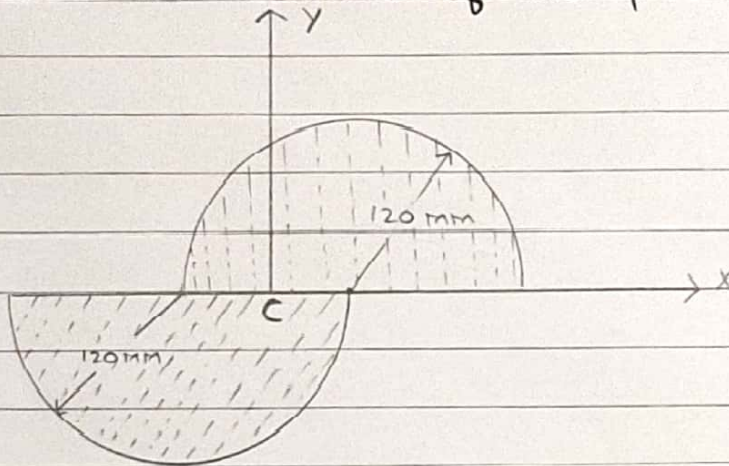
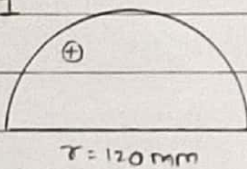
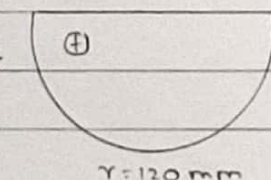
MechanicsAssignment no. 1-1 (centroid)

1-2-4) Locate the centroid of the plane area shown.



→	Component	Area (mm <sup>2</sup> )	x (mm)	y (mm)	Ax	Ay
1		$\frac{\pi r^2}{2}$ $= \frac{3.14 \times (120)^2}{2}$ $= 22608$	60	$\frac{4R}{3\pi}$ $= \frac{4 \times 120}{3\pi}$ $= 50.96$	1356480	1152103.68
2		22608	-60	-50.96	-1356480	-1152103.68
		$\Sigma A = 45,216$			$\Sigma Ax = 0$	$\Sigma Ay = 0$

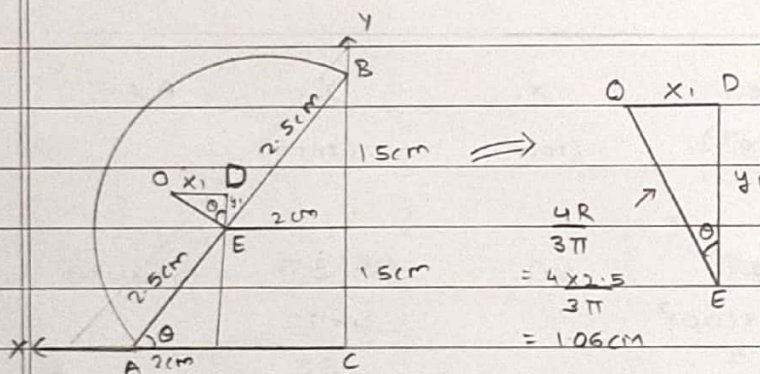
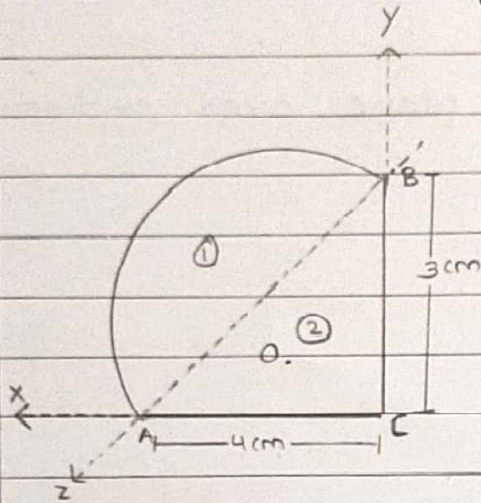
$$\therefore \bar{x} = \frac{\Sigma Ax}{\Sigma A} = 0, \quad \bar{y} = \frac{\Sigma Ay}{\Sigma A} = 0$$

