Nonlinear Adaptive Control of Active Suspensions

Nonlinear Dynamics and Chaos

Ayush Agrawal
Department of Mechanical Engineering
Indian Institute of Technology Bombay
Mumbai, India
170100044@iitb.ac.in

***Abstract*—In this report, a comparative study based on ride quality observed for various control algorithms applied to active suspensions has been made. This work is based on a paper by Andrew Alleyne and J. Karl Hedrick on Nonlinear Adaptive Control of Active Suspensions. In this report, simulations have showed that the active suspensions is better than the passive suspensions in terms of improving the ride quality of the vehicle. Further, a previously developed "sliding" control law applied to an electro-hydraulic suspensions system has been discussed. To reduce the errors in this model, a standard parameter adaptation scheme, based on Lyapunov analysis is introduced which takes into account the time varying nature of the parameter of electro-hydraulic actuator. The result of the this control scheme established a necessity of taking into account the dependence of parameters on the regions of state space. A modified adaptation scheme, which enables the identification of parameters whose value change with regions of state space is then presented. The performance is determined by the ability of the actuator output to track a specified force. Both of the adaptive scheme improve performance, with the modified scheme giving the greater improvement in the performance.**

I. Introduction

The active control of automobile suspensions is currently of great interest in both the academic and industrial fields. Most of the earlier research in the field concentrates on the optimal control law for the suspension and does not consider the dynamics of the actuator. In recent research, the dynamics of the active element been accounted for in the control algorithm. The paper by Andrew Alleyne and J. Karl Hedrick directly considers the nonlinear dynamics of an electro-hydraulic actuator in a quarter-car active suspension model and uses these dynamics to formulate a nonlinear control law.

The "sliding" control law used to control the active suspension requires an accurate model of the system dynamics to avoid large gains. To reduce the modeling error a parameter identification algorithm is coupled with the existing control law and the performance of the system is evaluated. In order to further reduce the modeling error and increase the system performance, a modified parameter identification algorithm is used. This algorithm is able to identify parameters of the electro-hydraulic actuator that vary with system states.

The goal of this investigation is to increase the ride quality of the automobile. Since the ride quality is linked to the acceleration felt m the passenger compartment, the goal becomes to reduce the acceleration while remaining within the constraints of the suspension system. The method used to achieve this goal is "skyhook damping". The performance of the controller is defined as the ability of the active element to track a given "skyhook" force, which is determined from the states of the system. The performance of the active systems, under various nonlinear controllers, are compared with each other and with a passive system.

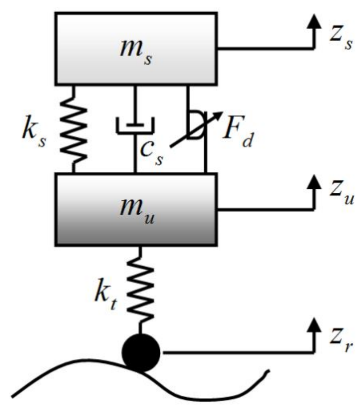

Fig. 1. Quarter car model

The state space equations for this model

$x_1 = Z_1 - Z_2 : suspension\ deflection$
$x_2 = \dot{Z}_2 : sprung\ mass\ velocity$
$x_3 = Z_u - Z_r : tyre$
$x_4 = \dot{Z}_u : unsprung\ mass\ velocity$
$x_5 = P_1 - P_2 : actuator\ load\ pressure\ (P_l)$
$x_6 : spool\ valve\ position\ (x_v)$

The rest of this paper is organized as follows. Section II gives the passive and active suspension system under consideration and describes the desired system dynamics. Section III reviews the sliding control method used for control of the active suspension. Section IV describes the standard adaptive algorithm and couples it with the nonlinear control law presented in Section III. Section V discusses the modified adaptive algorithm and combines it with the controller from Section III. The conclusion reviews and summarizes the main points of the paper.

