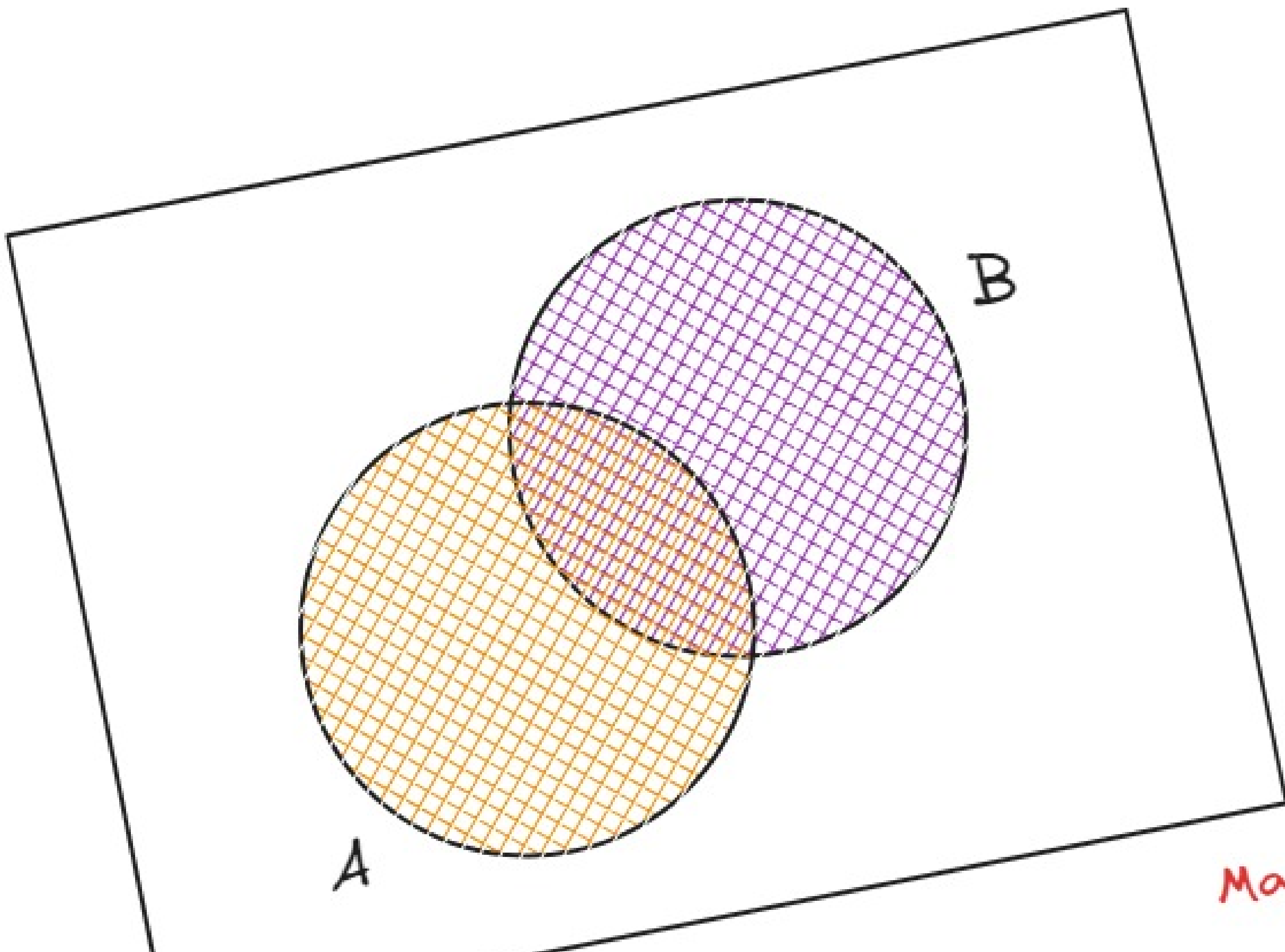
Probability in Machine Learning
Part-2



Marginalization

Prior

Prosterior

$$P(A|B) = P(B|A)*P(A)/P(B)$$

Likelihood

Contents

Contingency Table in Probability

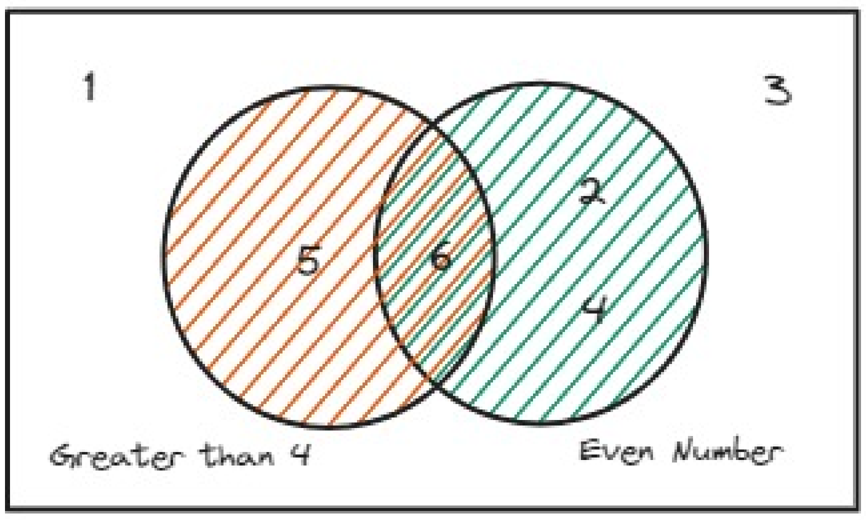
Contingency tables in probability are simple charts that display the frequency or proportion of observations within different categories or groups.

