



# Library Indian Institute of Science Education and Research Mohali



**DSpace@IISERMohali (/jspui/)**

**/ Thesis & Dissertation (/jspui/handle/123456789/1)**

**/ Master of Science (/jspui/handle/123456789/2)**

**/ MS-08 (/jspui/handle/123456789/270)**

Please use this identifier to cite or link to this item: <http://hdl.handle.net/123456789/310>

Title: Geodesic Conjugacy Rigidity of Nonpositively Curved Surfaces

Authors: Paul M, Jithin (/jspui/browse?type=author&value=Paul+M%2C+Jithin)

Issue Date: 26-Apr-2013

Abstract: It is a fundamental problem in Riemannian geometry to try and capture the geometry of a Riemannian manifold by certain of its geometric invariants. In this thesis we consider closed (compact without boundary) Riemannian manifolds  $M$  and the action of the geodesic flow  $gtM$  on the unit tangent bundle  $SM$ . It turns out that if  $M$  has negative sectional curvature then the geodesic flow  $gtM$  has significant influence on the geometry of  $M$ ; for instance, it is a well known fact that a typical geodesic in  $M$  is dense. This is in sharp contrast with the case of geodesics on the unit sphere in  $R^3$ , where every geodesic is a great circle; in particular none of the geodesics is dense. The classification theorem for surfaces says that a closed surface  $M$  in  $R^3$  is homeomorphic to either a sphere or a torus or a surface of higher genus. The genus of a surface determines its Euler characteristic, which is a topological invariant; more precisely, the Euler characteristic  $\chi(M)$  of a surface  $M$  of genus  $g$  is  $2 - 2g$ . The celebrated Gauss Bonnet theorem relates the Euler characteristic of a surface  $M$  to its Gaussian curvature  $K$  by the formula  $\int_M K dA = 2\pi\chi(M)$  where  $dA$  is the area form in  $M$ . A consequence of the Gauss Bonnet formula is that the sign of curvature on a given closed surface  $M$ , if the same sign holds at all points of  $M$ , is restricted to a single choice. For example on a sphere  $S^2$ , whose Euler characteristic is 2, a negative sign on the curvature at all of its point is not possible, whereas such a thing is possible on a surface of genus  $\geq 2$ . The classical uniformization theorem for surfaces precisely confirms this possibility. That is, a surface  $M$  of genus  $\geq 2$  admits a metric of constant negative curvature  $-1$ . The main theorem discussed in this thesis concerns metrics of non positive curvature on a surface  $M$  of genus  $\geq 2$  and proves that such metrics are determined up to isometry by the action of the geodesic flow  $gtM$  on  $SM$ . More precisely, we will discuss a proof of the following theorem. Theorem 0.0.1 (Croke, 1990). Let  $N$  be a closed surface of genus  $\geq 2$  with non positive sectional curvature and  $M$  be a compact surface whose geodesic flow is conjugate to  $N$  via  $F$ ; i.e.,  $F : SM \rightarrow SN$  is a  $C^1$ -diffeomorphism such that  $F \circ gtM = gtN \circ F$  for all  $t$  then  $F = gK N \circ df$ , where  $f$  is an isometry from  $M$  to  $N$  and  $K$  is a fixed number.


URI: <http://hdl.handle.net/123456789/310> (<http://hdl.handle.net/123456789/310>)

Appears in MS-08 (/jspui/handle/123456789/270)  
Collections:

Files in This Item:

File	Description	Size	Format	
MS-08026.pdf (/jspui/bitstream/123456789/310/1/MS-08026.pdf)		333.08 kB	Adobe PDF	<a href="#">View/Open (/jspui/bitstream/123456789/310/1/MS-08026.pdf)</a>

[Show full item record \(/jspui/handle/123456789/310?mode=full\)](#)

 [\(/jspui/handle/123456789/310/statistics\)](#)

Items in DSpace are protected by copyright, with all rights reserved, unless otherwise indicated.