

## ▼ Optimisation in ML

# Lab Assignment 1

-Ayush Abrol

---

## ▼ Question 3

```
R = 2000
```

```
import pandas as pd
```

```
import numpy as np
```

```
import matplotlib.pyplot as plt
```

```
data = pd.read_excel('3 columns.xls')
```

```
data
```

	area	bedrooms	price
0	3050	3	3920000
1	3100	2	2135000
2	4320	3	4690000
3	3060	3	3465000
4	2575	2	3290000
...	...	...	...
95	6600	4	9100000
96	2145	3	4200000
97	5200	4	2852500
98	4410	2	4200000
99	8400	3	5250000

100 rows × 3 columns

```
# We need to find the best fitting plane  $z = a_1x + a_2y + a_3$ 
```

```
# We can use the least squares method to find the best fitting plane
```

```
B = data.values
```

```
B
```

```
[ 3240, 2, 2450000],
```

```
[ 5400,      4, 2870000],
[ 6725,      3, 7580000],
[ 5960,      3, 8190000],
[ 3300,      3, 5530000],
[ 3500,      4, 4340000],
[ 4079,      3, 4200000],
[ 4600,      3, 8890000],
[ 4000,      3, 6790000],
[ 7482,      3, 8043000],
[ 2000,      2, 2660000],
[ 3150,      3, 2940000],
[ 2135,      3, 3500000],
[ 4120,      2, 4900000],
[ 3850,      3, 3115000],
[ 2145,      4, 3920000],
[ 4500,      3, 4007500],
[ 4080,      2, 3850000],

[ 6100,      3, 5110000],
[ 4080,      3, 4165000],
[ 6710,      3, 5390000],
[ 5010,      3, 4620000],
[ 3986,      2, 3150000],
[ 7320,      4, 5950000],
[ 8800,      3, 8575000],
[ 3480,      3, 2940000],
[ 10240,     2, 4760000],
[ 6360,      4, 7035000],
[ 3480,      2, 3745000],
[ 4880,      4, 8295000],
[ 3990,      3, 3500000],
[ 8250,      3, 3773000],
[ 8400,      4, 4550000],
[ 6235,      3, 7350000],
[ 4160,      3, 4830000],
[ 5885,      2, 4480000],
[ 5320,      3, 4550000],
[ 9800,      4, 5250000],
[ 3460,      3, 4025000],
[ 6100,      3, 5530000],
[ 4350,      3, 2835000],
[ 3880,      3, 4620000],
[ 5495,      3, 3129000],
[ 3650,      3, 3500000],
[ 6540,      4, 8540000],
[ 6360,      2, 4270000],
[ 3450,      1, 3150000],
[ 9000,      3, 6300000],
[ 11460,     3, 5873000],
[ 3792,      4, 3290000],
[ 3640,      3, 4200000],
[ 3450,      3, 3150000],
[ 3450,      2, 1820000],
[ 6600,      4, 9100000],
[ 2145,      3, 4200000],
[ 5200,      4, 2852500],
[ 4410,      2, 4200000],
[ 8400,      3, 5250000]], dtype=int64)
```

```
x = B[:,0]
```

```
y = B[:,1]
z = B[:,2]
```

```
ones = np.ones((len(x),1), dtype=float)
A = np.column_stack((x,y,ones))
A
```

```
[3.240e+03, 2.000e+00, 1.000e+00],
[5.400e+03, 4.000e+00, 1.000e+00],
[6.725e+03, 3.000e+00, 1.000e+00],
[5.960e+03, 3.000e+00, 1.000e+00],
[3.300e+03, 3.000e+00, 1.000e+00],
[3.500e+03, 4.000e+00, 1.000e+00],
[4.079e+03, 3.000e+00, 1.000e+00],
[4.600e+03, 3.000e+00, 1.000e+00],
[4.000e+03, 3.000e+00, 1.000e+00],

[7.482e+03, 3.000e+00, 1.000e+00],
[2.000e+03, 2.000e+00, 1.000e+00],
[3.150e+03, 3.000e+00, 1.000e+00],
[2.135e+03, 3.000e+00, 1.000e+00],
[4.120e+03, 2.000e+00, 1.000e+00],
[3.850e+03, 3.000e+00, 1.000e+00],
[2.145e+03, 4.000e+00, 1.000e+00],
[4.500e+03, 3.000e+00, 1.000e+00],
[4.080e+03, 2.000e+00, 1.000e+00],
[6.100e+03, 3.000e+00, 1.000e+00],
[4.080e+03, 3.000e+00, 1.000e+00],
[6.710e+03, 3.000e+00, 1.000e+00],
[5.010e+03, 3.000e+00, 1.000e+00],
[3.986e+03, 2.000e+00, 1.000e+00],
[7.320e+03, 4.000e+00, 1.000e+00],
[8.800e+03, 3.000e+00, 1.000e+00],
[3.480e+03, 3.000e+00, 1.000e+00],
[1.024e+04, 2.000e+00, 1.000e+00],
[6.360e+03, 4.000e+00, 1.000e+00],
[3.480e+03, 2.000e+00, 1.000e+00],
[4.880e+03, 4.000e+00, 1.000e+00],
[3.990e+03, 3.000e+00, 1.000e+00],
[8.250e+03, 3.000e+00, 1.000e+00],
[8.400e+03, 4.000e+00, 1.000e+00],
[6.235e+03, 3.000e+00, 1.000e+00],
[4.160e+03, 3.000e+00, 1.000e+00],
[5.885e+03, 2.000e+00, 1.000e+00],
[5.320e+03, 3.000e+00, 1.000e+00],
[9.800e+03, 4.000e+00, 1.000e+00],
[3.460e+03, 3.000e+00, 1.000e+00],
[6.100e+03, 3.000e+00, 1.000e+00],
[4.350e+03, 3.000e+00, 1.000e+00],
[3.880e+03, 3.000e+00, 1.000e+00],
[5.495e+03, 3.000e+00, 1.000e+00],
[3.650e+03, 3.000e+00, 1.000e+00],
[6.540e+03, 4.000e+00, 1.000e+00],
[6.360e+03, 2.000e+00, 1.000e+00],
[3.450e+03, 1.000e+00, 1.000e+00],
[9.000e+03, 3.000e+00, 1.000e+00],
[1.146e+04, 3.000e+00, 1.000e+00],
[3.792e+03, 4.000e+00, 1.000e+00],
[3.640e+03, 3.000e+00, 1.000e+00],
[3.450e+03, 3.000e+00, 1.000e+00],
```

```
[3.450e+03, 2.000e+00, 1.000e+00],
[6.600e+03, 4.000e+00, 1.000e+00],
[2.145e+03, 3.000e+00, 1.000e+00],
[5.200e+03, 4.000e+00, 1.000e+00],
[4.410e+03, 2.000e+00, 1.000e+00],
[8.400e+03, 3.000e+00, 1.000e+00]])
```

```
beta = np.dot(np.linalg.inv(np.dot(A.T,A)),np.dot(A.T,y.T))
print(beta)
```

```
[-4.33680869e-19  1.00000000e+00  0.00000000e+00]
```

```
print("Price of House: ", beta[0]*R + beta[1]*(R+3) + beta[2])
```

```
Price of House:  2002.999999999993
```

