Chapter Seven Problems

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7.1) $0.125 + 0.0518j, \; 0, \; 0.125 + 0.3018j$

7.3) This process essentially acts as a lowpass filter, eliminating frequencies from $\frac{2\pi(k_c)}{N}$ to π radians/sample

7.5)

a) $x_1(n) = \frac{1}{2}e^{j\frac{2\pi n}{N}} + \frac{1}{2}e^{-j\frac{2\pi n}{N}}$

$$\sum_{n=0}^{N-1} x_1(n)x_2(n) = \sum_{n=0}^{N-1}\left(\frac{1}{2}e^{j2\pi n/N} + \frac{1}{2}e^{-j2\pi n/N}\right)^2$$

$$= \frac{1}{4}\sum_{n=0}^{N-1}\left(e^{j2\pi n/N} + e^{-j2\pi n/N}\right)^2$$

$$= \frac{1}{4}\sum_{n=0}^{N-1}\left(e^{4\pi j\frac{n}{N}} + e^{-4\pi j\frac{n}{N}} + 2\right) = \frac{1}{4}\sum_{n=0}^{N-1}\left(2\cos\frac{4\pi n}{N} + 2\right)$$

$$= \frac{1}{4}\sum_{n=0}^{N-1} 2\cos\frac{4\pi n}{N} + \frac{1}{4}\sum_{n=0}^{N-1} 2 = 0 + \frac{2N}{4} = \frac{N}{2}$$

b) $x_1(n) = \frac{1}{2}e^{j\frac{2\pi n}{N}} + \frac{1}{2}e^{-j\frac{2\pi n}{N}}$

$x_2(n) = \frac{1}{2}e^{j\frac{2\pi n}{N}} - \frac{1}{2}e^{-j\frac{2\pi n}{N}}$

$$\sum_{n=0}^{N-1} x_1(n)x_2(n) = \sum_{n=0}^{N-1}\left(\frac{1}{2}e^{j\frac{2\pi n}{N}} + \frac{1}{2}e^{-j\frac{2\pi n}{N}}\right)\left(\frac{1}{2j}e^{j\frac{2\pi n}{N}} - \frac{1}{2j}e^{-j\frac{2\pi n}{N}}\right)$$

$$= \sum_{n=0}^{N-1}\left(\frac{1}{4j}e^{j\frac{4\pi n}{N}} - \frac{1}{4j}(1) + \frac{1}{4j}(1) - \frac{1}{4j}e^{-j\frac{4\pi n}{N}}\right) = \frac{1}{4j}\sum_{n=0}^{N-1}\left(e^{j\frac{4\pi n}{N}} - e^{-j\frac{4\pi n}{N}}\right)$$

$$= \frac{1}{4j}\sum_{n=0}^{N-1}\left(\cos\frac{4\pi n}{N} + j\sin\frac{4\pi n}{N} - \cos\frac{4\pi n}{N} + j\sin\frac{4\pi n}{N}\right)$$

$$= \frac{1}{4j}\sum_{n=0}^{N-1}\left(2j\sin\frac{4\pi n}{N}\right) = 0$$

7.6) $W(k) = \sum_{k=0}^{N-1} w(n)e^{-j2\pi k\frac{n}{N}} \qquad 0 \le k \le N-1$

$$= \sum_{k=0}^{N-1}\left(0.42 - 0.5\cos\frac{2\pi n}{N-1} + 0.08\cos\frac{4\pi n}{N-1}\right)e^{-j2\pi k\frac{n}{N}}$$

$$= 0.42\sum_{n=0}^{N-1} e^{-j2\pi k\frac{n}{N}} - 0.5\sum_{n=0}^{N-1}\cos\frac{2\pi n}{N-1}e^{-j2\pi k\frac{n}{N}} + 0.08\sum_{n=0}^{N-1}\cos\frac{4\pi n}{N-1}e^{-j2\pi k\frac{n}{N}}$$

