

AIP-Assignment 1

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1 Question 4

Algorithm : EWISTARS(Exponential Wavelet Iterative Shrinkage Thresholding Algorithm with Random Shift)

This uses following features :

1. ISTA
2. the sparse representation of exponent of wavelet transform(EWT)
3. the shift-invariance of random shifting (RS)

It is aimed at improving the reconstruction quality and accelerating computation.

Part A

1. Reconstruction Model

- (a) $y = Ux + e$, where U denotes the under sampling scheme in the k -space, namely, the incomplete Fourier transform.
 e is caused by either scanner imprecision's or the noise
- (b) $y = Q\omega + e$, where
 $Q=UW$, W is inverse sparse transform(EWT)
 ω denotes the sparsity coefficients
- (c) cost function definition
 $S(\omega) = \|y - Q\omega\|_2^2 + \lambda \|\omega\|_1$

2. Exponential Wavelet Transform Pseudocode of exponential wavelet transform (EWT) is as follows:

Step 1. Input the randomly under sampled MRI signal or image.

Step 2. Repeat k times.

Step 2.1. Carry out the wavelet transform (WT).

Step 2.2. Normalize wavelet coefficients to $[0 \ 1]$.

Step 3. Output the EWT coefficients.

k is the number of exponential iterations and T_E is the exponential wavelet transform. A single ($k=1$) EWT is implemented by

$$T_E(x, 1) = \frac{\exp(T_W(x)) - 1}{e - 1} \text{ where,}$$

$$T_W(x) = \omega \text{ here } T_W \text{ represents the wavelet transform } (T_W = W^{-1})$$

3. The iterative thresholding algorithm (ISTA) presents a sequence of estimates ω_n , which gradually approximates to the optimal result w^*

4. **AGORITHM EWISTA**

Input: ω_0
 $Y^{-1} \leftarrow \text{diag}(b)$
 Parameter Setting
 $t_0 = 1, n = 0, v_0 = \omega_0$
 Produce the sequence of EWT modified by RS
 $\{T_E(x, n)\}$
 Repeat
 $Q \leftarrow UW_n, A \leftarrow Q^H Q, a \leftarrow Q^H y, \{\text{step 1}\}$
 $\omega_{n+1} \leftarrow \Gamma_{\lambda b}(v_n + Y^{-1}(a - Av_n)) \{\text{step 2}\}$
 $t_{n+1} \leftarrow \frac{1 + \sqrt{1 + 4t_n^2}}{2}$
 $v_{n+1} \leftarrow \omega_{n+1} + \frac{(t_n - 1)}{t_{n+1}}(\omega_{n+1} - \omega_n) \{\text{step 3}\}$
 $n \leftarrow n + 1$
 Until termination criterion are met
 Output: ω_{n+1}
 $x^* \leftarrow W\omega_{n+1}$

Explanation:

Step 2

Γ is the shrinkage operator and b is the threshold:

$\Gamma_b(z) = \text{sgn}(z)(|z| - \min(b/2, |Z|))$ (same as ISTA step)

Step 3

t_n increases with the number of iteration. It basically fasten the process of reaching the termination criterion. It takes newly calculated ω_{n+1} in addition with $\frac{(t_n - 1)}{t_{n+1}}$ times the gap between newly calculated ω_{n+1} and ω_n .

Part b For paper link click here: [EWISTARS](#)

Part C Advantages over ISTA are :

1. It is aimed at improving the reconstruction quality (by using EWT) and accelerating computation (step 3).
2. EWISTARS algorithm takes advantage of the simplicity of ISTA, the sparse representation of EWT, and the shift-invariance of RS.
3. Applications in MRI

Part D Couldn't find an implementation of this algorithm.