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Robust H∞ Controller Design for Wheeled Mobile Robot with Time-delay

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Abstract

This paper introduces the modeling problem of wheeled mobile robots with nonholonomic restriction, and proposes a nonlinear mathematical model with uncertain disturbance and time-delay. Then we transform this nonlinear system into a linear one. Based on LMI, a local feedback H_∞ robust controller is designed and its validity is verified by simulation.

1.Introduction

Wheeled mobile robots(WMR) are widely used in lots of dangerous and heavy jobs, including the transportation of nuclear waste, fire control, lunar exploration etc. At present, another promising application is extended for the assistance of disabled, handicapped or elderly people[1-3].

In control field, WMR researchers have focused on establishing mathematical models, trajectory tracking and stabilization controller designing. Literature [4] applied UKF arithmetic in mobile robot control and gained better control effect, but its complexity limits its practical application. Literature [5][6] introduced intelligent arithmetic in this filed and avoided establishing the precise mathematical models, but the design of control rules are highly dependent on personal experience, and its preciseness and stability are not satisfactorily addressed and need to be further examined.

WMR is a complicated nonlinear coupling system with model uncertainties and time delay. In this paper, based on kinematic and kinetic analysis for the AS-R 3-Degree of freedom differential WMR,a nonlinear mathematical model with uncertain disturbance and time delay is established. An approximate linearizing algorithm based on balanced flow pattern is proposed to transform the model to linear control system. Then, based on LMI, a partial feedback H_∞ controller is designed. Finally, simulation is performed and the result shows the efficiency of the method.

2.Problem description

The WMR we discussed in this paper is AS-R robot made by Grandar Robotics Company. It is composed of three modules, including drive module, control module, and sensor module. The three components are connected and fixed through bolts. The robot keeps its balance by three wheels: two front wheels (body 1 and 2) are driving wheels and one rear wheel (body 3) is steering wheel. The geometrical model is schematically depicted in Fig.1.

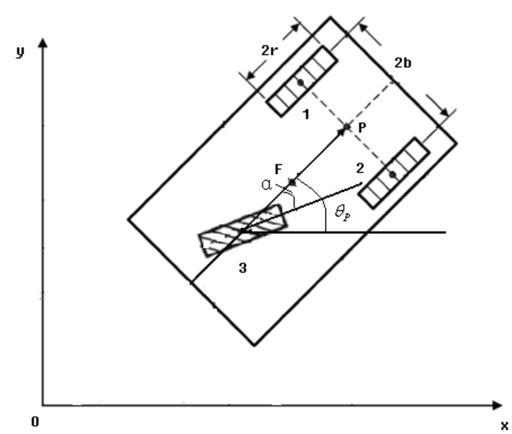

Fig. 1. Geometrical model of the mobile robot

Point F is the projection of the mass center C of the robot; Point P is the center of the two front wheels axle; 2b denotes the distance between the two front wheels; r is the radius of the wheels; l_F is the distance between point P and point F; θ_P is the heading angle of the robot; v_1 and v_2 are the speed of left and right driving wheels respectively; V_p denotes the speed of point P; . ω is the angular speed of the WMR. The relationship between v_P and v_1, v_2 is:

$$v_P = \frac{v_1 + v_2}{2} \tag{1}$$

The relationship between ω and v_1, v_2 is:

$$\omega = \frac{-v_1 + v_2}{2b} \tag{2}$$

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It is easy to see that:

$$\omega = \dot{\theta}_P \tag{3}$$

Coordinate components of v_P are:

$$\begin{cases} \dot{x}_P = \upsilon_P \cos\theta_P \\ \dot{y}_P = \upsilon_P \sin\theta_P \end{cases} \tag{4}$$

The relationship between point P and point F is:

$$\begin{cases} x_F = x_P - l_F \cos\theta_P \\ y_F = y_P - l_F \sin\theta_P \end{cases} \tag{5}$$