## ▼ Question 4

```
data = pd.read_excel('3 columns.xls')
data
```

	area	bedrooms	price
0	3050	3	3920000
1	3100	2	2135000
2	4320	3	4690000
3	3060	3	3465000
4	2575	2	3290000
...	...	...	...
95	6600	4	9100000
96	2145	3	4200000
97	5200	4	2852500
98	4410	2	4200000
99	8400	3	5250000

100 rows × 3 columns

```
B = data.values
```

```
B
```

```
[ 3240, 2, 2450000],
[ 5400, 4, 2870000],
[ 6725, 3, 7580000],
[ 5960, 3, 8190000],
[ 3300, 3, 5530000],
[ 3500, 4, 4340000],
```

```
[
    4079,      3,  4200000],
    4600,      3,  8890000],
    4000,      3,  6790000],
    7482,      3,  8043000],
    2000,      2,  2660000],
    3150,      3,  2940000],
    2135,      3,  3500000],
    4120,      2,  4900000],
    3850,      3,  3115000],
    2145,      4,  3920000],
    4500,      3,  4007500],
    4080,      2,  3850000],
    6100,      3,  5110000],
    4080,      3,  4165000],
    6710,      3,  5390000],
    5010,      3,  4620000],
    3986,      2,  3150000],

    7320,      4,  5950000],
    8800,      3,  8575000],
    3480,      3,  2940000],
    10240,     2,  4760000],
    6360,      4,  7035000],
    3480,      2,  3745000],
    4880,      4,  8295000],
    3990,      3,  3500000],
    8250,      3,  3773000],
    8400,      4,  4550000],
    6235,      3,  7350000],
    4160,      3,  4830000],
    5885,      2,  4480000],
    5320,      3,  4550000],
    9800,      4,  5250000],
    3460,      3,  4025000],
    6100,      3,  5530000],
    4350,      3,  2835000],
    3880,      3,  4620000],
    5495,      3,  3129000],
    3650,      3,  3500000],
    6540,      4,  8540000],
    6360,      2,  4270000],
    3450,      1,  3150000],
    9000,      3,  6300000],
    11460,     3,  5873000],
    3792,      4,  3290000],
    3640,      3,  4200000],
    3450,      3,  3150000],
    3450,      2,  1820000],
    6600,      4,  9100000],
    2145,      3,  4200000],
    5200,      4,  2852500],
    4410,      2,  4200000],
    8400,      3,  5250000]], dtype=int64)
```

```
x,y,z=B[:,0],B[:,1],B[:,2]
```

```
ones=np.ones((len(x),1), dtype=float)
```

```
A=np.column_stack((np.power(x,2),np.multiply(x,y),np.power(y,2),x,y,ones))
```

A

```
[2.9160000e+07, 2.1600000e+04, 1.6000000e+01, 5.4000000e+03,
 4.0000000e+00, 1.0000000e+00],
[4.5225625e+07, 2.0175000e+04, 9.0000000e+00, 6.7250000e+03,
 3.0000000e+00, 1.0000000e+00],
[3.5521600e+07, 1.7880000e+04, 9.0000000e+00, 5.9600000e+03,
 3.0000000e+00, 1.0000000e+00],
[1.0890000e+07, 9.9000000e+03, 9.0000000e+00, 3.3000000e+03,
 3.0000000e+00, 1.0000000e+00],
[1.2250000e+07, 1.4000000e+04, 1.6000000e+01, 3.5000000e+03,
