Mid Semester Evaluation:

Robust Controller Design Using H-infinity Loop-Shaping

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Overview

Background Study

- Preliminary mathematics
- MIMO framework
- Coprime factorisation and LFT
- Algebraic Riccati equation
- Loop shaping
- H_∞ control theory
- H_∞ controller design using loop shaping

Progress

- Preliminary mathematics
- Coprime factorisation : SISO
- MIMO framework: Beginner

Progress - Preliminary mathematics

Vector spaces

- Banach spaces
- Hilbert spaces
- l_2 and L_2 space in time
- L_2 and H_2 spaces (Laplace Transforms)
- L_∞ and H_∞ spaces

Induced norms and gains

- Matrix norm
- Induced norm
- G-induced norm (Transfer matrix, G)
- Measure of size of signal for performance specification

Progress - Coprime factorisation over RH_∞

For SISO systems

- Minimal realization and coprimeness
- Euclid’s algorithm for polynomials
- Bezout identity

For MIMO systems

- Bezout identity
- Left and right coprime factors
- Doubly coprime factorisation

Results

Solved a SISO system to find out normalised right coprime factorization for the following transfer function:

$$\frac{s-1}{(s+2)(s-3)}$$

$$N(s)'N(s) + M(s)'M(s) = 1$$

Also, singular value of column matrix of normalized coprime factors was calculated to be 1, which is obvious from the definition.

Progress - MIMO framework

Gain calculation

- Depends on input direction as well.
  - For same norms of input and output, the gain is different.
  - Proved this with examples.
- Singular value calculation gives a good measure of gain for different frequencies

Singular values and H_∞ norm

- Singular values from eigen values
- The H-infinity norm at a particular frequency is equal to the maximum singular value of the matrix at that frequency (for MIMO systems)
- This norm is also referred to as worst case attenuation

Results and simulations I

MATLAB Code SISO:

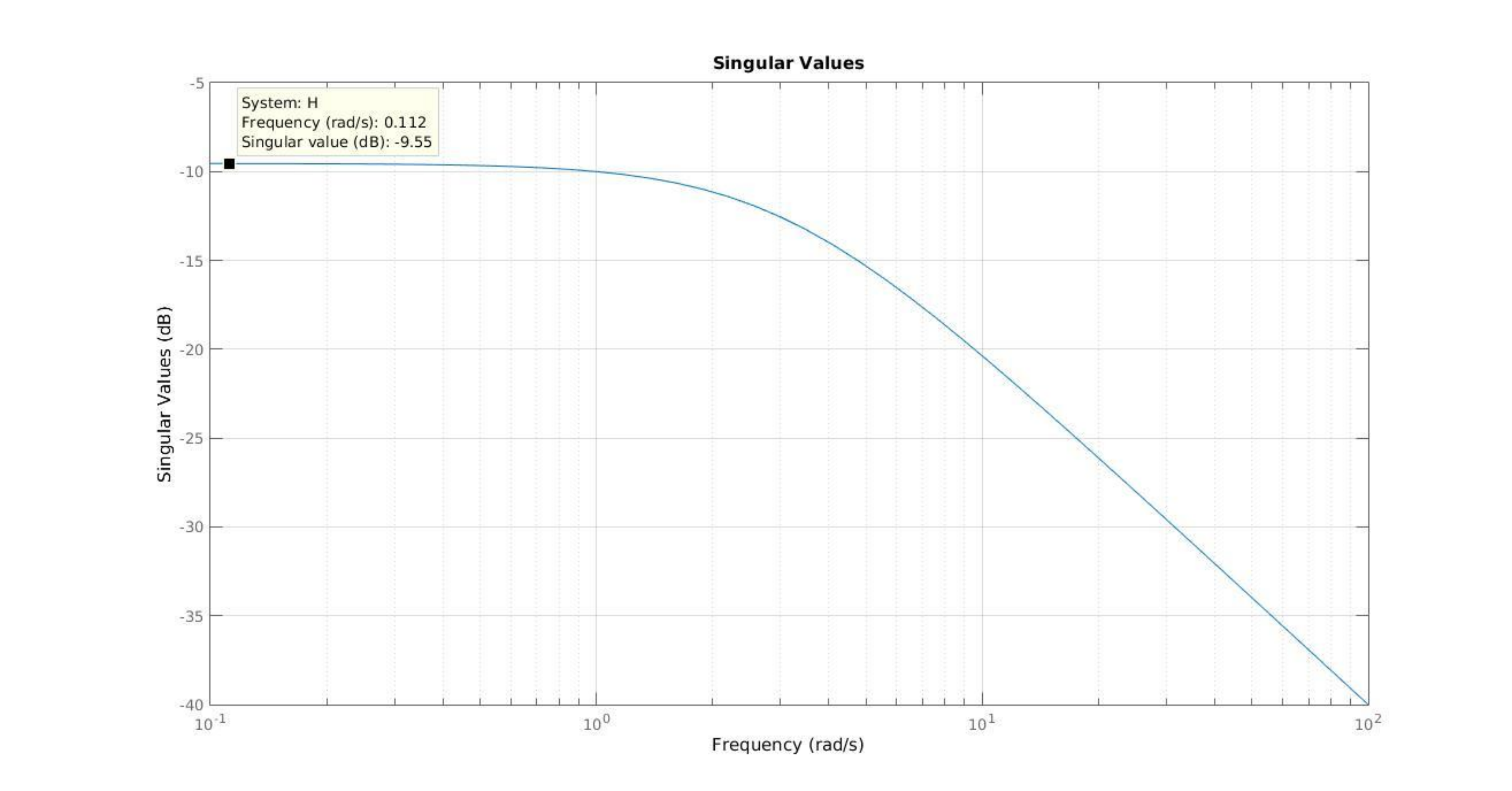
```
a=[1 0;4 -3];
b=[0;1];
c=[1 1];
d =0;
H=ss(a,b,c,d);
title('SISO Singular Values');
figure(1)
grid on
sigma(H)
```