II. Passive and Active suspensions

Passive suspension systems of conventional elements (springs and dampers) have limitations in respect of completely controlling the vehicle dynamics. They lack active control due to absence of force actuator in the system. Optimal values of spring constants and dampers are chosen for given sprung and unsprung masses in the system. The dynamics of passive suspensions is governed by the following equations:

The equation of motion of the sprung mass is:

$$M_s\frac{\mathrm{d}^2Z_s}{\mathrm{d}t^2} = -K_s(Z_s - Z_u) - B_s\left(\frac{\mathrm{d}Z_s}{\mathrm{d}t} - \frac{\mathrm{d}Z_u}{\mathrm{d}t}\right)$$

The equation of motion of unsprung mass is:

$$M_u\frac{\mathrm{d}^2Z_u}{\mathrm{d}t^2} = K_s(Z_s - Z_u) + B_s\left(\frac{\mathrm{d}Z_s}{\mathrm{d}t} - \frac{\mathrm{d}Z_u}{\mathrm{d}t}\right) - K_t(Z_u -$$

The values of the system parameter used in the simulations taken from the paper which were obtained both experiment and by literature research. The values are given below:
$M_s = 290\ kg$
$M_u = 59\ kg$
$K_t = 190000\ N/m$
$K_s = 16812\ N/m$
$B_s = 1000\ N/m/s$

The above system is a linear constant coefficient time invariant system. Assuming the initial values the states $Z_s\ Z_u,\ \dot{Z_s}\ and\ \dot{Z_u}$ are equal to zero, the above equation is easily converted to Laplace form and the following matrix equation is obtained:

$$\begin{pmatrix} (\mathrm{s}^2M_s + K_s + sB_s) & -(K_s + sB_s) \\ \text{-}(\mathrm{K}_s + sB_s) & (s^2M_u + K_s + sB_s) \end{pmatrix} \begin{pmatrix} \mathrm{Z}_s \\ \mathrm{Z}_u \end{pmatrix} = \begin{pmatrix} 0 \\ \mathrm{K}_t Z_r \end{pmatrix}$$

The above system can be simulated in the MATLAB environment and the variation of sprung mass acceleration with respect to time can be obtained.

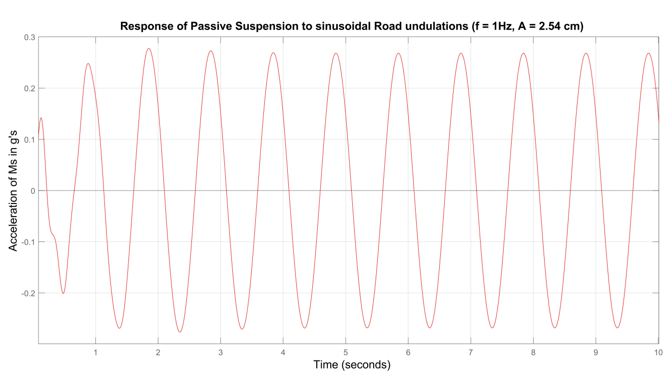

Fig. 2. Passive Sprung Mass Acceleration (Simulation) for K_t = 190000 N/m

The road input disturbance is a 2.54 cm (1 inch) sine wave with a frequency of 1 Hz. This frequency is used because it is approximately the body mode of the system. We can observe that the amplitude of sprung mass acceleration is $0.277g\ 2.71m/s^2$. Another intriguing observation can be made by increasing the value of K_t to 19000000 N/m. The variation of sprung mass acceleration will be as follows:

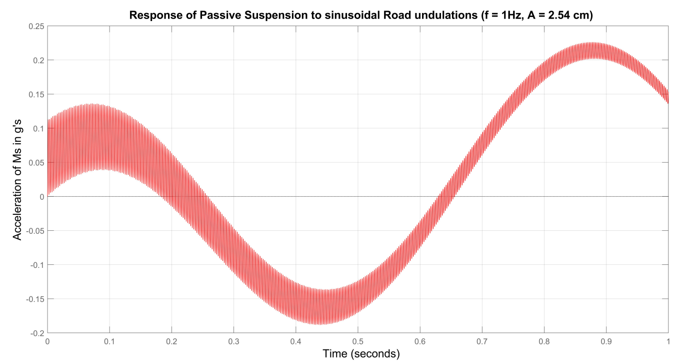

Fig. 3. Passive Sprung Mass Acceleration (Simulation) for K_t = 190000000 N/m

From this observations we can infer that installing softer tyres in the vehicle can improve the ride quality by eliminating the high frequency oscillatory part from the sprung mass acceleration.
Active suspension systems are intelligent in the sense that they employ controllable elements (actuators of some kind). These elements, using force feedback, are capable of generating forces which are linear or non-linear combinations of measured state variables. The desired dynamics of the active system is that of a passive system with a "skyhook damper" attached. The desired force output of the actuator consists of a term negatively proportional to the vertical sprung mass velocity. The control law that we have applied to this system is:

$$F_{des} = -C\frac{\mathrm{d}x_s}{\mathrm{d}t}$$

Here, the optimal value of C was determined to be 3000 N/m/s. With this control law the equations governing the dynamics of the active suspension system are:
The equation of motion of the sprung mass is:

$$M_s\frac{\mathrm{d}^2Z_s}{\mathrm{d}t^2} = -K_s(Z_s - Z_u) - B_s\left(\frac{\mathrm{d}Z_s}{\mathrm{d}t} - \frac{\mathrm{d}Z_u}{\mathrm{d}t}\right) - C\frac{\mathrm{d}x_s}{\mathrm{d}t}$$

The equation of motion of unsprung mass is:

$$M_u\frac{\mathrm{d}^2Z_u}{\mathrm{d}t^2} = K_s(Z_s - Z_u) + B_s\left(\frac{\mathrm{d}Z_s}{\mathrm{d}t} - \frac{\mathrm{d}Z_u}{\mathrm{d}t}\right) - K_t(Z_u - Z_r) + C\frac{\mathrm{d}x_s}{\mathrm{d}t}$$

The above system can be simulated using the shown control model in Simulink environment.
The variation of sprung mass acceleration and 'Skyhook'

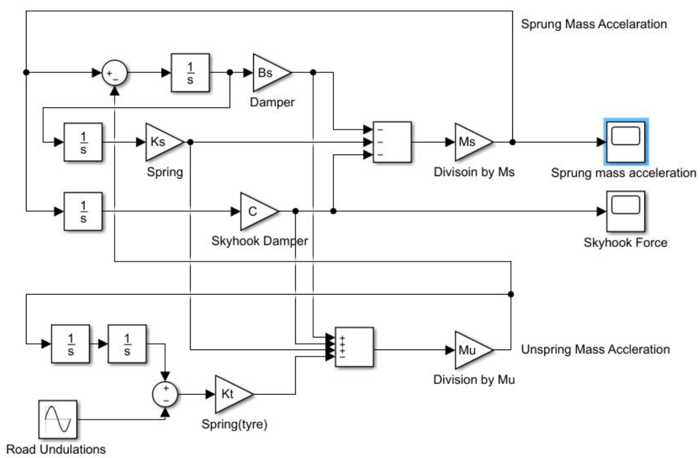

Fig. 4. Simulink Model for Active Suspensions

damping force with respect to time is shown in the following figures:

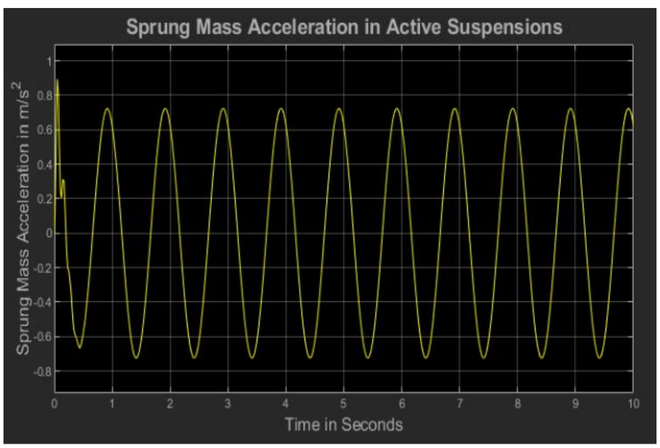

Fig. 5. Active Sprung Mass Acceleration (Simulation)

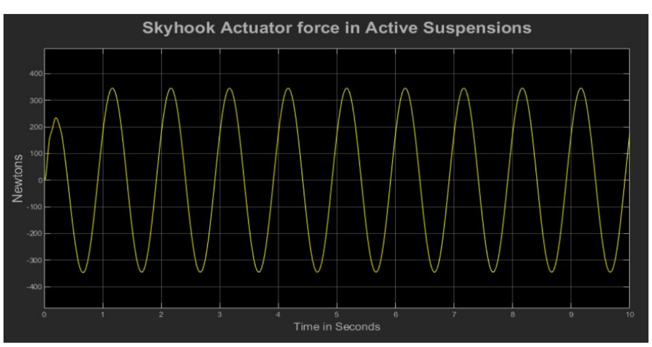

Fig. 6. Skyhook Actuator Force (Simulation)

When compared with Fig. 2, Fig. 5 shows that the system under skyhook damping will have sprung mass accelerations approximately one fourth of those of the passive system. This is an example of a desired force that we wish the controlled actuator to output. The performance of a controller is based on its ability to track the desired force trajectory. We saw that at lower frequencies like 1 Hz the active suspension improves the ride quality to a great extent. Apart from this, it has been observed that at higher frequency of road undulations the active response approaches the passive response.

III. System and Desired Dynamics

Till now we have modelled the suspensions by using equivalent spring and dampers in place of actual electro-hydraulic actuators. The electro-hydraulic actuator has a four way spool valve which is controlled by a flapper valve with a direct current input. The flapper valve uses the input current and a hydraulic assist to control the motion of the spool valve. The dynamics of the servovalve, including both the flapper and spool valves, are third order. However, two of the system poles have a very fast response and there is one slower pole that provides the dominant response mode. Consequently, we approximate the entire servovalve dynamics as a spool valvz having first order dynamics.

The variation of the pressure across the actuator cylinder i.e. is the load pressure is governed by the following equation:

$$\frac{V_t}{4\beta_e}\dot{P}_l = Q_l - C_{tm}P_l - A(\dot{Z}_s - \dot{Z}_u)$$

where
V_t total actuator volume
β_e effective bulk modulus
$\dot{Z}_s$ sprung maas velocity
$\dot{Z}_u$ unsprung mass velocity
A actuator ram area
C_{tm} coefficient of total leakage due to pressure

Using the equation for hydraulic fluid flow through an orifice, the relationship between spool valve displacement, X, , and the load flow QL:

$$Q_l = C_d w x_v \sqrt{\frac{P_s - sgn(x_6)x_5}{\rho}}$$

where
C_d discharge coefficient w spool valve area gradient C_{tm} total leakage coefficient
Combining the above two equations along with the equation of motion of the massess, results in the system state equations given below :

$$\dot{x_1} = x_2 - x_4$$

$$\dot{x_2} = \frac{1}{M_s}(-K_s x_1 - B_s(x_2 - x_4) + Ax_5 - F_f)$$

$$\dot{x_3} = x_4 - \dot{Z}_r$$

$$\dot{x_4} = \frac{1}{M_u}(K_s x_1 + B_s(x_2 - x_4) - K_t x_3 - Ax_5 + F_f)$$

$$\dot{x_5} = -\beta x_5 - \alpha A(x_2 - x_4) + \gamma x_6\sqrt{P_s - sgn(x_6)x_5}$$