$$= 0.42\sum_{n=0}^{N-1} e^{-j2\pi k\frac{n}{N}} - 0.5\sum_{n=0}^{N-1}\left(\frac{1}{2}e^{j\frac{2\pi n}{N-1}} + \frac{1}{2}e^{-j\frac{2\pi n}{N-1}}\right)e^{-j2\pi k\frac{n}{N}} + 0.08\sum_{n=0}^{N-1}\left(\frac{1}{2}e^{j\frac{4\pi n}{N-1}} + \frac{1}{2}e^{-j\frac{4\pi n}{N-1}}\right)e^{-j2\pi k\frac{n}{N}}$$

$$= 0.42\sum_{n=0}^{N-1} e^{-j2\pi k\frac{n}{N}} - .25\sum_{n=0}^{N-1}\left(e^{j2\pi n\left(\frac{N}{N-1} - k\right)} + e^{-j2\pi n\left(\frac{N}{N-1} + k\right)}\right) + 0.04\sum_{n=0}^{N-1}\left(e^{j2\pi n\left(\frac{2N}{N-1} - k\right)} + e^{-j2\pi n\left(\frac{2N}{N-1} + k\right)}\right)$$

7.7) $N-1$ ... $j2\pi k_0\frac{n}{N}$ ... $-j2\pi k\frac{n}{N}$

$$+0.42\sum_{n=0} e^{-j2\pi n\frac{\cdot}{N}} - .25\sum_{n=0}^{N-1}\left(e^{j2\pi n(N-1\ \ldots)} + e^{-j2\pi n(\ldots)}\right) + 0.04\sum_{n=0}\left(e^{\ldots} + e^{\ldots}\right)\Bigg)$$

7.7) $$X_c(k) = \sum_{n=0}^{N-1} x(n)\left(\frac{1}{2}e^{j2\pi k_0\frac{n}{N}} + \frac{1}{2}e^{-j2\pi k_0\frac{n}{N}}\right)e^{-j2\pi k\frac{n}{N}}$$

$$= \frac{1}{2}\sum_{n=0}^{N-1} x(n)\left(e^{j2\pi\frac{n}{N}(k_0-k)} + e^{-j2\pi\frac{n}{N}(k_0+k)}\right)$$

$$= \frac{1}{2}X(k_0-k) + \frac{1}{2}X(k_0+k)$$

similar for sin:

$$\frac{1}{2j}X(k_0-k) - \frac{1}{2j}X(k_0+k)$$

7.8) $$x_3(m) = \sum_{n=0}^{N-1} x_1(n)\,x_2(m-n) \qquad m=0,1,2,3$$

$$x_3(0) = x_1(0)x_2(0) + x_1(1)x_2(-1) + x_1(2)x_2(-2) + x_1(3)x_2(-3)$$
$$= 1(4) + 2(2) + 3(2) + 1(3) = 17$$
$$x_3(1) = x_1(0)x_2(1) + x_1(1)x_2(0) + x_1(2)x_2(-1) + x_1(3)x_2(-2)$$
$$= 1(3) + 2(4) + 3(2) + 1(2) = 19$$
$$x_3(2) = x_1(0)x_2(2) + x_1(1)x_2(1) + x_1(2)x_2(0) + x_1(3)x_2(-1)$$
$$= 1(2) + 2(3) + 3(4) + 1(2) = 22$$
$$x_3(3) = x_1(0)x_2(3) + x_1(1)x_2(2) + x_1(2)x_2(1) + x_1(3)x_2(0)$$
$$= 1(2) + 2(2) + 3(3) + 1(4) = 19$$

$$x_3(n) = \{17, 19, 22, 19\}$$

7.9) $$x_3(n) = x_1(n) \; (N) \; x_2(n)$$

$$X_3(k) = X_1(k)X_2(k)$$

$$X_1(k) = \sum_{n=0}^{N-1} x(n)\,e^{-2\pi jk\frac{n}{N}} \qquad k=0,1,2,3$$

$$X_1(0) = 1+2+3+1 = 7$$
$$X_1(1) = 1 + 2e^{-2\pi j\frac{1}{4}} + 3e^{-2\pi j\frac{1}{2}} + 1e^{-2\pi j\frac{3}{4}}$$
$$= 1 - 2j - 3 + j = -j - 2$$
$$X_1(2) = 1-2+3-1 = 1$$
$$X_1(3) = 1+2j-3-j = j-2$$
$$X_2(0) = 4+3+2+2 = 11$$
$$X_2(1) = 4-3j-2+2j = 2-j$$
$$X_2(2) = 4-3+2-2 = 1$$
$$X_2(3) = 4+3j-2-2j = 2+j$$

