Gaussian Eliminatation Method

Preliminaries:

Consider a matrix

```
M = {{8, 1, 7, 7}, {6, 9, 5, 6}, {0, 8, 0, 7}};
```

```
M // MatrixForm
```

$$\begin{pmatrix} 8 & 1 & 7 & 7 \\ 6 & 9 & 5 & 6 \\ 0 & 8 & 0 & 7 \end{pmatrix}$$

To get the entry of 2nd row and 3rd column of M

```
M[[2, 3]]
```

5

To get the entry of 3rd row and 4th column of M

```
M[[3, 4]]
```

7

To get 2nd row of M

```
M[[2]]
```

{6, 9, 5, 6}

To get 3rd row of M

```
M[[3]]
```

{0, 8, 0, 7}

Consider the following matrix A:

```
A = {{a11, a12, a13}, {a21, a22, a23}, {a31, a32, a33}};
```

```
A // MatrixForm
```

$$\begin{pmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{pmatrix}$$

To get submatrix consist of first two rows of the given matrix:

```
Take[A, 2] // MatrixForm
```

$$\begin{pmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \end{pmatrix}$$

To get a submatrix consists of first two rows and first two columns of the given matrix:

```
Take[A, 2, 2] // MatrixForm
```

$$\begin{pmatrix} a11 & a12 \\ a21 & a22 \end{pmatrix}$$

To get a submatrix consists of first two rows and last columns of the given matrix:

```
Take[A, 2, -1] // MatrixForm
```

$$\begin{pmatrix} a13 \\ a23 \end{pmatrix}$$

```
Take[A, 2, {3, 3}] // MatrixForm
```

$$\begin{pmatrix} a13 \\ a23 \end{pmatrix}$$

To get a submatrix consists of the second column of the given matrix:

```
Take[A, 3, {2, 2}] // MatrixForm
```

$$\begin{pmatrix} a12 \\ a22 \\ a32 \end{pmatrix}$$

```
Take[A, All, {2, 2}] // MatrixForm
```

$$\begin{pmatrix} a12 \\ a22 \\ a32 \end{pmatrix}$$

Gaussian Elimination Method:

Solve the following system of equations (without partial pivoting):

$$\begin{aligned} x_1 + 2\,x_2 + \;\;3\,x_3 &= 1 \\ 2\,x_1 + \;6\,x_2 + 10\,x_3 &= 0 \\ 3\,x_1 + 14\,x_2 + 28\,x_3 &= -8 \end{aligned}$$

```
Clear[A]
```

```
A = {{1, 2, 3}, {2, 6, 10}, {3, 14, 28}};
```

```
A // MatrixForm
```

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 6 & 10 \\ 3 & 14 & 28 \end{pmatrix}$$

```
x = {x1, x2, x3};
```

```
MatrixForm[x]
```

$$\begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix}$$

```
b = {{1}, {0}, {-8}};
```

```
b // MatrixForm
```

$$\begin{pmatrix} 1 \\ 0 \\ -8 \end{pmatrix}$$

Define the augmented matrix:

```
aug = ArrayFlatten[{{A, b}}];
```

```
aug // MatrixForm
```

$$\begin{pmatrix} 1 & 2 & 3 & 1 \\ 2 & 6 & 10 & 0 \\ 3 & 14 & 28 & -8 \end{pmatrix}$$

Now apply the following row operations:

```
aug[[2]] = aug[[2]] - 2 aug[[1]];
aug[[3]] = aug[[3]] - 3 aug[[1]];
aug // MatrixForm
```

$$\begin{pmatrix} 1 & 2 & 3 & 1 \\ 0 & 2 & 4 & -2 \\ 0 & 8 & 19 & -11 \end{pmatrix}$$

```
aug[[3]] = aug[[3]] - 4 aug[[2]];
aug // MatrixForm
```

$$\begin{pmatrix} 1 & 2 & 3 & 1 \\ 0 & 2 & 4 & -2 \\ 0 & 0 & 3 & -3 \end{pmatrix}$$

Then the reduced coefficient matrix Is

```
upper = Take[aug, 3, 3];
upper // MatrixForm
```