Taking the derivative of formula (5):

$$\begin{cases} \dot{x}_F = \dot{x}_P + l_F \omega \sin\theta_P \\ \dot{y}_F = \dot{y}_P - l_F \omega \cos\theta_P \end{cases} \tag{6}$$

The WMR is in the condition of nonholonomic restriction. And the restriction is expressed as:

$$\dot{x}_F \sin\beta - \dot{y}_F \cos\beta - l\dot{\beta} = 0 \tag{7}$$

Taking the derivative of formula (6):

$$\begin{cases} \ddot{x}_F = \ddot{x}_P + l_F \dot{\omega} \sin\theta_P + l_F \omega^2 \cos\theta_P \\ \ddot{y}_F = \ddot{y}_P - l_F \dot{\omega} \cos\theta_P + l_F \omega^2 \sin\theta_P \end{cases} \tag{8}$$

Taking the derivative of formula (4):

$$\begin{cases} \ddot{x}_P = \dot{v}_P \cos\theta_P - v_P \omega \sin\theta_P \\ \ddot{y}_P = \dot{v}_P \sin\theta_P + v_P \omega \cos\theta_P \end{cases} \tag{9}$$

Substitute formula (4) into formula (9):

$$\begin{cases} \ddot{x}_P = \dot{v}_P \cos\theta_P - \dot{y}_P \omega \\ \ddot{y}_P = \dot{v}_P \sin\theta_P + \dot{x}_P \omega \end{cases} \tag{10}$$

Substitute formula (10) into formula (8):

$$\begin{cases} \ddot{x}_F = \dot{v}_P \cos\theta_P - \dot{y}_P \omega + l_F \dot{\omega} \sin\theta_P + l_F \omega^2 \cos\theta_P \\ \ddot{y}_F = \dot{v}_P \sin\theta_P + \dot{x}_P \omega - l_F \dot{\omega} \cos\theta_P + l_F \omega^2 \sin\theta_P \end{cases} \tag{11}$$

Driving motors of the AS-R robot are RE36 Model made by the Maxon Company. They are hollow-cup-rotor DC electrical machines. T_1 and T_2 are driving torque of the left and right front wheels respectively. Let $u_1 = T_1 + T_2$, $u_2 = T_1 - T_2$. $u = [u_1 \quad u_2]^T$ is the input vector. Uncertain disturbance vector $w = [w_1 \quad w_2]^T$ denotes the disturbance put on the v and ω. Then kinetic equations can be expressed as[7]:

$$\begin{cases} \dot{v}_P = \beta_1 u_1 + \beta_2 w_1 \\ \dot{\omega} = \beta_3 u_2 + \beta_4 w_2 \end{cases} \tag{12}$$

Where $\beta_1 = \frac{1}{Mr}$, $\beta_2 = \frac{b}{Jr}$, $\beta_3 = \frac{1}{M}$, $\beta_4 = \frac{1}{J}$. M denotes the mass of the WMR, and J is the moment of inertia of the WMR.

Substitute formula (12) into formula (11):

$$\begin{cases} \ddot{x}_F = \beta_1 u_1 \cos\theta_P + \beta_3 w_1 \cos\theta_P - \dot{y}_P \omega \\ \qquad + l_F \beta_2 u_2 \sin\theta_P + l_F \beta_4 w_2 \sin\theta_P \\ \ddot{y}_F = \beta_1 u_1 \sin\theta_P + \beta_3 w_1 \sin\theta_P + \dot{x}_P \omega \\ \qquad - l_F \beta_2 u_2 \cos\theta_P - l_F \beta_4 w_2 \sin\theta_P \end{cases} \tag{13}$$

Time delay in WMB system appears in two situations: In local controller, position signal from robot to sensor and the execute signal from actor to robot cause time delay; In remote network controller, the signal transmission cause time delay. So a matrix A_d is used to express the time delay of the system.

