Modelling of Thermal hydraulic piston pump

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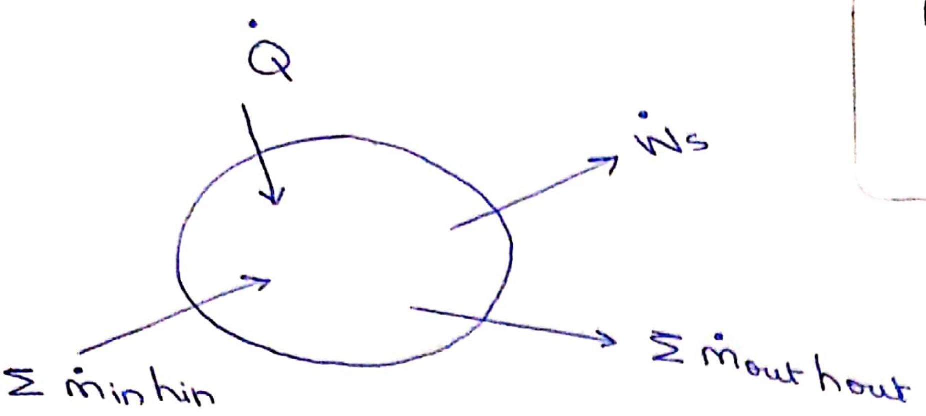


The first law of thermodynamics for a control volume –

$$\dot{E} = \Sigma \dot{m}_{in} h_{in} - \Sigma \dot{m}_{out} h_{out} + \dot{Q} - \dot{W} \quad \dots (I)$$

$$E = U + KE + PE$$

The kinetic and potential energy can be neglected so,

$$E = U = mu$$

$$\frac{dE}{dt} = \frac{d(mu)}{dt} = m\frac{du}{dt} + u\frac{dm}{dt} \quad \dots (II)$$

The continuity equation for 1D flow gives

$$\frac{dm}{dt} = \Sigma \dot{m}_{in} - \Sigma \dot{m}_{out} \quad \dots (III)$$

$$\frac{dh}{dt} = \left(\frac{\partial h}{\partial T}\right)_P \frac{dT}{dt} + \left(\frac{\partial h}{\partial P}\right)_T \frac{dP}{dt} \quad \dots (IV)$$

It can be recognized that,

$$C_p = \left(\frac{\partial h}{\partial T}\right)_P \quad \cdots\cdots (v)$$

also,

from the fundamental property relation,

$$dh = Tds + v\,dP$$

If temperature is constant then,

$$\left(\frac{\partial h}{\partial P}\right)_T = T\left(\frac{\partial s}{\partial P}\right)_T + v \quad \cdots\cdots (vi)$$

from the fundamental property relation for Gibbs energy,

$$dG = v\,dP - s\,dT$$

$$\left(\frac{\partial G}{\partial P}\right)_T = v \qquad \left(\frac{\partial G}{\partial T}\right)_P = -s$$

$$\frac{\partial^2 G}{\partial T\,\partial P} = \left(\frac{\partial v}{\partial T}\right)_P = -\left(\frac{\partial s}{\partial P}\right)_T$$

substituting back into

$$\left(\frac{\partial h}{\partial P}\right)_T = v - T\left(\frac{\partial v}{\partial T}\right)_P \quad \cdots\cdots (vii)$$

Taking

$$\left(\frac{\partial v}{\partial T}\right)_P = \alpha_p v \quad \text{, substituting back into (vii)}$$

$$\left(\frac{\partial h}{\partial P}\right)_T = v\left(1 - \alpha_p T\right) \quad \text{.... (viii)}$$

substituting (viii) into (iv)

$$\frac{dh}{dt} = C_p \frac{dT}{dt} + v\left(1 - \alpha_p T\right)\frac{dP}{dt} \quad \text{.... (ix)}$$

also, enthalpy is defined as

$$h = u + Pv$$

$$\frac{dh}{dt} = \frac{du}{dt} + P\frac{dv}{dt} + v\frac{dP}{dt}$$

$$\Rightarrow \frac{du}{dt} = \frac{dh}{dt} - P\frac{dv}{dt} - v\frac{dP}{dt} \quad \text{.... (x)}$$

combining (x), (iii) and (i) we get

$$\frac{dT}{dt} = \frac{1}{C_p m}\left[\Sigma \dot{m}_{in}(h_{in} - h) + \Sigma \dot{m}_{out}(h - h_{out}) + \dot{Q} - \dot{W} + P\frac{dV}{dt} + mT\alpha_p v\frac{dP}{dt}\right]$$

