Optimal configuration of Parametric Quantum Circuits for Quantum State Preparation

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Background

Quantum state preparation is a subroutine used in multiple algorithms where the system must be initialized in a certain state before the algorithm can be applied.

Parameteric quantum circuits (PQCs) are a powerful class of quantum circuits that are usable on near-term quantum hardware due to their constant depth and classical-quantum hybrid nature.

Using parametric circuits for quantum state preparation tasks adds a fixed depth to the circuit before using the quantum data in an algorithm. Intuitively, by virtue of the large number of parameters of the circuit, the PQC can be thought to span a substantial subspace of the Hilbert space to which the target state belongs to. Thus, finding the optimal set of parameters for which the prepared state is close to the target state can be reduced to an optimization problem.

Applications of Quantum State Preparation

Quantum state preparation is a subroutine often used to prepare a system in a given state before an algorithm can be used, such as in [3], [4], [5], [9], [11], [12], [8], [14], [6] and [2].

Parametric Quantum Circuits

Parametric Quantum Circuits are a class of circuits that typically consist of several layers of rotation and non-local gates. Under some complexity assumptions, PQCs cannot be simulated classically [13].

QSP with PQCs

Procedure to perform quantum state preparation with PQCs -

1. Decide on a circuit ansatz - which rotation gates, which entanglement policy, and the number of such layers. The parameters of the rotation gates (Θ) will be varied by the optimizer.
2. Randomly initialize Θ.
3. Capture the state prepared by this circuit. This can either be done by a simulator or on a real quantum computer using quantum state tomography [7].
4. Using the knowledge of the prepared state, compute the value of a cost function (such as infidelity).
5. Minimize this value over Θ using an appropriate optimizer.
6. The final Θ that is found by the optimizer can be plugged into the ansatz. This is the circuit that prepares the given quantum state.

Problem statement

For an N-dimensional datapoint

$$A = (a_0, a_1, \dots, a_{N-1})$$

with $\sum_k |a_k|^2 = 1$, we must prepare the target quantum state

$$|\psi_{target}\rangle = \sum_{k=0}^{N-1} a_k |k\rangle$$

from an initial state of $|0\rangle^{\otimes n}$, where $N = 2^n$.

Determine the configuration of the parametric quantum circuit to perform this task. Configuration involves the following choices -

1. Rotation gate(s)
2. Number of repetitions
3. Entanglement policy
4. Optimizer

Note that the first three of the above choices are in relation to the circuit ansatz, while the fourth choice is for the classical part of the workflow.

Methodology

We have analysed the optimal configuration for state preparation of two types of data - positive and real. The data is assumed to be normalised.

Let the set of parameters needed to define the rotation gates be denoted by Θ, and let the state prepared by this set of parameters be $|\psi\rangle$.

If only one rotation gate is used per layer (here, R_Y), the dimension of Θ will be $reps \times n$. The choice of gate is R_Y since we are only working with real data - thus there is no need to introduce imaginary components with other rotation gates.

We find the optimal Θ_{opt} such that $|\psi\rangle = |\psi_{target}\rangle$. This is done by minimizing the infidelity of the prepared state. Thus, the cost function is the infidelity $|\langle\psi_{prepared}|\psi_{target}\rangle$, where $|\psi_{prepared}\rangle$ is the state prepared by the circuit having rotation parameters Θ.

In order to find the best combination of *reps* and optimizer for state preparation, we perform the state preparation task for data of different sizes, and compare the combination on the following metrics -

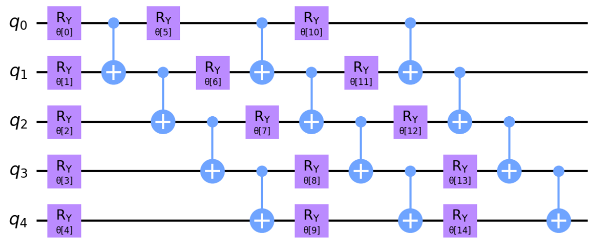
1. Number of qubits ($n = \log_2 N$)
2. Circuit depth
3. Number of parameters (Dimension of Θ)
4. Number of non-local gates
5. Fidelity of final prepared state
6. Time taken to find Θ_{opt}

We check for the best option between the optimizers 'BFGS' [1] and 'SLSQP' [10], both of which are Bayesian optimizers that are powerful for high dimensional optimization problems.

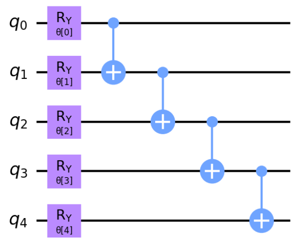
We are looking for a configuration of *reps* and *opt* that gives the lowest possible infidelity in the least time.