$\therefore$  Centroid C is at (0,0) mm





1.2.5) Find the centroid of area



$$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.87^\circ$$

$$\sin \theta = \frac{x_1}{1.06}$$

$$\cos \theta = \frac{y_1}{1.06}$$

$$\therefore \sin 36.87 = \frac{x_1}{1.06}$$

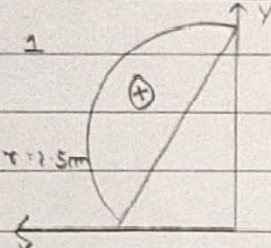
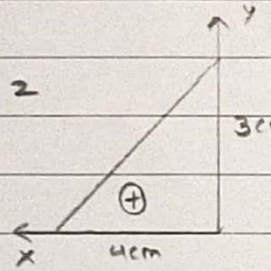
$$\cos 36.87 = \frac{y_1}{1.06}$$

$$\therefore x_1 = 0.64 \text{ cm}$$

$$\therefore y_1 = 0.85 \text{ cm}$$



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Component	Area (cm <sup>2</sup> )	x (cm)	y (cm)	Ax	Ay
	$\frac{\pi R^2}{2}$ $= \frac{\pi \times 2.5^2}{2}$ $= 9.82$	$2 + x_1$ $= 2 + 0.64$ $= 2.64$	$1.5 + y_1$ $= 1.5 + 0.85$ $= 2.35$	25.92	23.08
	$\frac{1}{2} \times 3 \times 4$ $= 6$	1.333	1	8	6
	$\Sigma A = 15.82$			$\Sigma Ax = 33.92$	$\Sigma Ay = 29.08$

$$\therefore \bar{x} = \frac{\Sigma Ax}{\Sigma A}$$

$$\bar{x} = \frac{33.92}{15.82}$$

$$\bar{x} = 2.14 \text{ cm}$$

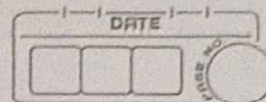
$$\therefore \bar{y} = \frac{\Sigma Ay}{\Sigma A}$$

$$= \frac{29.08}{15.82}$$

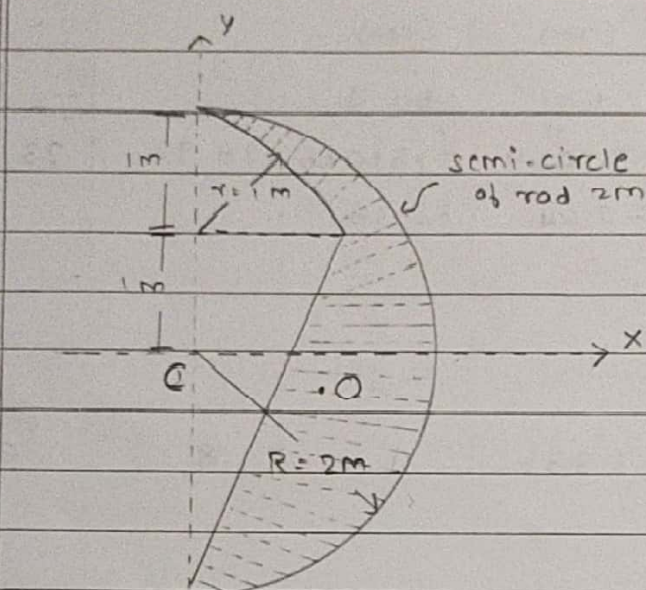
$$\bar{y} = 1.84 \text{ cm}$$

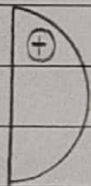
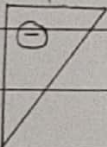
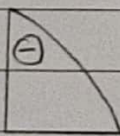
Centroid O is at (2.14, 1.84) cm





1.2.12) Find centroid.