$$\dot{x_6} = \frac{1}{\tau}(-x_6 + u)$$
$$y = Ax_5 - F_f$$

where

$$\gamma \ := \ \alpha C_d w\sqrt{\frac{1}{\rho}}$$
$$\beta := \alpha C_{tm}$$
$$\alpha := \frac{4\beta_e}{V_t}$$
$$\rho = hydraulic\ fluid\ velocity$$
$$u = input\ to\ servovalve$$

The values of the parameters used in experiments are $\alpha = 4.515e13N/m^2$, $\beta = 1.00$ and $\gamma = 1.545e9N/(m^{\frac{5}{2}}kg^{\frac{1}{2}}$, A = 3.35e-4 m^2 and P_s = 10342500 Pa.
This behavior of the system has been incorporated in the coming up analysis of nonlinear control schemes.

IV. Sliding Control

Our proposed model for plant need not accurately display the actual plant dynamics. This occurs because of the presence of noise and disturbances. To tackle this problem we need a robust controller. In order to obtain this we need to propose a ‘sliding control law’ for our system. Suppose we have a non linear system:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = f(x) + g(x)u$$

For the given, we want the behaviour to be as:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = v$$

The above function ’v’ has been obtained by pole placement method. So, in order to get this behaviour we load our system with the following control law:

$$u = \frac{1}{g(x)}(-f(x) + v)$$

In our suspension, we are using electro-hydraulic actuators whose dynamics is governed by the following equation :

$$A\frac{\mathrm{d}x_5}{\mathrm{d}t} = -A\beta_1 x_5 - \alpha_1 A^2(x_2 - x_4) + A\gamma_1 x_6\sqrt{P_s - sgn(x_6)x_5}$$

The above equation displays the desired dynamics of force, hence the control law for this system for proper choices of f(x) and g(x) will look like:

$$x_{6desired} =$$
$$\frac{1}{\gamma_1\sqrt{P_s - sgn(x_6)x_5}}\left(\beta_1 x_5 + \alpha_1 A(x_2 - x_4) + \frac{\dot{F_s}}{A} - ks\right)$$

Here, s is the sliding surface which is actually our error function:

$$s(x,t) = x_5 - x_{5desired}$$

The above control law was experimented on a setup made by keeping in mind the actual conditions of a car. Test were carried out on a Active Suspension test rig, and the following results were obtained for the force output and the sprung mass acceleration.

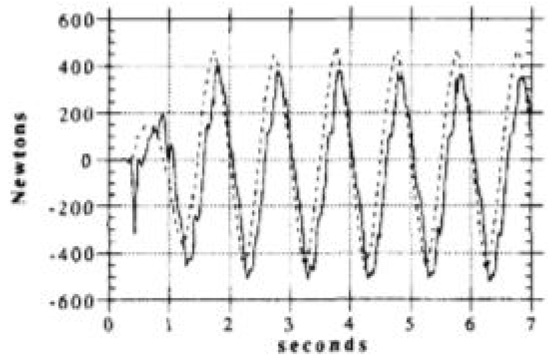

Fig. 7. Actuator force tracking (experimental): Desired : - - - Actual: —

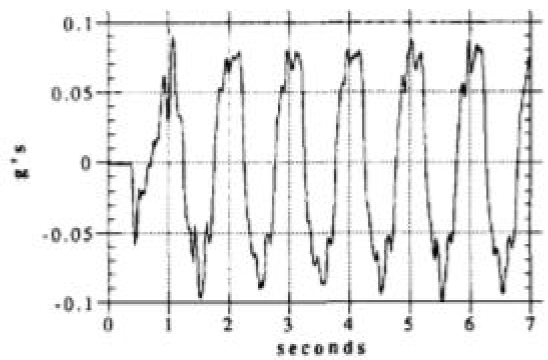

Fig. 8. Sprung Mass Acceleration (experiment)