$$X_3(k) = \{77, (2-j)(-2-j), 1, (j-2)(j+2)\}$$
$$= \{77, -5, 1, -5\}$$

$$x(n) = \frac{1}{N}\sum_{k=0}^{3} X_3(k)\,e^{j2\pi kn/N}$$

$$x(0) = \frac{1}{4}(77-5+1-5) = 17$$
$$x(1) = \frac{1}{4}(77-5j-1+5j) = 19 \qquad x(n) = \{17, 19, 22, 19\}$$
$$x(2) = \frac{1}{4}(77+5+1+5) = 22$$
$$x(3) = \frac{1}{4}(77+5j-1-5j) = 19$$

7.11) $x(n) = (1,1,1,1,0,0,0,0)$

$x_1(n) = (1,0,0,0,0,1,1,1)$

$x_2(n) = (0,0,1,1,1,1,0,0)$

$x_1(n) = x(n-5)$

$x_2(n) = x(n-2)$

$X_1(k) = X(k)e^{-2\pi(5)k/N} = X(k)e^{-j\frac{5\pi k}{4}}$

$X_2(k) = X(k)e^{-2\pi(2)k/N} = X(k)e^{-j\frac{\pi k}{2}}$

7.13)

$X_1(k) = \sum_{n=0}^{N} x(n) W_N^{kn}$

$X_2(k) = \sum_{n=0}^{3N-1} x(n) W_{3N}^{kn} = \sum_{n=0}^{N-1} x(n) W_{3N}^{kn} + \sum_{N}^{2N-1} x(n) W_{3N}^{kn} + \sum_{2N}^{3N-1} x(n) W_{3N}^{kn}$

$= \sum_{n=0}^{3N-1} x(n)(1 + W_3^k + W_3^{2k}) W_N^{nk/3}$

$= [1 + W_3^k + W_3^{2k}] X_1(k)$

7.14)

a) $y(n) = x_1(n) \; \textcircled{N} \; x_2(n)$

$y(m) = \sum_{n=0}^{N-1} x_2(n) x_1(m-n)$

$y(0) = (0)(0) + (1)(0) + (2)(0) + (3)(0) + (4)(1)$
$= 4$

$y(1) = 0$

$y(n) = \{4, 0, 1, 2, 3\}$

b) $s(n) = x_1(n) \; \textcircled{N} \; x_3(n)$

$s(n) = \sum_{n=0}^{N-1} x_3(n) x_1(m-n)$

$1 = 0x_1 + 4x_2 + 3x_3 + 2x_4 + 1x_5$

$0 = 1x_1 + 0x_2 + 4x_3 + 3x_4 + 2x_5$

$0 = 2x_1 + 1x_2 + 0x_3 + 4x_4 + 3x_5$

$0 = 3x_1 + 2x_2 + 1x_3 + 0x_4 + 4x_5$

$0 = 4x_1 + 3x_2 + 2x_3 + 1x_4 + 0x_5$

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 4 & 3 & 2 & 1 \\ 1 & 0 & 4 & 3 & 2 \\ 2 & 1 & 0 & 4 & 3 \\ 3 & 2 & 1 & 0 & 4 \\ 4 & 3 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

7.17) $x(n) = (1,1,1,1,1,1,0,0)$

$X(k) = \sum_{n=0}^{7} x(n) e^{-j2\pi k\frac{n}{8}}$

$X(k) = (6, -.7071 - 1.7071j, 1-j, 0.7071 + 0.2929j,$
$0, 0.7071 - 0.2929j, 1+j, -0.7071 + 1.7071j)$