Component	Area (m <sup>2</sup> )	x (m)	y (m)	Ax	Ay
1) 	$\frac{\pi R^2}{2}$ $= \frac{\pi \times 2 \times 2}{2}$ $= 6.28$	$\frac{4R}{3\pi}$ $= \frac{4 \times 2}{3\pi}$ $= 0.849$	0	5.33172	0
2) 	$\frac{1 \times 1 \times 3}{2}$ $= -1.5$	0.333	0	-0.4995	0
3) 	$-\frac{\pi R^2}{4}$ $= -\frac{\pi \times 1^2}{4}$ $= -0.785$	0.424	1.424	-0.33284	-1.11784
$\Sigma A = 3.995$				$\Sigma Ax = 4.49938$	$\Sigma Ay = -1.11784$



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$$\begin{aligned}\therefore \bar{x} &= \frac{\sum Ax}{\sum A} \\ &= \frac{4.49938}{3.995} \\ &= 1.126 \text{ m}\end{aligned}$$

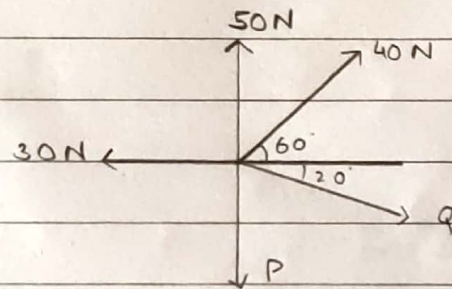
$$\begin{aligned}\therefore \bar{y} &= \frac{\sum Ay}{\sum A} \\ &= \frac{-1.11784}{3.995} \\ &= -0.28 \text{ m}\end{aligned}$$

$\therefore$  Centroid O is at  $(1.126, -0.28) \text{ m}$



### System of Coplanar forces

1.1.5) Five concurrent coplanar forces act on a body. Find the force P and Q such that the resultant of the five forces is zero.



→ According to given condition,

For Resultant (R) = 0

$$\therefore \sum F_x = 0, \sum F_y = 0$$

$$\therefore \sum F_x = 40 \cos 60 + Q \cos 20 - 30 = 0$$

$$\therefore Q \cos 20 = 30 - 40 \cos 60$$

$$\therefore Q = 10.6417 \text{ N}$$

$$\sum F_y = 0$$

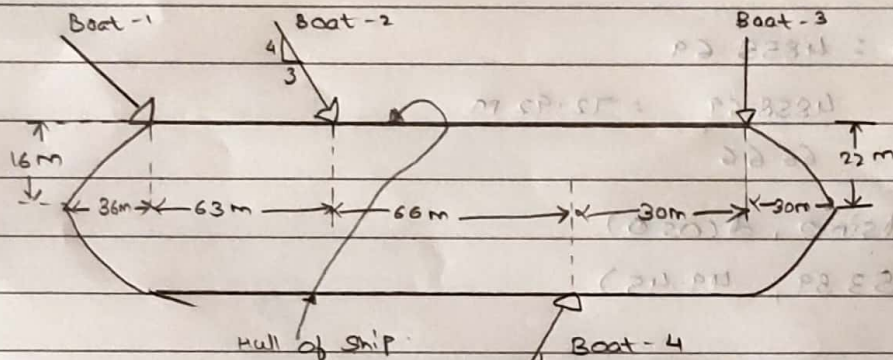
$$\therefore 40 \sin 60 - Q \sin 20 + 50 - P = 0$$

$$\therefore P = 40 \sin 60 - Q \sin 20 + 50$$

$$\therefore P = 81.0013 \text{ N}$$

$\therefore$  Force P is 81.0013 N and Force Q is 10.6417 N.

1.1.13) Four tugboats exert 25 kN to bring an ocean liner to pier. Determine the point on the hull where a single, more powerful tugboat should push to produce the same effect as the original four boats.



