CIS - 581

Homework 1: Convolution

Ques 2:

$$I = \begin{bmatrix} 0.0 & 1.0 & -1.0 \\ 2.0 & 1.0 & 0.0 \\ 0.0 & 3.0 & -1.0 \end{bmatrix} \qquad fx = \begin{bmatrix} -1.0 & 0.0 & 1.0 \end{bmatrix}$$

$$fy = \begin{bmatrix} 1.0 & 1.0 & 1.0 \end{bmatrix}^T$$

fx can also be written as (in 3x3 form)

$$fx = \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix} * (-1) + \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix}$$

(Considering fx_1) (Considering fx_2)

Similarly,

$$fy = \begin{bmatrix} 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix} + \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix} + \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \end{bmatrix}$$

Considering fy_1 fy_2 fy_3

2.1) $g_1 = I \otimes fx \otimes fy$

Computing $I \otimes fx$

$$I \otimes fx = I \otimes fx_1 + I \otimes fx_2$$

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f_{x_1} will shift the image by 1 unit on left and f_{x_2} will shift the image by 1 unit on right.

$$\Rightarrow I \otimes f_x = \begin{bmatrix} -1.0 & 1.0 & 0.0 \\ -1.0 & 0.0 & 0.0 \\ -3.0 & 1.0 & 0.0 \end{bmatrix} + \begin{bmatrix} 0.0 & 0.0 & 1.0 \\ 0.0 & 2.0 & 1.0 \\ 0.0 & 0.0 & 3.0 \end{bmatrix}$$

$$= \begin{bmatrix} -1.0 & 1.0 & 1.0 \\ -1.0 & 2.0 & 1.0 \\ -3.0 & 1.0 & 3.0 \end{bmatrix} \text{ (intermediate)}$$

Computing g_1

f_{y_1} will shift the intermediate image 1 unit upwards.
f_{y_2} will not make any change to the intermediate.
f_{y_3} will shift the intermediate 1 unit downwards.

$$\Rightarrow g_1 = I \otimes f_x \otimes f_y = \begin{bmatrix} -1.0 & 2.0 & 1.0 \\ -3.0 & 1.0 & 3.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix} + \begin{bmatrix} -1.0 & 1.0 & 1.0 \\ -1.0 & 2.0 & 1.0 \\ -3.0 & 1.0 & 3.0 \end{bmatrix} + \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ -1.0 & 1.0 & 1.0 \\ -1.0 & 2.0 & 1.0 \end{bmatrix}$$

$$\boxed{g_1 = \begin{bmatrix} -2.0 & 3.0 & 2.0 \\ -5.0 & 4.0 & 5.0 \\ -4.0 & 3.0 & 4.0 \end{bmatrix}}$$

$$f_{xy} = f_x \otimes f_y$$

$$= \underbrace{f_y \otimes f_{x_1}}_{\leftarrow} + \underbrace{f_y \otimes f_{x_2}}_{\rightarrow}$$

Shifts 1 unit on right

Shifts 1 unit on left

$$= \begin{bmatrix} -1.0 & 0 & 0 \\ -1.0 & 0 & 0 \\ -1.0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 1.0 \\ 0 & 0 & 1.0 \\ 0 & 0 & 1.0 \end{bmatrix}$$

$$\boxed{f_{xy} = \begin{bmatrix} -1.0 & 0 & 1.0 \\ -1.0 & 0 & 1.0 \\ -1.0 & 0 & 1.0 \end{bmatrix}}$$

$$g_2 = I \otimes f_{xy}$$

left movement ($\leftarrow$)

diagonally down ($\swarrow$)

$$= \begin{bmatrix} -1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 0 \\ -3 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 2 & 1 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix} +$$

diagonal up movement ($\nwarrow$)

diagonally up ($\nearrow$)

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix}$$

right ($\rightarrow$)

diagonally down ($\searrow$)

$$\boxed{\Rightarrow g_2 = \begin{bmatrix} -2 & 3 & 2 \\ -5 & 4 & 5 \\ -4 & 3 & 4 \end{bmatrix}}$$

$\because \quad g_1 = g_2$

$\Rightarrow (I \otimes f_x) \otimes f_y = I \otimes (f_x \otimes f_y)$

$\Rightarrow$ Convolution is associative

1.2) A total of 18 matrix multiplication steps are required to compute g_1 and g_2.

