Assignment - 2

Science-II

Due Date: 12th February, 2022

Instruction

In this assignment, you will be solving basic problems involving heat diffusions. You will also be required to answer some follow up questions. You are supposed to code these problems with python. Unless stated explicitly, you are only allowed to use Numpy and Matplotlib. **Plagiarism will be strictly penalised**. Submit a zip file (*Roll_Number.zip*) with 1 Jupyter notebook and one PDF file (with answers to theory questions; you can also type the answers on Jupyter notebook).

Question - 1: Diffusion Equation

1.1 - 1-D Diffusion (40 marks)

Question - 1.1.1 (10 marks) Solve the 1D diffusion equation numerically using some initial condition.
Question - 1.1.2 (10 marks) Explain analytically how change in the parameters involved in this equation influence the final results.
Question - 1.1.3 (10 marks) Plot the time evolution of the probability distribution as a function of time.
Question - 1.1.4 (10 marks) Calculate the mean and mean square displacement using your solution.

1.2 - 2-D Diffusion (60 marks)

Question - 1.2.1 (5 marks) Write down the two dimensional diffusion equation (axes: x and y) for a metal with diffusivity D.
Question - 1.2.2 (5 marks) Apply finite difference approximations to discretize the two dimensional diffusion equation.
Question - 1.2.3 (5 marks) Write down the maximum time step you can allow without the process becoming unstable. Comment on how you formulated the expression.
Question - 1.2.4 (10 marks) We will apply the 2-D diffusion equation to study the heat spread in a 2-dimensional metal plate. The initial temperature of the metal plate (T_i) is 300 K. The metal plate is kept aside a heat conducting material (circular disc) with temperature (T_j) 700 K. Assume that the size of the metal plate is 10 mm (width and height). Consider intervals in each direction (dx and dy) as 0.1 mm. Consider the dimensions of the circular disc as 2 mm centred at (5, 5) relative to the metal plate. Use the following information to quantify the initial conditions for the recurrence relation formed above. Vectorize your code properly.
Question - 1.2.5 (15 marks) We will consider 101 time steps. Using the stability condition formulated from Question 1.2.3, calculate the time interval between two time steps. Write a function to use this information to solve the recurrence relation for 101 time steps.
Question - 1.2.6 (20 marks) Using the recurrence relation obtained above, create a heat map animation to show the evolution of time steps for each of 20 time steps. Conduct the experiments for two different values of diffusivity (4.0 and 6.0) and comment your observations.