Robust H-infinity Control of Intelligent Autonomous Navigation Wheelchair

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***Abstract*—This paper proposes a robust control for a two-wheeled intelligent wheelchair using H-infinity (H_∞) control scheme in the presence of uncertainties. An optimal H_∞ controller design is given based on an iterative linear matrix inequality algorithm, which is robust to variance of the motor dynamics, sensor noise, and unmodeled vehicle dynamics. Finally, the simulation experiments show the effectiveness of the proposed scheme.**

I. Introduction

People with mobility impairments usually use wheelchair as a vehicle to travel. However, driving a wheelchair in domestic environments is not an easy task, even for a normal person. Especially, to the people with arms or hands impairments, which becomes even more difficult. The powered wheelchairs (Figure III) with high manoeuvrability and navigation is used, which is one of the great steps towards the intelligent autonomous control for severely physically disabled and mentally handicapped people [1], [2]. Simultaneously blind and paraplegic people must address two very important issues: navigation and autonomous control.

Intelligent wheelchairs are now expected to become the next generation of rehabilitation assist equipments for elderly and disabled people. By integrating an intelligent machine with a powered wheelchair, a robotic wheelchair has the ability to safely transport the user to a destination.

Previous work in autonomous mobile robot control generally involves path planning and path tracking control. At the beginning of the new millennium the PID controller continues to be the key component of industrial control, including robotics control. During this century many different structures of control have been proposed to overcome the limitations of PID controllers. The present- day structure of P, PI, and PID controllers is quite different from the original P, PI, PID controllers. Many algorithms such as anti-windup, auto-tuning, adaptive, and fuzzy fine tuning to improve their performances, but the basic actions remain the same. During the last two decades, the general reluctance of researchers to use PID controllers has begun to disappear [3], and there have been great advances in the theory of modern control. The design of robust uncertainty-tolerant multivariable feedback control systems has been resolved, at least partially [4]. The real problem in robust multivariable feedback control system is to synthesize a control law which maintains system response and error signals to within pre-specified tolerances despite the effects of uncertainty on the system [5]. Uncertainty may take many forms but among the most significant are noise, disturbance signals and transfer function modeling errors. Another source of uncertainty is unmodeled nonlinear distortion. Consequently people have adopted a standard quantitative measure for the size of the uncertainty, called the H_∞ norm. H_∞ control has been researched in several recent papers [3], [6].

Fig. 1. The Power Wheelchair (Invacare Nutron R-32)

The main structure is as follows; the section II interprets H_∞ design problems; The section III describes the model of two wheeled robotic system; The section IV, a robust H_∞ control scheme is presents; The section V presents the simulations; Finally, a summarize is given.

II. Interpreting H_∞ Design Problems

H_∞ control problems can be formulated in many ways, here is the most simplified interpretation of the problem is to find controller for the generalized plant such that infinity norm of the transfer function relating exogenous input $d(t)$ to performance output $z(t)$ is minimum. The minimum gain is denoted by γ^*. If the norm for an arbitrary stabilizing controller is $\gamma > \gamma^*$, then system is L_2 gain bounded. To

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solve the H_∞ problem we start with a value of γ and reduce it until γ^* is achieved.

Considering the input-output relations that are depicted by P, the relation between $[z, e]$ and $[w, u]$ could be written as given in 1

$$\begin{bmatrix} z \\ e \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \tag{1}$$

Considering the invariance of the plant P and the separability of the variables for the linear setting, the system has been partitioned into the state space for as given in 2

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1 w(t) + B_2 u(t) \\ z(t) &= C_1 x(t) + D_{11} w(t) + D_{12} u(t) \\ y(t) &= C_2 x(t) + D_{21} w(t) + D_{22} u(t) \end{aligned} \tag{2}$$

where $x \in R^n$ is the state, $u \in R^r$ is the control input, $e \in R^m$ is the observed output, $z \in R^q$ is the controlled output and $w \in R^p$ is the disturbance. (A, B_1) is stabilizable and (A, C_1) is detectable. (A, B_2) is controllable and (A, C_2) is observable, The control problem is: with a feedback $u = K(s)u$, find an admissible internally stabilizing control K which would be attenuating disturbances such that the norm of the stable closed loop system from the disturbances to the controlled outputs is less than γ (γ is equal to 1 for optimal and slightly greater than 1 for suboptimal control). The control objective is stated in the mathematical form in 3

$$\| F_l(P, K) \|_\infty = \sup_{\omega \in R} |F_l(P, K)(j\omega)| < \gamma \tag{3}$$

