Assignment 4 (Generalized plant, LFT, Structured singular value)

1) Using LFT obtain the robust stability conditions for
   a) Multiplicative output uncertainty
   b) Additive uncertainty
   c) Inverse multiplicative output uncertainty.

2) Consider the control system shown in the ~~figure~~ Fig 1 where the objectives are (a) input disturbance rejection and (b) robust stability against multiplicative output uncertainty.

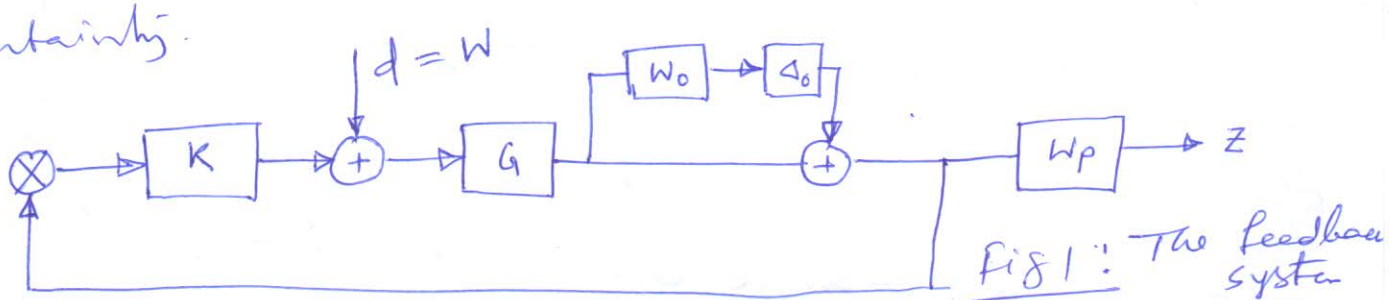

Fig 1: The feedback system

Represent the above control configuration in a general control block diagram ~~in a~~ ~~the~~ ~~form~~

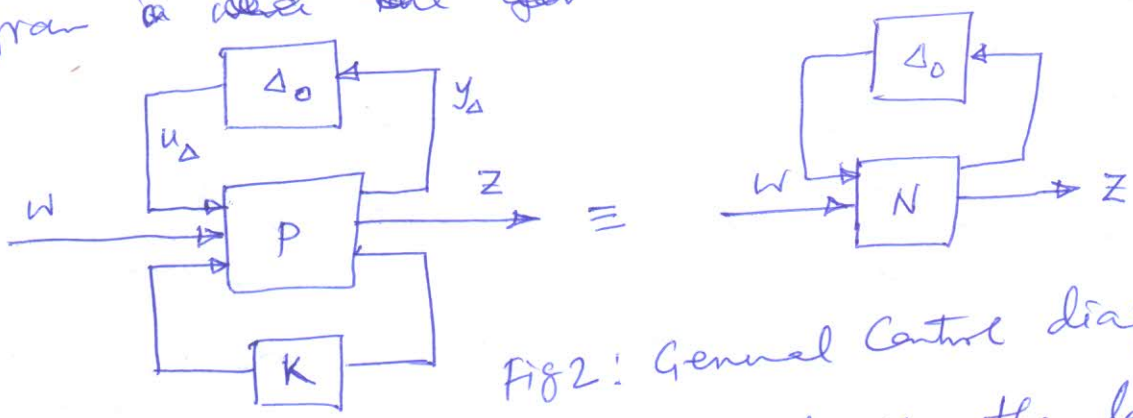

Fig 2: General control diagram

Derive ~~the~~ the generalized plant P and ~~the~~ the lower fractional transformation $N = F_e(P, K)$. Also derive N directly from the ~~block~~ diagram of Fig 1. From N, obtain the condition for robust stability and nominal performance.

3) Repeat the above when the disturbance ~~is~~ d appears at the output rather than the input.

