# Probabilities

We begin by making a note of all the information provided to us.
Probability of receiving correctly = 1 - $\delta$
Probability of receiving flipped = $\delta$
Also it is given that the probability of flipping is independent of the previous or future outcomes

Probability of a bit being '1' = 1-$p$
Probability of a bit being '0' = $p$

## Part A

As the probabilities linked with consecutive symbols are independent of each other, the probability that we are required to calculate is independent of the position (k)

$$P(k^{th}\, symbol\, being\, '0') = P(transmit\ =\ '0'\ \cap\ \overline{flipping})\ +\ P(transmit\ =\ '1'\ \cap\ flipping)$$
$$\Rightarrow\ P(required)\ =\ p.(1-\delta) + (1-p).\delta$$
$$\Rightarrow P(required) = p + \delta - 2p\delta$$

## Part B

In the given part, the encoding and decoding schemes for transmission and reception are changed.

$$\therefore\ P(receiving\, correct\, '0') = P(zero\, was\, decoded\, |\, zero\, was\, transmitted)$$
$$\Rightarrow P(required) = P(\overline{F1} \cap \overline{F2} \cap \overline{F2}) + P((F1 \cap \overline{F2} \cap \overline{F3}) \cup (\overline{F1} \cap F2 \cap \overline{F3}) \cup (\overline{F1} \cap \overline{F2} \cap F3))$$
$$\therefore$$
$$P(required) = (1-\delta)^3 + 3(1-\delta)^2\delta = (1-\delta)^2(1+2\delta)$$

## Part C

Let the decoding strategy $D : \{0, 1\} \to \{0, 1, \perp\}$ be such that we choose the output to be $\perp$ at every $N^{th}$ input at the destination. Now, we calculate the expectation of the penalty inccured as:

$$E(penalty) = \frac{2 \times P(Getting, wrong\, output) \times (N-1) + 1 \times (N - (N-1))}{N}$$

$$= \frac{2 \times (p * \delta + (1-p)\delta) \times (N-1) + 1}{N} = \frac{2\delta(N-1)+1}{N}$$

$$= 2\delta + (1-2\delta) \times \frac{1}{N}$$

The above obatined equation obtains its minima at $N \to \infty$ for $\delta \leq \frac{1}{2}$. As we can note, the above equation is independent of $p$, therefore the solution holds for every value of $p \in [0,1]$. This means that $\perp$ should never be the output for the given scenario. The decoding must be the simplest, i.e. giving out what is received at the input. Note that this will change if $\delta \geq \frac{1}{2}$.