# CV-Assignment 1

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# 1 Question 3

Given :

- Projection matrix
  $\mathbf{M}= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$
- Hyperbola $y = 1/x$

In parametric form, we represent a hyperbola as $\begin{bmatrix} t \\ 1/t \end{bmatrix}$

In homogeneous coordinate $\mathbf{X}= \begin{bmatrix} t \\ 1/t \\ 1 \end{bmatrix}$

Applying transform $\mathbf{M}$ on $\mathbf{X}$,we get

$\mathbf{Y}= \begin{bmatrix} 1 \\ 1/t \\ t \end{bmatrix} = \begin{bmatrix} 1/t \\ 1/t^2 \\ 1 \end{bmatrix}$

In parametric form in euclidean space $\mathbf{y}$ can be written as : $\begin{bmatrix} \text{1/t} \\ \text{1/t}^2 \end{bmatrix}$

So this give us the following equation : $y = x^2$

**Comments** : This represent a equation of parabola.Hence image of hyperbola $\mathbf{Y}$ will be a parabola under the transform $\mathbf{M}$.

# 2 Question 4

Assumption : Let (opx,opy) and (oqx,oqy) be the optical centre of the two cameras in their respective pixel cordinate system.

Given : vanishing points in p image :$(p_{1x}, p_{1y})$, $(p_{2x}, p_{2y})$, $(p_{3x}, p_{3y})$

Given : vanishing points in q image :$(q_{1x}, q_{1y})$, $(q_{2x}, q_{2y})$, $(q_{3x}, q_{3y})$

Focal lengths $f_p$ and $f_q$ of two cameras. For a point $(p_{1x}, p_{1y})$ we do following :

- From image plane to sensor coordinate system

  $$\begin{bmatrix} p_{1x} \\ p_{1y} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & opx \\ 0 & 1 & opy \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cp_{1x} \\ cp_{1y} \\ 1 \end{bmatrix}$$

  where ($cp_{1x}$,$cp_{1y}$) are pixel location of points ($p_{1x}$,$p_{1y}$)
  $cp_{1x}$=($p_{1x}$-opx)
  $cp_{1y}$=($p_{1y}$-opy)

  ($s_p$,$s_p$) as the aspect ratio for the image p. Therefore we get
  $cp_{1x}$=$s_p$.($p_{1x}$-opx)

$cp_{1y}$=$s_p$.($p_{1y}$-opy)

- from camera coordinate system to image plane

$$\begin{bmatrix} \mathrm{s}_p.(p_{1x}\text{-opx}) \\ \mathrm{s}_p.(p_{1y}\text{-opy}) \\ 1 \end{bmatrix} = \begin{bmatrix} \mathrm{f}_p & 0 & 0 & 0 \\ 0 & \mathrm{f}_p & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \mathrm{kp}_{1x} \\ \mathrm{kp}_{1y} \\ 1 \end{bmatrix}$$

$kp_1$=($s_p$.($p_{1x}$-opx),$s_p$.($p_{1y} - opx$), $f_p$)

Similarly for other vanishing points of image p and q:

$\mathrm{kp}_2 = (s_p.(p_{2x} - opx), s_p.(p_{2y} - opx), f_p)$
$\mathrm{kp}_3 = (s_p.(p_{3x} - opx), s_p.(p_{3y} - opx), f_p)$
$\mathrm{kq}_1 = (s_q.(q_{1x} - oqx), s_q.(q_{1y} - oqx), f_q)$
$\mathrm{kq}_2 = (s_q.(q_{2x} - oqx), s_q.(q_{2y} - oqx), f_q)$
$\mathrm{kq}_3 = (s_q.(q_{3x} - oqx), s_q.(q_{3y} - oqx), f_q)$

Now if we will assume that we know the intrinsic parameters i.e. $f_p, f_q, s_p, s_q, opx, opy, oqx, oqy$

**Rotation:**

Now let us convert these points into unit vectors (i.e directional vector) by normalizing them say $(kp'_1, kp'_2, kp'_3, kq'_1, kq'_2, kq'_3)$ . Then we can easily compute R (rotation matrix). By solving the below equations:

kp'$_1$ $= R * kq'_1$
kp'$_2$ $= R * kq'_2$
kp'$_3$ $= R * kq'_3$

**Note:** *We will be able to find R uniquely only if all three sets line in 3d space are NOT co-planer.*

**Translation:**

We have direction, so if we along that direction the vanishing point will remain same. As we know that for lines parallel to some direction will have same vanishing point. Therefore, from vanishing point we "cannot" extract the information about the translation. So not possible to find t$_x$, $t_y and t_z translation parameter$.