4) a) Let $M = \begin{bmatrix} a & a \\ b & b \end{bmatrix}$ and Δ be complex 2×2 matrices. Then prove that

$$\mu(M) = \begin{cases} |a+b| & \text{for } \Delta = \delta I \\ |a|+|b| & \text{for } \Delta = \text{diag}\{\delta_1, \delta_2\} \\ \sqrt{2|a|^2+2|b|^2} & \text{for } \Delta \text{ is a full matrix} \end{cases}$$

b) Let a, b, c and d be complex scalars. Show that for $\Delta = \text{diag}\{\delta_1, \delta_2\}$;

$$\mu\begin{bmatrix} ab & da \\ bc & cd \end{bmatrix} = \mu\begin{bmatrix} ab & ab \\ cd & cd \end{bmatrix} = |ab| + |cd|$$

5) Using the results of 4(b), obtain the robust performance condition for Questions 2 and 3.

6) Consider the example of distillation Process

$$G(s) = \frac{1}{75s+1}\begin{bmatrix} 87.8 & -86.4 \\ 108.2 & -109.6 \end{bmatrix}$$ and the inverse based controller $K(s) = \frac{0.7}{s}(75s+1)\begin{bmatrix} 87.8 & -86.4 \\ 108.2 & -109.6 \end{bmatrix}^{-1}$.

Also consider $W_p = \frac{s/2 + 0.05}{s}$, $W_o = \frac{s+0.2}{0.5s+1}$

Now for the control system block diagrams of Q-2 & 3 check i) Nominal stability (NS) ii) robust stability, and (iii) robust performance conditions use MATLAB if necessary.

7)

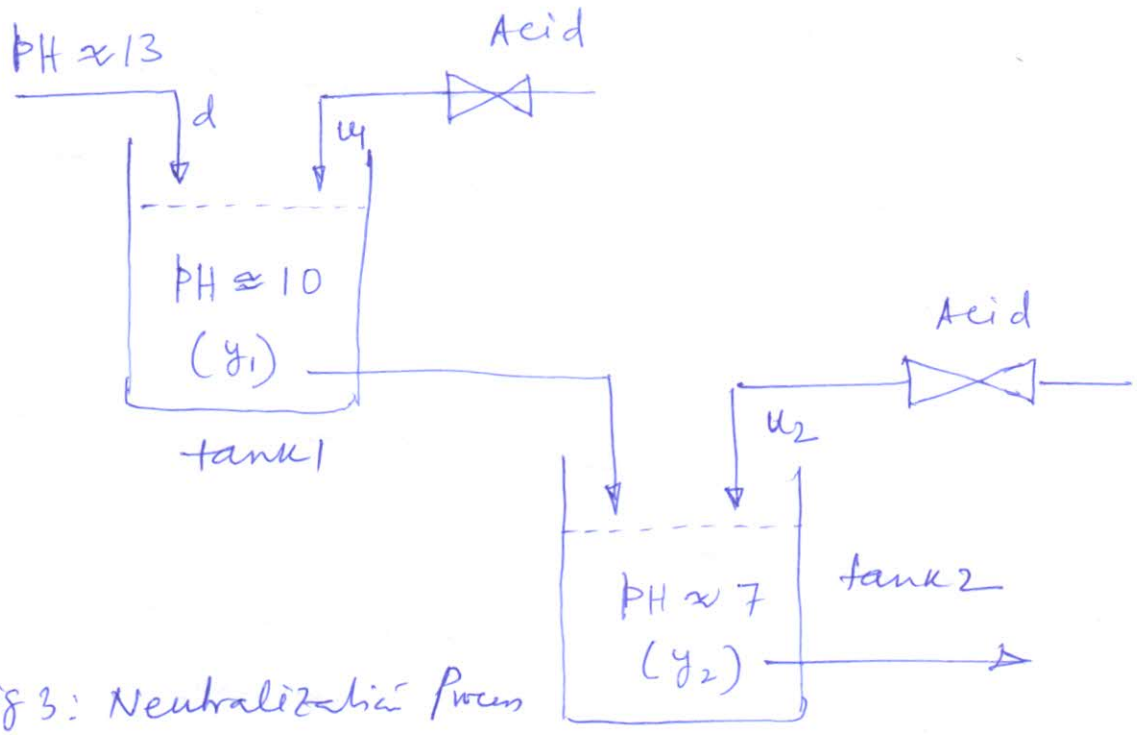

Fig 3: Neutralization Process

Consider the neutralization process shown in figure above where acid is added in two stages. Most of the neutralization takes place in tank 1 where a large amount of acid is used (input u_1) to obtain a pH of about 10 (measurement y_1). In tank 2 the pH is fine-tuned to about 7. (output y_2) by using a small amount of acid (input u_2). This description is just to give you some idea of a real process; all the information you need to solve the problem is given below..