The state variable $x = (x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6)^T = (x_F \quad y_F \quad \theta_P \quad \dot{x}_F \quad \dot{y}_F \quad \dot{\theta}_P)^T$ is used to describe the position and stance of the mobile robot. The six variables denote the X axis displacement, Y axis displacement, angular displacement, X axis speed, Y axis speed and angular speed of the mobile robot in the world coordinates.

Above all, mathematical model with uncertain disturbance and time delay is established as:

$$\begin{aligned} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{pmatrix} &= \begin{pmatrix} x_4 \\ x_5 \\ x_6 \\ -x_6 x_5 \\ x_4 x_6 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \beta_1 \cos x_3 & l_F \beta_2 \sin x_3 \\ \beta_1 \sin x_3 & -l_F \beta_2 \cos x_3 \\ 0 & \beta_2 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \\ &+ \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \beta_3 \cos x_3 & l_F \beta_4 \sin x_3 \\ \beta_3 \sin x_3 & -l_F \beta_4 \cos x_3 \\ 0 & \beta_4 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} + A_d X(t-\tau) \end{aligned} \tag{14}$$

The parameters of the mobile robot are showed in Table 1.

Table 1. The parameters of the mobile robot

Symbol	Parameter	Value	Unit
r	radius of front wheel	0.105	m
M	mass of robot	25	kg
J	moment of inertia of robot	0.5512	$kg.m^2$
2b	distance between the two front wheels	0.41	m
l_F	distance between point P and point F	0.09	m

From formula (14) we can see the WMB system is a complicated nonlinear coupling system. This makes it difficult for us to analyze and control the robot frequently and accurately. So we introduce an approximate linearizing algorithm[8] to transform the model to linear control system.

$$\begin{cases} \dot{x} = Ax + Ew + Bu + A_d x(t-\tau) \\ y = Cx \end{cases} \qquad (15)$$

3.Local feedback H_∞ controller design

For the facility application in engineering situation, we reduced the order of original system to 4 by the method introduced by [9].

$$\begin{cases} \dot{x} = A_1 x + A_d x(t-\tau) + B_1 w + B_2 u \\ z = C_1 x \end{cases} \qquad (16)$$

Where,

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -0.0175 & -0.01 \\ 0 & 0.0175 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 & 0 \\ 0.04 & 0 \\ 0 & -0.163 \\ 0 & 1.814 \end{bmatrix} \qquad B_2 = \begin{bmatrix} 0 & 0 \\ 0.381 & 0 \\ 0 & -0.319 \\ 0 & 3.542 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0.01 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathrm{A_d} = \begin{bmatrix} 0.3 & 0 & 0 & 0 \\ 0.1 & 0.2 & 0 & 0 \\ 0.1 & 0 & 0.2 & 0 \\ 0.1 & 0 & 0 & 0.2 \end{bmatrix}$$

If there exists matrix $X=X^T$ and matrix Q,Y, satisfied the following linear matrix inequality:

$$\begin{cases} \begin{bmatrix} (AX+B_2Y)^T + AX + B_2Y + Q & B_1 & (C_1X)^T & A_dX \\ B_1^T & -\gamma I & 0 & 0 \\ C_1X & 0 & -\gamma I & 0 \\ XA_d^T & 0 & 0 & -Q \end{bmatrix} < 0 \\ X > 0 \end{cases} \qquad (17)$$

Then, a state feedback robust H_∞ controller $u=YX^{-1}x(t)$ can be obtained to guarantee the local feedback system(as shown in Fig2.) progressive stability[9].

