Estimation of Term Structures using Nelson-Siegel and Nelson-Siegel-Svensson: A Case of a Zimbabwean Bank

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Abstract

The primary objective of the study was to determine the best parametric model that can be used for fitting yield curves for a bank between Nelson-Siegel model and Nelson-Siegel-Svensson. Nelson-Siegel and Nelson-Siegel-Svensson models using Ordinary Least Squares after fixing the shape parameters to make the models linear models. A t-test conducted is conducted on the adjusted R^2 of the two models and results showed that Nelson-Siegel-Svensson model fits better the yield curves of the Bank compared to Nelson-Siegel model. An analysis of the out-of-sample forecasting abilities of the two models using AR(1) conducted using E-views shows that the two parametric models have excellent out-of-sample forecasting abilities on all of their parameters. The time independent of Nelson-Siegel-Svensson model was found to be negative in most of the time and could not be interpreted as a long run yield of the Bank. It is also highlighted that the models produces very low levels of R^2 in many cases because of the high volatility that is found in the market interest rates of certificates of deposits. The estimated yield curve may be used as a reference curve for funds transfer pricing systems.

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1 Introduction

The term structure of interest rates if the relationship between yield to maturity of default free zero coupon securities and their maturities (Sundaresan 1997 p. 176) and it is usually represented by means of a yield curve indicating different rates for different maturities.The yield curve can take many different shapes and thus can be upward sloping downward sloping, flat or hump-shaped as explained by four main theories which happen to be not part of this study. Interest rate term structures have important applications in economics and finance especially for banks and governments.   An understanding of the term structure is very important for appraising the interest rate risk of banks because all funding or investing decisions resulting from liquidity gaps have an impact on interest rate risk. Funding or investing decisions require a choice of maturity and a choice of interest rate which can only be made with the interest rate structure as the basic input.

This study tries to contribute to the Asset and Liability Management (ALM) of banks in Zimbabwe by providing a solution to the estimation of bank specific term structures that are used in setting up of Fund Transfer Pricing systems. According to BIS (2005), all term structure estimation models can be broadly classified into parametric and spline based approaches. Nelson-Siegel and its extension the Nelson-Siegel-Svensson model are parametric model and are also known as function based models because they are specified as single-based functions that are defined over the entire maturity domain.

Zimbabwe's interbank market which is a key component of the money market and the starting point of the monetary transmission mechanism has suffered from the persistent liquidity challenges. The absence of money market instruments in form of government paper has been contributing to the poor functioning of the interbank market, because market players cannot trade without suitable and acceptable collateral instruments to cover counterparty risks. According RBZ (2013) the cost of capital has remained high as evidenced by high lending rates as well as high bank charges. The high lending rates have discouraged borrowing in the economy, while initiatives to meaningfully mobilize savings have been militated by high bank charges, thus undermining the intermediary role played by banks.

The estimations and calibrations in this study were focused only on two parametric models namely the Nelson-Siegel and Nelson-Siegel-Svensson. The study explored the estimation approaches that produce the best parameters and explored techniques for in-sample and out-of-sample forecasting that maintains goodness of fit and smoothness.

2   Literature on Estimation of Term Structures

The Nelson Siegel Model can express the yield curve at any point of time as a linear combination of the level, slope and curvature factors, the dynamics of which drive the dynamics of the entire yield curve as mentioned by Diebold et al (2004).

The most important factor in determining the movement of term structures is the level factor according to Litterman and Scheinkman (1994). The second factor tends to have an effect on short-term rates that is opposite to its effect on long term rates. The third factor is the curvature factor, because it causes the short and long ends to increase, while decreasing medium-term rates.

Diebold et al (2004) cited that the dynamics of the three factors drive the dynamics of the entire term structure. Nelson and Siegel in their initial model formulated the parameter

λsuch that it can change with time, but Diebold and Li (2003) argued that fixing the parameter for the entire time resulted in a very little loss of fit and therefore concluded that λt should be fixed at 0.0609 so that the estimation procedure is simplified economic intuition is sharpened.

Nelson and Siegel (1987) parameterised the Nelson Siegel model and computed the best fitting values of the coefficients using linear least squares. The procedure was repeated over a grid of values for λ (time constant) to produce the overall best fitting values. Annaert et al (2012) referred to this procedure as a grid search.   Nelson and Siegel (1987) found the best fitting values of λ within the range of 50-100 and discovered that small values of λ are able to fit the curvature at low maturities because they correspond to rapid decay in the regressors. Correspondingly large values of λ were found to produce slow decay in the regressors and fitted curvature over long maturity ranges though unable to follow extreme curvature at short maturities.

Nelson and Siegel (1987) obtained the best fit for US T-bills to be given by λ =40. Nelson and Siegel (1987) highlighted that the set values of parameters are not expected to fit the data perfectly because a highly parameterised model that could follow all the wiggles in the data is less likely to predict well than a parsimonious model that assumes more smoothness in the underlying relation than one observed in data. The Nelson Siegel model was able to account for a very large fraction of the yields with a median R^2 of 0.9159 and Nelson and Siegel (1987) empirically found that little is gained in practice by fitting λ to each data set individually.

Aljinovic et al (2012) compared the performance of Nelson-Siegel and Nelson-Siegel-Svensson models using yield data from the Croatian market. The main objective of Aljinovic et al (2012) was to find the best fit model for yield curve estimation in Croatia. Yield data that was used was collected on weekly basis and Aljinovic et al (2012) used Excel in estimating the parameters of the two models using OLS with quasi Newton. It cases where it was difficult to estimate the parameters, Aljinovic et al (2012) resorted to using the Simplex method in statistic that is found in StatSoft. Nelson-Siegel model and Nelson-Siegel-Svensson model were compared using R^2 that gives information about goodness of fit of a model. Aljinovic et al (2012) compared the determination coefficient for the two models and performed t-tests at a 1% level of significance and found out that Nelson-Siegel-Svensson model produced a better fit for Croatian term structure.

Annaert et al. (2012) compared the different estimation methods and evaluated the estimation procedures based on the mean absolute error of their forecasting performance (Mean Absolute Prediction Error/ MAPE) and considered the estimation procedure that produce the lowest MAPE to be the best method. Ridge regression produced the minimum MAPE out of the estimation methods that where evaluated. Rezende and Ferreira (2011) compared the modeling and forecasting ability of a class of Nelson-Siegel models that included the three factor Nelson Siegel (1987) model, Bliss's three factor Model (1997), Nelson-Siegel-Svensson Model (1994) and their Five Factor Model based on a Quantile Autoregression (QAR)forecasting approach and daily implicit yield data from the interbank market. Rezende and Ferreira (2011) used the same approach as used by Annaert et al (2012) of minimising the average of the root mean squared error in comparing the model fitness and found that Nelson-Siegel-Svensson QAR forecasts produce a smaller Root Mean Squared Error when compared with Nelson Siegel QAR forecasts.

Diebold and Li (2006) found out that <u>Nelson Siegel Model produces term structure forecasts that appear much more accurate at long horizons than various standard benchmark forecasts</u>. Nelson Siegel model is however inconsistent with the no-arbitrage property meaning that the consistency between the dynamic evolution of interest rates and the actual shape of the yield curve is not ensured at certain moments as argued by Bjork and Christensen (1999).

Elen (2010) empirically tested whether the Nelson Siegel parameters legitimately reflect the level, slope and curvature elements of a term structure by first constructing a level, slope and curvature from observed yield data and then comparing them with the estimated parameters of the model. 25 year yield was taken to be the level of the yield curve and the slope was defined as the difference between the 25 year and 3 month yields (straight line). The curvature was computed as 2 times 2 year yield minus the sum of the 3-month and 25 yields. Elen (2010) then created a time series of the three factors of Nelson Siegel found by ordinary least squares and found out that the estimated factors and the defined factors followed the same pattern thus concluding that based on Canadian yields, the three factors of Nelson Siegel were indeed level, curvature and slope.

The method of Svensson (1994) is more flexible and has a better fitting than the original method of Nelson& Siegel (1987) as noted by Laurini and Moura (2010). Gilli et al (2010) estimated the parameters of Nelson-Siegel-Svensson using the approach introduced by Diebold and Li (2003) of fixing λ1 and λ2 and then estimate the rest of the parameters using a least squares algorithm. Gilli et al (2010) pointed out that the need to have constraints when solving optimization problem of obtaining parameters to guarantee the getting reasonable values. Just like Nelson and Siegel (1987), Diebold and Li (2003) and others, Gilli et al (2003) used the least squares approach to obtain the parameters for Nelson-Siegel-Svensson model with constraints: $0<\beta1<15; -15<\beta2<30; -30<\beta3<30; -30<\beta4<30; 0<\lambda1<30$ and $0<\lambda2<30$. Gilli et al repeated the estimation procedure using other algorithms like MATLAB's fmin search that uses Nelder-Mead in obtaining parameters and observed that the yield fit was better than the parameter fit and most errors where of 10 basis points and concluded that Nelson-Siegel-Svensson works well in interpolating observed yields.

To avoid potential challenges in numerical optimization, Molenaars et al (2013) followed Diebold and Li (2003)'s approach by fixing $\lambda t=\lambda=0.0609$ and estimated the remaining parameters using ordinary least squares regression. The out of sample performance of forecasting procedures was evaluated using RMSE. The smaller the RMSE is, the better the forecasting ability of the model considered is. MATLAB was used in this study and Molenaars et al (2013) concluded that Nelson-Siegel model does not properly fit the yield curve at all dates because it imposes a functional form to the yield curve that can result in inferior yield estimates if the yield curve does not fit to the functional form.

Bliss (1996) demonstrated the dangers of using in-sample goodness of fit as the sole criterion for comparing term structure estimation method. Bliss used parametric and non-parametric tests in comparing five term structure estimation methods and concluded that the Unsmoothed Fama-Bliss is overally the best, but also recommended that users interested in fitting term structures parsimoniously need to consider either the Smoothed Fama-Bliss or the Extended Nelson Siegel methods. In-sample results give a distorted view of the performance of the term structure, because there is a danger of over-fitting the data and this can be eliminated by using out-of-sample tests evaluating estimation methods. Nelson and Siegel (1987) also argued that the criteria for a satisfactory yield

curve model is that it be able to predict yields beyond the maturity range of the sample used to fit it because a function may be flexible to fit data over a specific interval, but may have very poor properties when extrapolated outside that interval.