 4.0000000e+00, 1.0000000e+00],
[1.6638241e+07, 1.2237000e+04, 9.0000000e+00, 4.0790000e+03,
 3.0000000e+00, 1.0000000e+00],
[2.1160000e+07, 1.3800000e+04, 9.0000000e+00, 4.6000000e+03,
 3.0000000e+00, 1.0000000e+00],
[1.6000000e+07, 1.2000000e+04, 9.0000000e+00, 4.0000000e+03,
 3.0000000e+00, 1.0000000e+00],
[5.5980324e+07, 2.2446000e+04, 9.0000000e+00, 7.4820000e+03,
 3.0000000e+00, 1.0000000e+00],
[4.0000000e+06, 4.0000000e+03, 4.0000000e+00, 2.0000000e+03,
 2.0000000e+00, 1.0000000e+00],
[9.9225000e+06, 9.4500000e+03, 9.0000000e+00, 3.1500000e+03,
 3.0000000e+00, 1.0000000e+00],
[4.5582250e+06, 6.4050000e+03, 9.0000000e+00, 2.1350000e+03,
 3.0000000e+00, 1.0000000e+00],
[1.6974400e+07, 8.2400000e+03, 4.0000000e+00, 4.1200000e+03,
 2.0000000e+00, 1.0000000e+00],
[1.4822500e+07, 1.1550000e+04, 9.0000000e+00, 3.8500000e+03,
 3.0000000e+00, 1.0000000e+00],
[4.6010250e+06, 8.5800000e+03, 1.6000000e+01, 2.1450000e+03,
 4.0000000e+00, 1.0000000e+00],
[2.0250000e+07, 1.3500000e+04, 9.0000000e+00, 4.5000000e+03,
 3.0000000e+00, 1.0000000e+00],
[1.6646400e+07, 8.1600000e+03, 4.0000000e+00, 4.0800000e+03,
 2.0000000e+00, 1.0000000e+00],
[3.7210000e+07, 1.8300000e+04, 9.0000000e+00, 6.1000000e+03,
 3.0000000e+00, 1.0000000e+00],
[1.6646400e+07, 1.2240000e+04, 9.0000000e+00, 4.0800000e+03,
 3.0000000e+00, 1.0000000e+00],
[4.5024100e+07, 2.0130000e+04, 9.0000000e+00, 6.7100000e+03,
 3.0000000e+00, 1.0000000e+00],
[2.5100100e+07, 1.5030000e+04, 9.0000000e+00, 5.0100000e+03,
 3.0000000e+00, 1.0000000e+00],
[1.5888196e+07, 7.9720000e+03, 4.0000000e+00, 3.9860000e+03,
 2.0000000e+00, 1.0000000e+00],
[5.3582400e+07, 2.9280000e+04, 1.6000000e+01, 7.3200000e+03,
 4.0000000e+00, 1.0000000e+00],
[7.7440000e+07, 2.6400000e+04, 9.0000000e+00, 8.8000000e+03,
 3.0000000e+00, 1.0000000e+00],
[1.2110400e+07, 1.0440000e+04, 9.0000000e+00, 3.4800000e+03,
 3.0000000e+00, 1.0000000e+00],
[1.0485760e+08, 2.0480000e+04, 4.0000000e+00, 1.0240000e+04,
 2.0000000e+00, 1.0000000e+00],
[4.0449600e+07, 2.5440000e+04, 1.6000000e+01, 6.3600000e+03,
 4.0000000e+00, 1.0000000e+00],
[1.2110400e+07, 6.9600000e+03, 4.0000000e+00, 3.4800000e+03,
 2.0000000e+00, 1.0000000e+00],
[2.3814400e+07, 1.9520000e+04, 1.6000000e+01, 4.8800000e+03,
 4.0000000e+00, 1.0000000e+00],
```

```

beta=np.dot(np.linalg.inv(np.dot(A.T,A)),np.dot(A.T,y.T))
beta

array([ 5.50571416e-20, -2.49800181e-16,  1.98951966e-13,  9.71445147e-17,
        1.00000000e+00, -4.54747351e-13])

print("price of house",beta[0]*pow(R,2)+beta[1]*R*(R+3)+beta[2]
      *pow(R+3,2)+beta[3]*R+beta[4]*(R+3)+beta[5])

price of house 2003.0000007973103