**Intrinsic Parameters are Unknown**

- **Finding Image center i.e (opx,opy) (oqx,oqy)**
  We can find by using 3 vanishing point of same image subject to constraint

that they should not be collinear or say all 3 given sets of parallel line should not be coplaner in 3d space. By finding the **orthocenter** of the 3 vanishing point. That orthocenter will lie with the origin.

- **Finding $s_p$ and $s_q$**
  We know that all three sets of line are mutual perpendicular to each other. Therefore, dot product of any two line will be zero, so has to for their direction. Therefore we get:

kp'$_1$ $= \frac{kp_1}{||kp_1||}$

kp'$_2$ $= \frac{kp_2}{||kp_2||}$

(kp'$_1$)$^t * (kp'_2) = 0$

$$[s_p.(p_{2x} - opx), s_p.(p_{2y} - opx), f_p]^t \begin{bmatrix} s_p.(p_{2x} - opx) \\ s_p.(p_{2y} - opx) \\ f_p \end{bmatrix} = 0$$

By solving this, we will get "$s_p/f_p$" as some constant say c1. So we can write $s_p$ in terms of $f_p$ i.e:

$$s_p = c1 * f_p$$

Similarly for $s_q$ and $f_q$ for some constant c2 i.e

$$s_q = c2 * f_q$$

- **Finding $f_p$ and $f_q$**
  We cannot find $f_p$ and $f_q$ as information given is less. But we can find the out the direction of line without the knowledge of the $f_p$ and $f_q$.

  As we know:

$s_p = c1 * f_p (eq1)$

$s_q = c2 * f_q (eq2)$

By using eq1 and eq2, direction of line 1 is D1.

$$D1 = [\, kp_1 | 0] = \begin{bmatrix} s_p.(p_{2x} - opx) \\ s_p.(p_{2y} - opx) \\ f_p \\ 0 \end{bmatrix} = \begin{bmatrix} c1.f_p.(p_{2x} - opx) \\ c1.f_p.(p_{2y} - opx) \\ f_p \\ 0 \end{bmatrix} = \begin{bmatrix} c1.(p_{2x} - opx) \\ c1.(p_{2y} - opx) \\ 1 \\ 0 \end{bmatrix}$$

Similarly we can find out other direction.

## 3 Question 5.a

Prove (algebraically) that the projections (in image plane) of any two parallel lines L1 , L2 in $R^3$ have an intersection point, the vanishing point.
**Solution**
Prove : Let L1=A+tD and L2=B+tD where, D is the direction vector A, B is a point and "t" is any real number Since, lines are parallel they have the same direction vector D.
In homogeneous coordinate a point online L1, L2 can be written as :

$$\text{L1}_t = \begin{bmatrix} A_x + tD_x \\ A_y + tD_y \\ A_z + tD_z \\ 1 \end{bmatrix} = \begin{bmatrix} A_x/t + D_x \\ A_y/t + D_y \\ A_z/t + D_z \\ 1/t \end{bmatrix}$$

$$\text{L1}_t = \begin{bmatrix} B_x + tD_x \\ B_y + tD_y \\ B_z + tD_z \\ 1 \end{bmatrix} = \begin{bmatrix} B_x/t + D_x \\ B_y/t + D_y \\ B_z/t + D_z \\ 1/t \end{bmatrix}$$

Therefore point at infinity on line L1 and L2 will be L1$\infty$ and L$2_\infty$

$$\text{L1}_\infty = \begin{bmatrix} D_x \\ D_y \\ D_z \\ 0 \end{bmatrix} L2_\infty = \begin{bmatrix} D_x \\ D_y \\ D_z \\ 0 \end{bmatrix}$$

Projection of the infinity point will be the vanishing point and which only depends on the direction of the line.

$$v = PL1_\infty = PL2_\infty$$

## 4 Question 5.b

Let three different set parallel line are coplanar to plane A, and there is a plane B parallel to plane A and passing through the origin, it is evident that the plane B, contains the vanishing points associated with each sets of parallel line.

Thus the intersection of the image plane and B, which is a straight line, contains all the vanishing points associated with the parallel in A.