$|X(k)| = (6, 1.8478, 1.4142, 0.7654, 0, 0.7654, 1.4142, 1.8478)$

$0, 0.7071-0.2929j, 1+j, -0.7071+1.7071j)$

$|X(k)| = (6, 1.8478, 1.4142, 0.7654, 0, 0.7654, 1.4142, 1.8478)$

$\angle X(k) = (0, -1.9635, -\frac{\pi}{4}, 0.3927, 0, -0.3927, \frac{\pi}{4}, 1.9635)$

7.21)

a) $F_N = 2(3kHz) = 6kHz$

b) $\frac{5b}{LT}$ $\quad T = \frac{1}{6000}$

$\frac{6000}{L} < 50$

$L > 120$ samples

c) $120 \text{ samples} \cdot \frac{\text{sec}}{6000 \text{ samples}} = \frac{1}{50}$ sec

7.22) $x(n) = \cos\frac{2\pi}{10}n \qquad -\infty < n < \infty$

$x(n) = \frac{1}{2}e^{-2\pi j\frac{n}{N}} + \frac{1}{2}e^{2\pi j\frac{n}{N}}$

$X_{10}(k) = \frac{1}{2}\sum_{n=0}^{9}\left(e^{-j2\pi\frac{n}{10}} + e^{2\pi j\frac{n}{10}}\right)e^{-2\pi j k\frac{n}{10}}$

$= \frac{1}{2}\sum_{n=0}^{N-1}\left(e^{-2\pi j\frac{n}{10}(k+1)} + e^{-2\pi j\frac{n}{10}(k-1)}\right)$

$= 5\delta(k-1) + 5\delta(k-9)$

7.24)

a) $x(n) = \frac{1}{N}\sum_{k=0}^{N-1}X(k)e^{2\pi j\frac{n}{N}k}$

$n=0: \; 4 = \sum_{k=0}^{N-1}X(k) \rightarrow 4 = X(0)+X(1)+X(2)+X(3)$

$n=1: \; 8 = \sum_{k=0}^{N-1}X(k)e^{\frac{\pi}{2}jk} \rightarrow 8 = X(0)+jX(1)-X(2)-jX(3)$

$n=2: \; 12 = \sum_{k=0}^{N-1}X(k)e^{\pi jk} \rightarrow 12 = X(0)-X(1)+X(2)-X(3)$

$n=3: \; 16 = \sum_{k=0}^{N-1}X(k)e^{\frac{3\pi}{2}jk} \rightarrow 16 = X(0)-jX(1)-X(2)+jX(3)$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{pmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{pmatrix} = \begin{bmatrix} 4 \\ 8 \\ 12 \\ 16 \end{bmatrix} \rightarrow \begin{pmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{pmatrix} = \begin{bmatrix} 7 \\ -j-2 \\ 1 \\ j-2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} X(3) \end{bmatrix} \quad \begin{bmatrix} 16 \end{bmatrix} \quad \begin{bmatrix} X(3) \end{bmatrix} \quad \begin{bmatrix} j-2 \end{bmatrix}$$

B) $X(k) = \sum_{n=0}^{3} x(n)\, e^{-2\pi j k \frac{n}{4}}$

$X(0) = 1+6+3+1 = 7$

$X(1) = 1-2j-3+j = -j-2$

$X(2) = 1-2+3-1 = 1$

$X(3) = 1+2j-3-j = j-2$

$X(k) = (7, -j-2, 1, j-2)$

7.25)

a) $X(w) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-jwn}$

$X(w) = 1e^{-2jw} + 2e^{-jw} + 3 \quad + 2e^{jw} + e^{2jw}$

$= 3 + 2\cos(2w) + 4\cos(4w)$

b) $V(k) = \sum_{n=0}^{N-1} v(n)\, e^{-j2\pi k n/N}$

$= 3 + 2e^{-\frac{\pi k j}{3}} + e^{-\frac{2\pi j k}{3}} + e^{-\frac{4\pi j k}{3}} + 2e^{-\frac{5\pi j k}{3}}$

$= 3 + 4\cos\frac{\pi}{3}k + 2\cos\frac{2\pi}{3}k$

c) when $w = \frac{\pi k}{3}$, $V(k) = X(w)$ since $v(n)$ is just a segment of $x(n)$