```

## ▼ Question 7

```


data = pd.read_excel('Census data (Chandigarh).xls')
data = data.values

Year, Persons = data[:,0] ,data[:,1]
#z = log(Persons)
z = np.log(Persons)
ones=np.ones((len(Year),1), dtype=float)
A=np.column_stack((Year, ones))
beta=np.dot(np.linalg.inv(np.dot(A.T,A)),np.dot(A.T,z.T))

z_2021=beta[0]*2021+beta[1]
Pop_2021 = np.exp(z_2021)
print("Population of Chandigarh in year 2021 using Exponential fitting will be: ",Pop_2021)

Population of Chandigarh in year 2021 using Exponential fitting will be: 1799523.71

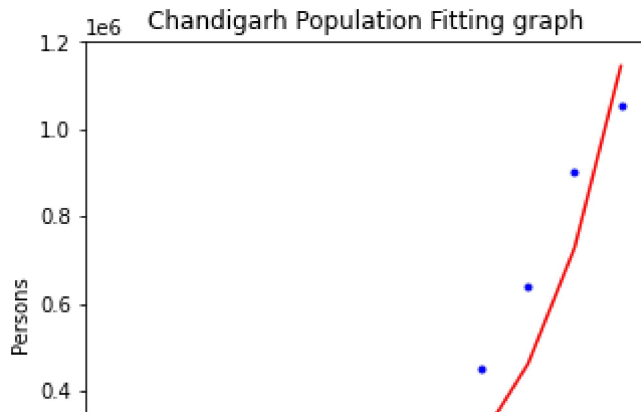
```



```

beta_0=np.exp(beta[1])
beta_1=beta[0]
#Graph
plt.figure(figsize = (5,5))
plt.title("Chandigarh Population Fitting graph")
plt.plot(Year, Persons, 'b.')
plt.plot(Year, beta_0*np.exp(beta_1*Year), 'r')
plt.xlabel('Year')
plt.ylabel('Persons')
plt.show()

