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Div - J1

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ESE - Physics-2

Q. 1) a

Step Index	Graded Index
1) Discontinuity of index profile at core-cladding junction.	1) R.I of core decreases ~~gradually~~ gradually to attain R.I of cladding at core-cladding junction.
2) R.I of core is constant.	2) R.I of core decreases nearly in parabolic manner.

→ Given: $d = 50\,\mu m$

$NA = 0.15$

$\lambda_0 = 0.75\,\mu m$

To find: N

Solution: $N = \frac{V^2}{2}$

Here $V_c = \frac{\pi d}{\lambda_0}(NA)$

$= \frac{\pi \times 50}{0.75}(0.15)$

$= 10\pi$

$= 31.416$

$\therefore$ Number of modes $= \frac{(31.416)^2}{2}$

$= \underline{\underline{493.48}}$

Q.1 b) For 0.5 cm $\longrightarrow$ 3000 lines

for 1cm $\longrightarrow$ 6000 lines

$\therefore (a+b) = \dfrac{1}{\text{Number of lines per cm}}$

$= \dfrac{1}{6000} = 1.667 \times 10^{-4}$ . . . (i)

$\therefore$ Angular seperation for 2^{nd} order and $\lambda_1 = 5893 \mathring{A}$

$\sin\theta_1 = \dfrac{2 \times 5893 \times 10^{-8}}{1.667 \times 10^{-4}} = 0.70701$

$\therefore \theta_1 = 44.9928'$ . . . . (2)

And for λ_2, $\therefore \sin\theta_2 = \dfrac{2 \times 5896 \times 10^{-8}}{1.667 \times 10^{-4}}$

$\therefore \sin\theta_2 = 0.707378$

$\therefore \theta_2 = 45.022$ . . . . (3)

$\therefore$ Angular seperation $= 0.0292$. (4)

$\therefore R.P = \dfrac{\lambda}{d\lambda} = m.N$

Now, $d\lambda = 5896 - 5893 = 3\mathring{A}$

and $\lambda = \dfrac{\lambda_1 + \lambda_2}{2} = \dfrac{5896+5893}{2} = 5894.5 \mathring{A}$

$\therefore RP = \dfrac{\lambda}{d\lambda} = \dfrac{5894.5}{3} = 1964.833$

$\therefore$ Number of lines per cm on grating which will just resolve in the second order is given by

$\therefore N = \dfrac{\lambda}{d\lambda} \cdot \dfrac{1}{m} = \dfrac{1964.833}{2} = 982.41$ . . . (5)

But number of lines per cm on grating is 6000

∴ Number of lines on grating per cm is much higher than the number of lines per cm needed for resolution.

∴ Both the lines i.e. 5893 Å and 5896 Å will be well resolved in 2nd order.

Scanned with CamScanner

Q 3 a)

→ Maxwell's fourth equation:

It states that, The line integral of magnetic field intensity $\vec{H}$ around a closed path is equal to the current enclosed by that path.

$$\therefore \oint \vec{H} \cdot d\vec{l} = I = \oint_s \bar{J} \cdot ds$$

where $\vec{J}$ = current density.

Using stokes theorem, $\oint \vec{H} \cdot d\vec{l} = \int (\vec{\nabla} \times \vec{H}) d\vec{s}$

$$\therefore \int_s (\vec{\nabla} \times \vec{H}) \cdot d\vec{s} = \oint_s (\vec{J} \cdot d\vec{s})$$

$$\therefore \boxed{\vec{\nabla} \times \vec{H} = \vec{J}} \quad \text{--- (i)}$$

This equation needs to be verified for time varying field as its validity holds for steady state only. Because if it is correct then conservation of charge will be voilated. The reason is very simple: div of curl is zero.

$$\text{div } J = \text{div}(\bar{\nabla} \times \bar{H}) = 0$$

This is in contrast with the continuity equation.

$$\text{div } \bar{J} + \frac{\partial \rho}{\partial t} = 0$$

$$\text{or div } \bar{J} = -\frac{\partial \rho}{\partial t}$$

To correct this, Maxwell suggested that total current density needs an additional component ie. $\bar{J}$

$\therefore$ div $\bar{J} = -\frac{\partial \rho}{\partial t}$

But, from Maxwell first equation,

$$\bar{\nabla} \cdot \bar{D} = \rho$$

$$\therefore \text{div } \bar{J} = \frac{\partial(\nabla \cdot \bar{D})}{\partial t} = \bar{\nabla} \cdot \frac{\partial \bar{D}}{\partial t}$$

Hence, $J' = \frac{\partial \bar{D}}{\partial t}$

$\therefore$ Maxwell's fourth equation is

$$\bar{\nabla} \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

→ The additional term $\frac{\partial \bar{D}}{\partial t}$ is called Maxwell's correction and it is known as displacement current.

