Tutorial Sheet - 4

Sol 1.)

$T(n) = 3T(n/2) + n^2$

$a = 3 \quad b = 2 \quad f(n) = n^2$

$\because$ a & b are constant and $f(n)$ is a +ve function.

$\therefore$ Master's theorem is applicable.

$c = \log_b a$

$= \log_2 3 = 1.58$

$\Rightarrow n^c = n^{1.58}$

which is $n^2 > n^{1.58}$

$\therefore$ Case 3 is applied here

$\boxed{T(n) = \theta(n^2)}$

Sol 2.)

$T(n) = 4T(n/2) + n^2$

$a = 4 \quad b = 2 \quad f(n) = n^2$

$\because$ a & b are const. and $f(n)$ is a +ve function.

$\therefore$ Masters' theorem is applicable.

$c = \log_b a$

$= \log_2 4 = \log_2 2^2 = 2\log_2 2 = 2$

$\Rightarrow n^c = n^2$

which is $n^2 = f(n)$

$\therefore$ Case 2 is applied here

$\boxed{Tn = \theta(n^2 \log n)}$

Sol 3.)

$T(n) = T(n/2) + 2^n$

$a = 1 \quad b = 2 \quad f(n) = 2^n$

$\because$ a & b are const. and $f(n)$ is a +ve function

$\therefore$ Master's theorem is applicable.

$c = \log_b a = \log_2 1$

$= 0$

$\Rightarrow n^c = n^0 = 1$

$\therefore f(n) > n^c$

$\therefore$ case 3 is applied here.

$\Rightarrow \boxed{T(n) = \Theta(2^n)}$

Sol 4.) $T(n) = 2^n\, T(n/2) + n^n$

$a = 2^n \quad b = 2 \quad f(n) = n^n$

$\because$ a ~~are~~ is not const, its value depends on n

$\therefore$ Master's theorem is not applicable here.

Sol 5.) $T(n) = 16\, T(n/4) + n$

$a = 16 \quad b = 4 \quad f(n) = n$

$\because$ a & b are const., and $f(n)$ is a +ve f^n

$\therefore$ Master's theorem is applicable here,

$c = \log_b a = \log_4 16 = \log_4 (4)^2$

$= 2 \log_4 4 = 2$

$\Rightarrow n^c = n^2$

$\because f(n) < n^c$

$\therefore$ Case 1 is applied here,

$\boxed{T(n) = \Theta(n^2)}$

Sol 6.) $T(n) = 2T(n/2) + n\log n$

$a = 2 \quad b = 2 \quad f(n) = n\log n$

$\because$ a & b are const. & f(n) is a +ve function

$\therefore$ $c = \log_b a$

$= \log_2 2 = 1.$

$n^c = n$

$n < n\log n \Rightarrow f(n) > n^c$

$\therefore$ Case 3 is applied

$\Rightarrow \boxed{T(n) = \Theta(n\log n)}$

Sol 7.) $T(n) = 2T(n/2) + n/\log n$

$a = 2 \quad b = 2 \quad f(n) = n/\log n$

$\because$ a & b are const. & f(n) is a +ve function

$c = \log_b a$

$= \log_2 2 = 1$

$n^c = n^1 = n$

$\because$ non-polynomial difference b/w f(n) & n^c

$\therefore$ Master's theorem is not applicable.

Sol 8.) $T(n) = 2T(n/4) + n^{0.51}$

$a = 2 \quad b = 4 \quad f(n) = n^{0.51}$

$\because$ a & b are const. & f(n) is a +ve function.

$\therefore$ Master's theorem is applicable

$c = \log_b a = \log_4 2 = 0.50$

$n^c = n^{0.50}$

$\because$ $f(n) > n^c$

$\therefore$ Case 3 is applicable

$\boxed{T(n) = \Theta(n^{0.50})}$

Sol 9.) $T(n) = 0.5 * T(n/2) + 1/n$

$a = 0.5 \quad b = 2 \quad f(n) = 1/n$

$\because a < 1$

$\therefore$ Master's theorem is not applicable.

Sol 10.) $T(n) = 16\,T(n/4) + n!$

$a = 16 \quad b = 4 \quad f(n) = n!$

$\because$ a & b are const. and f(n) is a +ve function.

$\therefore$ Master's theorem is applicable

$c = \log_b a$

$= \log_4 16 = \log_4 4^2 = 2\log_4 4 = 2$

$n^c = n^2$

$\because f(n) > n^c$

$\therefore$ case 3 is applied here

$\boxed{T(n) = \Theta(n!)}$

Sol 11.) $T(n) = 4T(n/2) + \log n$

$a = 4 \quad b = 2 \quad f(n) = \log n$

$\because$ a & b are constant & f(n) is a +ve f^n

$\therefore$ Master's theorem is applicable

$c = \log_b a = \log_2 4 = \log_2 2^2 = 2\log_2 2 = 2$

$n^c = n^2$

$\because f(n) < n^c$

$\therefore$ case 1 is applied.