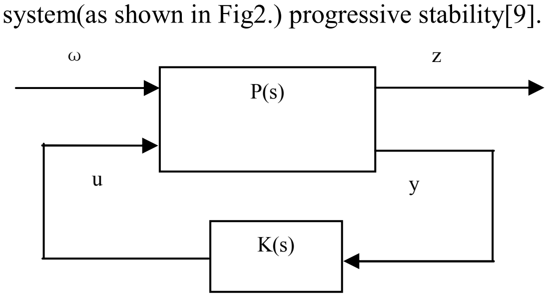


Fig.2. The frame of local feedback control system

Use LMI tool box in Matlab, we can get suited matrix X and matrix Y. Then, a proper local feedback H_∞ controller is obtained:

$$\mathrm{k} = \begin{bmatrix} -0.1620 & -0.0622 & -1.2221 & -0.1093 \\ -0.0617 & 0.0003 & -0.0971 & -0.0155 \end{bmatrix}$$

4. Simulation result

Applying the local feedback H_∞ controller represented in Section 3 to the target mobile robot, a local closed loop control system is formed, as shown in Fig.3.

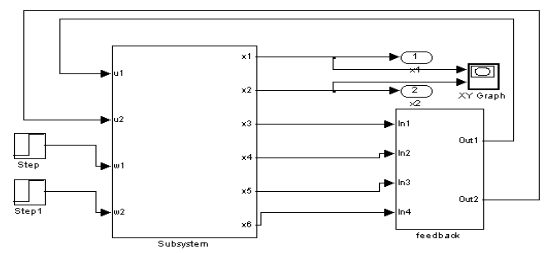
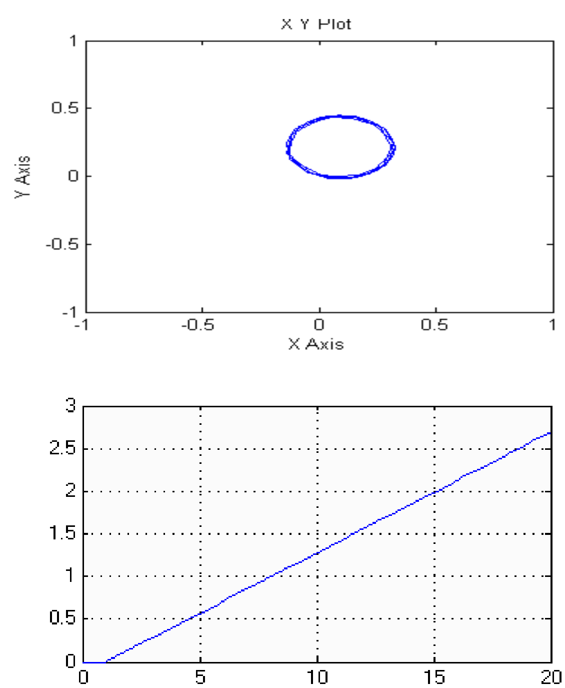


Fig 3. Simulation frame of feedback system

For open loop system, let u_1 and u_2 be constants, we obtain a circle contrail. The X-Y plot, angular speed and linear velocity plot are shown in Fig4.



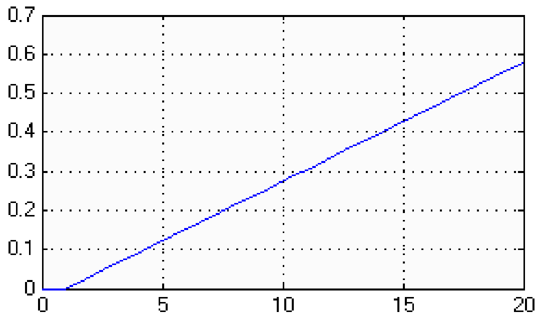


Fig4. The plots of open loop system
(a)X-Y plot (b)angular speed (c) linear velocity

When there exists time-delay in system, the according plots are shown in Fig.5.