3   Methodology

The researcher employed a quantitative research design in an attempt to answer the research questions of the study. The primary research question of this study was to find the most suitable method of fitting term structures for the Bank and suitability implies performing some statistical tests to compare the Nelson-Siegel model and the Nelson-Siegel-Svensson model.

3.1 Research Data

The researcher used daily yield data on certificates of deposits for the period starting March 2012 to March 2013. Yield data was annualized first before it was used in estimating the parameters of the term structure fitting models to ensure that yield data was based on the same temporal platform. The formula for calculating weighted annual effective rates is given below:

$$\text{Weighted AER}\left(M_j\right) = \frac{\sum_{i=1}^{n} \text{Cost}_i \text{AER}_i}{\sum_{i=1}^{n} \text{Cost}_i} \quad \text{for } \sum_{i=1}^{n} \text{Cost}_i > 0 \tag{1}$$

Where: j = maturity for certificate of deposits for $\forall j > 0$
                n = the number of deposits with the same maturity,
Cost_i is the size of deposit i with maturity M_j,
   AER_i is the Annual Effective Rate for deposit i with maturity M_j and it is calculated as:

$$\text{AER} = \left(1 + r_i \frac{j}{360}\right)^{\frac{360}{j}} - 1 \tag{2}$$

The weighted annual effective rates were regarded as the observed yields for the Bank and they were then used in the estimation of parameters for Nelson-Siegel model and Nelson-Siegel-Svensson models. Results from the two models were compared to determine the best fit model.

3.2 Term Structure Relationships

Term structure estimation methods are designed for the purpose of approximating one of the three equivalent representations of the term structure: spot rate curve, discount rate curve and the forward rate curve (Pooter, 2007). Assuming that all rates are continuously compounded, a forward rate $f_t(\tau, \tau^*)$ is the interest rate of a forward contract on an investment which is initiated τ periods in the future and matures τ^* periods after the date of initiation. The instantaneous forward rate $f_t(\tau)$ is obtained by letting the maturity of such a forward contract go to zero;

$$\lim_{\tau^* \downarrow 0} f_t(\tau, \tau^*) = f_t(\tau) \tag{3}$$

The spot rate (yield) with τ periods to maturity denoted by $y_t(\tau)$ can be obtained from the instantaneous forward rate. When the instantaneous forward rates are continuously compounded up to time to maturity (τ)and equally weighted, $y_t(\tau)$ is obtained:

$$y_t(\tau) = \frac{1}{\tau} \int_0^\tau f_t(m)dm \tag{4}$$

The discount function $P_t(\tau)$ denoting the present value of zero coupon bonds paying out a nominal amount of \$ after τ periods can be obtained from the spot rate$y_t(\tau)$:

$$P_t(\tau) = \exp[-\tau y_t(\tau)] \tag{5}$$

equivalently:

$$y_t(\tau) = -\frac{1}{\tau} \log P_t(\tau) \tag{6}$$

Linking forward rates directly to spot rates:

$$y_t(\tau) = \frac{1}{\tau} \int_0^\tau f_t(m)dm$$
$$\frac{d}{d\tau} y_t(\tau) = \frac{1}{\tau} f_t(m)|_0^\tau$$
$$\tau \frac{d}{d\tau} y_t(\tau) = f_t(\tau) - f_t(0)$$
$$f_t(0) = y_t(\tau) = \text{spot rate}$$
$$f_t(\tau) = \frac{d}{d\tau} y_t(\tau) + y_t(\tau) \tag{7}$$

3.3 Nelson Siegel Model

Nelson and Siegel (1987) introduced a simple parsimonious model for approximating the forward rate curve with mathematical approximating functions in the form of Laguerre functions. Nelson and Siegel (1987) assumed that the <u>instantaneous forward rate at maturity (τ) denoted $f_t(\tau)$ is given by the solution to a second order differential equation with real equal roots for spot rates:</u>

$$f_t(\tau) = \beta_{0,t} + \beta_{1,t} \left(\frac{-\tau}{\lambda_t}\right) + \beta_{2,t} \left(\frac{\tau}{\lambda_t}\right) \exp\left(\frac{-\tau}{\lambda_t}\right) \tag{8}$$

Averaging the forward rates

$$y_t(\tau) = \frac{1}{\tau} \left[\int_0^\tau \left[\beta_{0,t} + \beta_{1,t} \exp\left(\frac{-m}{\lambda_t}\right) + \beta_{2,t} \left(\frac{m}{\lambda_t}\right) \exp\left(\frac{-m}{\lambda_t}\right) \right] dm \right]$$

$$y_t(\tau) = \beta_{0,t} + \beta_{1,t}\left[\frac{1-\exp\left(-\frac{\tau}{\lambda_t}\right)}{\frac{\tau}{\lambda_t}}\right] + \beta_{2,t}\left[\frac{1-\exp\left(-\frac{\tau}{\lambda_t}\right)}{\frac{\tau}{\lambda_t}} - \exp\left(-\frac{\tau}{\lambda_t}\right)\right] \qquad (9)$$

The Nelson-Siegel Model that is widely used is the one for estimating spot rates given by equation (3.7) above. The model has four parameters that had to be estimated in order to estimate a yield for a given maturity(τ) which are $\beta_{0,t}, \beta_{1,t}, \beta_{2,t}$ and λ_t. The factors that make up the Nelson Siegel model are best interpreted by interpreting the factor loadings in the model because they exhibit different properties over a range of maturities. The first component in the Nelson-Siegel model is $\beta_{0,t}$ and its factor loading is 1 which is a constant that is independent of maturity, thus it is often interpreted as the level factor representing the long run yield.

The second component $\beta_{1,t}$ is interpreted as the slope and is weighted by a function of time to maturity (τ). The factor loading of $\beta_{1,t}$ is $\left[\frac{1-\exp\left(-\frac{\tau}{\lambda_t}\right)}{\frac{\tau}{\lambda_t}}\right]$ and when $\tau = 0$ it is unit and it exponentially decays to zero when $\tau \rightarrow \infty$ primarily affecting $\beta_{1,t}$ in the short run. The component can either have a downward ($\beta_1 > 0$) or upward ($\beta_1 < 0$) slope. The third component in the model $\beta_{2,t}$ is interpreted as the curvature and its factor loading is $\left[\frac{1-\exp\left(-\frac{\tau}{\lambda_t}\right)}{\frac{\tau}{\lambda_t}} - \exp\left(-\frac{\tau}{\lambda_t}\right)\right]$.

The function represent the medium term component in a yield curve because it starts at zero gradually increasing and decreases back to zero as $\tau \rightarrow \infty$ thereby adding a hump to the curve and is termed the medium term. The component can generate a hump if $\beta_{2,t} > 0$ or a trough if $\beta_{2,t} < 0$. The higher the absolute value of $\beta_{2,t}$, the more pronounced the trough or hump is.

The limiting behavior of the Nelson-Siegel model:

$$\lim_{\tau \to 0} y_t(\tau) = \beta_{0,t} + \beta_{1,t} \qquad (10)$$
$$\lim_{\tau \to} y_t(\tau) = \beta_{0,t} + \beta_{1,t} \qquad (11)$$

3.4 Nelson Siegel Svensson Model

Nelson-Siegel (1987) model has gone through several improvements to enhance flexibility and to capture a wider variety of curve shapes with the modifications mainly carried out by adding factors and decaying parameters. Svensson (1994) extended the Nelson Siegel model by incorporating for a second hump/trough in the model. Svensson (1994) added another exponential term more similar to the third term in Nelson-Siegel Model, but with a different decaying parameter λ_{2t}. The additional factor loading on the Nelson-Siegel-Svensson model is of β_{3t} and is given by the same Laguerre function as the one in Nelson-Siegel model that determines the curvature of the yield curve, but with a different decay parameter λ_{2t}. The additional factor loading is $\left[\frac{1-\exp\left(-\tau/\lambda_{2t}\right)}{\tau/\lambda_{2t}} - \exp\left(-\tau/\lambda_{2t}\right)\right]$. The limiting behavior of the Nelson-Siegel-Svensson model is the same as the limiting behavior of Nelson-Siegel model provided in (8) and (9).

$$y_t(\tau) =$$

$$\beta_{0t} + \beta_{1t}\left[\frac{1-\exp(-\tau/\lambda_{1t})}{\tau/\lambda_{1t}}\right] + \beta_{2t}\left[\frac{1-\exp(-\tau/\lambda_{1t})}{\tau/\lambda_{1t}} - \exp(-\tau/\lambda_{1t})\right] + \beta_{3t}\left[\frac{1-\exp(-\tau/\lambda_{2t})}{\tau/\lambda_{2t}} - \exp(-\tau/\lambda_{2t})\right] + \varepsilon_t^\tau$$

(12)

The NSS model therefore incorporates two decaying parameter which are λ_1 and λ_2 which means it has 6 parameters that will have to be estimated $\beta_{0,t}, \beta_{1,t}, \beta_{2,t}, \beta_{3,t}, \lambda_{1,t}$ and $\lambda_{2,t}$ compared to the 4 parameters in NS model. Bolder and Streliski (1999) discussed the multi-colinearity problem that came within the model if $\lambda_{1,t} = \lambda_{2,t}$. The multicolinearity problem is reduced by ensuring that $\lambda_{1,t} \neq \lambda_{2,t}$.

