

Q1

$$\mu_{\tilde{A}}(x) = \left\{ \frac{1.0}{0} + \frac{1.0}{1} + \frac{0.8}{2} + \frac{0.2}{5} + \frac{0.1}{7} + \frac{0.0}{9} + \frac{0.0}{10} \right\}$$

$$\mu_{\tilde{B}}(x) = \left\{ \frac{0}{0} + \frac{0}{1} + \frac{0}{2} + \frac{0.5}{5} + \frac{0.7}{7} + \frac{0.8}{9} + \frac{1.0}{10} \right\}$$

i) algebraic sum  $\mu_{\tilde{A}}(x) + \mu_{\tilde{B}}(x) - \mu_{\tilde{A}}(x) \cdot \mu_{\tilde{B}}(x)$

$$\boxed{\mu_{\tilde{A} + \tilde{B}}(x) = \mu_{\tilde{A}}(x) + \mu_{\tilde{B}}(x) - \mu_{\tilde{A}}(x) \cdot \mu_{\tilde{B}}(x)}$$

$$\begin{aligned} \mu_{\tilde{A} + \tilde{B}}(x) &= \left\{ \frac{1+0-0}{0} + \frac{1+0-0}{1} + \frac{0.8+0-0}{2} + \frac{0.7-0.1}{5} + \frac{0.8-0.7}{7} + \frac{0.8-0}{9} + \frac{1-0}{10} \right\} \\ &= \left\{ \frac{1}{0} + \frac{1}{1} + \frac{0.8}{2} + \frac{0.6}{5} + \frac{0.1}{7} + \frac{0.8}{9} + \frac{1}{10} \right\} \end{aligned}$$

ii) algebraic product

$$\mu_{\tilde{A} \cdot \tilde{B}}(x) = \mu_{\tilde{A}}(x) \cdot \mu_{\tilde{B}}(x)$$

$$\mu_{\tilde{A} \cdot \tilde{B}}(x) = \left\{ \frac{0}{0} + \frac{0}{1} + \frac{0}{2} + \frac{0.1}{5} + \frac{0.7}{7} + \frac{0}{9} + \frac{0}{10} \right\}$$

iii) bounded sum

$$\mu_{\tilde{A} \oplus \tilde{B}}(x) = \min \{ 1, \mu_{\tilde{A}}(x) + \mu_{\tilde{B}}(x) \}$$

$$\mu_{\tilde{A} \oplus \tilde{B}}(x) = \left\{ \frac{1}{0} + \frac{1}{1} + \frac{0.8}{2} + \frac{0.7}{5} + \frac{0.8}{7} + \frac{0.8}{9} + \frac{1}{10} \right\}$$

iv) bounded difference

$$\mu_{\tilde{A} \ominus \tilde{B}}(x) = \max \{ 0, \mu_{\tilde{A}}(x) - \mu_{\tilde{B}}(x) \}$$

$$\mu_{\tilde{A} \ominus \tilde{B}}(x) = \left\{ \frac{1.0}{0} + \frac{1.0}{1} + \frac{0.8}{2} + \frac{0}{5} + \frac{0}{7} + \frac{0}{9} + \frac{0}{10} \right\}$$

Q2

$$\underline{A} = \left\{ \frac{0.1}{30} + \frac{0.2}{60} + \frac{0.3}{90} + \frac{0.4}{120} \right\}$$

$$\underline{B} = \left\{ \frac{1}{1} + \frac{0.2}{2} + \frac{0.5}{3} + \frac{0.7}{4} + \frac{0.9}{5} + \frac{0}{6} \right\}$$

$$\underline{C} = \left\{ \frac{0.33}{100} + \frac{0.65}{2000} + \frac{0.92}{300} + \frac{0.21}{400} \right\}$$

i)  $\underline{R} = \underline{A} \times \underline{B}$

$$\mu_{\underline{R}} = \min \{ \mu_{\underline{A}}(x), \mu_{\underline{B}}(y) \}$$

$$\underline{R} = \begin{matrix} & \begin{matrix} b_1 & b_2 & b_3 & b_4 & b_5 & b_6 \end{matrix} \\ \begin{matrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{matrix} & \begin{bmatrix} 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0 \\ 0.3 & 0.2 & 0.3 & 0.2 & 0.3 & 0 \\ 0.4 & 0.2 & 0.4 & 0.4 & 0.3 & 0 \end{bmatrix} \end{matrix}$$

ii)  $\underline{S} = \underline{B} \times \underline{C}$

$$\underline{S} = \begin{matrix} & \begin{matrix} c_1 & c_2 & c_3 & c_4 \end{matrix} \\ \begin{matrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{matrix} & \begin{bmatrix} 0.33 & 0.65 & 0.92 & 0.21 \\ 0.2 & 0.2 & 0.2 & 0.2 \\ 0.33 & 0.5 & 0.5 & 0.21 \\ 0.33 & 0.65 & 0.7 & 0.21 \\ 0.3 & 0.3 & 0.3 & 0.21 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

iii) Max min composition

$$\underline{T} = \begin{matrix} & \begin{matrix} c_1 & c_2 & c_3 & c_4 \end{matrix} \\ \begin{matrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{matrix} & \begin{bmatrix} 0.1 & 0.1 & 0.1 & 0.1 \\ 0.2 & 0.2 & 0.2 & 0.2 \\ 0.3 & 0.3 & 0.3 & 0.2 \\ 0.33 & 0.4 & 0.4 & 0.21 \end{bmatrix} \end{matrix}$$

iv) Max product composition

$$\underline{T} = \begin{matrix} & \begin{matrix} c_1 & c_2 & c_3 & c_4 \end{matrix} \\ \begin{matrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{matrix} & \begin{bmatrix} 0.033 & 0.065 & 0.092 & 0.021 \\ 0.066 & 0.130 & 0.184 & 0.042 \\ 0.099 & 0.195 & 0.276 & 0.063 \\ 0.132 & 0.26 & 0.368 & 0.084 \end{bmatrix} \end{matrix}$$

Q3

$$R = A \times B = \begin{bmatrix} 1 & 0.2 & 0.3 & 0 \\ 0.2 & 0.4 & 0.5 & 0.6 \\ 0.3 & 0.4 & 0.6 & 0.9 \\ 0 & 0.2 & 0.9 & 1 \end{bmatrix}$$

$$S = B \times C = \begin{bmatrix} 1 & 0.5 \\ 1 & 0.5 \\ 0.5 & 1 \\ 0.5 & 1 \end{bmatrix}$$

$$R^T = \begin{bmatrix} 1 & 0.2 & 0.3 & 0 \\ 0.2 & 0.4 & 0.4 & 0.2 \\ 0.3 & 0.5 & 0.6 & 0.9 \\ 0 & 0.6 & 0.9 & 1 \end{bmatrix}$$

i) Max min composition

$$T = \begin{bmatrix} 0.3 & 0.5 \\ 0.4 & 0.4 \\ 0.5 & 0.9 \\ 0.6 & 0.9 \end{bmatrix}$$

ii) Max-product composition

$$T = \begin{bmatrix} 1 & 0.5 \\ 0.4 & 0.4 \\ 0.5 & 0.9 \\ 0.6 & 1 \end{bmatrix}$$