Task Sheet 4 - Dimension reduction and rate equations

Objective

A. Implementing the reduction of a high-dimensional swarm system to a 1D model

B. Understanding rate equations and delay equations

1 Dimension reduction and modeling

We implemented the locust simulation that was shortly mentioned in the lecture (continuous space, discrete-time). The locusts live on a ring of circumference $C = 1$ (positions $x_0 = 0$ and $x_1 = 1$ are identical). The locusts move with a speed of 0.001 either to the left ($v = -0.001$) or to the right ($v = +0.001$). They have a perception range of $r = 0.045$. A locust switches its direction in one of two situations:
1) The majority of locusts within its perception range have opposite direction to that of the considered locust.
2) A locust spontaneously switches its direction with a probability $P = 0.015$ per time step.
Initially, the locusts are uniformly randomly distributed and are moving left or right with equal probability. We simulate a swarm of size $N = 20$ for 500 time steps.

A. Implement and test your simulation. Plot the number of left-going locusts over time for one run.

B. We want to take the number of left-going locusts L as our modeling approach. Hence, we implement the dimension reduction of our model by averaging over many system configurations and summarizing them in groups of equal left-goer numbers. Create a histogram of the observed transitions $L_t \rightarrow L_{t+1}$ (change in the number of left-goers within one time step) With your simulation by doing 1000 sample runs of 500 time steps each. For example, you can use a 2-d array $A[\cdot][\cdot]$ of integers and an entry of this array $A[L_t][L_{t+1}]$ is increased by one whenever a transition $L_t \rightarrow L_{t+1}$ is observed. Plot the histogram.

C. In addition, count also the occurrences $M[L]$ of each model state L and use these to normalize the histogram entries: $A[i][j]/M[i]$. That way we get approximations of the transition probabilities. Use these approximations $P_{i,j} = A[i][j]/M[i]$ to sample evolutions of L_t over time t. Plot one such trajectory of L. How does this compare to the plot done in a)?

2 Rate Equations

We use the rate equations model of searching and avoiding from the lecture:

$$\frac{dn_s(t)}{dt} = -\alpha_r n_s(t)(n_s(t)+1) + \alpha_r n_s(t-\tau_a)(n_s(t-\tau_a)+1))$$
$$\frac{dm(t)}{dt} = -\alpha_p n_s(t)m(t)$$

A. Use a tool of your choice to calculate the temporal course of this system of ordinary differential equations (a simple forward integration in time can also be implemented from scratch for this system). Notice that we have delay equations. How should the delays be treated especially early in the simulation ($t < \tau_a$)? Use the following setting for the parameters: $\alpha_r = 0.6$, $\alpha_p = 0.2$, $\tau_a = 2$, $n_s(0) = 1$, $m(0) = 1$. Calculate the values of n_s and m for $t \in (0, 50]$ and plot them. Interpret your result.

B. Now we want to extend the model. In addition to *searching* and *avoiding* we introduce a third state: *homing* (n_h). Robots that have found a puck do a transition to the state *homing* in which they stay for a time $\tau_h = 15$. We assume that for unspecified reasons robots in state *homing* do not interfere with each other or with robots of any other state (assumption: no avoidance behavior for robots in state *homing* necessary). After the time of τ_h they have reached the home base and do a transition back to *searching*. Add an equation for n_h and edit the equation of n_s accordingly. Calculate the values of n_h, n_s and m for $t \in (0, 160]$ and plot them. In a second calculation, reset the ratio of pucks at time $t = 80$ to $m(80) = 0.5$ and plot the results. Interpret your result.

3 Your submission

- Please zip your submission in a single file
- Include the source code and a detailed readme. Optional but preferable: include a video where you go through and explain your code.
- For tasks 1 and 2, provide written solutions and plots.