Title:	Naively A 1 -Connected Components of Varieties
Authors:	Rani, Bandna
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Abstract:	A 1 -homotopy theory is a homotopy theory for schemes in which the affine line A 1 plays the role of the unit interval. The main objects of study are simplicial sheaves on the Nisnevich site of smooth schemes of finite type over a field. For these objects, one constructs analogues of various devices from the classical ho- motopy theory of topological spaces. One such device is the sheaf of A 1 -connected components of a simplicial sheaves. 1 For a general simplicial sheaf X , the sheaf π 0 A (X) of A 1 -connected components of X is generally hard to compute. However, one can attempt to study it by means of the sheaf of naively A 1 -connected components, denoted by S(X). The sheaf 1 S(X) may be viewed as a crude approximation to π 0 A (X), but it is easier to define and compute, at least when X is a sheaf of sets. The functor S is the main object of study in this thesis. When X is a sheaf of sets, the direct limit of the sheaves S n (X), which we denote by L(X) can be proved to be A 1 -invariant. In fact, this is the universal 1 A 1 -homotopic quotient of X . When π 0 A (X) is A 1 -invariant, it can be proved to 1 be isomorphic to L(X). A recent example of Ayoub has shown that π 0 A (X) is not always A 1 -invariant. However, we show that there is a natural bijection between 1 field valued points of the sheaves L(X) and π 0 A (X) for any sheaf of sets X . The sheaf L(X) is obtained by iterating S on a the sheaf X infinitely many times. Our second main result is to show that the infinitely many iterations are indeed necessary. We achieve this by constructing a family of sheaves {X n } n , indexed by the positive integers, such that S i (X n)/= S i+1 (X n) for any i < n. The third main result of this thesis is regarding retract rational varieties over an infinite field k. A result of Kahn and Sujatha shows that for a retract rational 1 variety X, the sheaf π 0 A (X) is the point sheaf. We strengthen this result by showing that S(X) is the point sheaf.
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