V. Adaptive Control

Sliding control can account for the errors present in plant modelling. But our hydraulic parameters are also time varying. If the structure of the governing equations of the model is known, but parameters in the model are uncertain, an adaptive controller can identify the parameters and improve tracking accuracy. Consider the hydraulic parameters defined in Section III. From (1.4). the actuator force ($A{*}x_5$) model can be written as:

$$A\frac{\mathrm{d}x_5}{\mathrm{d}t} = -A\beta_1 x_5 - \alpha_1 A^2(x_2 - x_4) + A\gamma_1 x_6\sqrt{P_s - sgn(x_6)x_5}$$

$\alpha_1, \beta_1\ and\ \gamma_1$ denote the estimated time varying nature of the

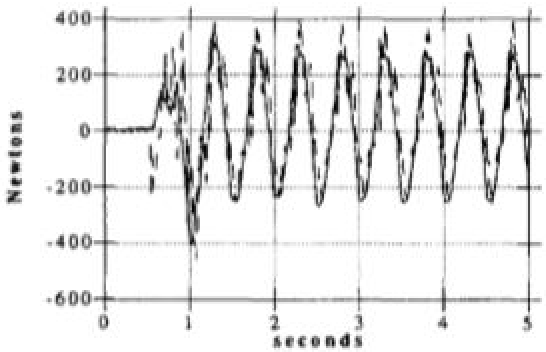

Fig. 9. Actuator force tracking (experimental): Desired : - - - Actual: —

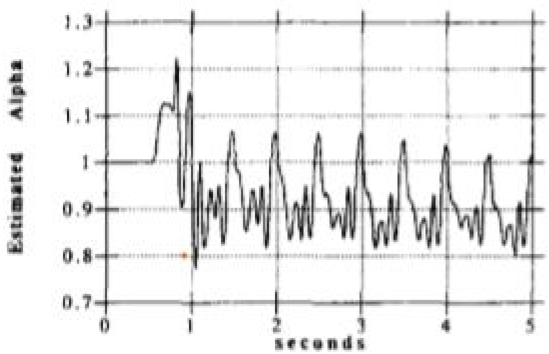

Fig. 10. Estimate of α

hydraulic parameters.

Let us define a Lyapunov function as a sum of square of hydraulic parameters and error function :

$$V = \frac{1}{2}\left(s_1^2 + \rho_1\gamma_2^2 + \rho_2\beta_2^2\rho_3\alpha_2^2\right)$$

$$\alpha_2 = \alpha - \alpha_1, \beta_2 = \beta - \beta_1 \; and \; \gamma_2 = \gamma - \gamma_1$$

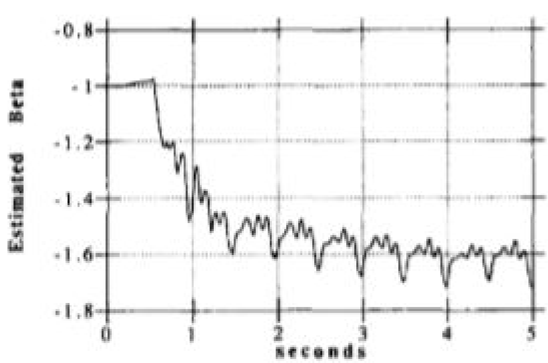

Fig. 11. Estimate of β

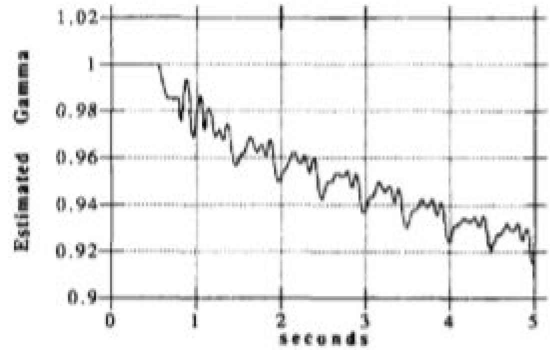

Fig. 12. Estimate of γ

Differentiating the Lyapunov function and using the fact $\dot{V} < 0$ necessary for asymptotic stability of the system, we get the following equations for displaying the dynamics of the hydraulic parameters:

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -ks_1^2$$

$$\frac{\mathrm{d}\beta_1}{\mathrm{d}t} = \frac{s_1}{\rho_2}Ax_5$$

$$\frac{\mathrm{d}\alpha_1}{\mathrm{d}t} = \frac{s_1}{\rho_3}A^2(x_2 - x_4)$$

$$\frac{\mathrm{d}\gamma_1}{\mathrm{d}t} = \frac{s_1\left(A\beta_1 x_5 + \alpha_1 A^2(x_2 - x_4) + \frac{\mathrm{d}F_{desired}}{\mathrm{d}t} - k_1 s_1\right)}{\rho_1\gamma_1}$$

The above equations were used to design an algorithm for force tracking. This algorithm to update the estimates of the parameters cannot guarantee that the estimates of the parameter will converge to the actual values. To guarantee parameter convergence, "persistence of excitation" (PE) is necessary. Qualitatively, persistence of excitation means that the desired force profile and its derivatives have to be "rich" enough in frequency that the only values of the estimates that will guarantee force tracking are the correct values. The variation of system parameters with time and the force tracking are shown in figure from Fig.9 to Fig. 12.

VI. Modified Adaptive Control

Although the adaptive control scheme took into account the time varying nature of the system parameters, it also showed that the parameters tend to converge to a value in the neighbourhood of the initial value. However, the estimates seem to oscillate between two sets of values. It was found that the parameters took different values depending upon the sign of the force output.It was found that the parameters depended on the sign of the spool valve displacement which leads to the modification of the parameter adaptation algorithm.With this thought, we divide the state space into separate regions, and in each of these regions parameters are constant or slow time varying.

$$\theta = \theta_+ \ if \ x_6 \geq 0 \ and \ \tilde{\theta_+} = \theta_+ - \hat{\theta_+} \ if \ x_6 \geq 0$$
$$\theta = \theta_- \ if \ x_6 < 0 \ and \ \tilde{\theta_-} = \theta_- - \hat{\theta_-} \ if \ x_6 < 0$$

Now consider the Lyapunov candidate,

$$V = \frac{1}{2}\left(s_1^2 + \rho_1 \tilde{\gamma_+}^2 + \rho_2 \tilde{\beta_+}^2 \rho_3 \tilde{\alpha_+}^2\right) + \frac{1}{2}\left(\rho_1 \tilde{\gamma_-}^2 + \rho_2 \tilde{\beta_-}^2 \rho_3 \tilde{\alpha_-}^2\right)$$

We will carry out our analysis for the case when spool valve displacement is positive. For the system to be stable the should be negative in the entire domain. This will give us four equations as follows:

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -ks_1^2 \leq 0$$
$$\frac{\mathrm{d}\tilde{\beta_+}}{\mathrm{d}t} = -\frac{s_1}{\rho_2} A x_5$$
$$\frac{\mathrm{d}\tilde{\alpha_+}}{\mathrm{d}t} = \frac{s_1}{\rho_3} A^2 (x_2 - x_4)$$
$$\frac{\mathrm{d}\tilde{\gamma_+}}{\mathrm{d}t} = \frac{s_1\left(A\tilde{\beta_+}x_5 + \tilde{\alpha_+}A^2(x_2 - x_4) + \dot{F}_{desired} - k_1 s_1\right)}{\rho_1 \tilde{\gamma_+}}$$

and

$$\dot{\tilde{\gamma_-}} = 0$$
$$\dot{\tilde{\beta_-}} = 0$$
$$\dot{\tilde{\alpha_-}} = 0$$