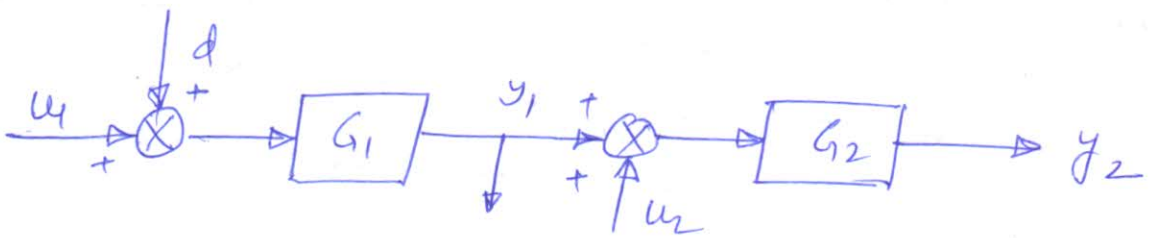

Fig 4: Block diagram of neutralization process

block diagram of the process is shown in Fig 4. It includes one disturbance, two inputs and two measurements

(y_1 and y_2). The main control objective is to keep $y_2 \approx r_2$. In addition, we would like to reset input 2 to its nominal value; that is, we want $u_2 \approx r_{u_2}$ at low frequencies. Note that there is no particular control objective for y_1.

(a) Define the general control problem: that is, find z, w, u, v and P.

(b) Define an H_∞ control problem based on P. Discuss briefly what you want the unweighted transfer function from d to z look like, and use this to say a little about how the performance weights should be selected.

(c)

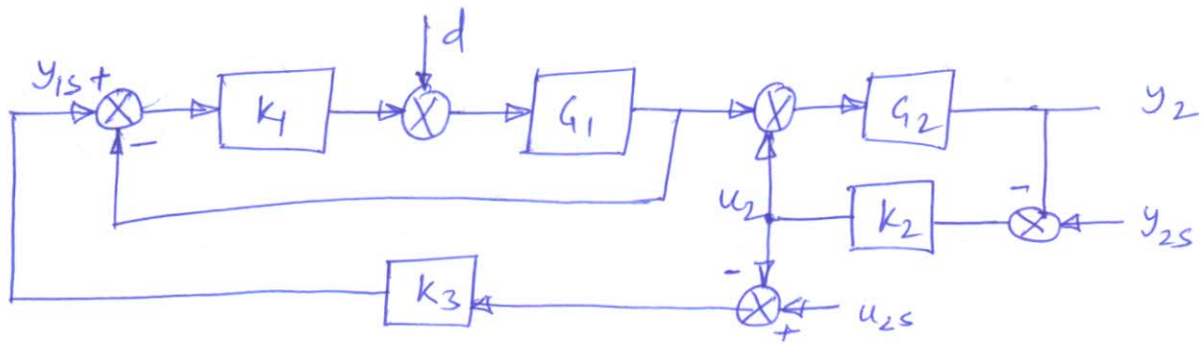

Fig 5: Proposed control structure for neutralization process

A simple practical solution based on single loops is shown in Fig 5. Explain briefly the idea behind this control structure, and find the interconnection matrix P and the generalized controller $K = \text{diag}\{K_1, K_2, K_3\}$. Note that u and y are different in this case, while w and z are the same.

8) a) Consider the plant $\dot{x} = a(1 + 1.5\delta_a)x + b(1 + 0.2\delta_b)u$, $y = x$. Where $|\delta_a| < 1$, $|\delta_b| < 1$. For a feedback controller $K(s)$ derive the interconnection matrix M for robust stability.

b) For the above case consider the condition $\bar{\sigma}(DMD^{-1}) < 1$ to check the robust stability (RS). What is D (give as few parameters as possible). Is RS tight in this case?