→

$$\tan \alpha = \frac{4}{3}$$

$$\tan \beta = 1$$

$$\therefore \beta = 45^\circ$$

$$\alpha = 53.13^\circ$$

$$\begin{aligned} \sum F_x &= 25 \cos 60^\circ + 25 \cos 53.13^\circ + 25 \cos 45^\circ \\ &= 45.17 \text{ kN} (\rightarrow) \end{aligned}$$

$$\begin{aligned} \sum F_y &= (-25 \sin 60^\circ - 25 \sin 53.13^\circ - 25 + 25 \sin 45^\circ) \\ &= -48.97 \text{ kN} \\ &= 48.97 \text{ kN} (\downarrow) \end{aligned}$$

$$R = \sqrt{F_x^2 + F_y^2} = \sqrt{(45.17)^2 + (48.97)^2} = 66.66 \text{ kN}$$

$$\theta = \tan^{-1} \left( \frac{\sum F_y}{\sum F_x} \right) = \tan^{-1} \left( \frac{48.97}{45.17} \right)$$

$$\therefore \theta = 47.3^\circ$$



$$\sum M_O = R \times d$$

$$\sum M_O = 25 \cos 60 \times 16 + 25 \sin 60 \times 36 + 25 \cos 53.13 \times 22 + 25 \sin 53.13 \times 99$$

$$- 25 \cos 45 \times 22 + 25 \sin 45 \times 16.5 + 25 \times 195$$

$$\sum M_O = 4858.69 \text{ kNm}$$

$$\therefore R \times d = 4858.69$$

$$d = \frac{4858.69}{66.66} = 72.92 \text{ m}$$

$$\therefore P = (d \sin \theta, d \cos \theta)$$

$$= (53.59, 49.45)$$

$$OB = \frac{\sum M_O}{\sum F_y} = \frac{4858.69}{48.97} = 99.218 \text{ m}$$

$$AC = 22 \text{ m}$$

$$BC = \frac{AC}{\tan \theta} = \frac{22}{\tan 47.3} = 20.295$$

Point on hull where a single resultant force can be applied is Point A

$$A = (OC, AC)$$

$$= (OB - BC, AC)$$

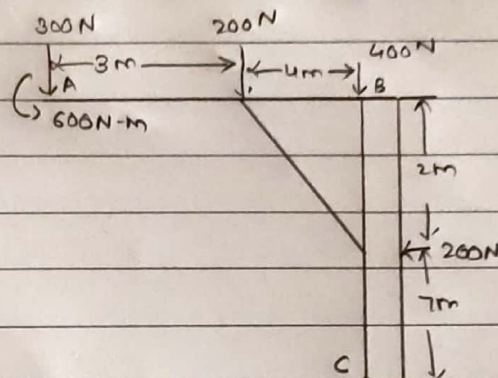
$$= (99.218 - 20.295, 22)$$

$$= [78.917, 22]$$

$\therefore [78.917, 22]$  is the point on hull where single more powerful tugboat  $[R = 66.66 \text{ kN}, \theta = 47.3]$  should push to produce same efforts as original 4 boats.



1.1.29) Replace the loading on the frame by a force and moment at point A.



$$\rightarrow \sum F_x = -200 \text{ N} = 200 \text{ N} (\leftarrow)$$

$$\sum F_y = -300 - 200 - 400 = -900 \text{ N} = 900 \text{ N} (\downarrow)$$

$$R = \sqrt{F_x^2 + F_y^2} = 921.95 \text{ N}$$

$$\theta = \tan^{-1} \left( \frac{F_y}{F_x} \right) = 77.47^\circ$$

$M_0$  = Moment about A (clockwise)

$$\therefore M_0 = 200 \times 3 + 400 \times 7 + 200 \times 2 - 600$$

$$= 3200 \text{ Nm} (\curvearrowright)$$

$\therefore$  The loading frame can be replaced by a force of 921.97 N and an angle of  $77.47^\circ$  and moment of 3200 N-m clockwise.