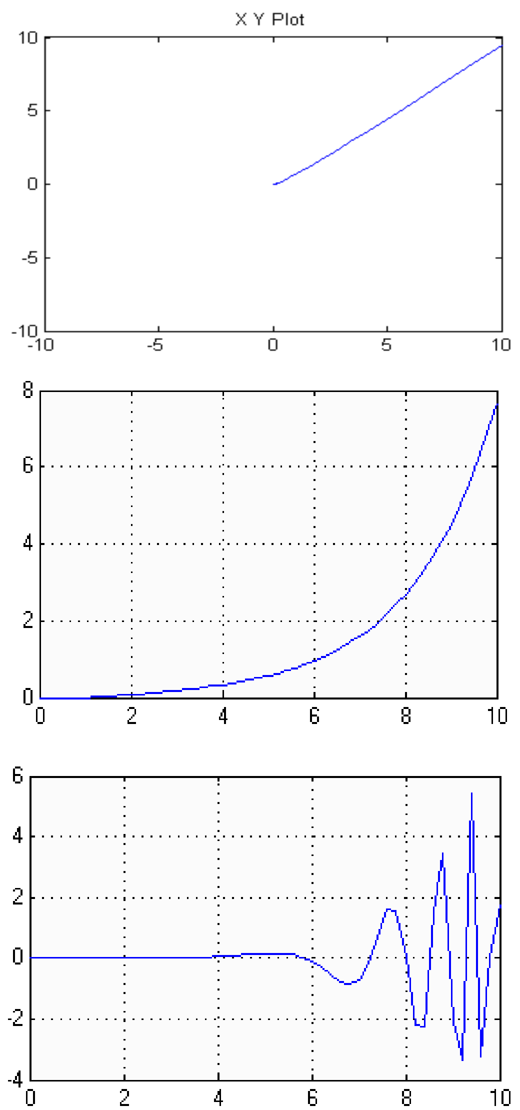


Fig5. The plots of system with time-delay
(a)X-Y plot (b)angular speed (c) linear velocity

From Fig.5.we can see that when there exists time-delay, the system is not convergent, so the control result is not satisfied.

According to the method introduced above, a robust H_∞ feedback controller is designed to improve the situation. The angular speed and linear velocity plots are shown in Fig.6.

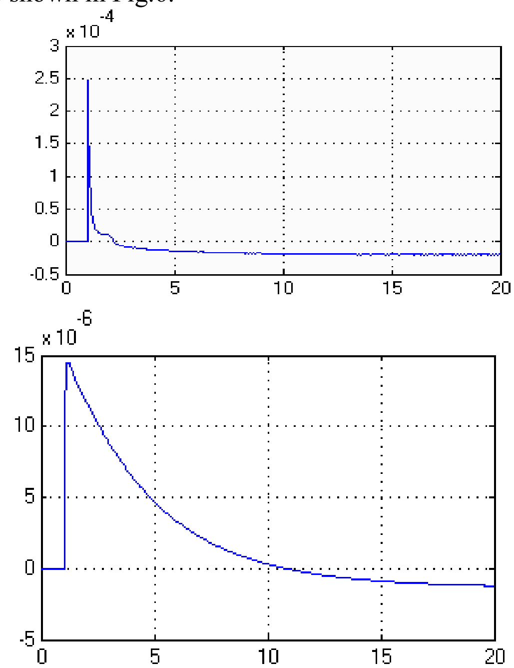


Fig.6.Plots of feedback system with time-delay
(a) angular speed plot (b) linear velocity plot

From the simulation results we can see, the linear velocity and angular speed are convergent to zero with the local robust H_∞ controller. So the system is progressive stability. Also we can see the controlled resolution is closed to 10-4 and 10-6 respectively with the controller we designed. In one word, the whole system is stable and robust with time-delay and uncertain disturbance.

5. Conclusions

In this paper, based on the kinematic and kinetic analysis for the AS-R 3-DOF differential wheeled mobile robot, a mathematical model with time delay and uncertain disturbance for such robots is proposed.. Based on model reduction, a partial output feedback H_∞ robust controller is designed via LMI. Simulation results show the whole system is reasonably stable. Our results can be extended to nonlinear systems widely existing in engineering situations.

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