3.5 Parameter Estimation

Ordinary Least Squares
The researcher fixed the λ_{1t} parameter as recommended by Diebold and Li (2004) and Elen(2010) to simplify the estimation procedure by reducing the model to a linear regression. The estimation of parameters was done by considering the following

$$Y_t(\tau) = X_t * \beta_t + \varepsilon_t$$

(13)

functional form for a regression:

where $Y_t(\tau)$ denotes the vector of interest rates (annual effective rates) at time t for n different times of maturity collected in vector τ,
ε_t is a vector of error terms at time t for n different estimates of interest rates,
X_t is a vector of the factor loadings of the Nelson Siegel Model which can be estimated given maturity (τ) and λ. The vector of the factor loadings can be interpreted as the vector for independent variables that explain the dependent variable yield. Using equation (3.9) for Nelson-Siegel model the following vector is obtained:

$$X_t = \begin{bmatrix} 1 & \frac{1-e^{-\kappa_t\tau_1}}{\kappa_t\tau_1} & \frac{1-e^{-\kappa_t\tau_1}}{\kappa_t\tau_1} - e^{-\kappa_t\tau_2} \\ 1 & \frac{1-e^{-\kappa_t\tau_2}}{\kappa_t\tau_2} & \frac{1-e^{-\kappa_t\tau_2}}{\kappa_t\tau_2} - e^{-\kappa_t\tau_2} \\ . & . & . \\ . & . & . \\ 1 & \frac{1-e^{-\kappa_t\tau_n}}{\kappa_t\tau_n} & \frac{1-e^{-\kappa_t\tau_n}}{\kappa_t\tau_n} - e^{-\kappa_t\tau_n} \end{bmatrix}$$

(14)

where $\frac{1}{\lambda_t}$ is replaced by κ_t for simplistic in writing the vector X_t and vector $\beta_t = \beta_{0,t}, \beta_{1,t}, \beta_{2,t}$
Equation 3.14 was extended with the inclusion of another column of the fourth loadings of Nelson-Siegel-Svensson model. Estimations were done for 252 different dates and the researcher built a model that used macro for ordinary least squares (algorithm for ordinary least squares) in Excel using Visual Basics in Excel. The VBA code to perform

the ordinary least squares regression and the analysis of variance was written using ordinary least squares formulas provided by Davidson and MacKinnon (1993) and the steps found in Rouah andVainberg (2007). The estimate of parameters using ordinary least squares is given by:

$$\hat{\beta}_{OLS} = (X^T X)^{-1} X^T Y \tag{15}$$

Where:

$\hat{\beta}_{OLS}$ = a vector of dimension k containing estimated parameters

$X = (\iota X_1 X_2 \ldots X_{k-1})$ = a design matrix of dimension n x k containing independent variable and the vector ι

ι = a vector of dimension n containing ones

The fitted values can then be obtained using the estimated vector of parameters $\hat{\beta}_{OLS}$

$$\hat{Y} = X\hat{\beta}_{OLS} \tag{16}$$

For the analysis, the total sum of squares (SSTO) is defined as the variability of the dependent variable about it mean. SSTO can be expressed in terms of the error sum of squares (SSE) and the regression sum of squares (SSR)

$$SSTO = SSE + SSR \tag{17}$$

Where:

$$SSTO = \sum_{i=1}^{n} (y_i - \bar{y})^2$$

$$SSE = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = (Y - \hat{Y})^T (Y - \hat{Y})$$

$$SSR = \sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2$$

y_i = elements of Y

$\hat{y}_i$ = elements of $\hat{Y}(i = 1,2,\ldots,n)$

$\bar{y} = \frac{1}{n}\sum_{i=0}^{n} y_i$ is the sample mean of the dependent variable

The mean square error (MSE) which is an estimate of the variance σ^2 is given as

$$\hat{\sigma}^2 = MSE = \frac{SSE}{n-k} \tag{18}$$

and the estimate of the standard deviation is given by $\hat{\sigma} = \sqrt{MSE}$. The coefficient of determination R^2 that measures the proportion of variability in the dependent variable that can be attributed to a linear model is given by the proportion of SSTO

$$R^2 = \frac{SSR}{SST\,0} \tag{19}$$

In order to incorporate for the increase in R^2 as a result of an increase in variables, adjusted R^2 is calculated and is given by

$$R_a^2 = 1 - \left(\frac{n-1}{n-k}\right)(1 - R^2) \tag{20}$$

The covariance matrix of the parameters is given by

$$Cov(\hat{\beta}_{OLS}) = MSE(X^TX)^{-1} \tag{21}$$

The t-statistic for each regression coefficient is given

$$t = \frac{\hat{\beta}_j}{SE(\hat{\beta}_j)} \tag{22}$$

where: $\hat{\beta}_j$= j-th element of $\hat{\beta}$
$SE(\hat{\beta}_j)$ = standard error of $\hat{\beta}_j$ obtained as the square root of the j-th diagonal element of (3.21). Once a *t-statistic* is obtained for a parameter, the two-tailed*p-value* can then be obtained from a Student *t-distribution* with *(n-k)* degrees of freedom to determine the statistical significance of the regression coefficient.

3.6 Model Comparison

The researcher compared the two models to determine which model best fits Zimbabwean data based on the yield data of certificates of deposits. The models were compared using the Coefficient of Determination (R^2) and the Root Mean Square Error (RMSE).
The formula for R^2 is given by:

$$R^2 = \frac{SS_R}{SS_T} \tag{23}$$

whereSS_R is the regression sum of squares calculated as $\sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2$ and SS_T is the total sum of squares calculated as $\sum_{i=1}^{n}(y_i - \bar{y})^2$. The t-test was conducted in Excel.
The formula for RMSE is given as:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n}} \tag{24}$$

3.7 Out-of-sample Forecasting

The researcher followed the works of Diebold and Li (2006) and De Pooter (2007) by specifying a first-order autoregressive process for forecasting the out-of-sample parameters for the Nelson-Siegel model and Nelson-Siegel-Svensson model. An autoregressive (AR) of order p is a process where the realisation β_t is a weighted sum of past realisations i.e.$\beta_{t-1}, \beta_{t-2}, ..., \beta_{t-p}$.The first-order autoregressive process (AR(1)) is

expressed as:

$$\beta_t = \mu + \phi\beta_{t-1} + v_t \tag{25}$$

where μ is the intercept and ϕ is the coefficient of the lagged value of β_t and v_t is the random error. The parameters of the AR (1) process were obtained using non-linear regression in E-views. The model was be used in forecasting $\beta_{0,t}, \beta_{1,t}, \beta_{2,t}$ parameters for Nelson-Siegel model and $\beta_{0,t}, \beta_{1,t}, \beta_{2,t}, \beta_{3,t}$ for Nelson-Siegel-Svensson model.
The researcher used the both the Dynamic and Static methods available in E-views in forecasting in order to determine the best approach.

4 Main Results

4.1 Grid Search for λ for Nelson-Siegel Model

A Grid search for λ was done using data for 25 March 2013 to find a value of λ that could be fixed for all the dates while producing a good fit. Increasing λ resulted in fitting a yield curve that increased yield in the short maturity and minimised the yield in the longer maturity showing signs on high decay parameter that results in a strong negative slope in the long run. Decreasing the shape parameter resulted in lower short term yields that were compensated by higher yields in the longer maturity. The evolution of the fitted yield curve to changes in the shape parameter is shown below:
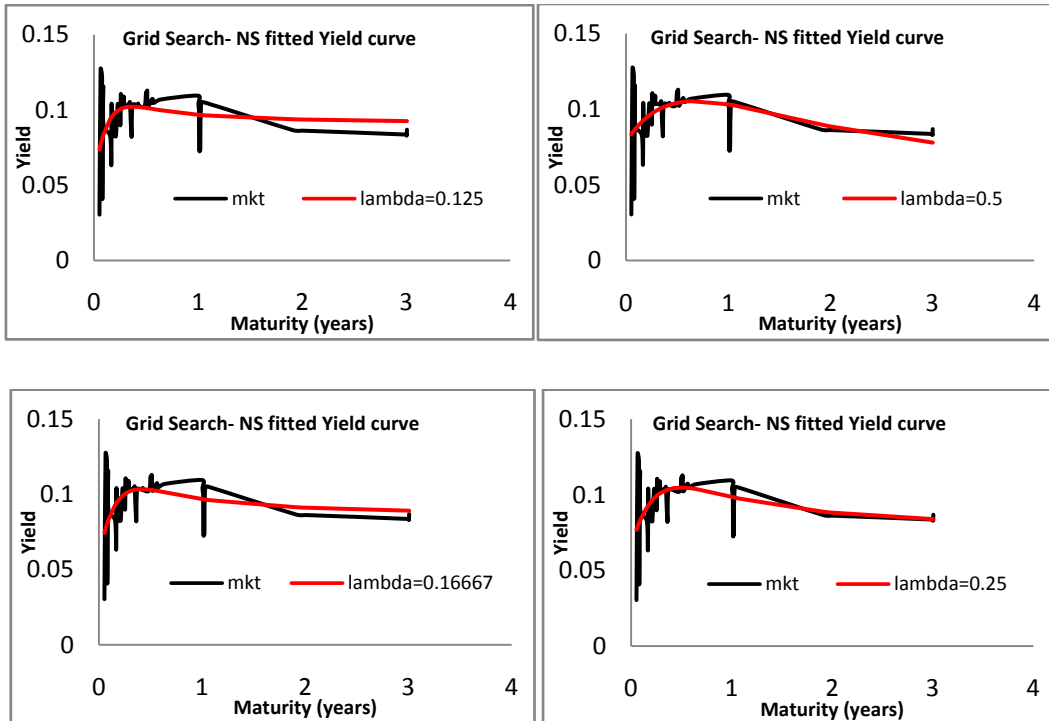
Figure 1: Fitted NS yield curves with different λs

The value of λ that was chosen is the one that fits the both the short maturities and long maturities better. λ =0.25 fitted the yield curve better and was considered as the shape parameter that was fixed for the whole year's daily estimates.

