Title:	Morse-Bott Theory
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Abstract:	A well-chosen function can give precise information on the topology of a manifold. To demonstrate this, very often 'the height function' on torus is used. These functions happen to be real-valued; moving in an increasing order along the values of the function, we note that the level sets all have the same topology until we meet an accident where the topology changes and subsequently remains the same until the next accident. The same holds for the 'sublevel sets', that is, what lies below a given level. The accidents are the critical values of the functions, which correspond to the critical points, those where the differential of the function is zero. One of the first results of this theory is a theorem due to Reeb; it asserts that a compact manifold on which there exists a function with only two critical points is homeomorphic to a sphere. If we assume the critical points to be 'nice' in certain sense, then modulo 2 the number of critical points equals the Euler characteristic of the manifold, the invariant that does not depend on the function but only on the manifold. Further, the concept of Witten spaces of trajectories allows us to present a finer invariant. It is clear that the torus and sphere are very different manifolds even if the two admit a function with four non degenerate critical points. This invariant is what is nowadays called the Morse Homology of the manifold. It is the homology of a complex, the Morse complex, constructed from the critical points of a Morse function by 'counting' its trajectories along a vector field that connects them. These trajectories are those of the gradient of the function. In case of the height function one could think of the trajectories of water drops flowing on the surface from one critical point to another. Ultimately, this homology depends only on the diffeomorphism type of the manifold. The remarks concerning the number of critical points can be expressed in terms of the famous "Morse Inequalities": the number c k of critical points of index k of a Morse function on a manifold satisfies c k ≥ dim HM(orse) k (M)
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