For an ansatz having one layer comprising of R_Y gates and linearly entangling $CNOT$ gates, we compare different optimizers and repetition values.

Here is an example of an ansatz with $n = 5$, $reps = 3$, and rotation gate R_Y-

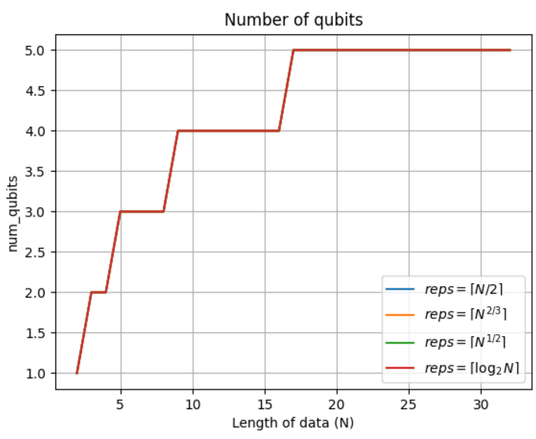


In this example, one layer is of the following form-

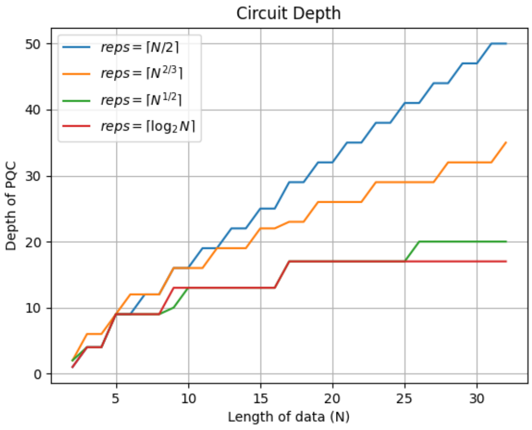


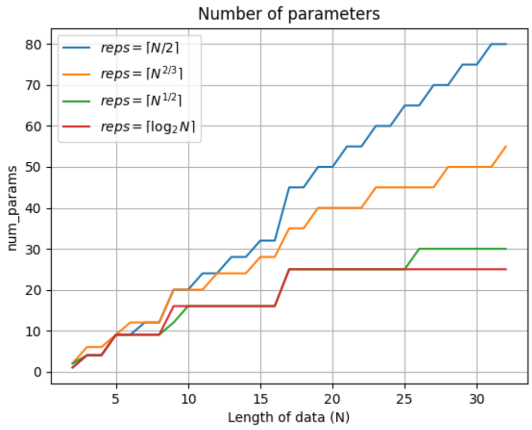
Results

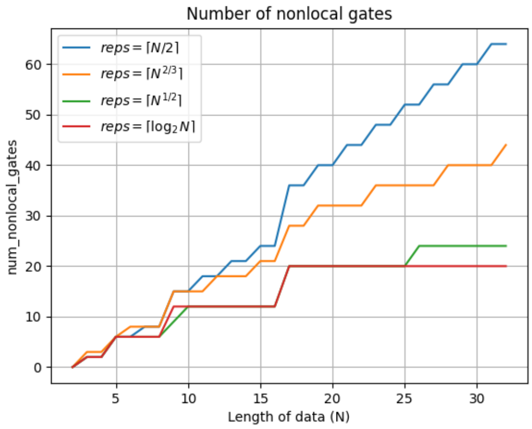
The first four metrics are independent of the nature of the data and the optimizer used, so we can compare their values for different values of *reps*.