The H_∞ solution involves two Hamiltonian matrices [7],

$$H_\infty : \; = \begin{bmatrix} A & \gamma^{-2} B_1 B_1' - B_1 B_1' \\ -C_1' C_1 & -A' \end{bmatrix} \tag{4}$$

$$J_\infty : \; = \begin{bmatrix} A' & \gamma^{-2} C_1' C_1 - C_1' C_1 \\ -B_1 B_1' & -A \end{bmatrix} \tag{5}$$

Theorem [7]: There exists an admissible controller such that $\| T_{zw} \|_\infty < \gamma$ iff the following three conditions hold. i. $H_\infty \in dom(Ric)$ and $X_\infty := Ric(H_\infty) \geq 0$. ii. $J_\infty \in dom(Ric)$ and $Y_\infty := Ric(J_\infty) \geq 0$. iii. $\rho(X\infty Y_\infty) < \gamma^2$

The proof of this theorem is seen in [7]. For these conditions being satisfied the suboptimal controller is

$$K_{sub}(s) = \left[\begin{array}{c|c} \hat{A}_\infty & -Z_\infty L_\infty \\ \hline F_\infty & 0 \end{array} \right] \tag{6}$$

where $\hat{A}_\infty := A + \gamma^{-2} B_1 B_1' X_\infty + B_2 F_\infty + z_\infty L_\infty C_2, F_\infty := -B_2' X_\infty, L_\infty := -Y_\infty C_2', Z_\infty := (I - \gamma^{-2} Y_\infty X_\infty)^{-1}$.

The resulting closed loop system with a feedback $u = K(s)u$ obtained by Linear Fractional Transformation is brought to the form

$$\begin{aligned} \dot{x}(t) &= A_c x_c(t) + B_1 w(t) \\ z(t) &= C_1 x_c(t) \end{aligned} \tag{7}$$

where x_c indicates the states of the closed loop system.

Definition: Given a scalar $\gamma > 0$, system 7 is said to be stable with disturbance attenuation γ if it satisfies the following conditions 1. A_c is a stable matrix 2. The transfer function from disturbance w to the controlled output z satisfies

$$\| C_1 (sI - A_c)^{-1} B_1 \| \infty < \gamma \tag{8}$$

Lemma: Let $\gamma > 0$ be given. The system 7 is stable with disturbance attenuation γ if and only if there exists a symmetric matrix X_∞ such that

$$A_c' X_\infty + X_\infty A_c + \gamma^{-2} X_\infty B_1 B_1' X_\infty + C_1' C_1 < 0 \tag{9}$$

Proof: Let the Lyapunov function for the system be $V(x_c(t)) = x_c(t)' X_\infty x_c(t)$. Making substitutions from 7 ,

$$\begin{aligned} \dot{V}(x_c(t))) = x_c'(t)[A_c' X_\infty + X_\infty A_c] x_c(t) + \\ x_c'(t) X_\infty B_1 w(t) + w(t)' B_1' X_\infty x_c(t) \end{aligned}$$

Let the performance measure corresponding to disturbance attenuation be

$$\begin{aligned} J &= \int_0^\infty [z(t)' z(t) - \gamma^2 w(t)' w(t)] dt \\ J &\leq \int_0^\infty [z(t)' z(t) - \gamma^2 w(t)' w(t) + \dot{V}(x_c(t))] dt \\ J &\leq \int_0^\infty \{x_c'(t) C_1' C_1 x_c(t) - \gamma^2 w(t)' w(t) \\ &+ x_c'(t)[A_c' X_\infty + X_\infty A_c] x_c(t) + x_c'(t) X_\infty \\ &\quad B_1 w(t) + w(t)' B_1' X_\infty x_c(t)\} dt \end{aligned}$$

with

$$\begin{aligned} & w(t)'(B_1' X_\infty x_c(t)) + (B_1' X_\infty x_c(t))' w(t) \\ & \leq \gamma^2 w(t)' w(t) + \gamma^{-2} x_c'(t) X_\infty B_1 B_1' X_\infty x_c(t) \end{aligned}$$

since $J \leq 0$ to be satisfied

$$A_c' X_\infty + X_\infty A_c + \gamma^{-2} X_\infty B_1 B_1' X_\infty + C_1' C_1 < 0$$

III. System Modeling

The mobility assistance robot has two differential driving wheels mounted on the same axis and two free castor front wheel, which has the better characteristics, such as big carrying capacity, fast moving speed, good moving stability and high energy efficiency. As illustrated in Figure III, the robot is located in a Cartesian coordinate system. There are two coordinate system in a 2D plane, a global coordinate system $X - Y$ and a local coordinate system $X_r - Y_r$. Before proceeding onto designing a controller, the system must be modelled mathematically. There are five key states: $v_{ox}, \dot{\alpha}, \alpha, x_g, y_g$. The controlled input variable in the wheelchair system is the torque $[M_{wl}, M_{wr}]^T$ supplied by the drive wheel motor. By controlling the drive wheel motor, the wheelchair can be led to different positions.

