Chapter One Problems

Thursday, November 24, 2022 5:00 PM

1.1)

a)
multi-channel (multiple stocks)
single-dimensional
discrete time
digital

b) multi-dimensional
multi-channel
continuous-time
analog

c) single-dimensional
single-channel
continuous-time
analog

d) multi-dimensional
single-channel
continuous-time
analog

1.2

a) periodic

$F_0 = \frac{.01\pi}{2\pi} = .005 \text{ hz}$

b) periodic

$F_0 = \frac{30\pi/105}{2\pi} = \frac{15}{105} = \frac{1}{7} \text{ hz}$

c) periodic

$F_0 = \frac{3\pi}{} = \frac{3}{} \text{ hz}$

d) not periodic

$$F_0 = \frac{3}{2\pi} hz$$

e) periodic

$$F_0 = \frac{62\pi/10}{2\pi} = \frac{31}{10} hz$$

1.3

a) periodic $F = \frac{5}{2\pi}$ $T_p = \frac{2\pi}{5}$

b) not periodic

c) $x(n) = 2e^{j(\frac{n}{6}-\pi)}$

not periodic

d) not periodic

e) periodic

$$F = \frac{\pi/2}{2\pi} = \frac{1}{4}, \frac{\pi/8}{2\pi} = \frac{1}{16}, \frac{\pi/4}{2\pi} = \frac{1}{8}$$

$$T = 4, 16, 8$$

$$T_0 = 16$$

1.4

a) $\alpha = GCD(k, N)$

$$N = N'\alpha$$

$$N' = \frac{N}{\alpha}$$

b) $N = 7$

$k = 0, 1, 2, 3, 4, 5, 6, 7$

c) $N = 16$

$k = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16$

$GCD = 16, 1, 2, 1, 4, 1, 2, 1, 8, 1, 2, 1, 4, 1, 2, 1, 16$

$N_p = 1, 16, 8, 16, 4, 16, 8, 16, 2, 16, 8, 16, 4, 16, 8, 16, 1$

GCD = 7, 1, 1, 1, 1, 1, 1, 1

$N_p = 1, 7, 7, 7, 7, 7, 7, 1$

1.5)

A)

B) $x(n) = 3 \sin \frac{\pi}{3} n$

$T = 6$ samples/cycle

c) 0 ... 5

$x(0) = 0$

$x(1) = \frac{3\sqrt{3}}{2}$

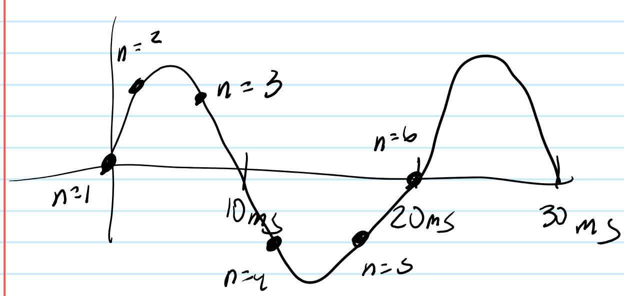
$x(2) = \frac{3\sqrt{3}}{2}$

$x(3) = 0$

$x(4) = -\frac{3\sqrt{3}}{2}$

$x(5) = -\frac{3\sqrt{3}}{2}$

$x(6) = 0$



20 ms

d) $\frac{.005\,s}{\text{sample}} = 200$ sample/sec

1.6)

A) $x_a(t) = A\cos(2\pi F_0 t + \theta) = A\cos\left(\frac{2\pi t}{T_p} + \theta\right)$

$x(n) = A\cos\left(2\pi \frac{T}{T_p} n + \theta\right)$

Since $\frac{T}{T_p} = \frac{k}{N}$, it must be rational, Since $\frac{1}{T_p} = \frac{k}{N}$, it must be rational, making $x(n)$ periodic

B) ?

1.7)

A) $\geq 20\,kHz$

B) $x_a(t) = A\cos(2\pi(5000)t + \theta)$

$$x(n) = A\cos\left(2\pi\left(\tfrac{5}{8}\right)n + \theta\right)$$

$$= A\cos\left(2\pi\left(\tfrac{3}{8}\right)n + \theta\right.$$

$$f = \tfrac{3}{8} \cdot 8khz = 3kHz$$

C) $x_a(t) = A\cos(2\pi(9000)t + \theta)$

$$x(n) = A\cos\left(2\pi\left(\tfrac{9}{8}\right)n + \theta\right)$$

$$= A\cos\left(2\pi\left(\tfrac{1}{8}\right)n + \theta\right)$$

$$f = \tfrac{1}{8} \cdot 8kHz = 1kHz$$

1.8)

A) $100Hz \cdot 2 = 200Hz$

B) $F_s = 250hz$

$$F_{max} = \tfrac{1}{2} \cdot 250Hz = 125Hz$$

1.9)

