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Maths - Tutorial 1

1) If $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$, find eigen values and eigenvectors for the following matrices

$A^T, A^{-1}, A^0, 4A^{-1}, A^2, A^2-2A+I, A^3+12I, Adj(A)$

→ Characteristic equation of A is:

$|A - \lambda I| = 0$

$\lambda^3 - 11\lambda^2 + (14+8+14)\lambda - 36 = 0$

$\lambda^3 - 11\lambda^2 + 36\lambda - 36 = 0$

$\lambda = 6, 3, 2$

For $\lambda_1 = 6$,

$(A - 6I) X_1 = 0$

$\cancel{\begin{bmatrix} -3 & -7 & -5 \\ -7 & -1 & \end{bmatrix}}$

$\begin{bmatrix} -3 & -1 & 1 \\ -1 & -1 & -1 \\ 1 & -1 & -3 \end{bmatrix} X_1 = 0 \Rightarrow$ ~~$R_3 = R_1 + R_3$~~ $R_3 = R_3 + R_1$ $\begin{bmatrix} -3 & -1 & 1 \\ -1 & -1 & -1 \\ -2 & -2 & -2 \end{bmatrix}$

$R_2 = R_2 + R_1$

$\begin{bmatrix} -3 & -1 & 1 \\ -4 & -2 & 0 \\ 1 & -1 & -3 \end{bmatrix}$

$R_3 = R_3 - 2R_2$, $R_2 = R_2 + R_1$

$\begin{bmatrix} -3 & -1 & 1 \\ -4 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\therefore -3x_1 - x_2 + x_3 = 0$

~~$-x_1 - x_2 - x_3 = 0$~~ $-4x_1 - 2x_2 = 0 \Rightarrow -2x_1 - x_2 = 0$

$x_2 = -2x_1$

For $\lambda_2 = 3$,

$(A - 3I) X_2 = 0$

$$\begin{bmatrix} 0 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 0 \end{bmatrix} X_2 = 0$$

$R_3 = R_3 + R_2$

$R_3 = R_3 + R_1$

$$\begin{bmatrix} 0 & -1 & 1 \\ -1 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix} X_2 = 0$$

$-x_2 + x_3 = 0$

$-x_1 + 2x_2 - x_3 = 0$

$\boxed{x_2 = x_3}$

$-x_1 + 2x_2 - x_2 = 0$

$-x_1 + x_2 = 0$

$\boxed{x_1 = x_2}$

$$\therefore X_2 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_2 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\therefore X_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ are eigen vector for $\lambda_2 = 3$

For $\lambda_2 = 6$

$|A - 6I|X_2 = 0$

$$\begin{bmatrix} -3 & -1 & 1 \\ -1 & -1 & -1 \\ 1 & -1 & -3 \end{bmatrix} X_2 = 0$$

$R_3 + R_1$

$$\longrightarrow \begin{bmatrix} -3 & -1 & 1 \\ -1 & -1 & -1 \\ -2 & -2 & -2 \end{bmatrix} X_2 = 0$$

$R_2 - R_3/2$

$$\longrightarrow \begin{bmatrix} -3 & -1 & 1 \\ 0 & 0 & 0 \\ -2 & -2 & -2 \end{bmatrix} X_2 = 0$$

$-3x_1 - x_2 + x_3 = 0$

$-\cancel{2(x_1+x_2)}\ -2x_1 - 2x_2 - 2x_3 = 0, \Rightarrow x_1 + x_2 + x_3 = 0$

$x_2 = -x_1 - x_3$

$x_3 = -x_1 - x_2$

$x_3 = -x_3 - x_2$

$2x_3 = -x_2$

$2x_3 = -x_2/$

$-3x_1 - (-x_1 - x_3) + x_3 = 0$

$-3x_1 + x_1 + x_3 + x_3 = 0$

$-2x_1 + 2x_3 = 0$

$\boxed{x_3 = x_1}$

$$\therefore X_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ -2x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$\therefore X_2 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ is eigen vector for $\lambda_2 = 6$