.... (xi)

$$\dot{W} = \dot{W}_s + \dot{W}_b$$

$$\dot{W}_b = P\frac{d\nu}{dt}$$

on substituting it back into (xi) $-\dot{W}_b$ cancels out $P\frac{dV}{dt}$

Also, it is assumed that the average enthalpy within the control volume is equal to the enthalpy with which it leaves,

$$\frac{dT}{dt} = \frac{1}{C_p m}\left[\Sigma \dot{m}_{in}(h_{in} - h) + \dot{Q} - \dot{W}_s + T\alpha_p \nu \frac{dP}{dt}\right]$$

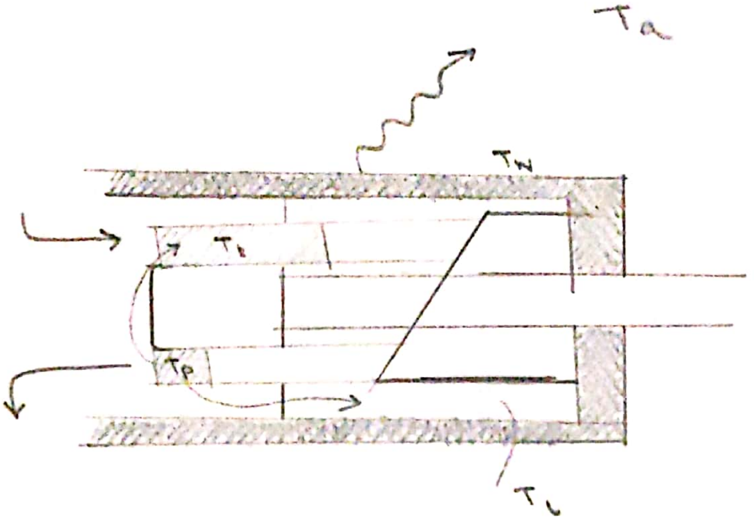
Integrating the fundamental relationship for enthalpy,

$$h_{in} - h = C_p(T_{in} - T) + (1 - \alpha_p \bar{T})\nu(P_{in} - P)$$

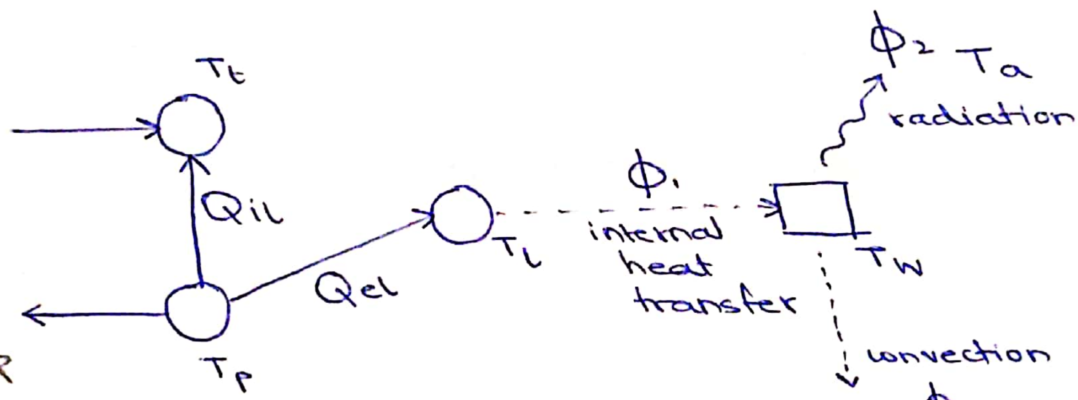
so,

$$\boxed{\frac{dT}{dt} = \frac{1}{C_p m}\left[\Sigma \dot{m}_{in}\left(C_p(T_{in} - \bar{T}) + (1-\alpha_p\bar{T})\nu(P_{in} - P)\right) + \dot{Q} - \dot{W}_s + T\alpha_p\nu\frac{dP}{dt}\right] \quad \ldots (xii)}$$

which is the general expression for a control volume.