→ Significance :

i) It describes that a changing electric field induces a magnetic field.

2) It plays a critical role in ~~prepa~~ propagation of electromagnetic waves.

Q.3 b) Given:

$\vec{W} = y\cos ax\ \vec{\omega_x} + (y + e^{2x})\vec{\omega_z}$

To find: curl $\vec{\omega}$ at origin.

Solution: In cartesian cordinates,

$$\text{Curl } \vec{\omega} = \vec{\nabla} \times \vec{\omega} = \begin{vmatrix} \vec{W_x} & \vec{W_y} & \vec{W_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \cos ax & 0 & y + e^{2x} \end{vmatrix}$$

$$= \frac{\partial}{\partial y}(y + e^{2x})\vec{\omega_x} - \vec{W_y}\left[\frac{\partial}{\partial y}(y + e^{2x}) - \frac{\partial}{\partial z}(y\cos ax)\right]$$
$$+ \frac{\partial}{\partial y}(-y\cos ax)\vec{\omega_z}$$

$\therefore$ Curl $\vec{\omega} = \vec{W_x} - 2e^{2x}\vec{W_y} - \cos ax\ \vec{\omega_z}$

$\therefore$ At the origin $(0,0,0)$

$\vec{\nabla} \times \vec{\omega} = \vec{\omega_x} - 2\vec{W_y} - \vec{\omega_z}$

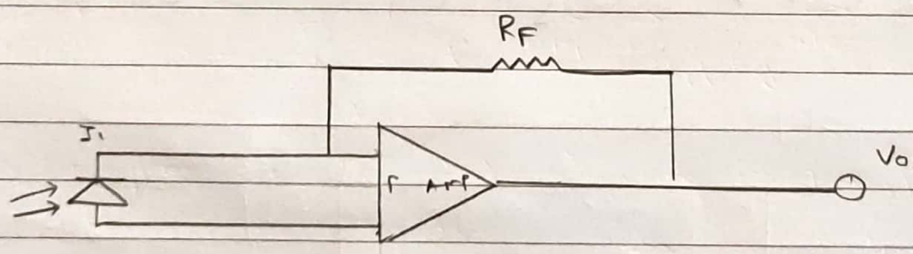
$\therefore$ Curl $\vec{\omega} = \vec{\omega_x} - 2\vec{\omega_y} - \vec{\omega_z}$

Q.4 a)

→ i) An optical sensor is a device that converts light rays into electronic signals. Similar to a photoresistor, it measures the physical quantity of light and translates it into a form read by the instrument. Usually, the optical system is a part of a larger system integrating a measuring device, a source of light and sensor itself. This is generally connected to an electrical trigger which reacts to a change in the signal within the light sensor.

ii) The operating modes of photodiode include 3 modes :

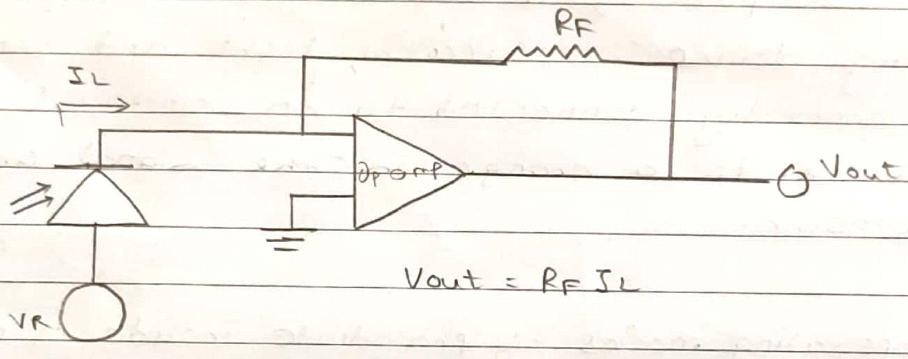
a) Photovoltaic mode: This mode is also known as zero-bias mode, in which a voltage is produced by the lightened photodiode. It gives a very small dyanamic range and non linear of voltage formed.



