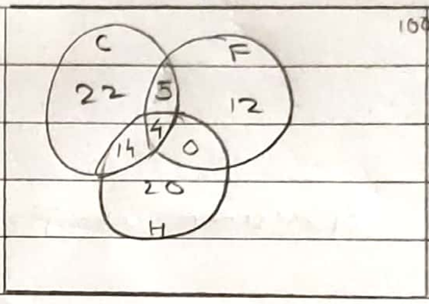
Name: Ayush Jain
SAP ID: 60004200132
Div: B1
Computer Engineering

21/02/22

Discrete Structures

Solutions:

→ 1 a) Let C = sportsmen who play cricket
F = sportsmen who play football
H = sportsmen who play hockey



∴ Total sportsmen = 100

∴ $n(C) = 45$
$n(F) = 21$
$n(H) = 38$
$n(C \cap H) = 18$
$n(C \cap F) = 9$
$n(F \cap H) = 4$

NONE = 23
∴ $|C \cup F \cup H| = 100 - 23 = 77$

∴ $\cancel{|C \cap F \cap H| = |C \cup F \cup H| = 77}$

~~den~~

∴ $|C \cup F \cup H| = n(C) + n(F) + n(H) - n(C \cap H) - n(C \cap F) - n(F \cap H) + |C \cap H \cap F|$

∴ $77 = 45 + 21 + 38 - 18 - 9 - 4 + |C \cap H \cap F|$

∴ $|C \cap F \cap F| = 4$

$\therefore$ Sportsmen who play only cricket : $|C| - |C \cap F| - |C \cap H| + |C \cap F \cap H|$

$= 45 - 9 - 18 + 4 \cancel{8}$

$= 22$

$\therefore$ Sportsmen who play only football : $|F| - |F \cap C| - |F \cap H| + |C \cap F \cap H|$

$= 21 - 9 - 4 + 4$

$= 12$

$\therefore$ Sportsmen who play only Hockey : $|H| - |C \cap H| - |F \cap H| + |C \cap F \cap H|$

$= 38 - 18 - 4 + 4$

$= 20$

$\therefore$ Sportsmen who play only one game

= only cricket + only football + only hockey

$= 22 + 12 + 20$

$= 54$

$\therefore$ Sportsmen who play exactly one of the game is 54

60004200132

→ 1 b) p : "Swimming at the New Jersey shore is allowed"
q : "Sharks have been spotted near the shore".

i) $\neg p \vee q$

Swimming at the New Jersey shore is not allowed or sharks have been spotted near the shore.

ii) $p \rightarrow \neg q$

If swimming at the New Jersey Shore is allowed then Sharks have not been spotted near the store.

iii) ~~$\neg p \rightarrow q$~~ $\neg q \rightarrow p$

If sharks have not been spotted near the shore then swimming at new Jersey store is allowed.

iv) $\neg p \rightarrow \neg q$

If swimming at New Jersey shore is not allowed then sharks have not been spotted near the shore.

v) ~~$p \leftrightarrow \neg q$~~ $p \leftrightarrow \neg q$

Swimming at New Jersey shore is allowed if and only if Sharks have not been spotted near the shore.

→Q. 3 a) Proof:

1) For $n=2$, we have

$5^n - 4n - 1 = 5^2 - 4(2) - 1 = 16$

$\therefore$ 16 is divisible by 16

2) Now we assume that $5^n - 4n - 1$ is divisible by 16

Hence, it can be written as $5^n - 4n - 1 = 16k$ for some natural k.

3) We need to prove that replacing $n \rightarrow n+1$, will result in the number that is divisible by 16.

$\therefore 5^{n+1} - 4(n+1) - 1$

$= 5^n \cdot 5 - 4n - 4 - 1$

From (2), we have

$5^n = 16k + 4n + 1$

Hence,

$= 5(16k + 4n + 1) - 4n - 5$

$= 80k + 20n + 5 - 4n - 5$

$= 80k + 16n$

$= 16(5k + n)$

$\because$ It is a 16 times an integer.

$\therefore 5^{n+1} - 4(n+1) - 1$ is divisible by 16.

$\therefore 5^n - 4n - 1$ is exactly divisible by 16.

Hence proved by mathematical induction.

6000420013 2

→ 3 b) Box contains : 4 white (W)

6 Black (B)

5 Red (R)

2 Balls are selected by a Person.

<u>To find</u>: a) Both are of same colour.

b) Both are not red.