4.2 Analysis of Time Series Results of Nelson-Siegel Model (λ = 0.25)

The researcher estimated and fitted the yield curve for 257 different days between March 2012 and March 2013. Analysis of variance was done on each day's estimations with t-statistics, p-values and betas (s.e.)[3] for the three parameters were calculated and analysed. R^2 Adjusted R^2, √MSE and ε2 was also calculated for the 257 days and a time series of these statistics was generated. The time series of the parameters and statistics is presented below by making use of graphs:

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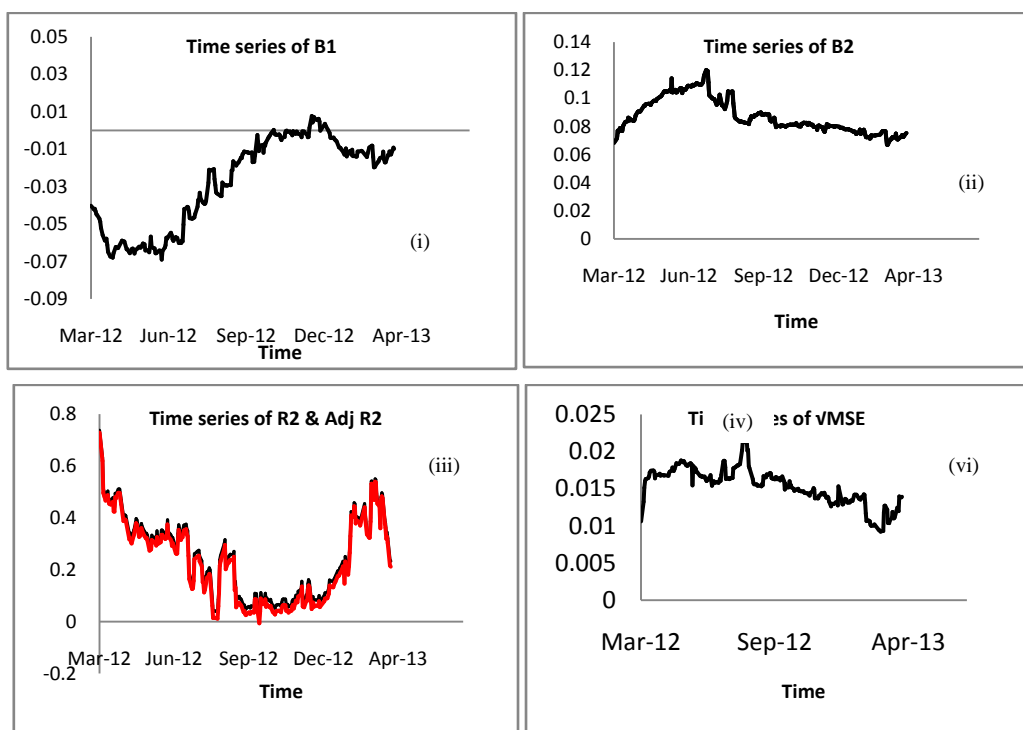
[3]Standard error of estimated parameters
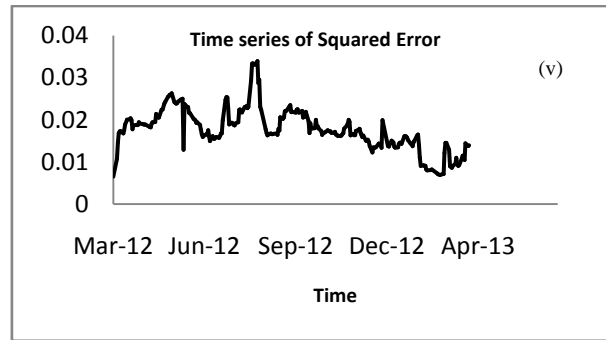
Figure 2: Time series of Nelson-Siegel Parameters

Table 1: Measures of central tendency for parameters and statistics of Nelson-Siegel model

	β_0	β_1	β_2	R^2	Adj R^2	$\sqrt{MSE}$	$\varepsilon 2$
Min	0.067008	-0.068990	-0.063170	0.018918	-0.006565	0.009233	0.006530
Max	0.120198	0.007775	0.217751	0.737676	0.728631	0.021569	0.033962
Mean	0.087544	-0.027450	0.072067	0.244388	0.224150	0.015346	0.017770
□	0.012451	0.024142	0.048620	0.152127	0.155248	0.002456	0.005000

The parameters of Nelson-Siegel exhibited an erratic behavior over the year that is mainly attributed to the erratic behavior in the economic environment in Zimbabwe. β_0 started at 0.0684 and increased steadily to reach a peak of 0.1202 in July 2012 before it started dropping and ended at 0.0752 in March 2013. β_0 is interpreted as the long run yield in literature (Diebold and Li, 2006) because the parameter remained positive over the year of study as expected in a market where there are no negative interest rates on long term loans data. β_1 was a little more volatile than β_2 and had a standard deviation of 2.41%. β_1 started at -0.0401 in March 2012 and increased gradually increased to reach a positive value of 0.00778 and started dropping to end at -0.00987 a higher value than the starting value. B_2 was the most volatile parameter of the Nelson-Siegel model with a standard deviation of 4.86%.

From the results and graphs presented above it was found the R^2 of the Nelson-Siegel model ranged from 1.9 % to 72.9% during the year depending on the structure of the market data that was used in making the estimates. The best fit was obtained on 3 March 2012 and the worst performance of the Nelson-Siegel model was observed on 1 October 2012. The overall average R^2 of the model is 24.4%. As earlier on argues at the beginning of the chapter, this small value of R^2 does not mean that Nelson-Siegel Model was unable to fit well a smooth curve that can be used for fund transfer pricing. The small value of R^2 is largely attributed to the noise in the market data based on certificate of deposits.

A correlation analysis of the parameters of Nelson-Siegel and R^2 showed that β_2 has a strong positive correlation with R^2 which implies that whenever the model has trouble in obtaining a good fit, it is reflected by a drop in the value β^2 that determines the weight of the curvature component of the mode. Thus any trouble in determining the curvature of the yield curve implies that the data is too volatile and the model will only manage to account for less of the proportion of variability. Correlation results are shown in Appendix 1. The diagram below shows the movements between β^2 and R^2
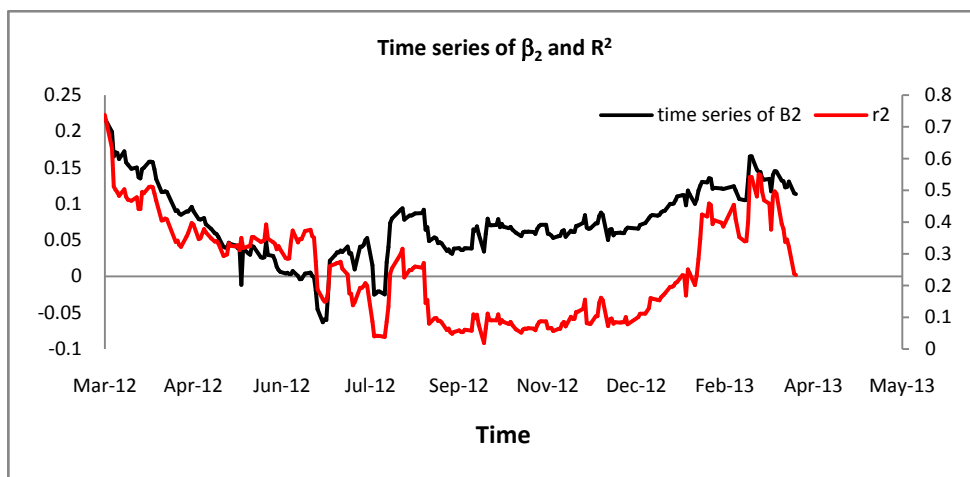
Figure 3: Correlation between β_2 and R^2

4.3 Nelson-Siegel-Svensson Model

Grid Search for λ_1 and λ_2 for Nelson-Siegel-Svensson Model

The Nelson-Siegel-Svensson required two shape parameters λ_1 and λ_2 so that the model can be estimated using OLS. The researcher conducted a grid search by varying λ_1 and λ_2 in the range (0, 5] and (0,5] respectively. The grid search was done on data for 31 August 2013 that posted one of the smallest R^2 using Nelson-Siegel model. 2500 estimates were done in the grid (0, 5] with variations of 0.1 on the estimations of shape parameters and the best fits that had a high R^2 were plotted to determine the evolution of the fitted yield curves as λ_1 and λ_2 changes. The diagram below shows some of the yield curves that were estimated in the grid search:
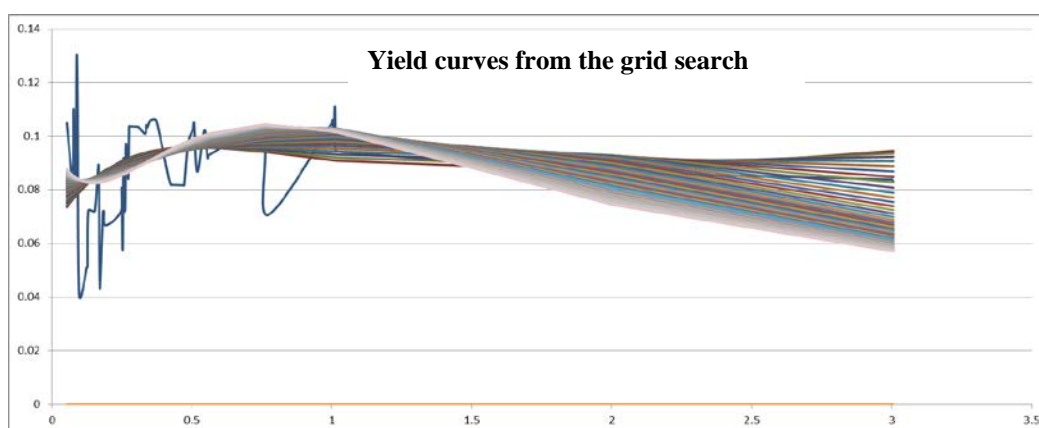
Figure 4: Different estimates of yield curve using varying shape parameters

The estimates were then further reduced making use of R^2 and observation and eliminated all the yield curves that did not produce a better fit and obtained the following 11 curves.
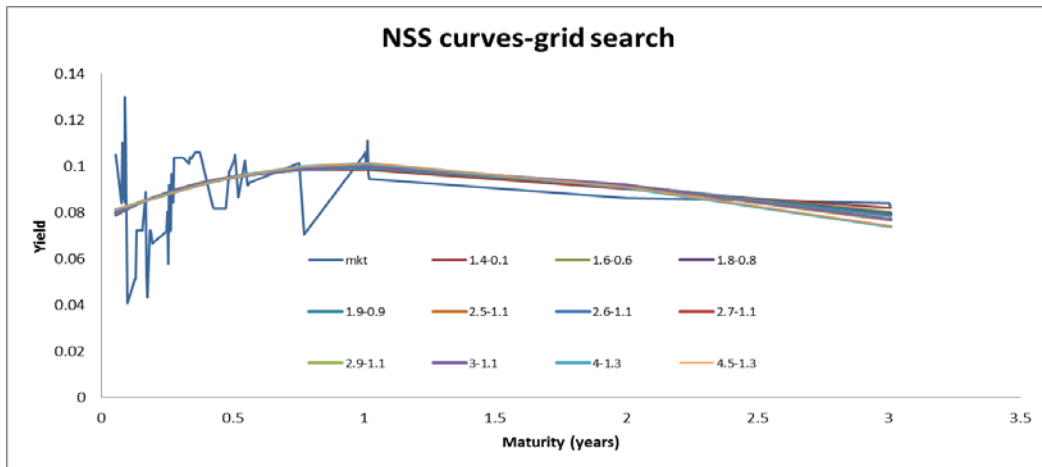
Figure 5: Reduced estimates of yield curve using varying shape parameters