$V_{out} = R_F I_L$

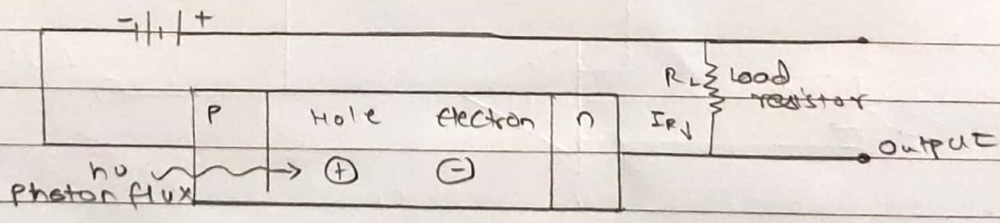
Photovoltaic mode

b) Photo conductive mode: The photodiode used in this mode is more usually reversed biased. The reverse voltage application will increase the depletion layer's width, which in turn decreases the response time and the junction capacitance. This mode is too fast and displays electronic noise.



$V_{out} = R_F I_L$

c) Avalanche Diode mode: It operates in a high resistance reverse bias condition, which permits the multiplication of an avalanche breakdown to each photo-produced electron hole pair. This outcome is an internal gain in the photodiode, which slowly increases the device response.

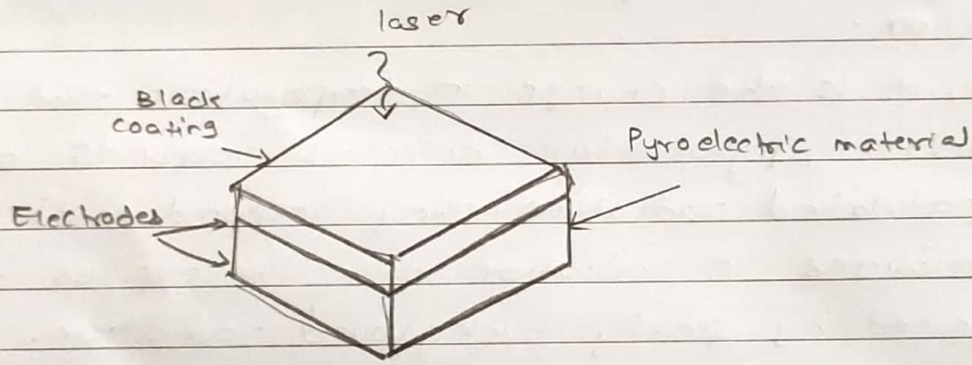


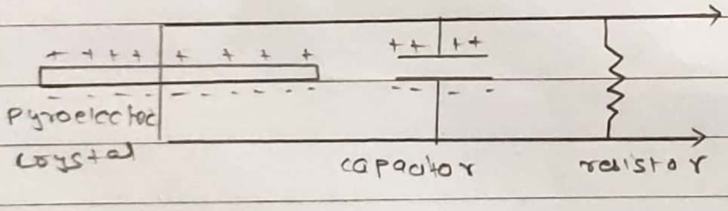
Q. 4 b)

→ i) Pyroelectric sensors:

Pyroelectric detectors are sensors for light which are based on the pyroelectric effect. They are widely used for detecting laser pulses often in the infrared spectral region, and with the potential for a very broad spectral response.

ii)





iii) Principle:

Pyroelectric crystals have a rare asymmetry due to their single polar axis. On the upper electrode of the crystal, an absorbing layer is applied. When this layer interacts with infrared radiation, the pyroelectric layer heats up and surface charge arises. If the radiation is switched off, a charge of the opposite polarity originates.

iv) Working:

1) A pyroelectric material which is usually crystalline, possesses an electric polarization, even in the absence of an applied voltage.
2) An incident ~~layer~~ laser pulse heats the crystal, which causes the material to expand and produce a change in the polarization. Charge builds up on opp surface of the crystal which generates a current flow that charges a capacitor.
3) Since, it is the change in temperature that produces the current, pyroelectric detectors responds only to pulsed or modulated radiation. They respond much more rapidly to variations in radiation than thermopiles and are unaffected by steady background radiation.

