# DNA Assignment-4

## Team Name: Collaborative Analysts

Team Id: 67

Q1) **Consider $R(A_1, A_2, ..., A_n)$ to be a relation with functional dependencies defined as follows**:

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$$A_2A_3 \rightarrow A_4A_5...A_nA_1$$

$$A_4A_5A_6 \rightarrow A_7A_8...A_nA_1A_2A_3$$

$$A_{\frac{(i-1)i}{2}+1} + A_{\frac{(i-1)i}{2}+2}...A_{\frac{(i-1)i}{2}+i} \rightarrow A_{\frac{(i-1)i}{2}+i+1}...A_nA_1...A_{\frac{(i-1)i}{2}}$$

$$for\ i > 3\ and\ till\ \frac{(i-1)i}{2} + i = n$$

Q 1.1) **For what values of $n$ is the above set of functional dependencies possible?**

Let $i$ be the number of functional dependencies, then clearly from the question given above the total number of attributes $n$ has to be of the form $\frac{i(i+1)}{2}$. We further observe that $i$ has to be greater than $2$. For $i = 1$, We have only $A_1$ and no other functional dependencies. In the case of $i = 2$, we have $A_1 \rightarrow A_2A_3$ and $A_2A_3 \rightarrow A_1$.

Q 1.2) **How many keys does the relation R have and what are they ?**

In the above question , we saw that there will be $m$ keys . So the left hand side attributes of every FD would be a key as every attribute in the relation R is a prime attribute .

Hence the keys would be $A_1, A_2A_3, A_4A_5A_6,$ ..... and so on .

No of candidate Keys: $\lfloor \frac{\lfloor \sqrt{1+8n} - 1 \rfloor}{2} \rfloor$

Q 1.3) **State the normal form of the above relation and normalize it to BCNF (if valid) using decomposition rules.**

1NF : A relation is in first normal form if every attribute in that relation is singled valued attribute.

We assume all the attributes are simple and indivisible and hence the relation is already in 1NF.

2NF:  A relation is in 2NF if it has No Partial Dependency, i.e., no non-prime attribute (attributes which are not part of any candidate key) is dependent on any proper subset of any candidate key of the table.

All the attributes are in relation R are prime attributes. Therefore , R is trivially in 2NF.

3NF: A relation is in third normal form, if there is no transitive dependency for non-prime attributes as well as it is in second normal form.

Similar to the previous argument, there are no non prime attributes in R. Therefore, no non prime attribute of R is transitively dependent on the primary key. Thus, R is in 3NF.

BCNF: A table is in BCNF is for every functional dependency $X \rightarrow Y$, X is the superkey of the table.

This is satisfied since every FD $X \rightarrow A$ , defined in relation R has X as key. Therefore, R is already in BCNF.

Q 1.4) **Find the minimal cover of the above relation and use it to normalize it to BCNF (if valid).**

To find the minimal cover we first need to have the $RHS$ of the functional dependencies with only one attribute. Thus the first functional dependency, $A_1 \rightarrow A_2A_3....A_n$ can be rewritten as $A_1 \rightarrow A_2$, $A_1 \rightarrow A_3$..... $A_1 \rightarrow A_n$. Let us take all of these new functional dependencies into our minimal cover set. The second functional dependency is given to be as $A_2A_3 \rightarrow A_4A_5.....A_nA_1$. This can be rewritten as $A_2A_3 \rightarrow A_4$, $A_2A_3 \rightarrow A_5$, ..... $A_2A_3 \rightarrow A_n$, $A_2A_3 \rightarrow A_1$. We see that as $A_2A_3 \rightarrow A_1$, then because $A_1 \rightarrow A_4$, Clearly there is a transitive dependency, thus we can omit $A_2A_3 \rightarrow A_4$.....$A_2A_3 \rightarrow A_n$ and take only $A_2A_3 \rightarrow A_1$. Similarly we can do that for every subsequent combined prime attribute. Thus the final minimal cover would be as follows :

$A_1 \rightarrow A_i$ , $\forall\ 2 <= i <= n$

$A_2A_3 \rightarrow A_1$

$A_4 A_5 A_6 \rightarrow A_1$

.....

$A_{\frac{i(i-1)}{2}+1} A_{\frac{i(i-1)}{2}+2} ..... A_n \rightarrow A_1$

Q2) **Consider $R(A_1, A_2, ..., A_n)$ to be a relation R with functional dependencies defined as follows:**

$A_i -> A_j \forall 1 <= i < j <= n$ and $A_i -> A_j \forall 1 <= i > j <= n$

Q2.1) **How many keys does the relation R have and what are they ?**

There will be k keys in total . All attributes in the relation R will be prime attribute and would also act as keys. Hence the keys are $A_1, A_2, A_3, A_4, A_5, A_6, .....$

Q2.2) **State the normal form of the above relation and normalise it to BCNF (if valid) using decomposition rules.**

1NF : A relation is in first normal form if every attribute in that relation is singled valued attribute.

We assume all the attributes are simple and indivisible and hence the relation is already in 1NF.

2NF : A relation is in 2NF if it has No Partial Dependency, i.e., no non-prime attribute (attributes which are not part of any candidate key) is dependent on any proper subset of any candidate key of the table.

3NF: A relation is in third normal form, if there is no transitive dependency for non-prime attributes as well as it is in second normal form.

Similar to the previous argument, there are no non prime attributes in R. Therefore, no non prime attribute of R is transitively dependent on the primary key. Thus, R is in 3NF.

BCNF: A table is in BCNF is for every functional dependency $X \rightarrow A$, X is the superkey of the table.

This is satisfied since every FD $X \rightarrow A$ , defined in relation R has X as key.

Q2.3) **Find the minimal cover of the above relation and use it to normalise it to BCNF (if valid).**

We can observe that RHS of all FDs will have only one single attribute. Therefore we need to only remove the transitive dependencies, lets take $A_1 \rightarrow A_2$, $A_2 \rightarrow A_3$, $A_3 \rightarrow A_4$, $A_{n-1} \rightarrow A_n$. We observe that we need to add one extra $A_n \rightarrow A_1$. Thus this is our final minimal cover. As it is already in BCNF, we do not have to normalise it again.