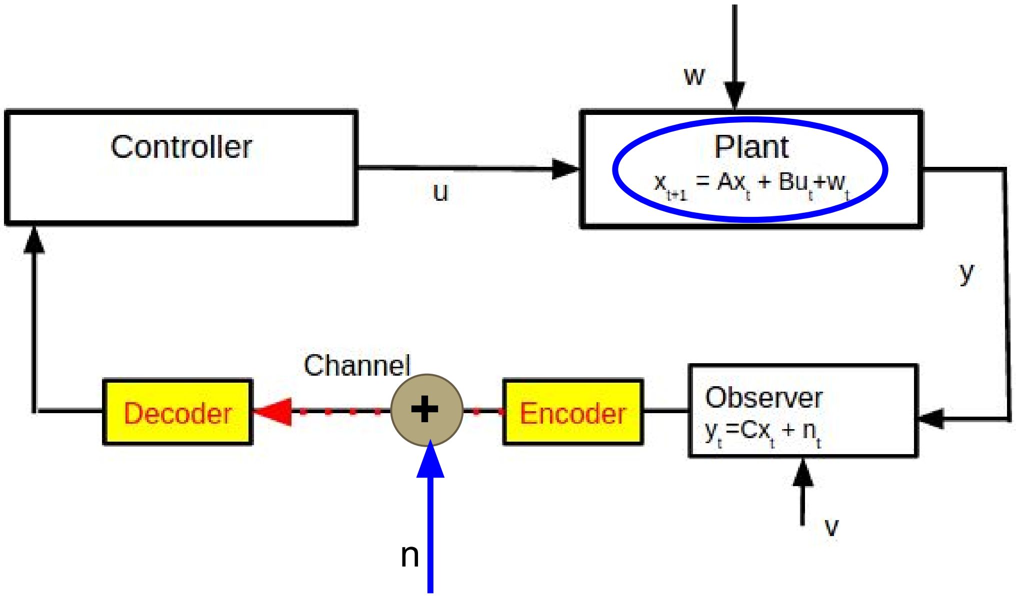
$$J(\cdot) = E\left(\sum_{t=0}^{N-1}\left(x_t^T Q x_t + u_t^T R u_t\right) + x_N^T Q x_N\right)$$

Find the optimal control policy that minimizes the cost.

Communication and Control



Three Important Questions



- What happens when the channel is noisy?
- What happens when the channel is rate-limited?
- What happens when the system has multiplicative uncertainties?

Control with Multiplicative Uncertainties

Question

To derive the stabilizability condition for:

$x_{t+1} = Ax_t + Bu_t + w_t$

$y_t = Cx_t + v_t$

where the system parameter A is uncertain (has a known probability distribution) and A > 1 (unstable system) with Gaussian ‘w’ & ‘v’.

Method

For $v_t = 0$, a scalar system (with B = C = 1), is stabilizable (Athans et al. 1977) if and only if,

$$\sigma_A^2 < 1$$

For the imperfect information case ($v_t \neq 0$)

- Attempted to lower bound the minimum cost.
- Showed that the estimation error covariance depends on the control law.
- Hence, inseparability occurs and we could not solve the problem analytically.

Control over AWGN Channel

Question

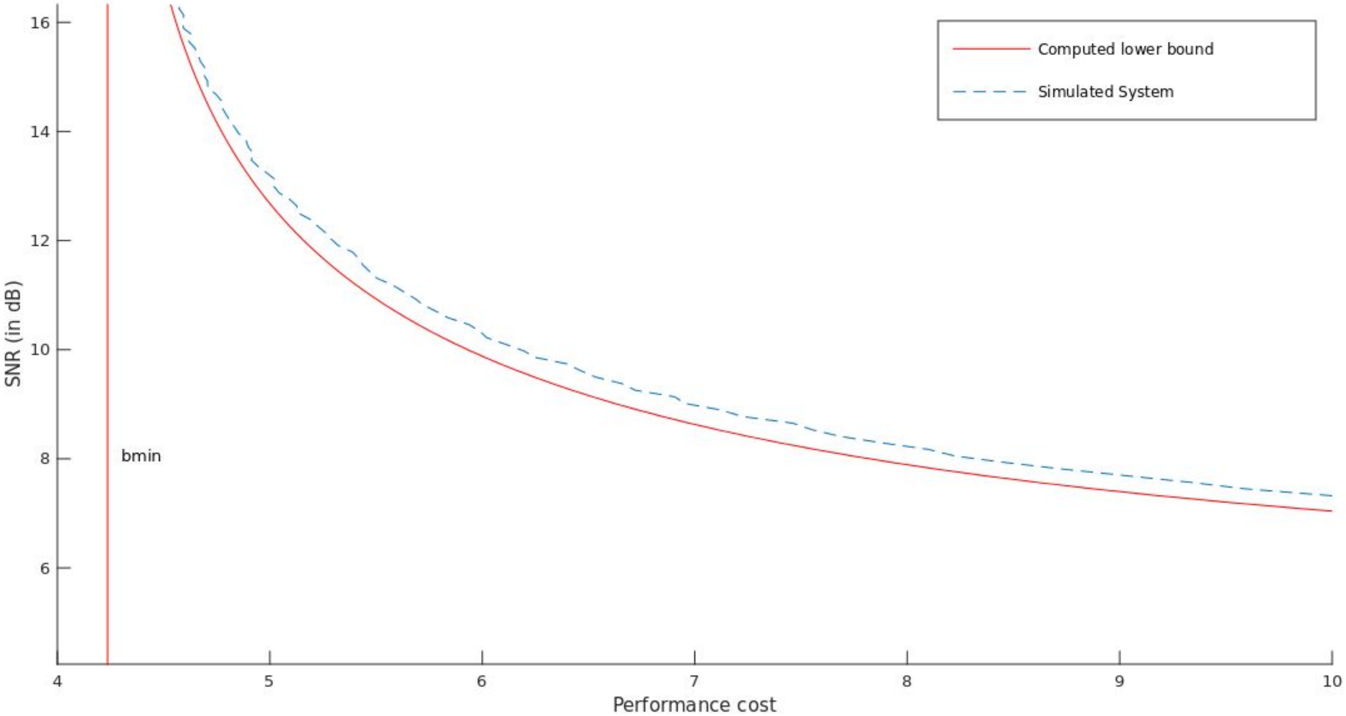
To study the tradeoff between the optimal LQG cost and channel SNR.