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Q. 6 a) i) $\phi = \cosh x^2 y^2 z^2$

$\therefore$ Gradient of ϕ :-

$$\bar{\nabla} \cdot \vec{\phi} = \frac{\partial \phi}{\partial x}\hat{i} + \frac{\partial \phi}{\partial y}\hat{j} + \frac{\partial \phi}{\partial z}\hat{k}$$

$$= 2xy^2z^2 + x^2y^2z^2\hat{i} + 2yx^2z^2 \cosh x^2y^2z^2\,\hat{j} + 2zx^2y^2 \sinh x^2y^2z^2\hat{k}$$

$$= 2xyz \sinh x^2y^2z^2 [yz\hat{i} + xz\hat{j} + xy\hat{k}]$$

ii) $\phi = x^2 + y + z^2$

$$\therefore \phi = \frac{\partial \phi}{\partial x}\hat{i} + \frac{\partial \phi}{\partial y}\hat{j} + \frac{\partial \phi}{\partial z}\hat{k}$$

$$= 2x\hat{i} + \hat{j} + 2z\hat{k}$$

$$= 2x\hat{i} + \hat{j} + 2z\hat{k}$$

$$B = x^2 z\,\vec{a_x} - y^2z^2\bar{a}_y + xy^2\,\vec{a_z}$$

$$\therefore \vec{\nabla} \cdot \vec{B} = \frac{\partial B}{\partial x}\hat{i} + \frac{\partial B}{\partial y}\hat{j} + \frac{\partial B}{\partial z}\hat{k}$$

$$= 2xz\,\overline{a_x} - 2yz^2\,\vec{a_y} + xy^2\,\vec{a_z}$$

$\therefore$ At $(2,-1,2)$ $\bar{\nabla} \cdot \vec{B} = 8\overline{a_x} + 8\overline{a_y} + 2\overline{a_z}$

Q.6 b)

→ 1) Piezo-electric oscillator:

working:

1) The plate current passing through coil L_2 set up magnetic field, which is linked with coil L. As the current through coil L_2 varies the magnetic flux, linked with the coil L changes an emf induced in it. The emf sends an a.c. current in the loop, which charges the parallel plates A and B.

2) As the plates are charged a changing electric field develops across the quartz crystal.

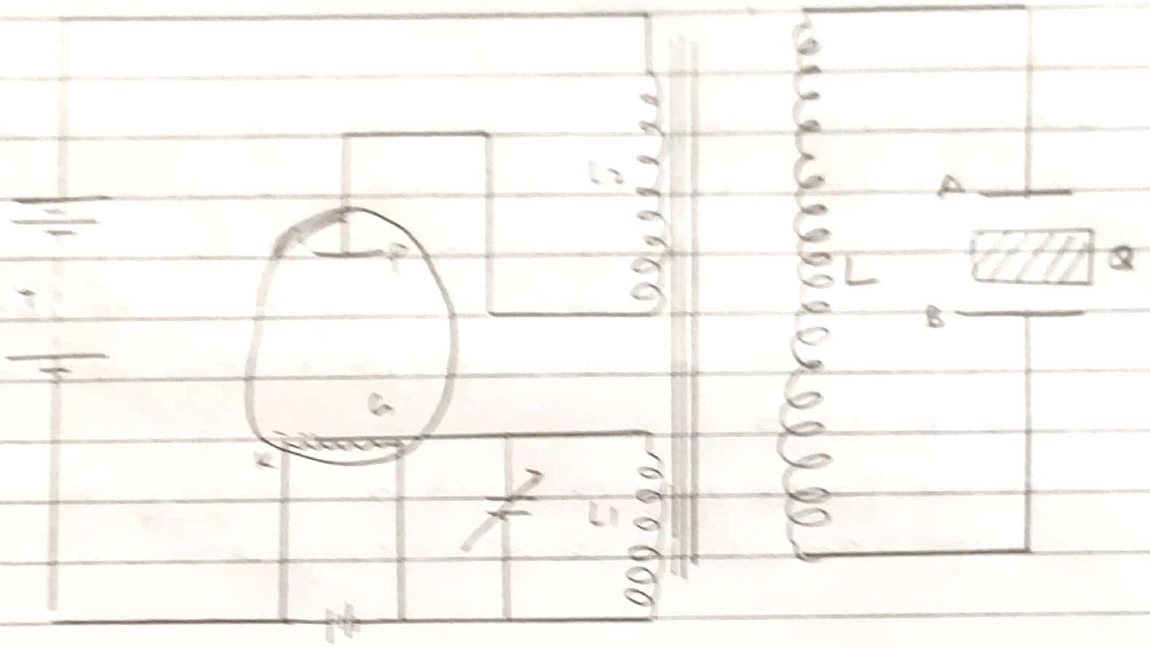
$$\therefore f = \frac{1}{2\pi\sqrt{LC}}$$

3) The capacity of variable condenser C is adjusted so that frequency of circuit is tuned to natural frequency of crystals.

$$\text{Thus} \quad n = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2l}\sqrt{\frac{4}{P}}$$

4) Now the quartz crystal is set into an mechanical vibrations and ultrasonic waves are produced.

Diagram:



Piezoelectric Oscillator.