MATLAB Code MIMO:

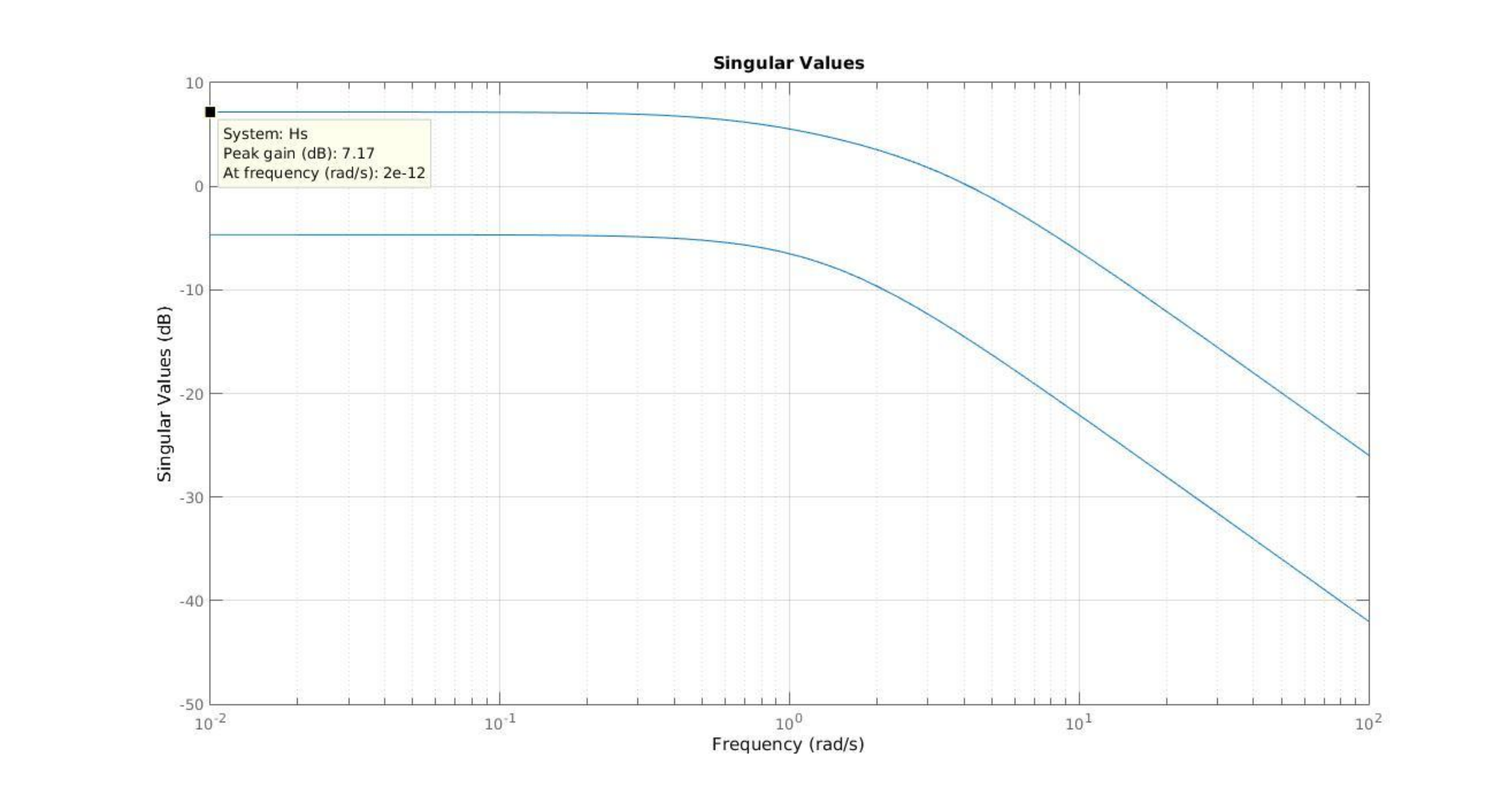
```
am=[-1 0 ;0 -3];
bm=[0 1;2 1];
cm=[1 2;1 0];
dm=0;
Hm=ss(am,bm,cm,dm);
title('MIMO Singular Values');
figure(2)
grid on
sigma(Hm)
```

The simulation results obtained were verified by hand calculations and the values obtained were same.

Results and Simulations II



Results and Simulations III



Schedule

11.10.15

Coprime factorisation over RH_∞ : MIMO systems

17.10.15

MIMO framework needed for robust control

10.11.15

Linear fractional transformation and Algebraic Riccati equations

20.12.15

H_∞ control theory and H_∞ loop-shaping controller design

Jan. ‘15

Problem statement to design the studied controller

Goals for end semester

1. Complete understanding using different examples and simulations of preliminaries to robust control
    a. Coprime factorisation
    b. LFT
2. Loop-shaping
3. Beginning with H_∞ control theory

References

Kemin Zhou et. al. *Robust and Optimal Control* 1995

Thank You