Let's create a contingency table for two events: rolling a number greater than 4 and rolling an even number.

Odd Numbers Less than equal to 4: {1, 3}
Odd Numbers Greater than 4: {5}
Even Numbers Less than equal to 4: {2,4}
Even Numbers Greater than 4: {6}

Contingency Table

	Less than equal to 4	Greater than 4
Odd number	2	1
Even number	2	1



Contingency tables and Venn diagrams serve similar purposes, aiding in probability determination.

Venn Diagram in Probability

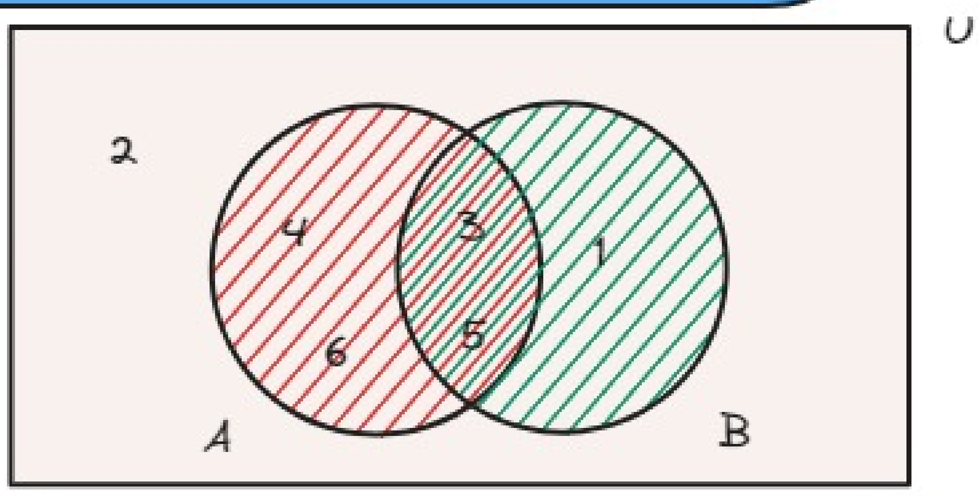
Venn Diagrams are like simple pictures that makes us understand relationships between different sets of things.

"In probability, Venn diagrams help us understand the relationships between probabilities and events. The universal set is represented by a rectangle, while events are depicted as circles within it.

In probability, the universal set has a probability of 1 since it encompasses all possible outcomes in the sample space. Understanding the relationships between events enables us to determine their probabilities."

For example, let's examine the following two events when rolling a single die:
a) Getting a number greater than 2
b) Getting an odd number

Favourable outcomes of the following events will be:
For U = 1,2,3,4,5,6
For A = 3,4,5,6
For B = 1, 3, 5



Let's calculate some probabilities with this example:

1) Probability of A: Possible outcomes = {3, 4, 5, 6} = 4/6 = 2/3 (Numbers in circle A)

2) Probability of A': Possible outcomes = {1, 2} = 2/6 = 1/3 (Numbers not in circle A)

3) Probability of B: Possible outcomes = {1, 3, 5} = 3/6 = 1/2 (Numbers in circle B)

4) Probability of A∪B = {1, 3, 4, 5, 6} = 5/6 (Numbers in either circle A or circle B)

5) Probability of A∩B = {3, 5} = 2/6 = 1/3 (Numbers common to both circle A and circle B)

6) Probability of U = {1, 2, 3, 4, 5, 6} = 6/6 = 1 (All numbers in the universal set)

Venn diagrams help us calculate event probabilities by showing how sets relate visually.

Types of Probability

There are major 3 types of Probability.

1. Joint Probability

The probability of two or more events occurring simultaneously.

2. Marginal Probability

The probability of a single event occuring without consideration for any other events.

3. Conditional Probability

The probability of an event occuring given that another event has already occured.

Joint Probability

Joint probability of random variable (X) & random variable (Y) is denoted as $P(X = x, Y = y)$, is the probability that X takes the value x & Y takes the value y at the same time.

Consider the following example:
- Let X represent the random variable associated with the mode of transport used by individuals.
- Let Y represent the random variable associated with the gender of the individuals using it.

	Male	Female	Total
Car	40	20	60
Public Transport	30	60	90
Total	70	80	150

Joint Probability

	Male	Female
Car	0.267	0.133
Public Transport	0.2	0.4

Joint Probability is Calculated by finding the likelihood of two or more events happening together.

