Chapter One Notes

Sunday, November 20, 2022 5:55 PM

Section 1.1

Signal: any physical quantity that varies with space, time, or other variables

↳ ex: speech can be represented as a function (very complex)

$$\sum_{i=1}^{N} A_i(t)\sin[2\pi F_i(t)t + \theta_i(t)]$$

A: amplitude θ: phase shift

F: frequency

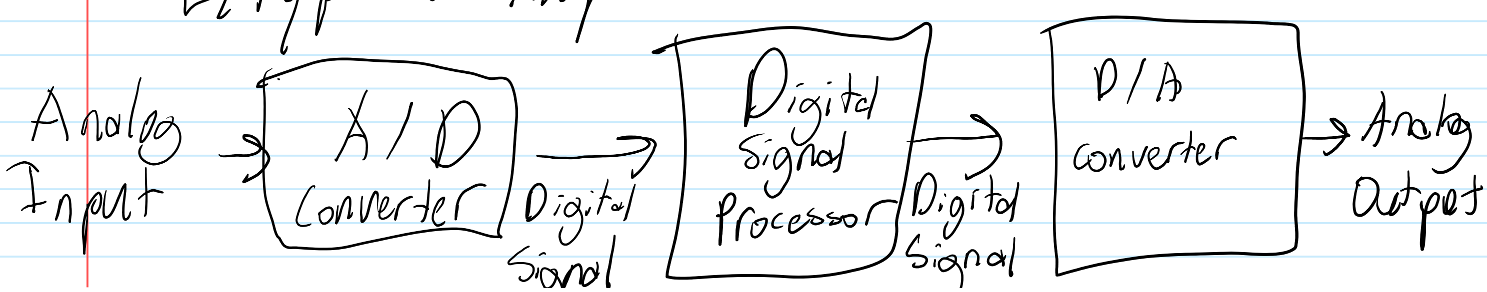
↳ images are also signals → function of position (2 independent variables)

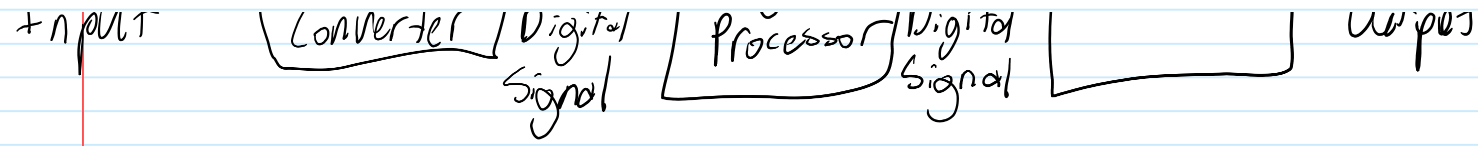
System: physical system/software that operates on a signal

↳ filters

- since most signals in nature are analog, they need to be converted to be digital in order to be processed

↳ typical implementation:





- benefits of digital processing over analog:
  - digital circuitry have a lower tolerance
  - less hardware redesign to change digital processor
  - easier to store digital signals
  - typically lower in cost

Section 1.2

multichannel signal: a vector of signals
↳ ex: EMG is typically a 12-channel signal

continuous-time signal: (also known as analog signal) a signal where the value is defined at every time step (or other independent variable)
↳ take on values in a continuous interval (a,b)

discrete-time signal: signal is only defined at specific values of time

Sampling: selecting values of an analog signal as discrete time steps

continuous-valued signal: signal that takes on all values in a finite or infinite range values in a finite or infinite range

discrete-valued signal: signal takes on values from a finite (typically equidistant) set of values

- for a signal to be processed digitally, it must be discrete-timed and discrete-valued
  - ↳ to convert an analog signal, it is typically sampled at discrete intervals and the sampled values are quantized into a set of discrete values

deterministic signal: can be defined by a mathematical model ⇒ all past, present, and future values of the signal are known