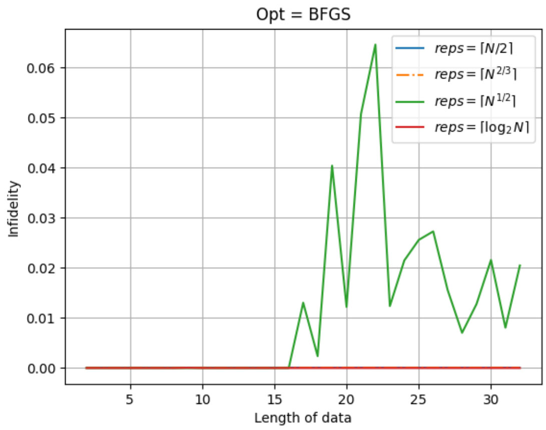


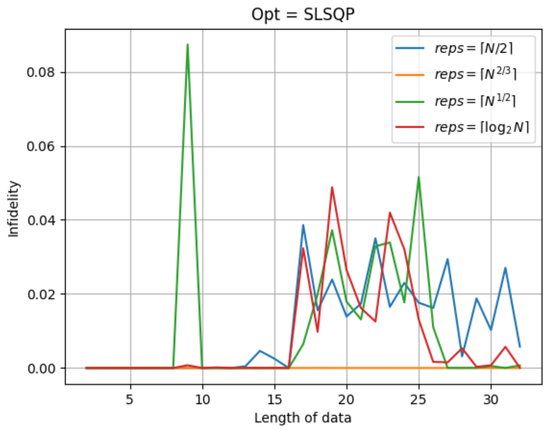
Positive valued array





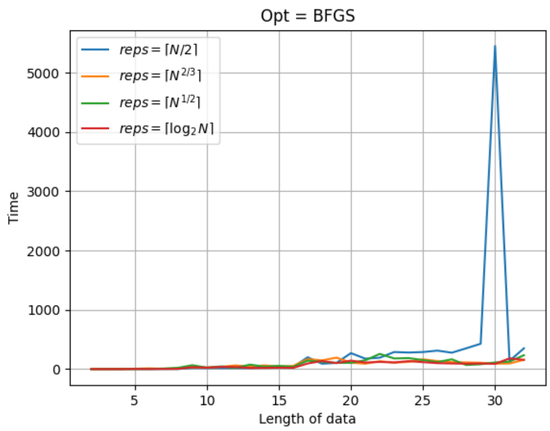


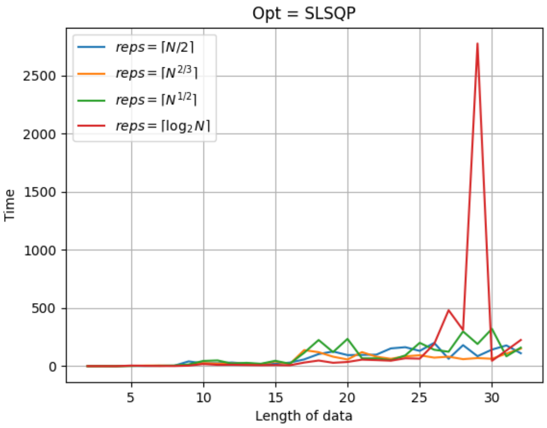




Note that the number of qubits is solely dependent on the length of the data. Circuit depth, number of parameters, and number of non-local gates are all functions of $reps$, and behave accordingly.

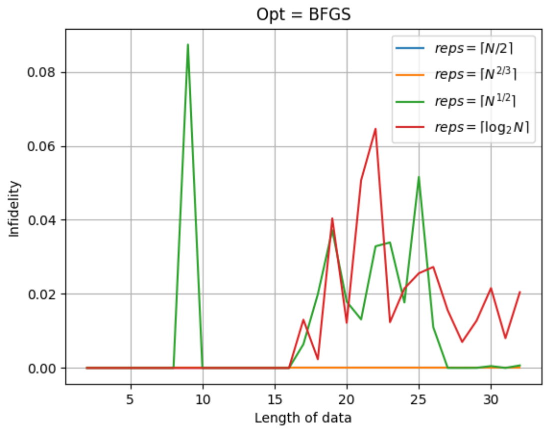
We now analyse the effect of the nature of the data on the final fidelity of the prepared state, and the time taken to find Θ_{opt}.

