Assignment 5--5

Solution) Here there are two cases given. One is the linear approximation of x_i & second is non-linear approximation of x_i.

Argument :→ second statement is true i.e. $E_L(v) >= E_N(v)$ i.e. non-linear approximation of x_i will give less or equal error as compared to linear approximation of x_i

Reason :→

Case 1 :→ Linear approximation of x will always choose the first K values of V corresponding to the first k eigen values for all points x_i.

Case 2 :→ Non linear approximation of x doesn't make such assumption of choosing first K values of V corresponding to the first k eigen values for all points x_i.

Input from the equation,

$$\alpha_i = \underset{c_i}{\operatorname{argmin}} \|x_i - Vc_i\|^2$$

We find c_i (with constraint that it can atmost have K non-zero elements), such that difference between x_i & Vc_i is minimum.

So non-approximation algo to calculate x_i gives us flexibility to choose different set of k columns of V for different points. Hence α_i choosen in such fashion will give minimum reconstruction error.

Hence, we have proven our argument.

Algo

(a) first find the $\alpha_i = V^T x_i$. α_i is the eigen coffiient it has d values. Value of α_i i.e. α_{ij} denotes the projection value of x_i on axis(vector)j.

(b) We will find c_i such that it has at most k largest value in α_i and rest to 0. (Greedy approch).

(c) For finding c_i what we can do is create max heap with node having value (α_{ij}, j), where j denotes index of α_{ij}. This can be done in O(d) time.

(d) for l = 1 to k, we will find reconstruction error i.e. $re_e = ||x_i - Vc_e||$ where c_e has 'l' largest absolute value α_i (use maxheap for this. α_i

~~for each~~

Operation $||x_i - Vc_e||^2$ will take $O(d^{2.7})$ for multiplication. O(d) for subshaction & summition ∴ total is $O(d^{2.7})$

∴ for k terms it is $O(k d^{2.7})$

Doing for every x_i will take $O(n \cdot k d^{2.7})$

Correctness.

α_{ij} denotes the projection value on jth axis. The smaller the value i.e. tending to zero it means we can drop that axis because it will not impact or introduce much error after reconstruction.

So its save to drop that axis.