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Title: The Leray-Serre Spectral Sequence and its Applications

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Abstract: The aim of this thesis is to understand the Leray-Serre spectral sequence of a fibration and use it to compute cohomology of some interesting manifolds. To do that, it is important to understand spectral sequences first. So the first part of my thesis is devoted to an introduction to spectral sequences and then some background in topology to understand the Leray-Serre spectral sequence of a fibration. A cohomology spectral sequence is a collection of differential bigraded R -modules $E^r_{p,q}$; $r = 1, 2, \dots$, where the differentials are all of bidegree $(r, 1 - r)$ such that for all r , the E^{r+1} -term is given as the cohomology of the E^r -term. Pictorially one can imagine this as a three dimensional lattice with each lattice point an R -module, the differentials as arrows between them and each page is obtained by taking the cohomology of the previous page. One can observe that, the knowledge of $E^r_{p,q}$ and d_r determines $E^{r+1}_{p,q}$, but not d_{r+1} . So if a differential is not known then one needs some other method to proceed. The first property that a spectral sequence admits is that it can be represented as an infinite tower of submodules of the E^2 -term and conversely. Thus one can define the limit term of this sequence which we call the E^1 -term. Now the ultimate goal is to compute this E^1 -term. It is interesting to note that if a spectral sequence 'collapses' at, say N , then the computation of E^1 -term becomes easy as the sequence becomes constant after $(N + 1)$ th page. Once we know what a spectral sequence is, the natural question that one can ask is how can we construct one? In this direction, there are two general algebraic settings in which spectral sequences arise naturally. First is a filtered differential graded module and second is an exact couple. In the first case, each filtered differential graded module A determines a spectral sequence with differential of bidegree $(r, 1 - r)$ and if the filtration is bounded then the spectral sequence converges to $H(A; d)$ (the homology of A with respect to d). This result first appeared in the work of Koszul [3] and Cartan [1]. There are also weaker conditions which ensure the convergence and uniqueness of the target. Thus if the filtration is exhaustive and weakly convergent, the same result will still hold true. Second case is that of an exact couple. This idea was introduced by Massey [5]. An exact couple also determines a spectral sequence of cohomological type. It is interesting to observe that one can also associate a tower of submodules of E and an E^1 -term to an exact couple, just as we can do for any spectral sequence. The next question one could ask is that if the two approaches are related in any way? and if yes then how do the two spectral sequences compare? The answer to the above question is yes and it is not very difficult to see that a filtered differential graded module gives rise to an exact couple. And in fact, the two spectral sequences, one associated to the filtered differential graded module and the other associated to the exact couple derived from the filtered differential graded module, turn out to be same. There is another algebraic object namely double complex which gives rise to two spectral sequences, which in turn help in the calculation of the homology of the total complex associated to a double complex. Double complexes offer an example of the filtered differential graded module construction of a spectral sequence. Finally, with enough background on spectral sequences, one can talk about fibrations and the spectral sequence associated to them. A map satisfying the homotopy lifting property with respect to all spaces is called a Hurewicz fibration (or just a fibration), while a map with the homotopy lifting property with respect to all n -cells is called a Serre fibration. It was Leray [4] who solved the problem of relating the cohomology rings of spaces making up a fiber space by developing a powerful computational gadget called a spectral sequence. This spectral sequence has many applications such as computation of cohomology of various Lie groups, homogeneous spaces and loop spaces.

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