The 11 curves were then analysed to obtain the curve that produced a better fit at the end of the curve because one that produces a lower cost at longer maturities can be misleading and very costly for the bank. The researcher realized that increasing the value of λ_1 resulted in a trough at the shortest maturity but this was compensated by significant loss of fit at the longer maturity as shown in Diagram 8. Based on the analysis of the results from the grid search, the researcher finally obtained the best yield curve that was estimated using $\lambda_1 = 0.714$ and $\lambda_2 = 10$. The yield curve is shown in the diagram below:
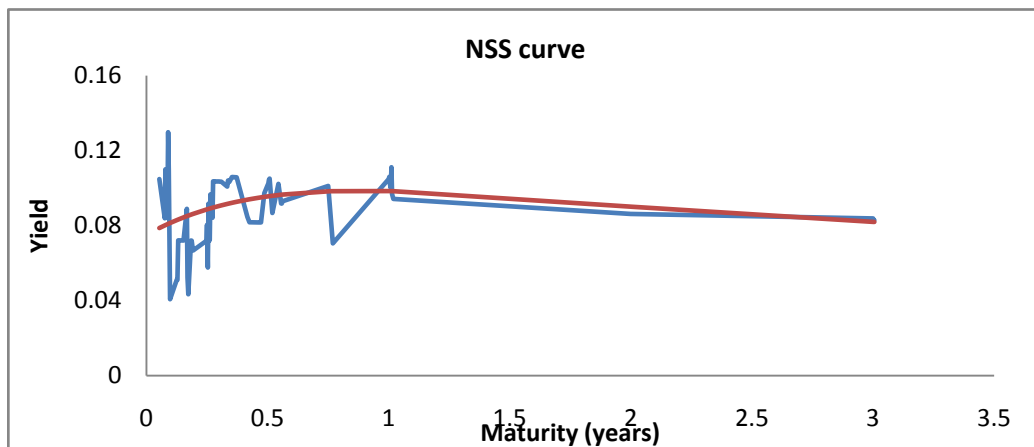
Figure 6: Best yield curve from the grid search

Analysis of time series results of Nelson-Siegel-Siegel Model ($\lambda_1=0.714$ and $\lambda_2=10$)
The researcher estimated and fitted the yield curves for the 257 different days between March 2012 and March 2013. Analysis of variance was done on each day's estimations with t-statistics, p-values and betas (s.e.) for the four parameters were calculated and analysed just like what was done for Nelson-Siegel Model. R^2, Adjusted R^2, $\sqrt{MSE}$ and ε^2 was also calculated for the 257 days and a time series of these statistics was generated. The time series of the parameters and statistics of Nelson-Siegel-Svensson model is presented below by making use of graphs:
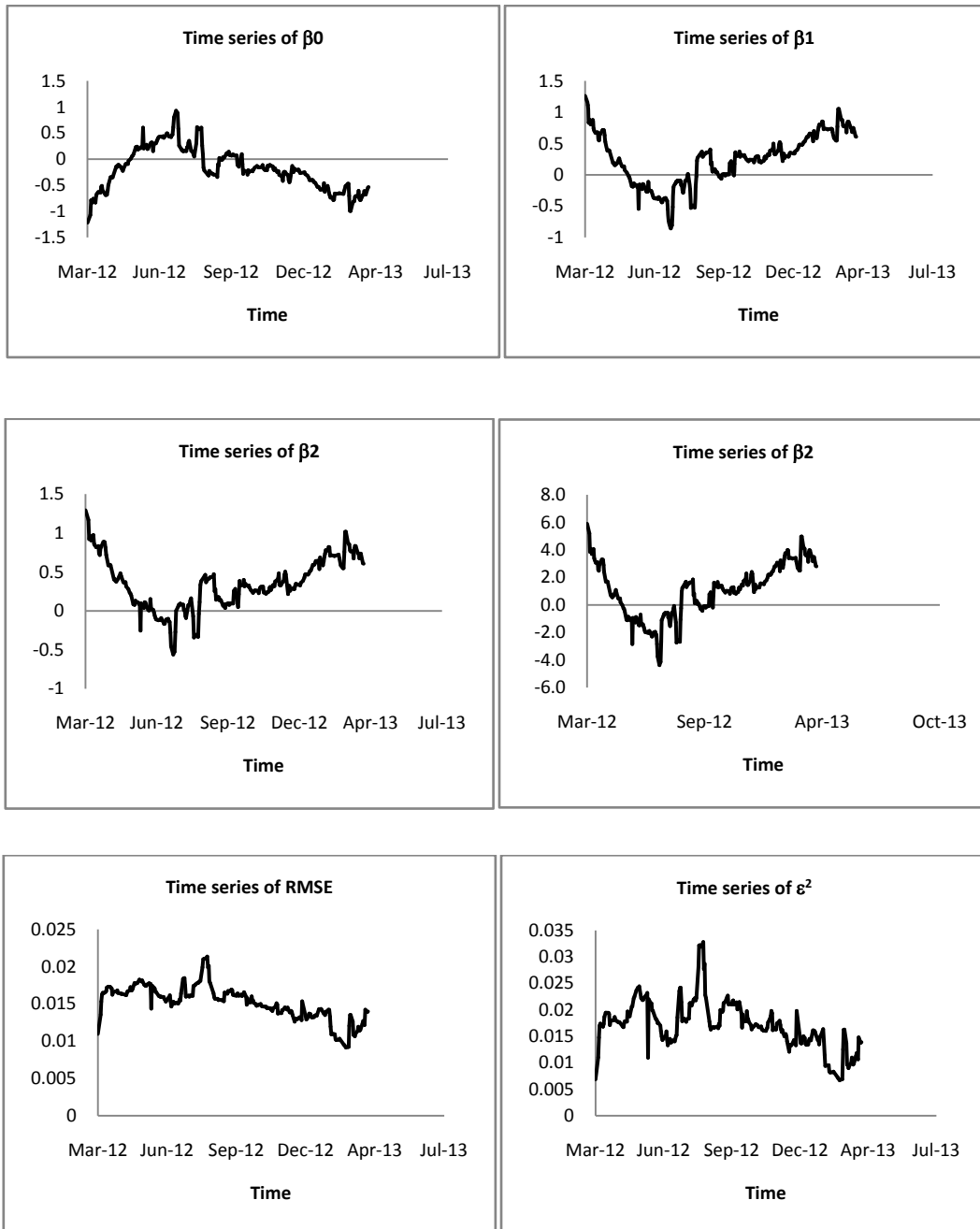
Figure 7: Time series of Nelson-Siegel-Svensson Parameters and Statistics

Table 2: Measures of central tendency for parameter and statistics of
Nelson-Siegel-Svensson model

	β_0	β_1	β_2	β_3	R^2	Adj R^2	$\sqrt{MSE}$	$\varepsilon2$
Min	-1.21683	-0.85409	-0.56155	-4.36781	0.03868	0.00073	0.00915	0.00670
Max	0.92950	1.25956	1.29225	5.90104	0.72439	0.70989	0.02135	0.03282
Mean	-0.17147	0.23748	0.33615	1.00433	0.26425	0.23435	0.01520	0.01719
☐	0.40473	0.40485	0.33236	1.96836	0.15327	0.15785	0.00225	0.00456

β_0 started at -1.2168 in March 2012 and increased steadily to reach a peak of 0.9295 in July 2012 before it started dropping and ended at -0.53452 in March 2013. This trend was also identified with β_0 of Nelson-Siegel model but the difference is that parameters of in Nelson-Siegel the parameter was never positive and was easily interpreted as representing the long run yield in the Zimbabwean market. β_0 is thus not interpreted as the long run yield for the market understudy as it is interpreted in literature (Diebold and Li, 2006) because of its negative values. The parameter β_0 also showed a very large standard deviation of 40.47% which is much higher that of Nelson-Siegel of 1.25%. β_1 of Nelson-Siegel-Svensson was unstable compared to that of Nelson-Siegel Model with a standard deviation of40.48%. β_1 moved in a perfect opposite direction with β_0 as shown by a high negative correlation of 99.9%.

β_2 was the less volatile compared to the rest of Nelson-Siegel-Svensson parameters and moved in the same direction with β_3 and β_1 with correlations of 97.42% and 97.6% respectively. The fourth parameter of Nelson-Siegel-Svensson model that is responsible for fitting a second hump to a yield curve had a high negative correlation with ☐ of 99.9%. The p-values of the model parameters were below 0.05 in most of the estimates showing that they were statistically significant at a 95% significance level.

From the results and graphs presented above it was found the R^2 of the Nelson-Siegel model ranged from 3.8% to 72.4% during the year depending on the structure of the market data that was used in obtaining the estimates. The best fit was obtained on 3 March 2012 and the worst performance of the Nelson-Siegel-Svensson model was observed on 1 October 2012 and these dates correspond to the best and worst performance of Nelson-Siegel model. This shows that the performance of the two models depended on the structure of the market data. The overall average R^2 of the Nelson-Siegel-Svensson model is 26.4

4.4 Model Comparisons

Coefficient of Determination
A t-test was conducted on the adjusted coefficients of determination (adj R^2) of Nelson-Siegel and Nelson-Siegel-Svensson so that results that incorporated a penalty of additional variables are used in comparing the model.Results of a t-test at 5% significance level are shown in the Table 3 below.