1.3) Due to the convolution, the image is becoming blurred,

<u>Ques 2:</u> $I = \begin{bmatrix} 1 & 5 & 2 \\ 7 & 8 & 6 \\ 3 & 9 & 4 \end{bmatrix}$ $\quad g = \begin{bmatrix} 29 & 43 & 10 \\ 62 & 52 & 30 \\ 15 & 45 & 20 \end{bmatrix}$

As can be seen on comparing I and g, most of the elements in g seem to be multiple of 5 ~~of~~ of elements in I.
Hence,

$$g = I \otimes f_1 + I \otimes f_2$$

where $f_1 = 5 \times \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$\Rightarrow I \otimes f_2 = \begin{bmatrix} 24 & 18 & 0 \\ 27 & 12 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

which indicates that elements of image I have been shifted by 1 unit in top-left dirn and are multiple of 3.

$$\Rightarrow f_2 = 3 \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{\Rightarrow f = 5 \times \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + 3 \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix}}$$

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Ques 3: $I_Q = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $I_T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$

$$I_m = \begin{bmatrix} 0.25 & 0.5 & 0.5 & 0.25 \\ 0.5 & 1 & 1 & 0.5 \\ 0.5 & 1 & 1 & 0.5 \\ 0.25 & 0.5 & 0.5 & 0.25 \end{bmatrix}$$

3.1) Considering 3x3 f_1 as $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

$\Rightarrow$ flipped $f_1 = \begin{bmatrix} i & h & g \\ f & e & d \\ c & b & a \end{bmatrix}$

$\because$ $I_Q \otimes f_1 = I_m$

Solving eq^n simuntaneonsly.

① $\boxed{i = 0.25}$ ② $i + f = 0.5 \Rightarrow \boxed{f = 0.25}$

③ $i + f + c = 0.5$ $\Rightarrow \boxed{c = 0}$ ④ $i + h = 0.5$ $\Rightarrow \boxed{h = 0.25}$

⑤ $i + h + g = 0.5$ $\Rightarrow \boxed{g = 0}$ ⑥ $a + b + c + d + e + f + g + h + i = 1$ $\Rightarrow a + b + d + e = 0.25$ $\Rightarrow \boxed{\begin{matrix} a = 0 \\ d = 0 \end{matrix}}$

⑦ $i + h + f + e + c + b = 1$ $\Rightarrow b + e = 0.25$ $\Rightarrow \boxed{b = 0}$ ⑧ $i + h + f + e = 1$ $\Rightarrow \boxed{e = 0.25}$

Hence, $\boxed{f_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.25 & 0.25 \\ 0 & 0.25 & 0.25 \end{bmatrix}}$

Considering 3x3 f_2 as $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

$\rightarrow$ flipped $f_2 = \begin{bmatrix} i & h & g \\ f & e & d \\ c & b & a \end{bmatrix}$

$\because$ $I_T \otimes f_2 = I_m$.

Solving eq^n simultaneously.

