Assignment - 1
Random Walk and Monte Carlo Simulation

Science-II

Due Date: 9th Feb, 2022

1 Random Walk

A **Random Walk** is a random process, that describes a path that consists of a succession of random steps on a mathematical space, mostly integers. A random walk can be of multiple dimensions, the easiest one and the one we are going to simulate is 1 dimensional (Random Walk on integer number line, $\mathbb{Z}$).

It starts at 0 and at each step it moves +1 or -1 with equal probability. It can be mapped with fair coin toss experiment, where if it is heads we move right and if it is tails we move left or vice-versa.

Formally we can define this walk as:

$$X(t+1) = X(t) + S(t)$$

$$S(t) = \delta_{C(t),Heads} - \delta_{C(t),Tails}$$

where we have $C(t)$ as the coin toss at time t and δ is the kronecker-delta function.

1.1 Task 0: Simulate a random walk for n timesteps

Make a function that takes n as an input and runs a 1-D random walk for n timesteps, and returns us the coordinates of the point at all timesteps. Also plot the graph of position x vs t.

1.2 Task 1: Simulate a random walk with two people

Two drunks start out together at the origin, each having equal probability of making a step to the left or right along the X-axis. They make their steps simultaneously. Simulate the experiment appropriate number of times and answer the following parts:

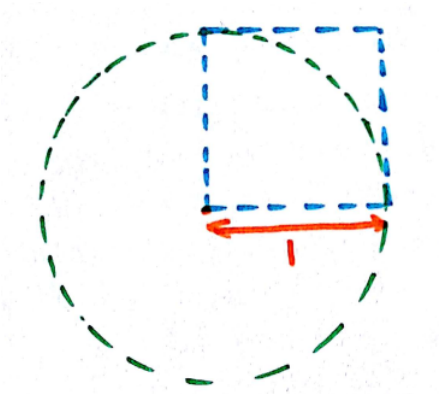
- **Part 1:** Find the Probability that they meet again after N timesteps by running the simulation multiple times and plot the probability for different values of N.
- **Part 2:** Simulate the random walk of a single drunk person and find the **probability** that he/she returns back to the **Origin** after N timesteps. Plot the probabilities for different values of N.
- **Part 3:** Find the Mean and Mean Squared Displacement for the drunk after N steps. Perform the experiment multiple times and find these central tendencies for N time steps.

2 Monte Carlo Simulation

Monte Carlo methods, or Monte Carlo experiments, are a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results. The underlying concept is to use randomness to solve problems that might be deterministic in principle. We will use it to calculate the value of π

2.1 Task

Consider a unit circle and unit square as shown in the figure.



You have been given N point pebbles. Simulate an experiment to determine the value of π using these inputs. And write a code to find π.

3 Instructions

- Language allowed: Python
- Libraries allowed (unless stated otherwise): NumPy, Matplotlib
- Submit a single Python notebook (**RollNumber.ipynb**) with both questions and their written explanations wherever needed. State your assumptions clearly.
- Code should be vectorized using NumPy (Refer the tutorial on Numpy).
- **Plagiarism will be strictly penalised.**