Table 3: t-test for the means of adj R2 of Nelson-Siegel and Nelson-Siegel-Svensson models

t-Test: Two-Sample Assuming Unequal Variances

	NS Model	NSS Model
Mean	0.224149822	0.234357317
Variance	0.024102045	0.024919111
Observations	257	257
Hypothesized Mean Difference	0	
df	512	
t Stat	-0.739084265	
P(T<=t) one-tail	0.230097259	
t Critical one-tail	-1.647835164	
P(T<=t) two-tail	0.460194518	
t Critical two-tail	1.964608113	
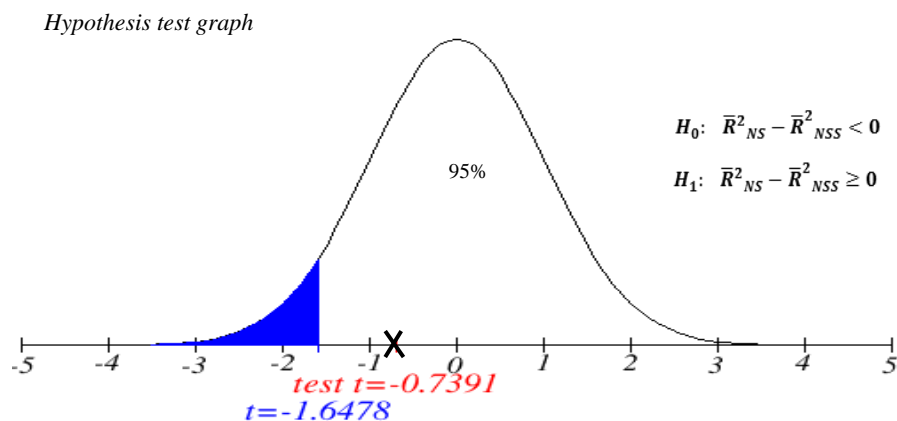
Figure 8: Hypothesis t-test graph

A test statistic of -0.7391 was obtained against a 5% critical t-value of -1.6478. The test results failed to reject the notion that the proportion of variability of Nelson-Siegel-Svensson model was smaller than that of Nelson-Siegel model i.e. adj R^2 of Nelson-Siegel-Svensson model is statistically higher than that of Nelson-Siegel. The results therefore concludes that Nelson-Siegel-Svensson model is the better model of fitting a term structure for the Bank if the models are compared based on their proportion of variability. See Figure 15 below for graphical comparison of adj R^2.

Root Mean Squared Error
Daily comparisons of the RMSE from the two models were conducted to determine the model that fitted the term structure with precision on each day of the year. Summarized results are in the table below.

Table 4: RMSE comparisons

Model	Frequency
Nelson-Siegel	118
Nelson-Siegel-Svensson	139
Total	257

The assessment of the estimate of the standard deviation (RMSE) showed that Nelson-Siegel fitted the yield curve with more precision than its extension, Nelson-Siegel-Svensson model on 118 occasions out of the 257 days estimated. Nelson-Siegel-Svensson proved superior on 139 occasions which corresponds to 54% of the time.
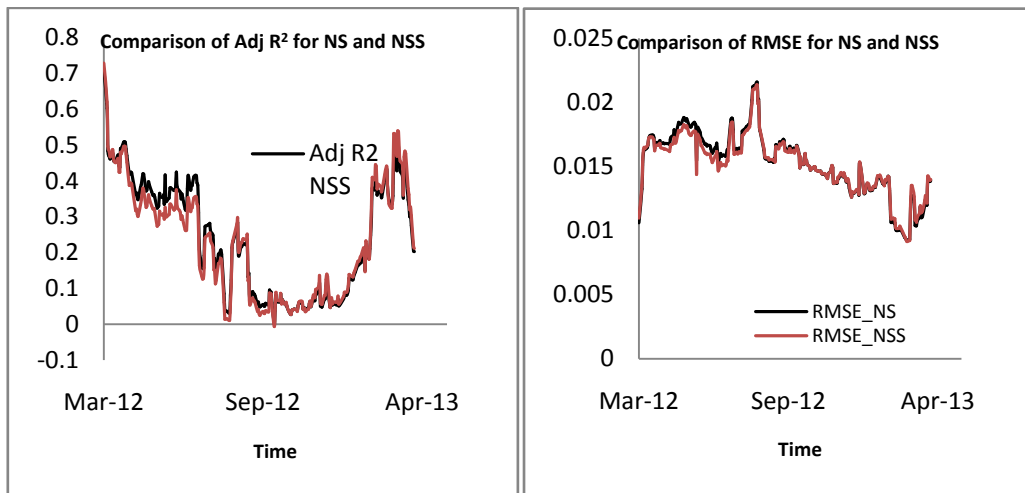
Figure 15: Comparisons of Adj R2 and RMSE

4.5   In-sample and Out-of-sample forecasting

The researcher obtained the following summarized results for the AR(1) models for the different parameters of Nelson-Siegel model and Nelson-Siegel-Svensson model. Results were estimated using data observations obtained from daily estimations of the models for the period 01 Mar 2012 – 31 Oct 2012. The rest of the observations from 01 Nov 2012 – 26 Mar 2013 were excluded from parameter estimation and were treated as hold out sample.   Results are presented in the format of the specification of an AR (1) model presented in equation (3.25):

$$\beta_t = \mu + \phi\beta_{t-1} + \nu_t$$

Results for Nelson-Siegel model parameters:

$$\begin{pmatrix} \beta_{0t} \\ \beta_{1t} \\ \beta_{2t} \end{pmatrix} = \begin{pmatrix} 0.003283 \\ 0.000245 \\ 0.002697 \end{pmatrix} + \begin{pmatrix} 0.965910\beta_{0t-1} \\ 1.000530\beta_{1t-1} \\ 0.937058\beta_{2t-1} \end{pmatrix} + \begin{pmatrix} v_t \\ v_t \\ v_t \end{pmatrix}$$

Results for Nelson-Siegel-Svensson parameters:

$$\begin{pmatrix} \beta_{0t} \\ \beta_{1t} \\ \beta_{2t} \\ \beta_{3t} \end{pmatrix} = \begin{pmatrix} 0.006538 \\ -0.002582 \\ 0.008314 \\ -0.021603 \end{pmatrix} + \begin{pmatrix} 0.936134\beta_{0t-1} \\ 0.937684\beta_{1t-1} \\ 0.933942\beta_{2t-1} \\ 0.937250\beta_{3t-1} \end{pmatrix} + \begin{pmatrix} \upsilon_t \\ \upsilon_t \\ \upsilon_t \\ \upsilon_t \end{pmatrix}$$

Detailed results for the estimation of parameters are found in Appendix 2 and 3. The in-sample and out-of-sample forecasts are plotted in the diagrams below:
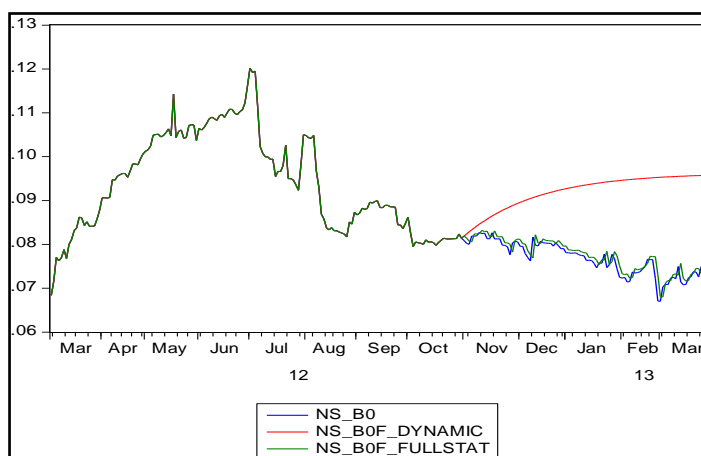
Forecast: B0_NSF	
Actual: B0_NS	
Forecast sample: 11/01/2012 3/26/2013	
Included observations: 95	
Root Mean Squared Error	0.001599
Mean Absolute Error	0.001211
Mean Abs. Percent Error	1.600144
Theil Inequality Coefficient	0.010352
Bias Proportion	0.206612
Variance Proportion	0.004158
Covariance Proportion	0.789230

Figure 9: In-sample and out-of sample forecasts for β_0 (NS)
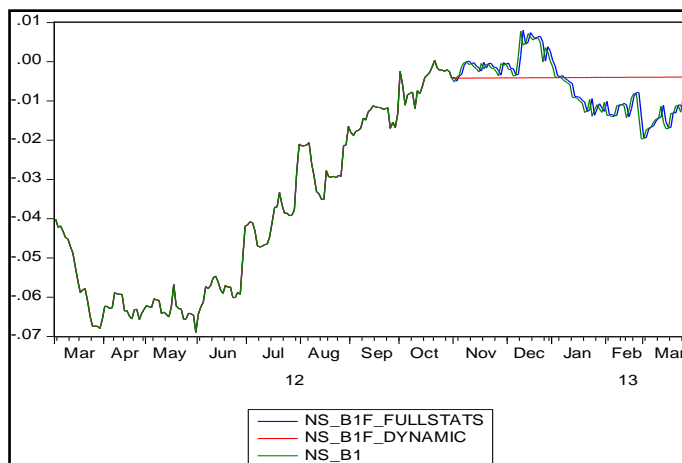
Forecast: B1_NSF	
Actual: B1_NS	
Forecast sample: 11/01/2012 3/26/2013	
Included observations: 95	
Root Mean Squared Error	0.002115
Mean Absolute Error	0.001530
Mean Abs. Percent Error	142.4685
Theil Inequality Coefficient	0.112411
Bias Proportion	0.020246
Variance Proportion	0.000000
Covariance Proportion	0.979754

Figure 10: In-sample and out-of sample forecasts for β_1 (NS)
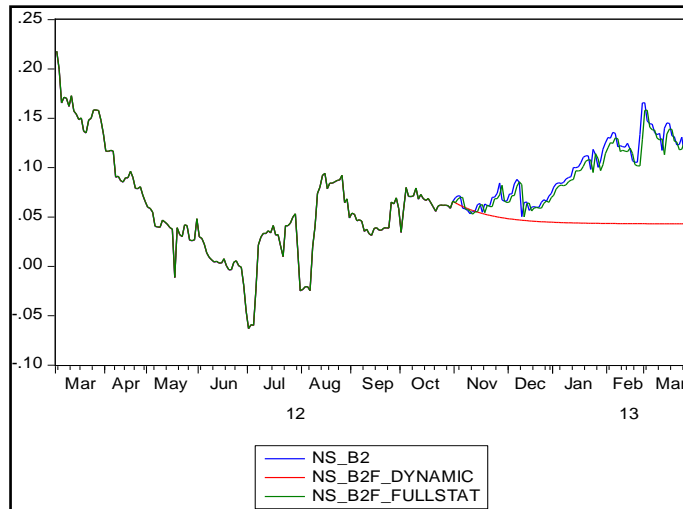
Forecast: B2_NSF	
Actual: B2_NS	
Forecast sample: 11/01/2012 3/26/2013	
Included observations: 95	
Root Mean Squared Error	0.009795
Mean Absolute Error	0.007071
Mean Abs. Percent Error	7.522035
Theil Inequality Coefficient	0.049544
Bias Proportion	0.153105
Variance Proportion	0.034741
Covariance Proportion	0.812155

