Q-4) a)

To prove Shift Theorem: $R(g(x-x_0, y-y_0))(\rho,\theta)$
$= R(g(x,y))(\rho - x_0\cos\theta - y_0\sin\theta, \theta)$

L.H.S.

$$R(g(x-x_0, y-y_0))(\rho,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x-x_0, y-y_0)\,\delta(x\cos\theta + y\sin\theta - \rho)\,dx\,dy$$

let $x' = x - x_0$
$y' = y - y_0$

$$R(g(x-x_0, y-y_0))(\rho,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x',y')\,\delta((x'+x_0)\cos\theta + (y'+y_0)\sin\theta - \rho)\,dx\,dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x',y')\,\delta(x'\cos\theta + y'\sin\theta - (\rho - x_0\cos\theta - y_0\sin\theta))\,dx\,dy$$

$$= R(g(x,y))(\rho - x_0\cos\theta - y_0\sin\theta, \theta)$$

Hence Proved

(4) b) **To prove:** $R(g')(\rho,\theta) = R(g)(\rho, \psi_0 - \theta)$

$x = r\cos\psi$, $y = r\sin\psi$. $\hat{x} = r\cos(\psi+\psi_0)$ $\hat{y} = r\sin(\psi+\psi_0)$

we can define new cordinate system as:→

$$\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = \begin{bmatrix} \cos\psi_0 & \sin\psi_0 \\ -\sin\psi_0 & \cos\psi_0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\psi_0 & -\sin\psi_0 \\ \sin\psi_0 & \cos\psi_0 \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix}$$

Note: It is a fact that if a function is rotated by an angle ψ then its radon transform is also rotated by same angle.

L.H.S. $R(g')(\rho,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x',y')\,\delta(x\cos\theta + y\sin\theta - \rho)\,dx\,dy$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x',y')\,\delta(\hat{x}\cos\psi_0\cos\theta - \hat{y}\sin\psi_0\cos\theta + \hat{x}\sin\psi_0\sin\theta + \hat{y}\cos\psi_0\sin\theta - \rho)\,dx\,dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x',y')\,\delta(\hat{x}(\cos\psi_0\cos\theta + \sin\psi_0\sin\theta) + \hat{y}(-\sin\psi_0\cos\theta + \cos\psi_0\sin\theta) - \rho)\,dx\,dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x',y')\,\delta(\hat{x}\cos(\theta-\psi_0) + \hat{y}\sin(\theta-\psi_0) - \rho)\,d\hat{x}\,d\hat{y}$$

$= R(g)(\rho, \theta-\psi_0)$ <u>Hence proved</u>

$$d\hat{x}\,d\hat{y} = \begin{vmatrix} \frac{\partial\hat{x}}{\partial x} & \frac{\partial\hat{y}}{\partial x} \\ \frac{\partial\hat{x}}{\partial y} & \frac{\partial\hat{y}}{\partial y} \end{vmatrix} dx\,dy = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} dx\,dy = dx\,dy.$$