According to Newton's Second Law and the relation between global coordinate system and local coordinate, we can

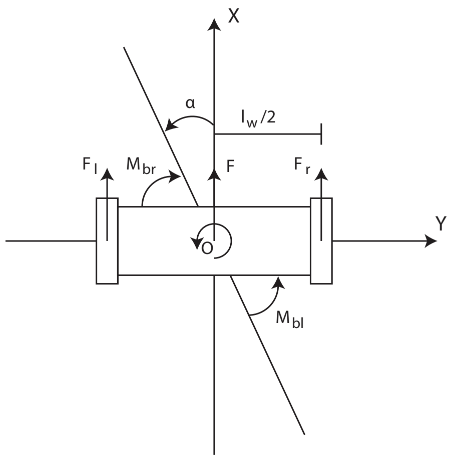

Fig. 2. The planar model in $x-y$ plane

TABLE I
MODEL'S QUANTITIES OF SYSTEM

Quantities	Description
m_{2w}	Mass of two wheels
m_b	Mass of the body
l_b	Distance between the body's center of mass and the rotation point O
l_w	Distance between two wheels
I_{pitch}	Pitch inertia of the body about y axis
I_{yaw}	Yaw inertia of the whole system about z axis
R_w	Radius of the wheels
v_{ox}	Velocity of point O in x direction
x_g	Velocity of in X direction
Y_g	Velocity of in Y direction
α	Yaw angle from the x_p axis to the yaw axis
F_b	Input force on the body through the wheels
M_{wl}	Moment on the left wheel caused by the motor
M_{wr}	Moment on the right wheel caused by the motor

get

$$\dot{v}_{ox} = \frac{(I_{pitch} + m_b l_b^2)F_b}{m_b l_b^2 (m_{2w}) + I_{pitch}(m_b + m_{2w})} \tag{10}$$

$$\dot{x}_g = \dot{x}\cos\alpha \tag{11}$$

$$\dot{y}_g = \dot{x}\sin\alpha \tag{12}$$

$$\ddot{\alpha} = (-M_{wl} + M_{wr})\frac{l_w}{2R_w I_{yaw}} \tag{13}$$

The state of the system is chosen as $\mathbf{x} = [v_{ox},\, \dot{\alpha},\, \alpha,\, x_g,\, y_g]^T$, and the complete state space representation for the non-linear system as follows:

$$\dot{\mathbf{x}} = \begin{bmatrix} \frac{(I_{pitch}+m_b l_b^2)F_b}{m_b l_b^2(m_{2w})+I_{pitch}(m_b+m_{2w})} \\ (-M_{wl}+M_{wr})\frac{l_w}{2R_w I_{yaw}} \\ \dot{\alpha} \\ \dot{x}\cos\alpha \\ \dot{x}\sin\alpha \end{bmatrix} \tag{14}$$

$$\mathbf{y} = Diag(5)\cdot\mathbf{x} \tag{15}$$

where $Diag(5)$ is 7×7 unit matrix, $u = [M_{wl},\, M_{wr}]^T$ is the input of the system.

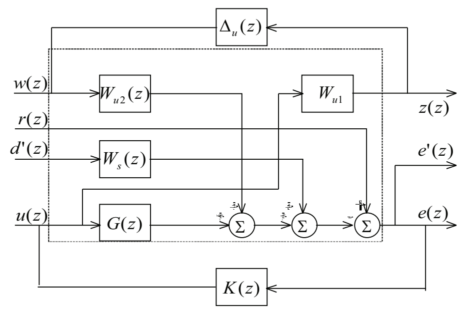

Fig. 3. A 2×2 block diagram

IV. ROBUST H_∞ CONTROL SCHEME

Considering the additive uncertainty of the system, W_u makes an upper bound of the uncertainty model, which has been implemented as a constant W_{u1} and a boundary weight $W_{u2}(z) = \frac{1}{3}W_u(z)$. $M(z), S_o(z)$ are the control sensitivity and output sensitivity respectively under additive uncertainty and weighted ($W_s(z)$) disturbance $d'(z)$. In order to derive the robust controller, the inputs and outputs are grouped together. such as Figure IV, which illustrates a 2×2 block problem.