Figure 11: In-sample and out-of sample forecasts for β_2 (NS)
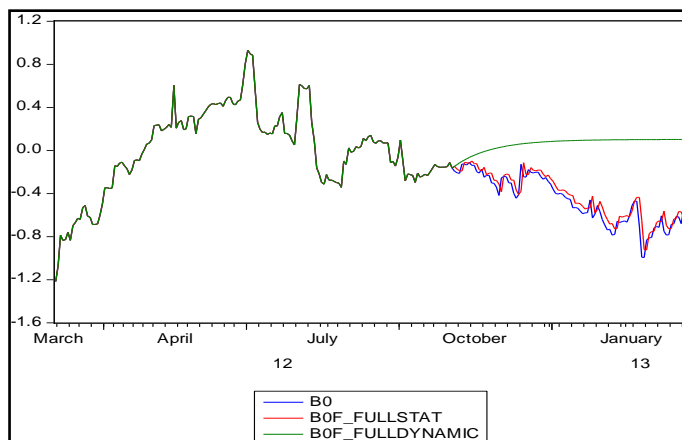
Forecast: B0F_NSS
Actual: B0
Forecast sample: 11/01/2012 3/26/2013
Included observations: 95
Root Mean Squared Error   0.083125
Mean Absolute Error   0.062235
Mean Abs. Percent Error   16.29405
Theil Inequality Coefficient 0.084343
   Bias Proportion   0.228133
   Variance Proportion   0.022526
   Covariance Proportion   0.749340

Figure 12: In-sample and out-of sample forecasts for β_0 (NSS)
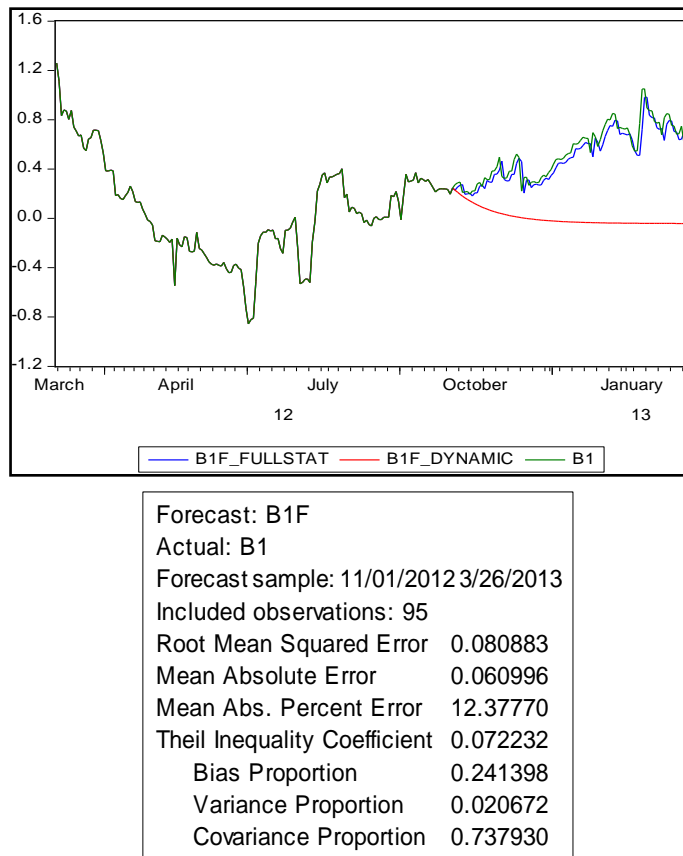
Figure 13: In-sample and out-of sample forecasts for β_1 (NSS)
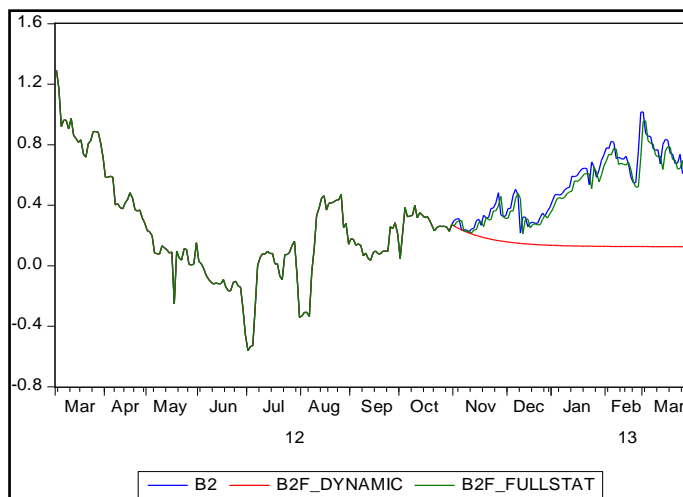
Forecast: B2F_NSS
Actual: B2
Forecast sample: 11/01/2012 3/26/2013
Included observations: 95

Root Mean Squared Error	0.072147
Mean Absolute Error	0.052685
Mean Abs. Percent Error	10.70053
Theil Inequality Coefficient	0.065194
Bias Proportion	0.173213
Variance Proportion	0.028709
Covariance Proportion	0.798079

Figure 14: In-sample and out-of sample forecasts for β_2 (NSS)

Forecast: B3F_NSS	
Actual: B3	
Forecast sample: 11/01/2012 3/26/2013	
Included observations: 95	
Root Mean Squared Error	0.395885
Mean Absolute Error	0.298125
Mean Abs. Percent Error	13.45923
Theil Inequality Coefficient	0.076509
Bias Proportion	0.240649
Variance Proportion	0.020546
Covariance Proportion	0.738806
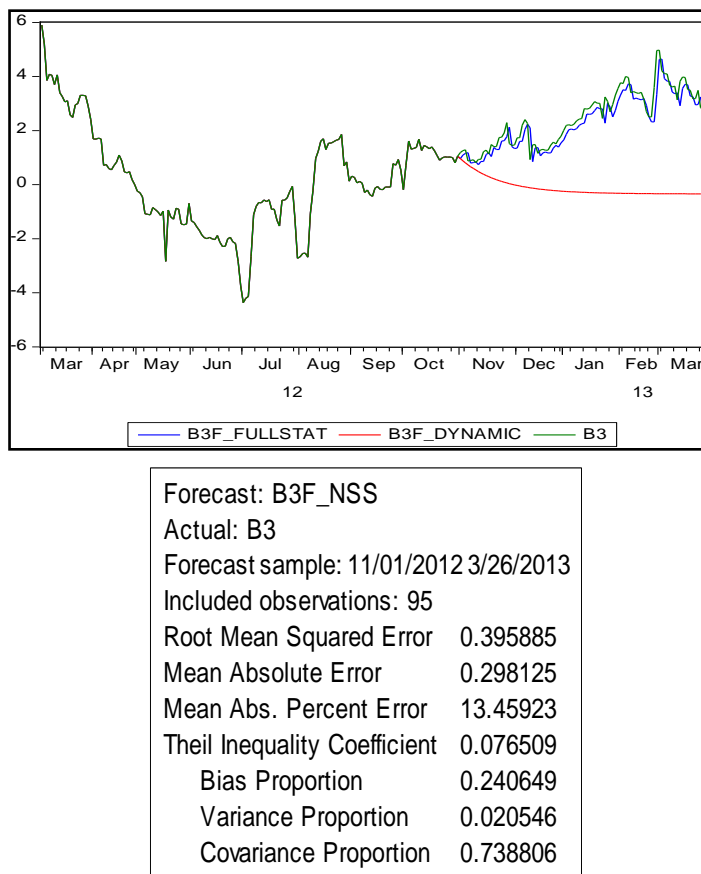
Figure 15: In-sample and out-of sample forecasts for β_3 (NSS)

A graphical analysis of the in-sample(green trends) and out-of-sample (blue trends) forecasts shows that both Nelson-Siegel model and Nelson-Siegel-Svensson models have great out-of-sample forecasting capabilities as shown by the blue trends that are almost in line with the green trends. A statistical analysis of the results of each parameter that was forecasted using the Static method in E-views confirms that the models have very good out-of-sample forecasting abilities using AR(1). The variation proportion of the out-of-sample forecasts for all parameters (NS and NSS) measures the difference between variation in forecasts and variation of actual data and in all cases this value is very small confirming that the variation of out-of-sample forecasts is almost equal to the variation of actual data. The bias component that measures the extent to which the mean of the out-of-sample forecasts is different to the mean of the actual data is also low on all the forecasts. Root mean squared Error that is an estimate of the standard deviation of the out-of-sample is also low on most cases with the exception of forecasts for β3 of NSS where it is 0.396.

5   Conclusion

The results presented in this chapter showed that the shape parameter for the Nelson-Siegel model should be fitted at 0.25 and the shape parameters for Nelson-Siegel-Svensson Model should be fitted at 0.714 and 10 respectively to obtain better fits. Analysis of time series parameters and statistics of the two models showed that the models are good in fitting the term structure of a Zimbabwean bank but the coefficients of variability were significantly affected by the noise that was prevalent in the market data of savings certificates used in the study. The t-test that was conducted to test the adjusted R^2 of the models showed that statistically Nelson-Siegel-Svensson model fits the market data better that Nelson-Siegel model. The average R^2 of Nelson-Siegel and Nelson-Siegel-Svensson model was 24.4% and 26.4% respectively. Despite the low levels of R^2 the researcher concludes that the models are fit for fitting the yield curves for a bank because a large proportion of variability of the model was lost due to the volatility nature of the market data used in the study. The two models showed that they have exceptional out-of sample forecasting capabilities when using AR (1) as the forecasting model.

5.1 Prototype developed

The prototype (TeSffim) is a tool that can be used in the estimation and forecasting of daily term structures for a bank. The design of the prototype used the results obtained from the research titled "*Estimation of Term Structures using Nelson-Siegel and Nelson-Siegel-Svensson Models: Case of a Zimbabwean Bank*" The prototype is in the form of a Macro-Enabled Excel File (.xlsm) and it was designed using Visual Basics in Excel 2010. The Prototype uses data from an Exposure Limits report of a bank and the deliverables are a fitted smooth yield curve that can be used as a reference in building Fund Transfer Pricing System. The prototype should therefore be run on machines with Office 2010 or later versions for maximum performance and Macros must be enabled for the prototype to function.

