Title:	Introduction to Riemann Surfaces
Authors:	Jana, Barnali
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Abstract:	I have done my masters thesis on Riemann Surfces . This article provides an introduc- tion to Riemann surfaces, which are locally open sets in the complex plane. The definition is made precise by defining complex charts and structures, and examples of compact Rie- mann surfaces, including the Projective Line P 1 , complex tori, and smooth plane curves, are presented. To determine if a function defined near a point on a Riemann surface is holomor- phic, complex charts are used to transport the function to the neighborhood of a point in the complex plane, and this process is made precise for a variety of properties. The concept of singularity type (removable, pole, essential) for functions of a single variable extends read- ily to functions on a Riemann surface. Several theorems concerning holomorphic maps, including the open mapping theorem, identity theorem, and discreteness of preimages, are immediate consequences of the corresponding theorem for holomorphic functions. Holo- morphic maps between two Riemann surfaces have a standard normal form in some local coordinates, where essentially every map looks like a power map. Holomorphic maps be- tween compact Riemann surfaces exhibit several beautiful properties, including constancy of degree map. The article provides a proof that the sum of orders of a non-constant mero- morphic function on a Riemann surface is zero. The constancy of the degree of a holomor- phic map between compact Riemann surfaces, combined with the theory of Euler numbers, gives an important formula known as Riemann-Hurwitz's formula. The article also covers gluing of Riemann surfaces, with hyperelliptic Riemann surfaces as an important example, and identifies all automorphism groups of holomorphic functions between complex tori. In the last section, the article discusses group actions on Riemann surfaces, and introduces the basic construction of Riemann surfaces by dividing a known Riemann surface by the action of a group.
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