Where $N(z)$ is the transfer function from outputs to inputs and the disturbance input $\bar{d}'(z)$ is a column vector given by $\bar{d}'(z) = [d'\ r]^T$. The 2×2 block diagram for the H_∞ control problem is shown in Figure . The transfer function $N(z)$ can be found as

$$\begin{bmatrix} z \\ e' \\ e \end{bmatrix} = N(z)\begin{bmatrix} w \\ d' \\ r \\ u \end{bmatrix} \tag{16}$$

Thereby the transfer function $N(z)$ must be given by

$$N(z) = \begin{bmatrix} 0 & 0 & 0 & W_{u1} \\ -W_{u2} & -W_s & 1 & -G \\ -W_{u2} & -W_s & 1 & -G \end{bmatrix} \tag{17}$$

The state space matrices for $N(z)$ must be transformed into $N(s)$ and divided into the different types of inputs and outputs. The discrete state space representation is given by

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned} \tag{18}$$

These matrices can then be split further

$$\left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right] \tag{19}$$

According to the 2×2 block diagram shown in , The obtained matrices can the used to get a robust controller via the command in Matlab software. Which is an iterative process, the disturbance weight $W_s(s)$ must be tuned according the compromise between performance and robustness. The detailed procedure was described in the simulations.

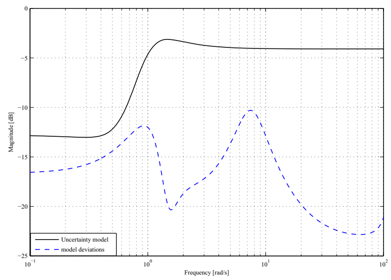

Fig. 4. Uncertainty model and the additive model deviations

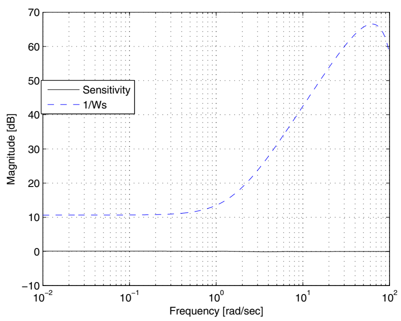

Fig. 5. Output sensitivity

V. Simulations

The uncertainty model W_u is shown together with the additive model deviations $\Delta_a(z)$ in Figure V. In order to tune the disturbance weight W_s it must be decided whether to prioritize either performance or stability. It is assumed that the behavior of the Wheelchair can be approximately described as a second order system. The damping and natural frequency are $\zeta = 1$ and $\omega_n = 0.78$ rad/s.

The output sensitivity $S_o(z)$ and control sensitivity $M(z)$ with the inverse upper bound of the disturbance weight $W_s(z)$ and the inverse upper bound of the uncertainty model $W_u(z)$ have been plotted in Figure V and Figuire V respectively. The disturbance weight $W_s(z)$ has been illustrated together with the uncertainty model $W_u(z)$ in Figure V. For the simulation a step from 0.5 m/s to 0 m/s has been used, although the direction of the step is unimportant for linearized systems. The simulated velocity, with and without a controller, with a step from 0.5 m/s to 0 m/s, is shown in Figure V, from which it can be seen that the undershoot, in velocity, has been removed.

VI. Conclusion

In this paper the robust control design of an intelligent wheelchair was presented. Modeling of the system was done taking into account the kinematics and dynamics to achieve a state space model for systems. The robust internally stabilizing H_∞ controller was obtained such that the closed loop system was internally stable and the effect of disturbances and model uncertainties on some of the outputs was attenuated.

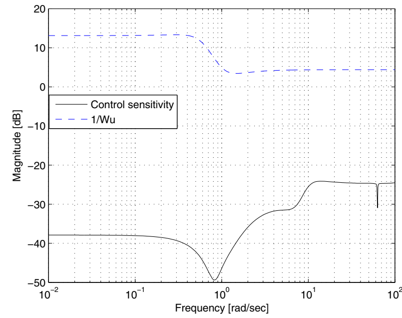

Fig. 6. Control sensitivity

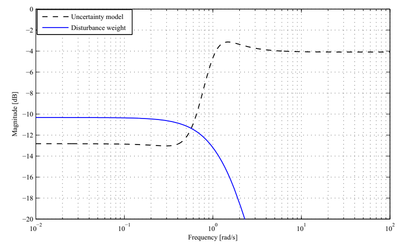

Fig. 7. Disturbance weight and uncertainty model

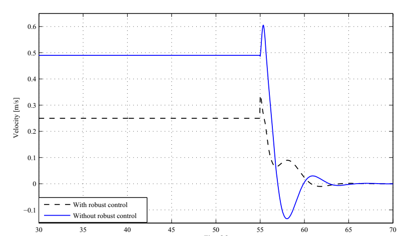

Fig. 8. Simulated velocity of the wheelchair 2

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