Title:	Fusion Modules of Classical Lie Algebras and Image of Multilinear Polynomials on Nilpotent Lie Algebras
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Abstract:	In the thesis we explore two distinct problems in the theory of Lie algebras and its repre- sentations. Given a finite-dimensional simple Lie algebra g, one can associate with it a graded Lie algebra g[t], which is called the current algebra of g. Fusion modules constitute a family of cyclic finite-dimensional graded representations of g[t], introduced by Feigin and Loktev in 1999. It has been proven that several important classes of finite-dimensional g[t]-modules are, in fact, Fusion modules. In the initial segment, we delve into the fusion product modules associated with current Lie algebras of types A, B, C, and D. In the case where g is a classical Lie algebra of rank n, given two dominant weights λ and μ of g, we define a cyclic g[t] module F λ,μ via generators and relations. By associating a series of short exact sequences with F λ,μ , we obtain a graded decomposition of the module F λ,μ when λ and μ are both multiples of either the first or n th fundamental weight of g. This enables us to establish that for the mentioned choice of weights (λ, μ), the module F λ,μ is indeed a fusion product module. Furthermore, our methods aid in determining the graded character for these fusion modules. In the second segment, we consider the image of a multilinear Lie polynomial of degree 2 when evaluated on a class of nilpotent Lie algebras. A non-zero multilinear Lie polynomial in two variables is defined as a non-zero scalar multiple of the Lie bracket of these variables. It has been proved that for a large class of simple Lie algebras G defined over fields whose cardinality is sufficiently large, the image set of multilinear Lie polynomials of degree 2 when evaluated on G is G itself. However, the image set of multilinear Lie polynomials of degree 2 when evaluated on nilpotent Lie algebras is not always a vector space. We show that if L is a nilpotent Lie algebra over a field k, and L 0 = [L, L] is its derived subalgebra, then the image of multilinear polynomials of degree 2 when evaluated on L is equal to L 0 whenever char k is not equal to 2 and dim k L 0 ≤ 3. Furthermore, when L is a nilpotent Lie algebra over a finite field k of odd characteristic, and the dimension of L 0 over k is equal to 4, we obtain the necessary and sufficient conditions under which the images of multilinear polynomials of degree 2 when evaluated on L are not equal to L 0 ; in this case, we provide an explicit description of the image set of the multilinear polynomial.
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