4 different control volumes can be identified, the pump inlet, pump outlet, the wall and the leakage volume, which can be considered as nodes



fluid temp

$$\boxed{\phi_{convec} = k_{fw} A_{in} (T_f - T_w)}$$

ambient temp

$$\boxed{\phi_{rad} = \varepsilon \sigma A_{wa} (T_w^4 - T_a^4)}$$

using eq XII that was derived previously, the rate of change of temperature can be written for all 4 nodes, looking at the figure above

inlet node

$$\frac{dT_t}{dt} = \frac{1}{c_p m_p}\Big[\rho Q_{iL}\big(c_p(T_p - T_t) + (1 - \alpha_p (T_p + T_T)/2) \times (\nu)(P_p - P_T)\big) + \rho \omega D \big(c_p (T_{in} - T)\big) + T_T \alpha_p \nu_T \frac{dP_T}{dt}\Big] \quad \ldots\ldots M1$$

$\Sigma \dot{m}_{in}$

D is volume of fluid displaced per rotation outlet node

$$\frac{dT_p}{dt} = \frac{1}{c_p m_p}\Big[\rho w D\big(c_p(T_T - T_p) + (1-\alpha_p(T_p+T_T)/2) \times \nu(P_T - P_p)\big) + D w(P_p - P_T) + T_p \alpha_p \nu_p \frac{dP_p}{dt}\Big]$$

.... M2

leakage node

$$\frac{dT_l}{dt} = \frac{1}{c_p m_l}\Big[\rho Q_{el}\big(c_p(T_p - T_l) + (1-\alpha_p(T_p+T_l)/2) \times (P_p - P_l)\nu\big) - k_{fw}A_{fw}(T_l - T_w) + T_l \alpha_p \nu_L \frac{dP_l}{dt}\Big]$$

..... M3

mass node

$$\frac{dT_w}{dt} = \frac{1}{c_p m_l}\Big[k_{fw}A_{fw}(T_l - T_w) - k_{wa}A_{wa}(T_w - T_a) - \varepsilon\sigma A_{wa}(T_w^4 - T_a^4) + wD(P_p - P_T)(1-\eta_m)/\eta_m\Big]$$

..... M4

Also

$$\frac{dP}{dt} = \left(\frac{\partial P}{\partial T}\right)_{\nu} \frac{dT}{dt} + \left(\frac{\partial P}{\partial \nu}\right)_{T} \frac{d\nu}{dt}$$

If ν is assumed to be constant

$$\frac{dP}{dt} = k \frac{dT}{dt}$$

Thus,

$$\frac{dP_T}{dt} = k_1 \frac{dT_T}{dt} \quad \ldots\ldots M5$$

$$\frac{dP_P}{dt} = k_2 \frac{dT_P}{dt} \quad \ldots\ldots M6$$

$$\frac{dP_L}{dt} = k_3 \frac{dT_L}{dt} \quad \ldots\ldots M7$$

hence $M_1, M_2, M_3, M_4, M_5, M_6, M_7$ are the equations that model the temperatures of the different nodes of the piston pump.

Refrences

Base paper : Thermal-hydraulic Modeling and Simulation of Piston Pump
- LI Cheng-gong, JIAO Zong-xin

supporting papers : i) Thermal-hydraulic Modeling and Simulation of the Hydraulic system Based on the Electro-hydrostatic Actuator
- Kai Li, Zhong Lv, Kun Lu, Ping Yu

ii) Thermal-hydraulic modeling and analysis of hydraulic system by pseudo-bond graph
- HU Jun-Ping, LI Ke-jun