Aip-Assign4-2

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1 Question 2

Paper name: Blind Compressive Sensing Dynamic MRI
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1.1 Some important points about the paper

1. Mentioned algorithm is proposed to be considerably faster than approaches that alternates between sparse coding and dictionary estimation, as well as the extension of K-SVD dictionary learning scheme.

2. **Summary of the proposed Algorithm:** In the BCS representation, the signal matrix Γ is modeled as the product $\Gamma = UV$, where U is the sparse coefficient matrix V is the temporal dictionary. The recovery is formulated as a constrained optimization problem, where the criterion is a linear combination of the data consistency term and a sparsity promoting l_1 prior on U, subject to a Frobenius norm (energy) constraint on V. Paper solve U and V using a majorize–minimize framework. Specifically, it decompose the original optimization problem into three simpler problems. An alternating minimization strategy is used, where we cycle through the minimization of three simpler problems.

3. Main goal of the paper is to recover the dynamic dataset $\gamma(x,t) : Z^3 \rightarrow C$ from its under-sampled Fourier measurements.
   $\gamma(x,t) = \Sigma_{i=1}^{R} u_i(x)v_i(t)$
   Here, $u_i(x)$ corresponds to the ith column of U and is termed as the ith spatial weight. Similarly, $v_i(t)$ corresponds to the ith row of and is the ith temporal basis function. The rows of are constrained to be sparse, which imply that there are very few nonzero entries.

1.2 Main objective function

$\{\hat{U}, \hat{V}\}$ =argmin$_{U,V}[\Sigma_{i=1}^{N} \parallel A_i(UV) - b_i \parallel_2^2] + \lambda \parallel U \parallel_{l_1}$such that $\parallel V \parallel_F^2 <= c$.

1. first term in the objective function ensures data consistency.
2. $U_{M\times R}$ is sparse coefficient matrix
   $V_{R\times N}$ is a dictionary of temporal basis functions
3. The second term is the sparsity promoting norm on the entries of U defined as the absolute sum of its matrix entries:$U :\parallel U \parallel_{l_1} = \sigma_{i=1}^{M} \sigma_{j=1}^{R} |u(i,j)|$
4. λ is the regularization parameter, and is a constant that is specified a priori.
5. The Frobenius norm constraint on V is imposed to make the problem well posed. If this constraint is not used, the optimization scheme can end up with coefficients that are arbitrarily small in magnitude.
6. It is observed that the specific choice of c is not very important. If c is changed, the regularization parameter λ also has to be changed to yield similar results.

1.3 The optimization technique

The Lagrangian of the constrained optimization problem in above objective function is specified by
L(U,V,η)=$[\Sigma_{i=1}^{N} \parallel A_i(UV) - b_i \parallel_2^2] + \lambda \parallel U \parallel_{l_1} + \eta(\parallel V \parallel_F^2 - c); \eta >= 0$(eq 1)
where $\eta$is the Lagrange multiplier.

1. l_1 penalty on the coefficient matrix is a non differentiable function, we approximate it by the differentiable Huber induced penalty which smooths the l_1 penalty
   $\psi_B(U) = \Sigma_{i=1}^{M} \Sigma_{j=1}^{R} \psi_B(u_{i,j})$
   where u_i, j are the entries of U and $\psi_B(u)$ is defined as
   $\psi_B(x) = |x| - 1/2\beta, if|x| >= \frac{1}{\beta}$
   $\psi_B(x) = \beta|x|^2/2, else$
2. The Lagrangian function obtained by replacing the l_1 in (eq 1) by ψ_B is
   $D_B(U, V, \eta) = [\Sigma_{i=1}^{N} \parallel A_i(UV) - b_i \parallel_2^2] + \lambda\psi_B(U) + \eta(\parallel V \parallel_F^2 - c); \eta >= 0$(eq 2)
3. Paper rely on the majorize–minimize framework to realize a fast algorithm. It starts by majorizing the Huber norm in (eq 2) as
   $\psi(U) = min_L \frac{\beta}{2} \parallel U - L \parallel_F^2 + \parallel L \parallel_{l_1}$(eq 3)
   where L is an auxiliary variable.
   Substituting (eq 3) in (eq 2), we obtain the following modified Lagrange

function, which is a function of four variables: U,V,L, and η

$D_B(U,V,L,\eta) = [\Sigma_{i=1}^{N} \parallel A_i(UV) - b_i \parallel_2^2] + \eta(\parallel V \parallel_F^2 - c) + [\parallel U - L \parallel_F^2 + \parallel L \parallel_{l_1}]$(eq 4)

4. At each step, paper solve for a specific variable, assuming the other variables to be fixed; we systematically cycle through these subproblems until convergence. The subproblems are specified below

   (a) $L_{n+1} = argmin_L \parallel U_N - L \parallel_2^2 + \frac{2}{\beta} \parallel_{l_1}$

   Can be solved analytically as:
   $L_{n+1} = \frac{U_n}{|u_n|}(|U_n - \frac{1}{\beta}|)_+$

   (b) $U_{n+1} = argmin_U \Sigma_{i=1}^{N}[\parallel A_i(UV_n) - b_i \parallel_2^2] + \frac{\lambda\beta}{2} \parallel U - L_{n+1} \parallel_2^2$ we solve it using conjugate gradient (CG) algorithms.

   (c) $V_{n+1} = argmin_V \Sigma_{i=1}^{N}[\parallel_i (U_{n+1}V) - b_i \parallel_2^2] + \eta_n(\parallel V \parallel_F^2 - c)$ we solve it using conjugate gradient (CG) algorithms.

5. paper use a steepest ascent rule to update the Lagrange multiplier at each iteration
   $\eta_{n+1} = (\eta_n + \parallel v_{n+1} \parallel_F^2 - c)_+$
   where "+" represents the operator defined as $(\tau)_+ = max\{0, \tau\}$, which is used to ensure the positivity constraint on η

1.4 paper deals with following application

1. paper illustrate the usefulness of the proposed blind compressing scheme for **MR image reconstruction**.

2. Since the dictionary is learned from the measurements, authors observe superior reconstructions compared to compressive sensing schemes that assume fixed dictionaries.