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MAEER's MIT

Discrete Structures

Tutorial 4

Solutions:

1) $f(n) = n^2$ and $g(n) = 2^n$

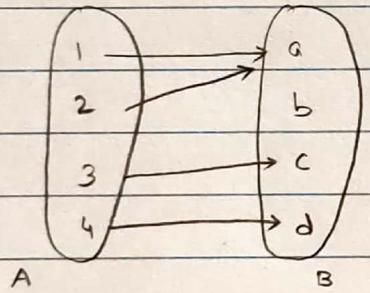
$\therefore fof = f(f(n)) = f(n^2) = n^4$

$\therefore gog = g(g(n)) = g(2^n) = 2^{2^n}$

$\therefore fog = f(g(n)) = f(2^n) = 2^{2^n}$

$\therefore gof = g(f(n)) = g(n^2) = 2^{n^2}$

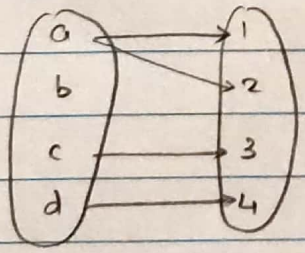
2) $f = \{(1,a), (2,a), (3,c), (4,d)\}$



Here f is a function as every element of set A has a unique value and every element is mapped.

$\therefore f^{-1} : B \rightarrow A$

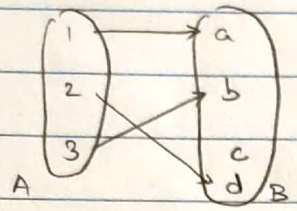
$\therefore f^{-1} = \{(a,1), (a,2), (c,3), (d,4)\}$



f^{-1} is not a function as 'b' is not mapped and 'a' has two values.

$\therefore$ f is a function and f^{-1} is not a function.

3) (i) $f = \{(1,a), (2,d), (3,b)\}$

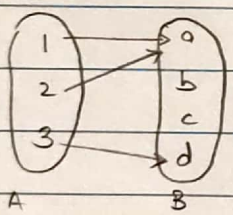


f is a function as every element of set A is mapped and has unique value.

It is not a surjective function as range of f ≠ co-domain. Hence, it is not bijective also.

It is a one-to-one function as no two elements of A point towards the same element of B.

(ii) $g = \{(1,a), (2,a), (3,d)\}$



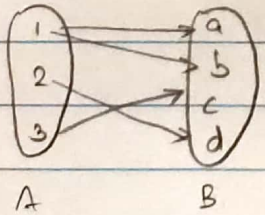
g is a function as every element of set A is mapped and has unique value.

∴ It is not a onto function as range of g ≠ co-domain.

It is not a one-to-one function as '1' and '2' of set A point towards one element 'a'. It is not bijective also.

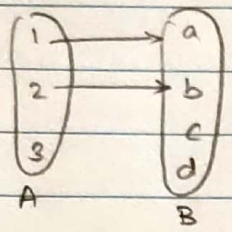
It is a into function and everywhere defined function as domain of g = A

(iii) $h = \{(1,a), (1,b), (2,d), (3,c)\}$



h is not a function as '1' does not have a unique value, it is mapped to two elements of set B (a, b)

(iv) $j = \{(1,a),(2,b)\}$



j is not a function as '3' is not mapped with any element of set B.

4) $f(x) = x^3$, $g(x) = 4x^2 - 1$, $h(x) = 7x - 2$

$hog = h(g(x)) = h(4x^2-1) = 7(4x^2-1) - 2 = 28x^2 - 7 - 2$

$\therefore hog = 28x^2 - 9$

~~gof~~ $go(hof) = go[h(f(x))]$

$= g[h(x^3)]$

$= g[7x^3 - 2]$

$= 4(7x^3-2)^2 - 1$

$= 4(49x^6 - 28x^3 + 4) - 1$

$= 196x^6 - 112x^3 + 16 - 1$

$\therefore go(hof) = 196x^6 - 112x^3 + 15$

Let $hog = s$, where s is function $= 28x^2 - 9$

and $go(hof) = r$, where r is a function $= 196x^6 - 112x^3 + 15$

$\therefore hog$ of $go(hof) = s \circ r = s(r(x)) = s(196x^6 - 112x^3 + 15)$

$= 28(196x^6 - 112x^3 + 15)^2 - 9$

$\therefore (hog)$ of $go(hof) = 28(196x^6 - 112x^3 + 15)^2 - 9$