Tutorial - 4

Q1. $T(n) = 3T(n/2) + n^2$

→ $a = 3, b = 2, f(n) = n^2$

$c = \log_b a = \log_2 3 \approx 1.53$

$\because n^2 > n^{1.53}$

$\therefore T(n) = \Theta(n^2)$

Q2. $T(n) = 4T(n/2) + n^2$

→ $a = 4, b = 2, f(n) = n^2$

$c = \log_b a = \log_2 4 = \log_2 2^2 = 2$

$n^2 = n^2$

$\therefore T(n) = \Theta(n^2 \log n)$

Q3. $T(n) = T(n/2) + 2^n$

→ $a = 1, b = 2, f(n) = 2^n$

$c = \log_2 1 = 0$

$2^n > n^0$

$\therefore T(n) = \Theta(2^n)$

Q4. $T(n) = 2^n T(n/2) + n^n$

→ $f(n) = n^n$, $a = 2^n$, $b = 2$

$c = \log_2 2^n = n$

$n^n = n^n$

$\therefore T(n) = \Theta(n^n \log n)$

Master theorem is not applicable.

Q5. $T(n) = 16\,T(n/4) + n$

→ $f(n) = n$, $a = 16$, $b = 4$

$c = \log_4 16 = \log_4 4^2 = 2 \Rightarrow n^c = n^2$

$n^2 > n$

$\Theta(n) = \Theta(n^2)$

Q6. $T(n) = 2T(n/2) + n \log n$

→ $a = 2$, $b = 2$, $f(n) = n \log n$

$c = \log_2 2 = 1 \Rightarrow n^c = n$

$n \log n > n$

$\therefore T(n) = \Theta(n \log n)$

Q7. $T(n) = 2T(n/2) + n/\log n$

$f(n) = \frac{n}{\log n} = n * \frac{1}{\log n} = n \log \log n$

$a = 2$, $b = 2$

$c = \log_2 2 = 1 \Rightarrow n^c = n$

$n \log\log n > n$

$\therefore T(n) = \Theta(\log\log n)$

Q8. $T(n) = 2T(n/4) + n^{0.51}$

→ $f(n) = n^{0.51}$, $a = 2$, $b = 4$

$c = \log_4 2 = 0.51$

$n^c = n^{0.51}$

$n^{0.51} = n^{0.51}$

$T(n) = \Theta(n^{0.51} \log n)$

Q9. $T(n) = 0.5\,T(n/2) + 1/n$

Here $a = 0.5$ which is less than 1

$\therefore$ Master Theorem is not applicable.

Q10. $T(n) = 16T(n/4) + n!$

$a = 16$, $b = 4$, $f(n) = n!$

$c = \log_4 16 = 2$

$n^c = n^2$

$n! > n^2$

$\therefore T(n) = \Theta(n!)$

Q11. $T(n) = 4T(n/2) + \log n$

$a = 4, b = 2, f(n) = \log n$

$c = \log_2 4 = 2$

$n^c = n^2$

$n^2 > \log n$

$\therefore T(n) = \Theta(n^2)$

Q12. $T(n) = \text{sqrt}(n)\, T(n/2) + \log n$

Master theorem not applicable as a is not constant.

Q13. $T(n) = 3T(n/2) + n$

→ $a = 3, b = 2, f(n) = n$

$c = \log_2 3 \simeq 1.5$

$n^c = n^{1.5}$

$n^{1.5} > n$

$\therefore T(n) = \Theta(n^{1.5})$

Q14. $T(n) = 3T(n/3) + \sqrt{n}$

$a = 3, b = 3, f(n) = \sqrt{n}$

$c = \log_3 3 = 1$

$n^c = n^1 = n$

$\sqrt{n} < n$

$\therefore T(n) = \Theta(n)$

Q15. $T(n) = 4T(n/2) + cn$

$a = 4, b = 2, f(n) = cn$

$c = \log_2 4 = 2$

$n^c = n^2$

$n^2 > cn$

$\therefore T(n) = \Theta(n^2)$

Q16. $T(n) = 3T(n/4) + n \log n$

$a = 3, b = 4, f(n) = n \log n$

$n^c = n^{\log_4 3} = n^{0.79}$

$n^{0.79} < n \log n$

$\therefore T(n) = \Theta(n \log n)$

Q17. $T(n) = 3T(n/3) + n/2$

$a = 3, b = 3, f(n) = n/2$

$n^c = n^{\log_3 3} = n$

~~$n > n/2$~~ $\Theta(n) = \Theta(n/2)$

$\therefore T(n) = \Theta(n \log n)$

Q18. $T(n) = 6T(n/3) + n^2 \log n$

$a = 6, b = 3, f(n) = n^2 \log n$

$n^{\log_b a} = n^{\log_3 6} = n^{1.63}$

$n^{1.63} < n^2 \log n$

$\therefore T(n) = \Theta(n^2 \log n)$

Q19. $T(n) = 4T(n/2) + n / \log n$

$\rightarrow a = 4, b = 2, f(n) = n / \log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > n / \log n$

$T(n) = \Theta(n^2)$

Q20. $T(n) = 64T(n/8) - n^2 \log n$

Master theorem is not applicable.

Q21. $T(n) = 7T(n/3) + n^2$

→ $a = 7, b = 3, f(n) = n^2$

$n^{\log_b a} = n^{\log_3 7} = n^{1.7}$

$n^{1.7} < n^2$

$\therefore T(n) = \Theta(n^2)$

Q22. $T(n) = T(n/2) + n(2 - \cos n)$

Master theorem is not applicable.