- most signals are random signals, not deterministic in nature

Section 1.3

- continuous-time sinusoidal signals

$x_a(t) = A\cos(\Omega t + \theta)$

or

$x_a(t) = A\cos(2\pi F t + \theta)$

where

A: amplitude $\quad \Omega$: frequency (rad/s)

F: frequency (hz) $\quad \theta$: phase shift

- signals carry over to complex coordinates
  - $x_a(t) = Ae^{j(\Omega t + \theta)}$
    - ↳ based on $e^{\pm j\phi} = \cos\phi \pm j\sin\phi$
- $x_a(t) = A\cos(\Omega t + \theta) = \frac{A}{2}e^{j(\Omega t+\theta)} + \frac{A}{2}e^{-j(\Omega t+\theta)}$
  - sin cancels out, cos add
- **Phasors**: equal amplitude, complex-conjugate exponential signals
- discrete-time sinusoidal signal

  $x(n) = A\cos(2\pi f n + \theta)$

  where n is an integer variable (sample number)

  ↳ only periodic if its frequency is a rational number

  periodic if $x(n+N) = x(n)$ for all n

  where N is the period

  ↳ sinusoids with frequencies separated by an integer multiple of 2π are the same

  ↳ $\cos([\omega_0 + 2\pi]n + \theta) = \cos(\omega_0 n + 2\pi n + \theta) = \cos(\omega_0 n + \theta)$

  ↳ as a result, $x_k(n) = A\cos(\omega_k n + \theta)$

  where $\omega_k = \omega_0 + 2k\pi \quad -\pi \leq \omega_0 \leq \pi$

- **Fourier series**:
  - representation of a signal as the summation

Fourier Series:

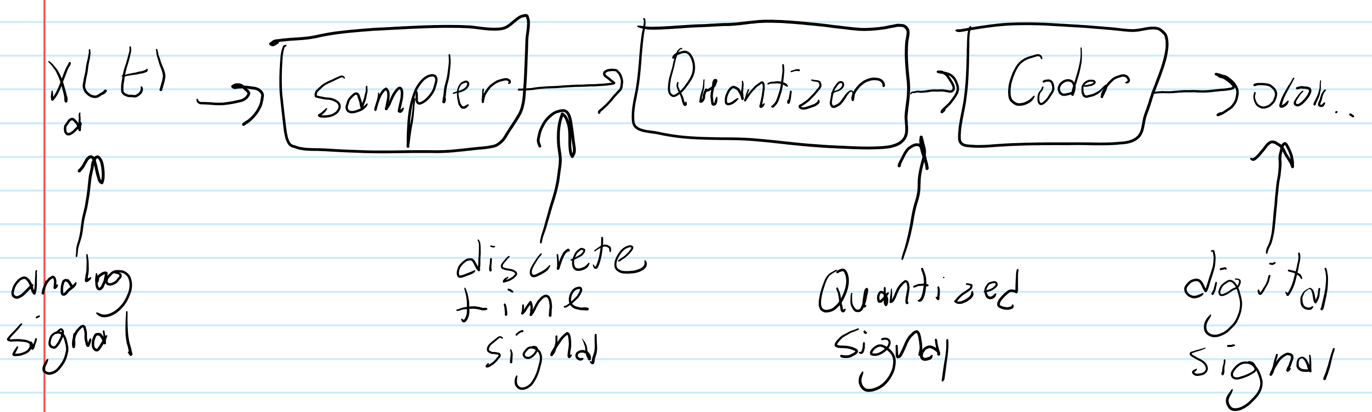
- representation of a signal as the summation of harmonically related complex exponentials

↳ $x_a(t) = \sum_{k=-\infty}^{\infty} c_k e^{j\Omega_0 t}$ analog

↳ $x(n) = \sum_{k=0}^{N-1} c_k e^{j2\pi k \frac{n}{N}}$

Section 1.4

- A/D Converter:



- sampling:

$x(n) = x_a(nT)$

where $x(n)$ is a discrete-time signal,

$x_a(t)$ is continuous-time signal

and T is the sampling period/interval

sampling frequency $= \frac{1}{F}$

- relative or normalized frequency:

$f = \frac{F}{F_s}$ → used to convert sampling

$f = \frac{F}{F_s}$ → used to convert sampling frequency to analog frequency

↳ $-\frac{1}{2} < f < \frac{1}{2}$

→ $-\frac{F_s}{2} < F < \frac{F_s}{2}$

meaning $F_s > 2F$ to properly analyze the analog signal

Relations Between Frequencies

Continuous	Discrete
$\Omega = 2\pi F$	$\omega = 2\pi f$
$\frac{rad}{sec}$ hz	$\frac{rad}{sample}$ $\frac{cycles}{sample}$
	$-\pi < \omega \leq \pi$
	$-\frac{1}{2} < f < \frac{1}{2}$
	(because of aliasing and equivalence)

$-\infty < \Omega < \infty$

$-\infty < F < \infty$

$\omega = \Omega T,\ f = \frac{F}{F_s}$ →

$-\frac{\pi}{T} < \Omega < \frac{\pi}{T}$

$-\frac{F_s}{2} < F < \frac{F_s}{2}$

← $\Omega = \frac{\omega}{T}$, $F = f \cdot F_s$

Example 1.4.1

$x_1(t) = \cos 2\pi(10)t$
$x_2(t) = \cos 2\pi(50)t$ } analog signals

$F_s = 40hz$

$x_1(n) = \cos 2\pi \frac{10}{40} t$
$x_2(n) = \cos 2\pi \frac{50}{40} t$ } discrete-time signals

$x_2(n) = \cos 2\pi \frac{10}{40} t$

$x_1(n) = x_2(n)$

Example 1.4.2

A) $x_a(t) = 3\cos 100\pi t = 3\cos 2\pi(50)t$

$F = 50hz$

$F_s \geq 100\ hz$

B) $F_s = 200\ hz$

$x(n) = 3\cos 2\pi \frac{50}{200} t = 3\cos \frac{\pi}{2} t$

C) $x(n) = 3\cos 2\pi \frac{50}{75} t = 3\cos \frac{4}{3}\pi t \rightarrow 3\cos \frac{2\pi}{3} t$

d) $f = \frac{1}{3}$ $\quad F = f \cdot F_s = \frac{1}{3} \cdot 75hz = 25hz$

50hz

Example 1.4.4

A) $F_{max} = 6000hz$ $\quad$ Nyquist rate: 12000 hz

2π $\quad 6\pi$ $\quad 12\pi$

B) $x(n) = 3\cos\frac{2\pi}{5}t + 5\sin\frac{6\pi}{5}t + 10\cos\frac{12\pi}{5}t$

$= 3\cos\frac{2\pi}{5}t - 5\sin\frac{4\pi}{5}t + 10\cos\frac{2\pi}{5}t$

$= 13\cos\frac{2\pi}{5}t - 5\sin\frac{4\pi}{5}t$

c) $f = \frac{1}{5}, \frac{2}{5}$

$F = F_s \cdot f = 1000, 2000$

$x_a(t) = 13\cos 2\pi(1000)t - 5\sin 2\pi(2000)t$

- Quantization:

$x_q(n) = Q[x(n)]$

where $x_q(n)$ is the quantized discrete-time signal

error:

$e_q(n) = x_q(n) - x(n)$

↳ instantaneous error cannot exceed half of the quantization step when rounding

$$-\frac{\Delta}{2} \leq e_q(n) \leq \frac{\Delta}{2}$$

where

$$\Delta = \frac{x_{max} - x_{min}}{L-1}$$

- if F_s is above the nyquist rate, the quantization of the signal should be the only source of error