- Using the scheme in (Anatoly, Victoria and Babak).
- Using rate-distortion bounds (Victoria and Babak)

Method

- Computed the optimal cost for the scheme.
- Verified the results by simulating a scalar system.
- Using channel capacity, expressed the rate-distortion bound for the AWGN channel.
- Studied the tightness of the bound so obtained for simulation of system with non-Gaussian disturbance ‘w’.

Results : Control over AWGN Channel



The SNR of the channel with optimal cost achieved for a scalar system, with n= 1, A=2, B = Q = R = 1, V = W = 1.

Control over Rate-Limited Channel

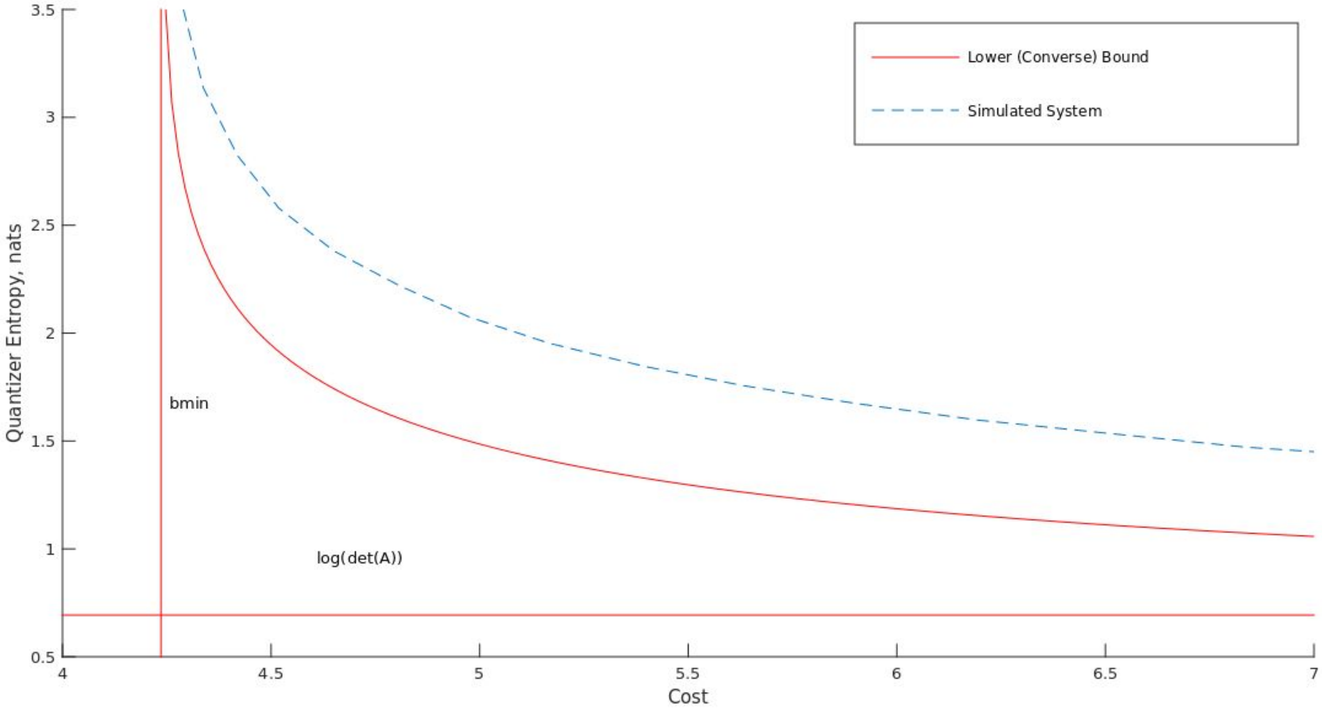
Question

To study the tightness of the rate-distortion bounds. (Victoria and Babak)

