Title:	Hyperbolicity and Cannon-Thurston maps for complexes of spaces.
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Keywords:	Thurston maps. Hyperbolicity. Cannon-Thurston.
Issue Date:	Dec-2023
Publisher:	IISER Mohali
Abstract:	The concept of Cannon-Thurston maps in Geometric Group Theory was intro- duced by Mitra in [Mit98a] motivated by the fundamental work of Cannon and Thurston (see [CT85,CT07]). Given Gromov hyperbolic groups H < G (see [Gro87]) one asks if the inclusion map i : H → G naturally induces the Cannon-Thurston (CT) map ∂i : ∂H → ∂G which is characterized by the property that for any se- quence {h n } in H and ξ □ ∂H, h n → ξ implies h n → ∂i(ξ). It is well-known that such a map is continuous when it exists, but it may not, in general, exist (see [BR13]). In the first part of the thesis, among other things, we show the existence of CT maps for a pair of hyperbolic groups H < G where (1) G is the fundamental group of a graph of hyperbolic groups (G, Y), say, satisfying qi embedded condition such that G is hyperbolic (see [BF92]), (2) H is the fundamental group of a subgraph of hy- perbolic subgroups (H, Z), say, of (G, Y), (3) for any vertex v of Z, the inclusion of the vertex groups H v → G v of (H, Z) and (G, Y) admits the CT map and (4) for any edge e of Z, the edge group H e of (H, Z) is same as the corresponding edge group G e of (G, Y). (One is refered to [Bas93, Corollary 1.14] for the definition of a sub- graph of subgroups of a graph of groups.) This result is deduced by first proving an existence theorem for CT maps for certain morphisms of trees of hyperbolic metric spaces, which generalizes earlier results of M. Mitra ([Mit98b]), and (a special cases of) M. Kapovich and P. Sardar ([KS22, Theorem 8.11]). Moreover, in the course of this work, we also found a nonexistence theorem for CT maps which is similar to that of Baker-Riley ([BR13]) but is conceptually somewhat easier to understand. In the second part of the thesis, we prove a combination theorem for trees of metric bundles extending the combination theorems for trees of hyperbolic metric spaces due to Bestvina-Feighn ([BF92]) and metric bundles due to Mj-Sardar ([MS12]). More precisely, we prove that if π B : B → T is a tree of hyperbolic metric spaces whose edge spaces are points and π X : X → B is a 1-Lipschitz surjective map then X is hyperbolic if the following holds: (1) The fibers of π B ◦ π X are hyperbolic metric spaces which are nonelementary (i.e., their barycentric maps are coarsely surjective as in [MS12, Section 2]) and are all uniformly properly embedded in X. (2) B is hyperbolic. −1 (3) For all vertex u of T , let B u = π B −1 (u) and X u = π X (B u). Then the restriction of π X to X u gives a metric bundle X u → B u as defined by [MS12]. (4) Suppose e is the edge in T joining two vertices u, v. Let e B denote the (isometric) lift of e in B joining b u □ B u and b v □ B v . Then π X restricted −1 to π X (e B) is a tree of metric spaces with the qi embedded condition over e B = [b u , b v] as defined in [Mit98b]. 12 ABSTRACT (5) The parameters of (1), the bundles in (3) and the trees of metric spaces in (4) are uniform. (6) Bestvina-Feighn's hallway flaring condition holds for qi lifts in X of geodesics in B. This theorem is then used to prove a combination theorem for certain complexes of hyperbolic groups. References [Bas93] Hyman Bass, Covering theory for graphs of groups, J. Pure Appl. Algebra 89 (1993), 3–47. [BF92] M. Bestvina and M. Feighn, A Combination theorem for Negatively Curved Groups, J. Differential Geom., vol 35 (1992), 85–101. [BR13] O. Baker and T. R. Riley, Cannon-Thurston maps do not always exist, Forum Math., vol 1, e3 (2013). [CT85] J. Cannon and W. P. Thurston, Group Invariant Peano Curves, preprint, Princeton (1985). [CT07] , Group Invariant Peano Curves, Geom. Topol. 11 (2007), 1315–1355. [Gro87] M. Gromov, Hyperbolic Groups, in Essays in Group Theory, ed. Gersten, MSRI Publ.,vol.8, Springer Verlag (1987), 75–263. [KS22] Michael Kapovich and Pranab Sardar, Trees of hyperbolic spaces, https://arxiv.org/abs/2202.09526 (2022). [Mit98a] M. Mitra, Cannon-Thurston Maps for Hyperbolic Group Extensions, Topology 37 (1998), 527–538. [Mit98b] , Cannon-Thurston Maps for Trees of Hyperbolic Metric Spaces, J. Differential Geom. 48 (1998), 135–164. [MS12] Mahan Mj and Pranab Sardar, A combination theorem for metric bundles, Geom. Funct. Anal. Vol. 22 (2012), 1636–1707.
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