Title:	Algebraic structures in knot theory
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Abstract:	Knot theory is the study of embedded circles in the 3-sphere. A central problem in the subject is to develop computational invariants that can distinguish two knots. One such almost complete invariant that surfaced independently in the works of Matveev and Joyce in 1982 is what is called a link quandle, which is basically a minimal algebraic structure that encodes the three Reidemeister moves of planar diagrams of links in the 3-sphere. One of the fundamental results is that two non-split tame links have isomorphic link quandles if and only if there is a homeomorphism of the 3-sphere that maps one link onto the other, not necessarily preserving the orientations of the ambient space and that of links. Many classical topological, combinatorial and geometric knot invariants such as the knot group, the knot coloring, the Conway polynomial, the Alexander polynomial and the volume of the complement in the 3-sphere of a hyperbolic knot can be retrieved from the knot quandle. Thus, understanding of knot quandles is of fundamental importance for the classification problem for knots. The first and major component of the thesis is a fusion of ideas from combinatorial group theory into the theory of quandles. More precisely, we introduce residual finiteness and or- derability in quandles. One of our main results is that every link quandle is residually finite, a proof of which uses the idea of subgroup separability in fundamental groups of 3-manifolds. As immediate consequences of this result, it follows that the word problem is solvable for link quandles, and that every link admits a non-trivial coloring by a finite quandle. We also develop a general theory of orderability of quandles with a focus on link quandles and give some general constructions of orderable quandles. We prove that knot quandles of many fibered prime knots are right-orderable, whereas link quandles of many non-trivial torus links are not right- orderable. We prove that link quandles of certain non-trivial positive (or negative) links are not bi-orderable, which includes some alternating knots of prime determinant and alternating Montesinos links. The results show that orderability of link quandles behave quite differently than that of corresponding link groups. Viewing classical knots as knots in the thickened 2-sphere, it is natural to explore knot theory in thickened surfaces of higher genera. This idea led to what is now known as virtual knot theory, a subject pioneered by Kauffman in 1999 with a completely different set-up. Though many invariants from the classical knot theory extend to the virtual setting, a lot is still unknown, and the second component of the thesis focuses on this theme. We define virtually symmetric representations of virtual braid groups by automorphism groups. We prove that many known representations of these groups such as the generalized Artin representation, the Silver-Williams representation, the Boden-Dies representation and the Wada representation are equivalent to virtually symmetric representations. We use one such representation to define new virtual link groups which are extensions of link groups known due to Kauffman. Finally, we introduce marked Gauss diagrams as a generalization of Gauss diagrams and extend the definition of virtual link groups to marked Gauss diagrams
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