Chapter Four Notes

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Note: certain sections in chapter 4 were omitted as they weren't as relevant to developing intuition on the DFT

Big idea:

- most signals can be expressed as the sum is sinusoidal signals (fourier series)

Section 4.1

- linear combination of harmonically related complex exponentials:

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi k F_0 t}$$

- $e^{j2\pi k F_0 t}$ becomes the "building blocks" or basis for the analyzed signal

- equation for fourier coefficients:

$$c_l = \frac{1}{T_p} \int_{T_p} x(t) e^{-j2\pi l F_0 t} dt$$

- Fourier synthesis:

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi k F_0 t}$$

- Fourier Analysis:

$$c_k = \frac{1}{T_p} \int_{T_p} x(t) e^{-j2\pi k F_0 t} dt$$

Section 4.2

Frequency analysis of discrete time

- frequency analysis of discrete-time signals
- frequency over the range $(-\pi, \pi)$ or $(0, 2\pi)$
  - frequency components separated by $\frac{2\pi}{N} \frac{\text{radians}}{\text{sample}}$ or $\frac{1}{N}$ cycles/sample

    → at most N frequencies
- fourier series of $x(n)$ consists of N harmonically related exponential functions

  → basis: $e^{j2\pi k \frac{n}{N}}$ $k = 0, 1, \ldots, N-1$

  → series: $x(n) = \sum_{k=0}^{N-1} c_k e^{j2\pi k \frac{n}{N}}$
- to derive the fourier coefficients (c_k):

$$c_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-j2\pi k \frac{n}{N}}$$

- synthesis:

$$x(n) = \sum_{k=0}^{N-1} c_k e^{j2\pi k \frac{n}{N}}$$

→ using $e^{-j2\pi (k) \frac{n}{N}}$, $\{c_k\}$, $k = 0, 1, \ldots, N-1$ shows the amplitude of each frequency present in $x(n)$

where $f = \frac{k}{N}$ where $0 < f < 1$

and $\omega_k = \frac{2\pi k}{N} \frac{\text{radians}}{\text{sample}}$ where $0 < \omega_k < 2\pi$

→ due to the periodic nature of this analysis, $c_k = c_{k+N}$

analysis, $c_k = c_{k+N}$

$\rightarrow$ proof: $c_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n)\, e^{-j(2\pi)k\frac{n}{N}}$

$$c_{k+N} = \frac{1}{N}\sum_{n=0}^{N-1} x(n)\, e^{-j(2\pi)(k+N)\frac{n}{N}}$$

$$= \frac{1}{N}\sum_{n=0}^{N-1} x(n)\, e^{-j(2\pi)k\frac{n}{N}} \cdot e^{-j(2\pi)N\cdot\frac{n}{N}}$$

$$= \cos(2\pi n) + j\sin(2\pi n) = 1$$

$$= \frac{1}{N}\sum_{n=0}^{N-1} x(n)\, e^{-j(2\pi)k\frac{n}{N}}$$

$\therefore c_k = c_{k+N}$