Sol Assignment 1

Question 5

a). Let

$L_1: \; x_0 + t l_x = x$ $\qquad$ $L_2: \; x = x_1 + t l_x$

$y_0 + t l_y = y$ $\qquad$ $y = y_1 + t l_y$

$z_0 + t l_z = z$ $\qquad$ $z = z_1 + t l_z$

Projection on the image plane:

$$L_1 : \; x = f\frac{X}{Z} = f \cdot \frac{x_0 + t l_x}{z_0 + t l_z} \; ; \; y = f \frac{y_0 + t l_y}{z_0 + t l_z}$$

$$\text{V.P: as} \; \lim_{t \to \infty} (x, y) = \left(f\frac{l_x}{l_z}, \; f\frac{l_y}{l_z} \right)$$

similarly, for L_2

$$\text{VP}: \; (x, y) = \left(f\frac{l_x}{l_z}, \; f\frac{l_y}{l_z} \right).$$

Clearly, L_1 & L_2 have the same vanishing point owing to the fact that VP depends only on the slopes of the lines.

b) Let l^1, l^2, & l^3 be the slopes of the three lines.

$\because$ l^1, l^2 & l^3 are coplanar,

$l^3 = t_1 l^1 + t_2 l^2$, for some t_1, t_2.

$$\text{or, } \begin{pmatrix} l_x^3 \\ l_y^3 \\ l_z^3 \end{pmatrix} = t_1 \begin{pmatrix} l_x^1 \\ l_y^1 \\ l_z^1 \end{pmatrix} + t_2 \begin{pmatrix} l_x^2 \\ l_y^2 \\ l_z^3 \end{pmatrix} \quad — ①$$

Now, find the slopes b/w the VP of $L_1 L_2$ & $L_1 L_3$.

$$\text{slope b/w } L_1 L_2 = \frac{l_y^2 l_z^1 - l_y^1 l_z^2}{l_x^2 l_z^1 - l_x^1 l_z^2} \quad — ②$$

$$\text{similarly, for } L_1 L_3 : \frac{l_y^3 l_z^1 - l_y^1 l_z^3}{l_x^3 l_z^1 - l_x^1 l_z^3} \quad — ③$$

Replacing ① in ③, we'll find the same slope as in ②. Thus, proving that the three VPs are collinear in image plane.