- Probability of Male using Car: 40/150 = 0.267
- Probability of Female using Car: 20/150 = 0.133
- Probability of Male using Public Transport: 30/150 = 0.2
- Probability of Female using Public Transport: 60/150 = 0.4

The sum of the combinations of possible outcomes in joint probability will equal 1.

Marginal Probability

Marginal probability of a random variable is a simply the probability of that variable taking the certain value, regardless of the value of other variables.

Taking the same example:
- Let X represent the random variable associated with the mode of transport used by individuals.
- Let Y represent the random variable associated with the gender of the individuals using it.

	Male	Female	Total
Car	40	20	60
Public Transport	30	60	90
Total	70	80	150

Marginal Probability

	Male	Female	
Car	0.267	0.133	0.4
Public Transport	0.2	0.4	0.6
	0.467	0.533	

Marginal Probability is calculated by summing up the probabilities of one event across all possible outcomes.

- Probability of being male using any transport: 70/150 = 0.467 (0.267 + 0.2)
- Probability of being female using any transport: 80/150 = 0.533 (0.133 + 0.4)
- Probability of using a car regardless of gender: 60/150 = 0.4 (0.267 + 0.133)
- Probability of using public transport regardless of gender: 90/150 = 0.6 (0.2 + 0.4)

The sum of all possible outcomes of an individual event in marginal probability will equal 1.

Conditional Probability

Conditional Probability of random variable is event of interest is A and event B is already occur

Considering the same example

	Male	Female	Total
Car	40	20	60
Public Transport	30	60	90
Total	70	80	150

Conditional Probability (Given Event A (Gender) is already chosen)

P(A|B) = P(A∩B)/P(B)

Conditional Probability (Given Event B (Trabsport) is already chosen)

	Male	Female
Car	0.572	0.25
Public Transport	0.428	0.75

	Male	Female
Car	0.668	0.332
Public Transport	0.333	0.667

Conditional probability is calculated by finding the probability of one event happening given that another event has already happened.
Considering the case where the selection of gender has already occurred (A|B):
- Probability of using a car given that the gender is male: 0.267/0.467 = 0.572
- Probability of using public transport given that the gender is male: 0.2/0.467 = 0.428
- Probability of using a car given that the gender is female: 0.133/0.533 = 0.25
- Probability of using public transport given that the gender is female: 0.4/0.533 = 0.75

Considering the case where the selection of gender has already occurred (B|A):
- Probability of gender being male given that a car is used: 0.267/0.4 = 0.668
- Probability of gender being female given that a car is used: 0.133/0.4 = 0.332
- Probability of gender being male given that public transport is used: 0.2/0.6 = 0.333
- Probability of gender being female given that public transport is used: 0.4/0.6 = 0.667

The sum of all possible outcomes of the same event, given that another event has already occurred, in conditional probability will equal 1.

Independent vs Mutually Exclusive Event

Independent Event

Independent events are events where the occurence of one event does not affect the occurence of another

Eg: Flipping a coin & rolling a die, Drawing a card with replacement

For independent event:

$P(A|B) = P(A)$

$P(B|A) = P(B)$

$P(A \cap B) = P(A)*P(B)$

Dependent Event

Dependent events are events where the occurence of one event affect the occurence of another

Eg: Drawing a card without replacement

For dependent event:

$P(A|B) = P(A \cap B)/P(B)$

Mutually Exclusive Event

Mutually exclusive events are those that cannot occur simultaneously. If one event happens, the other cannot.

Eg: Rolling a die and getting an odd number or rolling a die and getting an even number are mutually exclusive events, as they cannot happen simultaneously.

For mutually exclusive event:

$P(A \cup B) = 0$

$P(A|B) = 0$

Bayes Theorem

Bayes' Theorem is a fundamental concept in the field of probability & statistics that describes how to to update the probabilities of hypotheses when given evidence.

Formula

$$\underbrace{P(A|B)}_{\text{Posterior}} = \underbrace{P(A)}_{\text{Prior}} * \underbrace{P(B|A)}_{\text{Likelihood}} / \underbrace{P(B)}_{\text{Marginal}}$$

Mathematical Proof of Theorem

Proof comes from the formula of the conditional probability

$$P(A|B) = P(A \cap B) \,/\, P(B)$$

$$P(B|A) = P(B \cap A) \,/\, P(A)$$

$$P(A \cap B) = P(B \cap A)$$

$$P(A|B) * P(B) = P(B|A) * P(A)$$

$$\boxed{P(A|B) = P(A) * P(B|A)/P(B)}$$

Example Usage: Consider a group of 5 people, consisting of 3 males and 2 females. Among the males, 2 died and 1 survived, while among the females, 1 died and 1 survived. We want to determine the probability of survival given that an individual is male.

We need to find P(Survive | Male)
Here,
P(Male) = 3/5 = 0.6
P(Survive) = 2/5 = 0.4
P(Male | Survive) = 1/2 = 0.5

Using Bayers Theorm: P(Survive | Male) = P(Survive) * P(Male | Survive) / P(Survive)

= (0.4)*(0.5)/(0.6)
= 0.333
His survival chance is 33% given that he is Male.

The famous machine learning algorithm "Naive Bayes" is based on this theorem.

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