<u>Solution</u>: Total balls = 15

$P(W) = \frac{4}{15}$, $P(R) = \frac{5}{15}$, $P(B) = \frac{6}{15}$

a) P(Event of 2 balls) = $^{15}C_2$

P(2 white balls) = $^{4}C_2$

P(2 red balls) = $^{5}C_2$

P(2 black balls) = $^{6}C_2$

$\therefore$ P (both of same color) = $\frac{^{4}C_2 + {^{5}C_2} + {^{6}C_2}}{^{15}C_2}$

$= \frac{6 + 10 + 15}{105}$

$= \frac{31}{105}$

b) P(Both are not red) = 1 - P(Both are red)

$$= 1 - \frac{5C_2}{15C_2}$$

$$= 1 - \frac{2}{21}$$

$$= \frac{19}{21}$$

→ Q. 2) $A = \{1, 2, 3, 4, 5\}$

$R = \{(1,4), (2,1), (2,5), (2,4), (4,3), (5,3), (3,2)\}$

As W_R is a 5×5 matrix, $n = 5$. We need to compute till W_5

Let M_R denote matrix representation of R.

$$W_0 = M_R = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 0 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 1 & 1 \\ 3 & 0 & 1 & 0 & 0 & 0 \\ 4 & 0 & 0 & 1 & 0 & 0 \\ 5 & 0 & 0 & 1 & 0 & 0 \end{array}$$

Column 1 → $p_i = 2$

Row 1 → $q_i = 4$

So we put 1 in $(p_i, q_i) = (2,4)$

$$\therefore W_1 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Column 2 → $p_i = 3$

Row 2 → $q_i = 1, 4, 5$

So we put 1 in (p_i, q_i):

$(3,1), (3,4), (3,5)$

$$\therefore W_2 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Column 3 → $p_i = 4, 5$

Row 3 → $q_i = 1, 2, 4, 5$

So we put 1 in (p_i, q_i):

$(4,1), (4,2), (4,4), (4,5),$

$(5,1), (5,2), (5,4), (5,5)$

60004200132

$$\therefore W_3 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Column 4 → p_i = 1, 2, 3, 4, 5

Row 4 → q_i = 1, 2, 3, 4, 5

So we put 1 in (p_i, q_i):

(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (2,5), (3,1), (3,2), (3,3), (3,4), (3,5), (4,1), (4,2), (4,3), (4,4), (4,5), (5,1), (5,2) (5,3), (5,4), (5,5)

$$\therefore W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Here, since all elements are 1

$W_5 = W_4$

$\therefore W_5 = W_4$

Thus, the transitive closure of R is:

{(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (2,5), (3,1), (3,2), (3,3), (3,4), (3,5), (4,1), (4,2), (4,3), (4,4), (4,5), (5,1), (5,2), (5,3), (5,4), (5,5)}

5 a) The given recurrence relation is

$a_r + 6a_{r-1} + 12a_{r-2} + 8a_{r-3} = 0$ —— (i)

This is third order linear homogeneous recurrence relation with constant coefficients.

Let $a_r = r^n$ be solution of (1)

The characteristic equation is

$r^3 + 6r^2 + 12r + 8 = 0$

$(r+2)^3 = 0$

$\therefore r = -2, -2, -2$

The roots are real and repeated.

Hence, the general solution be,

$a_n = b_1\alpha^n + b_2 n \alpha^n + b_3 n^2 \alpha^n$

Here $\alpha = -2$

$\therefore a_n = b_1(-2)^n + b_2(-2)^n n + b_3 n^2 (-2)^n$

→ 5 b) Let $a_r = b_r + c_r$

where $b_r = 3^r$

and $c_r = 4^{r+1}$

Let $A(z), B(z), C(z)$ be the generating functions of a, b and c

For $b_r = 3^r$, the corresponding generating function is

$$B(z) = \frac{1}{1-3z}$$

For $c_r = 4^{r+1}$

$$C(z) = \frac{4}{1-4z}$$

$\therefore$ $A(z) = B(z) + C(z)$

$$A(z) = \frac{1}{1-3z} + \frac{4}{1-4z}$$

$$A(z) = \frac{(5-16z)}{(1-3z)(1-4z)}$$

$\therefore$ Generating function of $a_r = 3^r + 4^{r+1}$ is $\frac{(5-16z)}{(1-3z)(1-4z)}$