$\boxed{T(n) = \Theta(n^2)}$

Sol 12.) $\sqrt{n}\, T(n/2) + \log n$.

$a = \sqrt{n}$ $b = 2$ $f(n) = \log n$

$\because$ a is not constant

$\therefore$ Master's theorem is not applicable.

Sol 13.) $T(n) = 3T(n/2) + n$

$a = 3$ $b = 2$ $f(n) = n$

$\because$ a & b are const. & f(n) is a +ve f^n

$\therefore$ Master's theorem is applicable

$c = \log_b a = \log_2 3 = 01.58$

$n^c = n^{1.58}$

$\because f(n) < n^c$

$\therefore$ case 1 is applied here.

$\boxed{T(n) = \theta(n^{1.58})}$

Sol 14.) $T(n) = 3T(n/3) + \sqrt{n}$

$a = 3$ $b = 3$ $f(n) = \sqrt{n}$

$\because$ a & b are const. & f(n) is a +ve f^n

$\therefore$ Master's theorem is applicable.

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

$\because f(n) < n^c$

$\therefore$ case 1 is applicable

$\boxed{T(n) = \theta(n)}$

Sol 15:) $T(n) = 4T(n/2) + c \cdot n$

$a = 4 \quad b = 2 \quad f(n) = c \cdot n$

$\because$ a & b are constant & f(n) is a +ve f^n

$\therefore$ Master theorem is applicable here.

$c = \log_b a = \log_2 4 = \log_2 2^2 = 2\log_2 2 = 2$

$n^c = n^2$

$\because f(n) < n^c$

$\therefore$ Case 1 is applied here

$\Rightarrow \boxed{T(n) = \theta(n^2)}$

Sol 16:) $T(n) = 3T(n/4) + n\log n$

$a = 3 \quad b = 4 \quad f(n) = n\log n$

$\because$ a & b are constant & f(n) is a +ve function

$\therefore$ Master's theorem is applicable here.

$c = \log_b a = \log_4 3 = 0.79$

$n^c = n^{0.79}$

$\because f(n) > n^c$

$\therefore$ case 3 is applicable here

$\Rightarrow \boxed{T(n) = \theta(n\log n)}$

Sol 17:) $T(n) = 3T(n/3) + n/2$

$a = 3 \quad b = 3 \quad f(n) = n/2$

$\because$ a & b are const. & f(n) is a +ve f^n.

$\therefore$ Master's theorem is applicable here.

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

$\therefore f(n) = n^c$

$\therefore$ Case 2 is applied here

$\Rightarrow \boxed{T(n) = n \log n}$

Sol 18.)

$T(n) = 6T(n/3) + n^2 \log n$

$a = 6 \quad b = 3 \quad f(n) = n^2 \log n$

$\because$ a & b are const and $f(n)$ is a +ve f^n

$\therefore$ Master's Theorem is applicable here

$c = \log_b a = \log_3 6 = 1.63$

$n^c = n^{1.63}$

$\because f(n) > n^c$

$\Rightarrow$ case 3 is applied here

$\Rightarrow \boxed{T(n) = \Theta(n^2 \log n)}$

Sol 19.)

$T(n) = 4T(n/2) + n/\log n$

$a = 4 \quad b = 2 \quad f(n) = n/\log n$

$\because$ a & b are const. and $f(n)$ is a +ve f^n

$\therefore$ Master's theorem is applicable here

$c = \log_b a$

$= \log_2 4$

$= \log_2 2^2 = 2 \log_2 2 = 2$

$n^c = n^2$

$\because f(n) < n^c$

$\therefore$ Case 1 is applied here

$\Rightarrow \boxed{T(n) = \Theta(n^2)}$

Sol 20.)

$$T(n) = 64T(n/8) - n^2 \log n$$

∵ a & b are const. but ~~are~~ $f(n)$ is a $-ve$ f^n

∴ Master's theorem is not applicable.

Sol 21.)

$$T(n) = 7T(n/3) + n^2$$

$a = 7 \quad b = 3 \quad f(n) = n^2$

∵ a, b are const. & $f(n)$ is a $+ve$ f^n

∴ Master's Theorem is applied here

$\Rightarrow \quad c = \log_b a = \log_3 7 = 1.77$

$n^c = n^{1.77}$

∵ $f(n) > n^c$

∴ Case 3 is applied here

$$\boxed{T(n) = \Theta(n^2)}$$

Sol 22.)

$$T(n) = T(n/2) + n(2 - \cos n)$$

∵ $f(n)$ is not regular function

∴ Master's theorem ~~does~~ can not be applied here.