# 5 Question 6

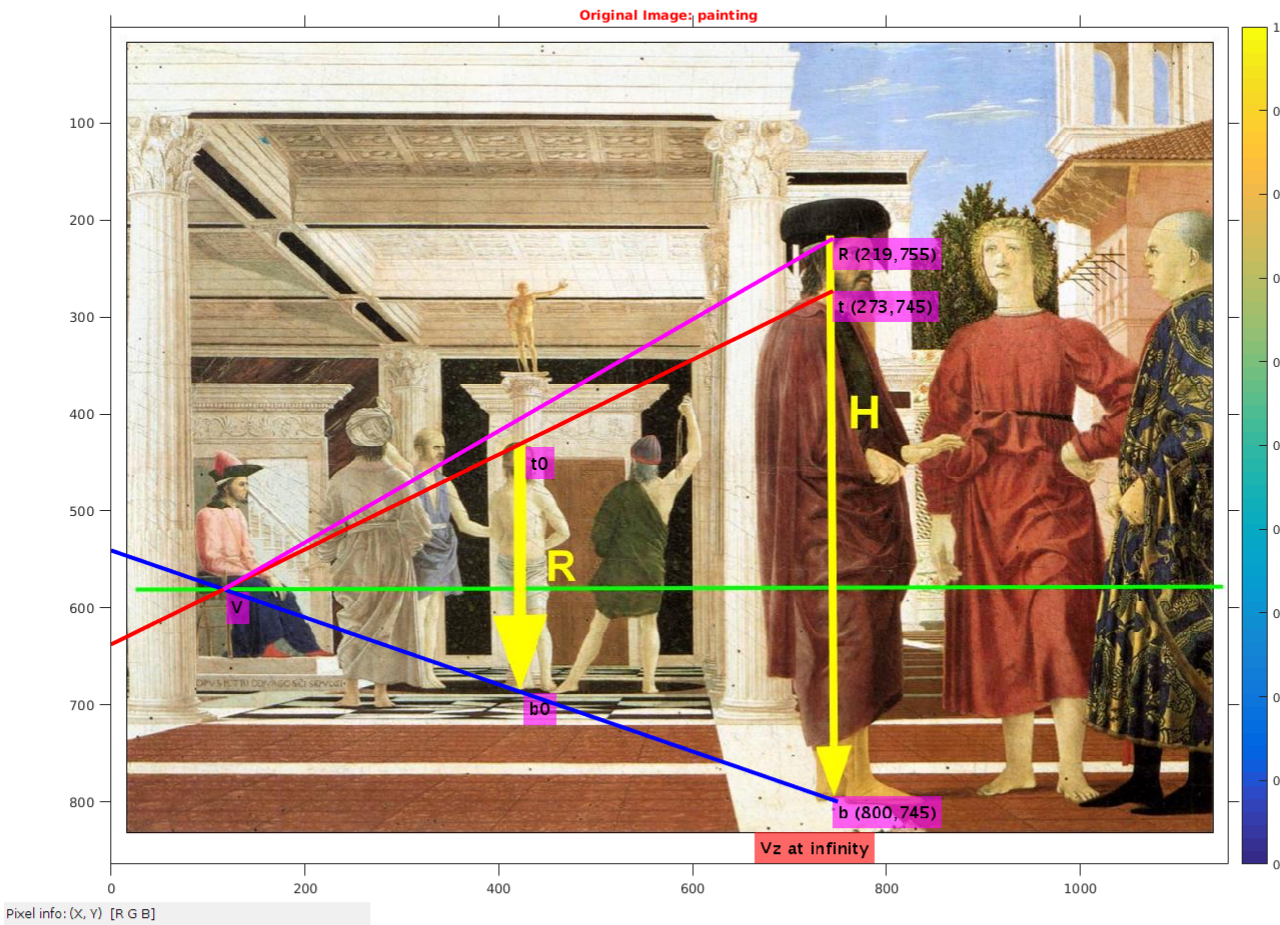


Figure 1: point mapping

- To Determine : height of the person (R) from the image.
- Points b and $b_0$ are both on the ground plane. Consider a line (say L) passing through t0and parallel to line $bb_0$.
- The images of L and $bb_0$will intersect at a point on the horizon at a point v.
- Now line $vt_0$ (i.e. L) is parallel to $bb_0$ in 3D space. Also line $vt_0$ intersects the reference direction at point t.
- Hence in 3D space, tb=$t_0b_0$= known height R which is 180cm

- Now using cross-ratio invariance property to find H,we use
  $\frac{|r-b||t-v_z|}{|r-v_z||t-b|} = \frac{H}{R}$
- Putting values in the equation, we get following equation
  $\frac{581}{527} = \frac{H}{180}$
  H=198.44
  Hence height H is equal to 198.44 cm