60004200132

→ 6 a) $f(x) = x^3+1$, $g(x) = \sqrt[3]{x-1}$, $f, g : R \to R$

$(g o f)(x) = g(f(x))$

$= \sqrt[3]{f(x)-1}$

$= \sqrt[3]{(x^3+1)-1}$

$= \sqrt[3]{x^3}$

$\therefore \boxed{(g o f)(x) = x}$

$\therefore (g o f)(1) = g(f(1))$

Now, $f(1) = (1)^3+1 = 2$

$\therefore g(f(1)) = g(2) = \sqrt[3]{2-1}$

$= \sqrt[3]{1}$

$= 1$

$\therefore \boxed{(g o f)(1) = 1}$

$(g o f)(2) = g(f(2))$

Now, $f(2) = 2^3+1 = 8+1 = 9$

$g(f(2)) = g(9) = \sqrt[3]{9-1} = \sqrt[3]{8} = 2$

$\therefore \boxed{(g o f)(2) = 2}$

$(g o f)(3) = g(f(3))$

Now, $f(3) = 3^3+1 = 27+1 = 28$

$g(f(3)) = g(28) = \sqrt[3]{28-1} = \sqrt[3]{27} = 3$

$\therefore \boxed{(g o f)(3) = 3}$

→ 6 b) $A = \{1, 2, 3, 4, 6, 9, 12\}$

$R = \{(a,b) : a, b \in A$, b is exactly divisible by a $\}$

$\therefore R = \{(1,1), (1,2), (1,3), (1,4), (1,6), (1,9), (1,12), (2,2), (2,4), (2,6), (2,12), (3,3), (3,6), (3,9), (3,12), (4,4), (4,12), (6,6), (6,12), (9,9), (12,12)\}$

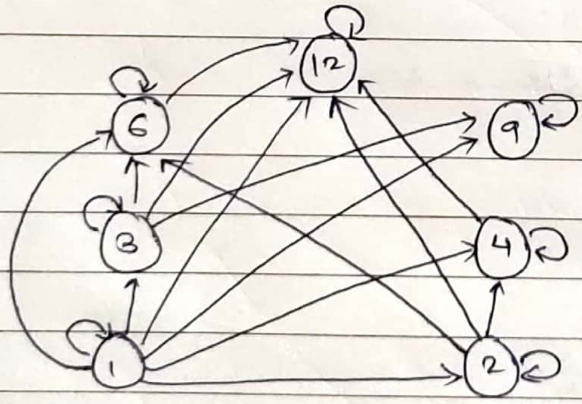
(i) Relation R is reflexive, since (1,1), (2,2), (3,3), (4,4), (6,6), (9,9), (12,12) exist in R.

(ii) Relation R is anti-symmetric, since for any x, y, if xRy and yRx are in set R, they x=y only.

(iii) Relation R is transitive, since if x divisible y and y divides z, then x will divides z for all x, y, z.

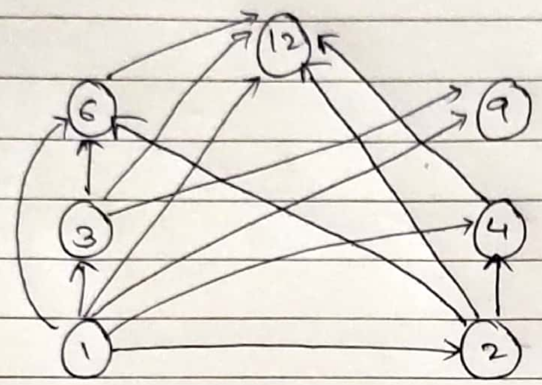
Hence, relation R is Partially ordered set, i.e. POSET

HASSE DIAGRAM

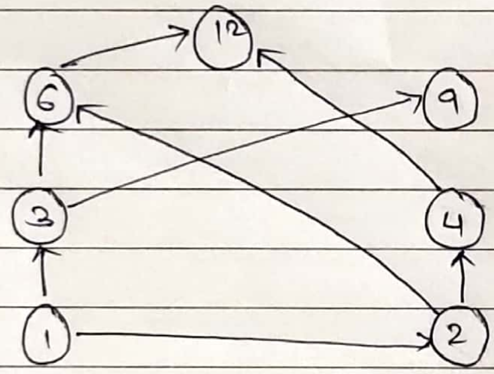


6000420013 2

1. Deleting all reflexive edges



2. Deleting all transitive edges.



3. Replacing circles with dots and omitting arrows.