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 3 \end{pmatrix}$$

and the constant matrix is

```
c = Take[aug, 3, -1];
```

```
c // MatrixForm
```

$$\begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix}$$

```
upper.x == c
```

```
{x1 + 2 x2 + 3 x3, 2 x2 + 4 x3, 3 x3} == {{1}, {-2}, {-3}}
```

Thus the corresponding solution is

```
Solve[upper.x == c]
```

```
{{x1 → 2, x2 → 1, x3 → -1}}
```

Solve the following system of equations (with partial pivoting):

$$2\,x_1 + 6\,x_2 + 10\,x_3 = 0$$
$$x_1 + 3\,x_2 + 3\,x_3 = 2$$
$$3\,x_1 + 14\,x_2 + 28\,x_3 = -8$$

```
Clear[A, x, b, c, aug]

A = {{2, 6, 10}, {1, 3, 3}, {3, 14, 28}};

A // MatrixForm
```

$$\begin{pmatrix} 2 & 6 & 10 \\ 1 & 3 & 3 \\ 3 & 14 & 28 \end{pmatrix}$$

```
x = {x1, x2, x3};

MatrixForm[x]
```

$$\begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix}$$

```
b = {{0}, {2}, {-8}};

b // MatrixForm

aug = ArrayFlatten[{{A, b}}];
aug // MatrixForm
```

$$\begin{pmatrix} 2 & 6 & 10 & 0 \\ 1 & 3 & 3 & 2 \\ 3 & 14 & 28 & -8 \end{pmatrix}$$

To determine the first pivoting element

```
Max[Abs[Take[aug, 3, 1]]]
```

3

```
row1 = aug[[3]];
row2 = aug[[1]];
aug[[1]] = row1;
aug[[3]] = row2;
aug // MatrixForm
```

$$\begin{pmatrix} 3 & 14 & 28 & -8 \\ 1 & 3 & 3 & 2 \\ 2 & 6 & 10 & 0 \end{pmatrix}$$

```
aug[[2]] = aug[[2]] - (1 / 3) aug[[1]];
aug[[3]] = aug[[3]] - (2 / 3) aug[[1]];
aug // MatrixForm
```

$$\begin{pmatrix} 3 & 14 & 28 & -8 \\ 0 & -\frac{5}{3} & -\frac{19}{3} & \frac{14}{3} \\ 0 & -\frac{10}{3} & -\frac{26}{3} & \frac{16}{3} \end{pmatrix}$$

To get the second pivoting element:

```
Max[Abs[Take[aug, {2, 3}, {2, 2}]]]
```

$$\frac{10}{3}$$

```
r1 = aug[[2]];
r2 = aug[[3]];
aug[[2]] = r2;
aug[[3]] = r1;
aug // MatrixForm
```

$$\begin{pmatrix} 3 & 14 & 28 & -8 \\ 0 & -\frac{10}{3} & -\frac{26}{3} & \frac{16}{3} \\ 0 & -\frac{5}{3} & -\frac{19}{3} & \frac{14}{3} \end{pmatrix}$$

```
aug[[3]] = aug[[3]] - (aug[[3, 2]] / aug[[2, 2]]) aug[[2]];
aug // MatrixForm
```

$$\begin{pmatrix} 3 & 14 & 28 & -8 \\ 0 & -\frac{10}{3} & -\frac{26}{3} & \frac{16}{3} \\ 0 & 0 & -2 & 2 \end{pmatrix}$$

```
U = Take[aug, 3, 3];
U // MatrixForm
```

$$\begin{pmatrix} 3 & 14 & 28 \\ 0 & -\frac{10}{3} & -\frac{26}{3} \\ 0 & 0 & -2 \end{pmatrix}$$

```
H = Take[aug, 3, -1];
```

```
H // MatrixForm
```

$$\begin{pmatrix} -8 \\ \frac{16}{3} \\ 2 \end{pmatrix}$$

```
U.x == H
```

$$\left\{3\,x1 + 14\,x2 + 28\,x3,\ -\frac{10\,x2}{3} - \frac{26\,x3}{3},\ -2\,x3\right\} == \left\{\{-8\},\ \left\{\frac{16}{3}\right\},\ \{2\}\right\}$$

```
Solve[U.x == H]
{{x1 → 2, x2 → 1, x3 → -1}}
```