1. $\boxed{a = 0.25}$
2. $a + d = 0.5 \Rightarrow \boxed{d = 0.25}$
3. $a + d + g = 0.5 \rightarrow \boxed{g = 0}$
4. $a + b = 0.5 \Rightarrow \boxed{b = 0.25}$
5. $a + b + c = 0.5 \Rightarrow \boxed{c = 0}$.
6. $a + b + d + e = 1 \Rightarrow \boxed{e = 0.25}$
7. $a + b + d + e + g + h = 1 \Rightarrow \boxed{h = 0}$
8. $a + b + c + d + e + f + g + h + i = 1 \Rightarrow \boxed{f = i = 0}$

Hence, $\boxed{f_2 = \begin{bmatrix} 0.25 & 0.25 & 0 \\ 0.25 & 0.25 & 0 \\ 0 & 0 & 0 \end{bmatrix}}$

3.2) $f = \frac{f_1 + f_2}{2} = \begin{bmatrix} 0.125 & 0.125 & 0 \\ 0.125 & 0.25 & 0.125 \\ 0 & 0.125 & 0.125 \end{bmatrix}$

$I_Q' = I_Q \otimes f.$

flipped $f = f.$ (matrix is same about central element)

$$\Rightarrow I_Q' = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \otimes 0.125 \times \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= 0.125 \times \begin{bmatrix} 5 & 6 & 4 & 1 \\ 6 & 8 & 6 & 2 \\ 4 & 6 & 5 & 2 \\ 1 & 2 & 2 & 1 \end{bmatrix}$$

$$\boxed{I_Q' = \begin{bmatrix} 0.625 & 0.75 & 0.5 & 0.125 \\ 0.75 & 1 & 0.75 & 0.25 \\ 0.5 & 0.75 & 0.625 & 0.25 \\ 0.125 & 0.25 & 0.25 & 0.125 \end{bmatrix}}$$

$I_T' = I_T \otimes f.$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \otimes 0.125 \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix} = 0.125 \begin{bmatrix} 1 & 2 & 2 & 1 \\ 2 & 5 & 6 & 4 \\ 2 & 6 & 8 & 6 \\ 1 & 4 & 6 & 5 \end{bmatrix}$$

$$\Rightarrow \boxed{I_T' = \begin{bmatrix} 0.125 & 0.25 & 0.25 & 0.125 \\ 0.25 & 0.625 & 0.75 & 0.5 \\ 0.25 & 0.75 & 1 & 0.75 \\ 0.125 & 0.5 & 0.75 & 0.625 \end{bmatrix}}$$

$$D(I_Q, I_T) = \left((1^2+1^2+1^2+0^2) + (1^2+0^2+0^2+1^2) + (1^2+0^2+0^2+1^2) + (0^2+1^2+1^2+1^2) \right)$$

$$\boxed{D(I_Q, I_T) = 10}$$

$$D(I_Q', I_T') = \left((0.5^2+0.5^2+0.25^2+0^2) + (0.5^2+0.375^2+0^2+0.25^2) + (0.25^2+0^2+0.375^2+0.5^2) + (0^2+0.25^2+0.5^2+0.5^2) \right)$$

$$\boxed{D(I_Q', I_T') = 2.03125}$$

Due to the ~~[illegible]~~ convolution operation, the distances or the differences in the individual elements of the query image and barrel template has reduced.

Ques 4:

4.1) Output image (a) corresponds with the kernel (2)

Output image (b) corresponds with the kernel (6)

Output image (c) corresponds with the kernel (3)

4.2) Output image (d) corresponds with the kernel (4)

Output image (e) corresponds with the kernel (1)

Output image (f) corresponds with the kernel (5)

Output image (g) corresponds with the kernel (7)

Ques 5:

$$I = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 1 & 1 \end{bmatrix} ; \quad f = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$I \otimes f = \begin{bmatrix} 6 & 9 & 7 \\ 8 & 12 & 9 \\ 6 & 9 & 7 \end{bmatrix} \times \frac{1}{9} ; \quad I \otimes f \otimes f = \begin{bmatrix} 35 & 51 & 37 \\ 50 & 73 & 58 \\ 35 & 51 & 37 \end{bmatrix} \times \frac{1}{81}$$

As can be seen from above, the more we convolve image I with the kernel f, the individual elements of the convolved matrix keeps on reducing. Since the elements are less than 1, over time as $f \to \infty$, the elements would decrease to zero.

$$\boxed{\therefore g \text{ would be } \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}$$