These equations were used to find the estimates of system parameters. These estimates are shown below:

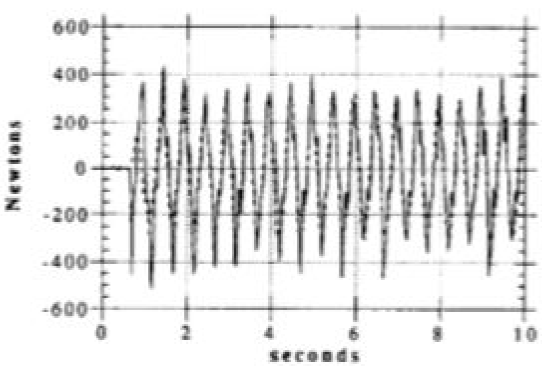

Fig. 13. Actuator force tracking (experimental): Desired : - - - Actual: —

The above system was experimented with road input that is a 2Hz, 1.27 cm sine wave. Figures 9 and 10 show the parameters converging to two distinct sets of normalized values, as was predicted, and Fig 8 shows the tracking convergence. It can be observed that the β, which is the hydraulic cylinder leakage term, converges to negative values which is physically impossible. However, the force error converges, which is all that the analysis guarantees.

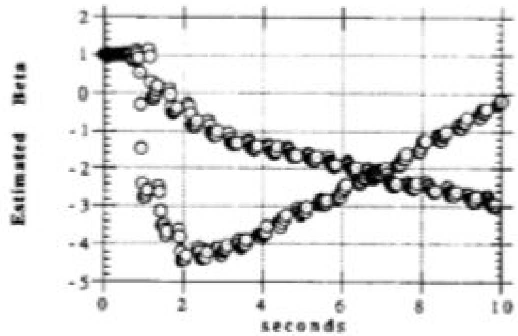

Fig. 14. Estimate of beta (experimental)

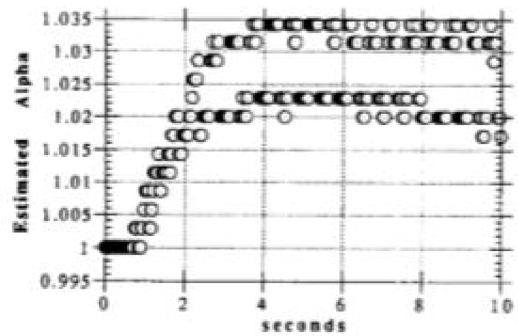

Fig. 15. Estimate of alpha (experimental)

VII. Conclusions

In this report, linear and nonlinear control of suspensions is reviewed. The system was solved using state space equations where a relative two degree system was converted to two relative one degree problems. A standard parameter adaptation scheme based on Lyapunov analysis is also reviewed. The result of this control scheme lead to the motivation behind developing a modified adaptation scheme. The performance of the system, defined by the ability of the actuator to track a desired force, under various controllers was examined. It was shown that active suspensions reduces the sprung mass acceleration four times as compared to the the passive suspensions when the road input frequency was near the body mode of the vehicle. Further, when the system was analyzed under the nonlinear sliding controller, the force tracking by the system improved.The use of the modified adaptation scheme further improved the performance of the system. As verified by experiment, the modified scheme would have beneficial effects whenever the parameters of the system change with states of the system that can be measured.

References

[1] Samy Aly Hassan, Thesis on "Fundamental Studies of Passive, Active and Semi-Active Automotive Suspension Systems," The University of Leeds, Dept. Mechanical Engineering, July 1986.

[2] Andrew Alleyne and J.K.Hedrick, "Nonlinear Adaptive Control of Active Suspensions", IEEE transactions on Control Systems Technology, vol. 3. March 1995, pp.94–101.