A) $x_a(t) = \sin(2\pi(240)t) + 3\sin(2\pi(360)t)$

$F_{max} = 360hz$

$Nyquist = 2 \cdot F_{max} = 720hz$

$_{max}$

$Nyquist = 2 \cdot F_{max} = 720hz$

B) $F_s = 600\ hz$

$$F_{fold} = \frac{F_s}{2} = 300\ hz$$

C) $x(n) = \sin(2\pi(\frac{2}{5})n) - 3\sin(2\pi(\frac{2}{5})n)$

$$= -2\sin(2\pi(\frac{2}{5})n)$$

$$f = \frac{2}{5}\ \text{cycles}/\text{sample} \cdot \frac{2\pi\ \text{radians}}{\text{sample}} = \frac{4\pi}{5}\ \text{rad}/\text{sample}$$

D) $F = f \cdot F_s = \frac{2}{5} \cdot 600 = 240hz$

$$y_a(t) = -2\sin(2\pi(240)t)$$

1.10)

A) bits per sample $= \log_2 1024 = 10$

$$\frac{10{,}000\ \text{bits}}{s} \cdot \frac{\text{sample}}{10\ \text{bits}} = 1000\ \text{samples/sec}$$

$$F_{fold} = \frac{F_s}{2} = 500Hz$$

B) $x_a(t) = 3\cos(2\pi(300)t) + 2\cos(2\pi(900)t)$

$$F_{max} = 900\ hz$$

$$Nyquist = 2 \cdot F_{max} = 1800\ hz$$

C) $x(n) = 3\cos(2\pi(\frac{3}{10})n) + 2\cos(2\pi(\frac{1}{10})n)$

$$f = \frac{3}{10}, \frac{1}{10}$$

D) $x_{max} = 5 \qquad x_{min} = -5$

$$\Delta = \frac{x_{max} - x_{min}}{L-1} = \frac{5-(-5)}{1024-1} = \frac{10}{1023}$$

1.11)

$x_a(t) = 3\cos(2\pi(50)t) + 2\sin(2\pi(125)t)$

$F_s = \frac{1}{5ms} = \frac{1}{100s} = 200\ Hz$

$x(n) = 3\cos(2\pi(\frac{1}{4})n) - 2\sin(2\pi(\frac{3}{8})n)$

$F_s' = \frac{1}{1ms} = 1000hz$

$y_a(t) = 3\cos(2\pi(250)t) - 2\sin(2\pi(375)t)$

1.12)

A) $x_a(t) = 3\cos(2\pi(25)t) + 10\sin(2\pi(150)t) - \cos(2\pi(50)t)$

$x(n) = 3\cos(2\pi(\frac{1}{12})n) + 10\sin(2\pi(\frac{1}{2})n) - \cos(2\pi(\frac{1}{6})n)$

$= 3\cos(\frac{\pi}{6}n) - \cos(\frac{\pi}{3}n)$

B) $y_a(t) = 3\cos(\frac{5000\pi}{3}t) - \cos(\frac{10000\pi}{3}t)$

1.13)

A) $x_{max} = 6.35 \quad x_{min} = -6.35$

$\Delta = \frac{x_{max} - x_{min}}{L-1} \rightarrow .1 = \frac{6.35-(-6.35)}{L-1}$

$L = \frac{12.7}{.1} + 1 = 128$

$\log_2 128 = 7$ bits

B) $\Delta = \frac{x_{max} - x_{min}}{L-1} \rightarrow .02 = \frac{12.7}{L-1}$

$L = \frac{12.7}{.02} + 1 = 636$

$\log_2(636) = 9.31 \rightarrow 10$ bits

1.14) $\frac{20 \text{ samples}}{\text{sec}} \cdot \frac{8 \text{ bit}}{\text{sample}} = 160$ bits/sec

$$F_{max} = \frac{F_s}{2} = 10\,hz$$

$$\Delta = \frac{x_{max} - x_{min}}{L-1} = \frac{1}{2^8 - 1} = 0.0039$$

1.15)

A) $F_0 = .5 ; x(n) = \sin(2\pi(\frac{1}{10})n)$

$F_0 = 4.5 ; x(n) = -\sin(2\pi(\frac{1}{10})n)$

$F_0 = 2 : x(n) = \sin(2\pi(\frac{2}{5})n)$

$F_0 = 3 ; x(n) = -\sin(2\pi(\frac{2}{5})n)$

$F_0 = .5, 4.5$ are just phase shifts of eachother

Same can be said for $F_0 = 2, 3$

B)

1) $x(n) = \sin(2\pi(\frac{1}{25})n)$

$f_0 = \frac{1}{25}$

2) $F_s = 25\,kHz$

$x(n) = \sin(2\pi(\frac{2}{25})n)$

yes $\rightarrow f_0$ is a rational number

$f_0 = \frac{2}{25}$