```



## ▼ Question 8

```
population = pd.read_excel('population1.xls')
population = population.values
```

```
Year, Populations = population[:,0] ,population[:,1]
z=np.ones((len(Year),1), dtype=float)
```

```
def polynomial_fit(x,y,p):
    temp = []
    for i in range(p,0,-1):
        temp.append(np.power(x,i))
    temp.append(z)
    temp = tuple(temp)
    A = np.column_stack(temp)
    beta=np.dot(np.linalg.inv(np.dot(A.T,A)),np.dot(A.T,y.T))
    arr = beta[0]*np.power(x,p)
    for i in range(p-1,0,-1):
        arr += (beta[p-i]*np.power(x,i))
    arr += beta[p]
    len_x = len(x)
    res = np.dot(A,beta)-y
    avg_loss = np.dot(res.T,res)/(2*len_x)
    print("Average Loss for Degree ",p," is:",avg_loss)
    #Figure for degree = 3
    if(p==3):
        plt.figure(figsize = (5,5))
        plt.title("Plot for Degree "+str(p))
        plt.plot(x, y, 'b.')
        plt.plot(x, arr, 'r')
        plt.xlabel('x')
        plt.ylabel('y')
        plt.show()
    return avg_loss
```

```
min_loss = polynomial_fit(Year,Populations,1)
degree = 1
```



```

for i in range(2,21):
    loss = polynomial_fit(Year,Populations,i)
    if loss<min_loss:
        min_loss = loss
        degree = i

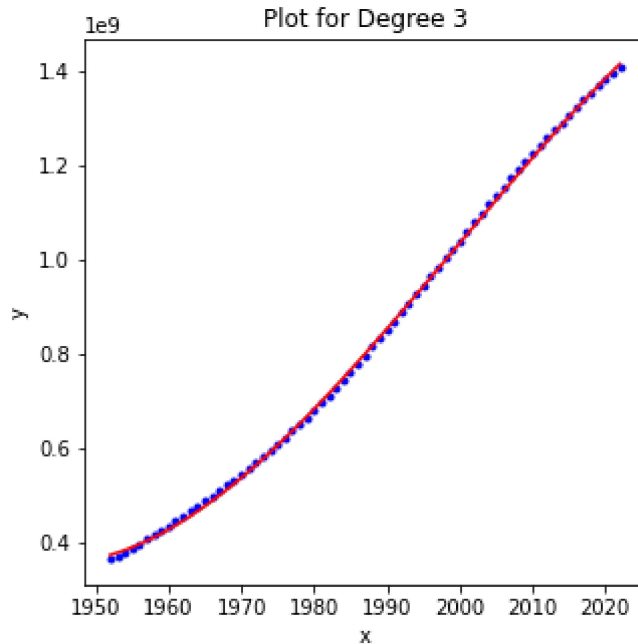
print("Lowest average loss of ", min_loss," is obtained at degree: ", degree)

```

```

Average Loss for Degree 1 is: 523860937459408.3
Average Loss for Degree 2 is: 109243277742810.06
Average Loss for Degree 3 is: 17767906501445.42

```



```

Average Loss for Degree 4 is: 9738430412788124.0
Average Loss for Degree 5 is: 1.0886643273209038e+16
Average Loss for Degree 6 is: 4.7833446584639384e+16
Average Loss for Degree 7 is: 1864605519128191.8
Average Loss for Degree 8 is: 9349668764966870.0
Average Loss for Degree 9 is: 1.1365397575924876e+16
Average Loss for Degree 10 is: 1600056954355572.5
Average Loss for Degree 11 is: 2.2725839884268028e+16
Average Loss for Degree 12 is: 7664308530118347.0
Average Loss for Degree 13 is: 4757888654965746.0
Average Loss for Degree 14 is: 5.432956356516732e+16
Average Loss for Degree 15 is: 1.1452466803916235e+17
Average Loss for Degree 16 is: 1.106633295731981e+16
Average Loss for Degree 17 is: 8.27496792636314e+16
Average Loss for Degree 18 is: 6.422318986556853e+16
Average Loss for Degree 19 is: 1.301322473349929e+16
Average Loss for Degree 20 is: 5081460991010505.0
Lowest average loss of 17767906501445.42 is obtained at degree: 3

```