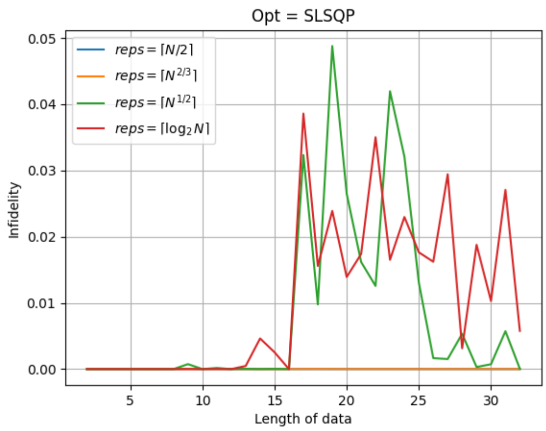


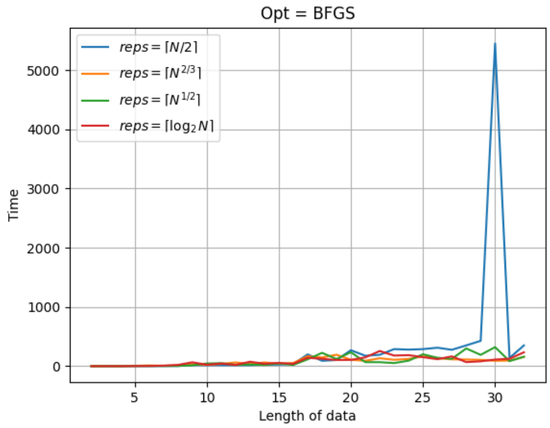


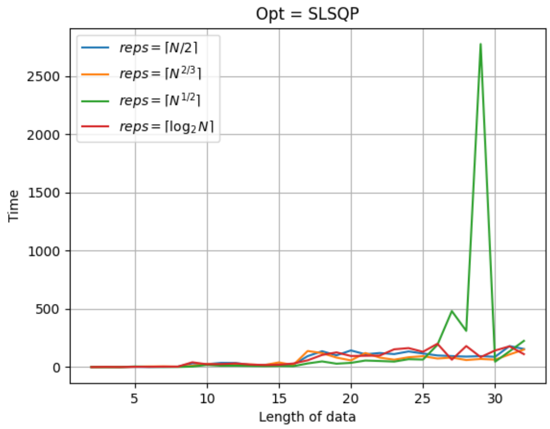
The combination of the BFGS optimizer and the function $\log_2 N$ as *reps* is the best combination as both the circuit depth and time taken to find the optimal value increase at a slowly rate with increase in number of elements.

Real valued array









We can consider optimizer SLSQP with $N^{2/3}$ as the *reps* function for data of length less than 16, and optimizer BFGS with $N^{2/3}$ as the *reps* function for data of lengthgreater than 16.

Future work

1. A metric to identify the best configuration can be developed - which takes into account the weights of every factor (eg. infidelity, time)

2. In order to use this method on a real quantum system, quantum state tomography must be used. In this case, one more factor of error can be analysed.

3. More data types (complex valued, encoding-specific values) can be analysed.

References

[1] *Numerical Optimization*. Springer New York, 2006.

[2] Dominic W. Berry, Andrew M. Childs, Richard Cleve, Robin Kothari, and Rolando D. Somma.

Simulating hamiltonian dynamics with a truncated taylor series. *Phys. Rev. Lett.*, 114:090502, Mar 2015.

[3] Dominic W. Berry, Andrew M. Childs, and Robin Kothari. Hamiltonian simulation with nearly optimal dependence on all parameters. In *2015 IEEE 56th Annual Symposium on Foundations of Computer Science*, pages 792–809, 2015.

[4] Andrew M. Childs. On the relationship between continuous- and discrete-time quantum walk. *Communications in Mathematical Physics*, 294(2):581–603, oct 2009.

[5] Andrew M. Childs, Dmitri Maslov, Yunseong Nam, Neil J. Ross, and Yuan Su. Toward the first quantum simulation with quantum speedup. *Proceedings of the National Academy of Sciences*, 115(38):9456–9461, sep 2018.

[6] B. D. Clader, B. C. Jacobs, and C. R. Sprouse. Preconditioned quantum linear system algorithm. *Phys. Rev. Lett.*, 110:250504, Jun 2013.

[7] Marcus Cramer, Martin B. Plenio, Steven T. Flammia, Rolando Somma, David Gross, Stephen D. Bartlett, Olivier Landon-Cardinal, David Poulin, and Yi-Kai Liu. Efficient quantum state tomography. *Nature Communications*, 1(1), dec 2010.

[8] Aram W. Harrow, Avinatan Hassidim, and Seth Lloyd. Quantum algorithm for linear systems of equations. *Phys. Rev. Lett.*, 103:150502, Oct 2009.

[9] Iordanis Kerenidis and Anupam Prakash. Quantum recommendation systems, 2016.

[10] Dieter Kraft. A software package for sequential quadratic programming, ein software-paket zur sequentiellen quadratischen optimierung, forschungsbericht. deutsche forschungs- und versuchsanstalt für luft- und raumfahrt, dfvlr. Technical report, Institut für Dynamik der Flugsysteme, Deutsche Forschungs- und Versuchsanstalt für Luft- und Raumfahrt $DFVLR$, Oberpfaffenhofen, Köln, 1988.

[11] Seth Lloyd, Masoud Mohseni, and Patrick Rebentrost. Quantum principal component analysis. *Nature Physics*, 10(9):631–633, jul 2014.

[12] Guang Hao Low and Isaac L. Chuang. Hamiltonian simulation by qubitization. *Quantum*, 3:163, jul 2019.

[13] Barbara M. Terhal and David P. DiVincenzo. Adaptive quantum computation, constant depth quantum circuits and arthur-merlin games. 2002.

[14] Nathan Wiebe, Daniel Braun, and Seth Lloyd. Quantum algorithm for data fitting. *Phys. Rev. Lett.*, 109:050505, Aug 2012.