Q-5). Given: vector e is the eigen vector of matrix C, with the highest eigen value

To Prove: The direction f perpendicular to e for which $f^t C f$ is maximized, is the eigen vector of C with the second highest eigenvalue.

Objective function:→ $\vec{f}^T C \vec{f}$

Constraints: (i) $\vec{f}^t \vec{f} = 1$

(ii) $\vec{f}^t \vec{e} = 0$

Using method of Lagrange multipliers, we get

$$\tilde{J}(\vec{e},\vec{f}) = \vec{f}^T C\vec{f} - \lambda(\vec{f}^T\cdot\vec{f} - 1) - \mu(\vec{f}^T\cdot\vec{e})$$

Taking derivative of $\tilde{J}(\vec{e},\vec{f})$ w.r.t. $\vec{f}$

$$2C\vec{f} - 2\lambda\vec{f} - \mu\vec{e} = 0$$

$$C\vec{f} = \lambda\vec{f} + \frac{\mu}{2}\vec{e}$$

Multiplying by $\vec{f}^T$ on both sides.

$$\vec{f}^T C\vec{f} = \lambda\vec{f}^T\vec{f} + \frac{\mu}{2}\vec{f}^T\vec{e}$$

$$\vec{f}^T C\vec{f} = \lambda I + 0 \qquad \because \vec{f}^T\vec{f} = 1 \;\&\; \vec{f}^T\vec{e} = 0$$

Therefore $\vec{f}$ is the eigenvetor with eigenvalue λ. As we have to maximize $\vec{f}^T C\vec{f}$, $\vec{f}$ has to be a eigen vector with highest eigen value. But as we know $\vec{e}$ eigen vector has highes eigen value, & to satisfy this constraint $\vec{f}^t\vec{e}$, $\vec{f}$ has to be eigen vector with second highest eigen value.

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