Assignment 5-1

Image 1 - $g_1 = f_1 + h_2 * f_2 \rightarrow g_1(x,y) = f_1(x,y) + (h_2 * f_2)(x,y)$

Image 2 - $g_2 = f_2 + h_1 * f_1 \rightarrow g_2(x,y) = f_2(x,y) + (h_1 * f_1)(x,y)$

Applying fourier transform to both the images :-

$$G_1(u,v) = F_1(u,v) + H_2(u,v) F_2(u,v) \quad \text{—} ①$$

$$G_2(u,v) = F_2(u,v) + H_1(u,v) F_1(u,v) \quad \text{—} ②$$

from eq ②

$$F_2(u,v) = G_2(u,v) - H_1(u,v) F_1(u,v)$$

Putting value of F_2 in eq ①

$$G_1(u,v) = F_1(u,v) + H_2(u,v)\left(G_2(u,v) - H_1(u,v) \cdot F_1(u,v)\right)$$

$$G_1(u,v) = F_1(u,v) + H_2(u,v) G_2(u,v) - H_2(u,v) H_1(u,v) \cdot F_1(u,v)$$

$$G_1(u,v) = F_1(1 - H_2(u,v) H_1(u,v)) + H_2(u,v) \cdot G_2(u,v)$$

$$F_1(u,v)(1 - H_2(u,v) H_1(u,v)) = G_1(u,v) - H_2(u,v) G_2(u,v)$$

$$F_1(u,v) = \frac{G_1(u,v) - H_2(u,v) G_2(u,v)}{1 - H_2(u,v) H_1(u,v)} = \hat{F}_1(u,v)$$

Similary we can determine for $F_2(u,v)$ by putting F_1 in eq ②
we get,

$$F_2(u,v) = \frac{G_2(u,v) - H_1(u,v) G_1(u,v)}{1 - H_2(u,v) H_1(u,v)} = \hat{F}_2(u,v)$$

Now, $f_1(x,y)$ & $f_2(x,y)$ can be obtained by taking inverse fourier transform

i.e. $f_1(x,y) = F_1^{-1}(\hat{F}_1(u,v))$

$f_2(x,y) = F_2^{-1}(\hat{F}_2(u,v))$

Now, since H_1 & H_2 are low pass filter we H_1 & H_2 for low frequency almost equal to 1. Hence ~~our~~ in this case our solution is undefined for the low frequency where $H_1(u,v)\ H_2(u,v)=1$.

Here reconstruction of higher frequencies is fine but lower frequence (especially the DC component) cannot be recovered robustly.