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Appendix

A1: Correlations

Correlation Analysis-Nelson Siegel			
	B$_o$	**B$_1$**	**B$_2$**	**r^2**
Bo	1	-0.69996	-0.78795	-0.01891
B1	-0.69996	1	0.12006	-0.61416
B2	-0.78795	0.12006	1	0.56956
r^2	-0.01891	-0.61416	0.56956	1

Correlation Analysis-Nelson Siegel Svensson Model				
	β$_o$	**β$_1$**	**β$_2$**	**β$_3$**	**r^2**
β$_o$	1	-0.99921	-0.98388	-0.99886	-0.26194
β$_1$	-0.99921	1	0.976083	0.99992	0.227363
β$_2$	-0.98388	0.976083	1	0.974269	0.411925
β$_3$	-0.99886	0.99992	0.974269	1	0.220185
r^2	-0.26194	0.227363	0.411925	0.220185	1

A2: Results for Nelson-Siegel Model parameters
i.)   Estimation statistics for β_0
Dependent Variable: NS_B0
Method: Least Squares
Date: 04/23/14   Time: 19:27
Sample (adjusted): 3/05/2012 10/31/2012
Included observations: 161 after adjustments
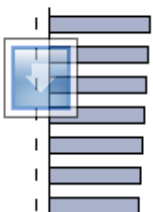
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.003283	0.001559	2.105897	0.0368
NS_B0(-1)	0.965910	0.016476	58.62535	0.0000

R-squared	0.955783	Mean dependent var		0.094005
Adjusted R-squared	0.955505	S.D. dependent var		0.011258
S.E. of regression	0.002375	Akaike info criterion		-9.235427
Sum squared resid	0.000897	Schwarz criterion		-9.197149
Log likelihood	745.4519	Hannan-Quinn criter.		-9.219884
F-statistic	3436.931	Durbin-Watson stat		1.865832
Prob(F-statistic)	0.000000			

Date: 04/23/14 Time: 19:33
Sample: 3/01/2012 3/26/2013
Included observations: 257

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.979	0.979	249.39	0.000
		2	0.959	-0.003	489.48	0.000
		3	0.939	-0.015	720.31	0.000
		4	0.918	-0.023	941.78	0.000
		5	0.899	0.039	1155.1	0.000
		6	0.883	0.082	1362.0	0.000
		7	0.868	-0.005	1562.7	0.000

ii.)   Estimation statistics for β_1
Dependent Variable: NS_B1
Method: Least Squares
Date: 04/23/14   Time: 21:02
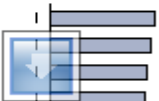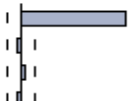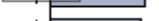Sample (adjusted): 3/05/2012 10/31/2012
Included observations: 161 after adjustments
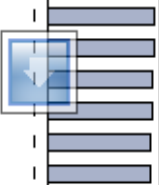
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000245	0.000449	0.545134	0.5864
NS_B1(-1)	1.000530	0.009848	101.6016	0.0000

R-squared	0.984831	Mean dependent var		-0.039740
Adjusted R-squared	0.984736	S.D. dependent var		0.022141
S.E. of regression	0.002736	Akaike info criterion		-8.952638
Sum squared resid	0.001190	Schwarz criterion		-8.914360
Log likelihood	722.6874	Hannan-Quinn criter.		-8.937096
F-statistic	10322.89	Durbin-Watson stat		1.680162
Prob(F-statistic)	0.000000			

Date: 04/23/14 Time: 19:58
Sample: 3/01/2012 3/26/2013
Included observations: 257

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.993	0.993	256.39	0.000
		2	0.984	-0.113	509.39	0.000
		3	0.977	0.063	759.40	0.000
		4	0.968	-0.067	1006.1	0.000
		5	0.961	0.071	1249.9	0.000
		6	0.954	0.045	1491.3	0.000

i.)   Estimation statistics for β_2
Dependent Variable: NS_B2
Method: Least Squares
Date: 04/23/14   Time: 20:16
Sample (adjusted): 3/05/2012 10/31/2012
Included observations: 161 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.002697	0.001531	1.760985	0.0802
NS_B2(-1)	0.937058	0.019757	47.42995	0.0000

R-squared	0.933987	Mean dependent var		0.056911
Adjusted R-squared	0.933571	S.D. dependent var		0.050177
S.E. of regression	0.012932	Akaike info criterion		-5.845804
Sum squared resid	0.026593	Schwarz criterion		-5.807526
Log likelihood	472.5872	Hannan-Quinn criter.		-5.830262
F-statistic	2249.600	Durbin-Watson stat		1.800307
Prob(F-statistic)	0.000000			

Date: 04/23/14   Time: 20:21
Sample: 3/01/2012 3/26/2013
Included observations: 257

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.951	0.951	235.15	0.000
		2	0.902	-0.028	447.42	0.000
		3	0.858	0.036	640.54	0.000
		4	0.814	-0.036	814.85	0.000
		5	0.775	0.034	973.39	0.000
		6	0.746	0.083	1120.9	0.000
		7	0.719	0.016	1258.6	0.000

A3: Results for Nelson-Siegel-Svensson Model parameters
i.)   Estimation statistics for β_0
Dependent Variable: B0
Method: Least Squares
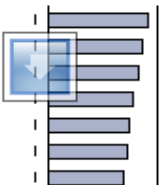Date: 04/23/14   Time: 15:39
Sample (adjusted): 3/05/2012 10/31/2012
Included observations: 161 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.006538	0.007807	0.837440	0.4036
B0(-1)	0.936134	0.020042	46.70929	0.0000

R-squared	0.932073	Mean dependent var		0.006490
Adjusted R-squared	0.931646	S.D. dependent var		0.378895
S.E. of regression	0.099061	Akaike info criterion		-1.773826
Sum squared resid	1.560266	Schwarz criterion		-1.735548
Log likelihood	144.7930	Hannan-Quinn criter.		-1.758283
F-statistic	2181.758	Durbin-Watson stat		1.799652
Prob(F-statistic)	0.000000			

Date: 04/23/14 Time: 18:55
Sample: 3/01/2012 3/26/2013
Included observations: 257

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.961	0.961	240.35	0.000
		2	0.922	-0.026	462.46	0.000
		3	0.888	0.038	669.03	0.000
		4	0.853	-0.026	860.33	0.000
		5	0.821	0.031	1038.4	0.000
		6	0.797	0.085	1207.0	0.000
		7	0.776	0.017	1367.1	0.000
		8	0.757	0.043	1520.4	0.000

ii.)   Estimation statistics for β_1
Dependent Variable: B1
Method: Least Squares
Date: 04/23/14   Time: 18:47
Sample (adjusted): 3/05/2012 10/31/2012
Included observations: 161 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.002582	0.007652	-0.337409	0.7363
B1(-1)	0.937684	0.019662	47.68934	0.0000

R-squared	0.934656	Mean dependent var	0.053246
Adjusted R-squared	0.934245	S.D. dependent var	0.374162
S.E. of regression	0.095945	Akaike info criterion	-1.837731
Sum squared resid	1.463677	Schwarz criterion	-1.799452
Log likelihood	149.9373	Hannan-Quinn criter.	-1.822188
F-statistic	2274.273	Durbin-Watson stat	1.799141
Prob(F-statistic)	0.000000		

Date: 04/23/14 Time: 19:08
Sample: 3/01/2012 3/26/2013
Included observations: 257

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.952	0.952	235.66	0.000
		2	0.903	-0.031	448.69	0.000
		3	0.861	0.045	643.11	0.000
		4	0.819	-0.032	819.40	0.000
		5	0.781	0.035	980.40	0.000
		6	0.753	0.086	1130.8	0.000
		7	0.728	0.019	1272.1	0.000

iii.)   Estimation statistics for β_2
Dependent Variable: B2
Method: Least Squares
Date: 04/23/14   Time: 19:00
Sample (adjusted): 3/05/2012 10/31/2012
Included observations: 161 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.008314	0.008234	1.009688	0.3142
B2(-1)	0.933942	0.020321	45.95998	0.0000

R-squared	0.929997	Mean dependent var		0.215383
Adjusted R-squared	0.929556	S.D. dependent var		0.329472
S.E. of regression	0.087446	Akaike info criterion		-2.023247
Sum squared resid	1.215841	Schwarz criterion		-1.984969
Log likelihood	164.8714	Hannan-Quinn criter.		-2.007704
F-statistic	2112.319	Durbin-Watson stat		1.806542
Prob(F-statistic)	0.000000			

Date: 04/23/14 Time: 19:08
Sample: 3/01/2012 3/26/2013
Included observations: 257

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.952	0.952	235.66	0.000
		2	0.903	-0.031	448.69	0.000
		3	0.861	0.045	643.11	0.000
		4	0.819	-0.032	819.40	0.000
		5	0.781	0.035	980.40	0.000
		6	0.753	0.086	1130.8	0.000
		7	0.728	0.019	1272.1	0.000

Estimation statistics for β₃

Dependent Variable: B3
Method: Least Squares
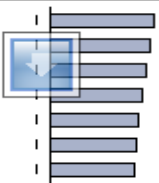Date: 04/23/14   Time: 19:11
Sample (adjusted): 3/05/2012 10/31/2012
Included observations: 161 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.021603	0.037404	-0.577556	0.5644
B3(-1)	0.937250	0.019988	46.88961	0.0000

R-squared	0.932560	Mean dependent var	0.105718
Adjusted R-squared	0.932135	S.D. dependent var	1.817027
S.E. of regression	0.473351	Akaike info criterion	1.354384
Sum squared resid	35.62566	Schwarz criterion	1.392662
Log likelihood	-107.0279	Hannan-Quinn criter.	1.369926
F-statistic	2198.636	Durbin-Watson stat	1.797856
Prob(F-statistic)	0.000000		

Date: 04/23/14   Time: 19:19
Sample: 3/01/2012 3/26/2013
Included observations: 257

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.961	0.961	240.23	0.000
		2	0.922	-0.028	462.01	0.000
		3	0.886	0.034	667.93	0.000
		4	0.851	-0.026	858.30	0.000
		5	0.819	0.032	1035.3	0.000
		6	0.795	0.088	1202.7	0.000
		7	0.773	0.018	1361.7	0.000