Method

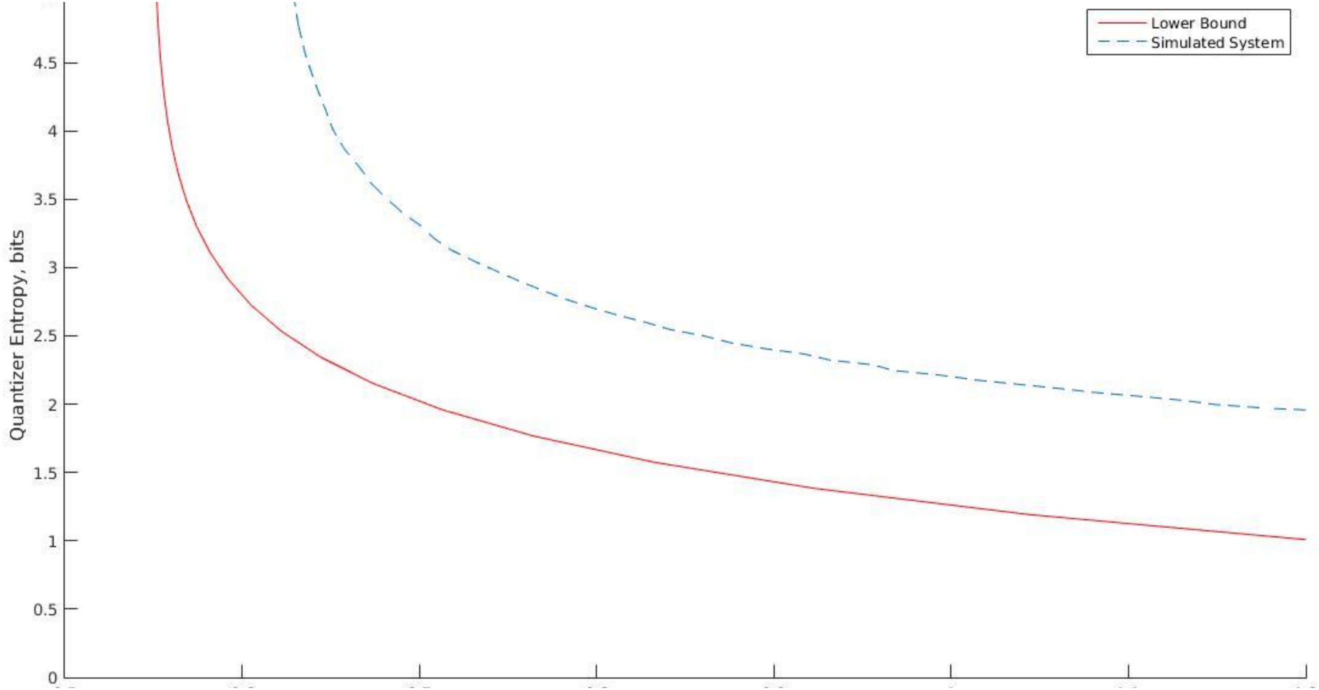
- Simulated a simple, variable-rate, uniform quantization scheme.
- Used entropy-coding to calculate the entropy of output of the quantizer.
- Compared the simulated system performance with the bounds
  - For Gaussian 'w'
  - For Laplacian 'w'

Results : Rate-Limited Channel



The minimum quantizer entropy with optimal cost achieved for a scalar system, with n= 1, A=2, B = Q = R = 1, V = W = 1.

Results : Rate-Limited Channel with Multiplicative Uncertainty in the System



The minimum quantizer entropy with optimal cost achieved for a scalar system, with n= 1, gaussian A with mean =2 and Variance = 0.36, B = Q = R = 1, V = W = 1.

Conclusions

Control with Multiplicative Uncertainty

We studied the problem in detail and tried to tackle it by finding a lower bound to the minimum achievable cost. We showed why an analytical solution is not possible.

For three different scenarios, the scalar uniform quantizer performs close to the lower bound :

- Control over AWGN channels for system with non-Gaussian disturbances.
- Control over rate-limited channel.
- Control with multiplicative uncertainties and rate-limited channel.

Future Work

Control with Multiplicative Uncertainty

For the imperfect information case, the optimal problem remains unsolved.

Control over AWGN Channels

It would be interesting to look at vector channel in this setting for the rate-matched case.

Control over Rate-Limited Channels

A better upper bound to the rate-distortion function could be possible. Also, better quantization strategies than a uniform quantizer would be interesting to look at.

References

[1] Anatoly Khina, Victoria Kostina, and Babak Hassibi. “Multi-Rate control over AWGN channels : An analog joint source-channel coding perspective”. In: *Conference on Decision and Control* (2016)

[2] Victoria Kostina and Babak Hassibi. “Rate-distortion tradeoffs in control”.

[3] Stephen Boyd. “Linear Quadratic Stochastic Control with Partial State Observation”. In: (2008)

[4] Victoria Kostina et al. “Rate-limited control of systems with uncertain gain”.

Acknowledgements

Thank you for your attention.