For $\lambda_3 = 2$,

$(A - 2I)X_2 = 0$

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 1 \end{bmatrix} X_2 = 0$$

$R_3 - R_1$

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 3 & -1 \\ 0 & 0 & 0 \end{bmatrix} X_2 = 0$$

$\therefore x_1 - x_2 + x_3 = 0 \implies x_2 = x_1 + x_3$

$-x_1 + 3x_2 - x_3 = 0 \implies x_2 = 0$

$\therefore x_1 = -x_3$

$$X_3 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -x_3 \\ 0 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$\therefore X_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ are eigenvector for $\lambda_3 = 2$

(i) A^T will have same eigen values and eigen vectors as that of A

$\therefore$ Eigen values of A^T = 6, 3, 2

Eigen vectors of A^T : $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

(ii) A^{-1} will have eigen values as $\frac{1}{\lambda}$

$\therefore$ Eigen values of A^{-1} = $\frac{1}{6}, \frac{1}{3}, \frac{1}{2}$

Eigen vector will be same as A

$\therefore$ Eigen vector of A^{-1} = $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

(iii) $A^{\theta} = (\bar{A})^T$

Eigen values of A^{θ} will be ~~as~~ same as λ

$\therefore$ Eigen values of A^{θ} : 6, 3, 2

Eigen vectors will be same as of A

$\therefore$ Eigen vectors of A^{θ} = $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

(iv) $4A^{-1}$ will have eigen value as $4/\lambda$

$\therefore$ Eigen value of $4A^{-1} = \frac{4}{6}, \frac{4}{3}, \frac{4}{2}$

Eigen vector $= \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

(v) A^2 will have eigen values λ^2

$\therefore$ Eigen value of $A^2 = 36, 9, 4$

Eigen ~~values~~ vector of $A^2 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

(vi) $A^2 - 2A + I$

It will have eigen value as $\lambda^2 - 2\lambda + I$

$\therefore$ eigen value of $A^2 - 2A + I = 238, 39, 20$

Eigen vector of $A^2 - 2A + I = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

(vii) $A^3 + 12I$

It will have eigen value as $\lambda^3 + 12$

$\therefore$ Eigen values are $228, 39, 20$ for $A^3 + 12I$

Eigen vectors for the same are $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

(viii) Adj(A)

It will have eigen values as $\frac{|A|}{\lambda}$

$\therefore$ Eigen values are 6, 12, 18 for Adj(A)

Eigen vector are $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ for Adj(A)

2) Find the characteristic equation of $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$

and hence, find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$ by using Cayley - Hamilton theorem

→ The characteristic equation of A

$\lambda^3 - (5)\lambda^2 + (2 + 3 + 2)\lambda - 3 = 0$

$\lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0$

By ~~cla~~ Cayley - Hamilton theorem,

$A^3 - 5A^2 + 7A - 3I = 0$

Now,

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$$\cancel{\lambda^3-5\lambda^2+7\lambda-3 \;\big)\; \lambda^8-5\lambda^7+7\lambda^6-3\lambda^5+\lambda^4-5\lambda^3+8\lambda^2-2\lambda+1}$$

$$\begin{array}{r|l}
 & x^5+x \\
x^3-5x^2+7x-3 & x^8-5x^7+7x^6-3x^5+x^4-5x^3+8x^2-2x+1 \\
 & -\;x^8-5x^7+7x^6-3x^5 \\
\hline
 & \qquad\qquad x^4-5x^3+8x^2-2x+1 \\
 & \qquad\qquad -\;x^4-5x^3+7x^2-3x \\
\hline
 & \qquad\qquad\qquad x^2+x+1
\end{array}$$

$\therefore$ By division Algorithm,

$$(x^8-5x^7+7x^6-3x^5+x^4-5x^3+8x^2-2x+1) = (x^3-5x^2+7x-3)(x^5+x) + (x^2+x+1)$$

$$= 0 + x^2+x+1$$

$$\therefore A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I = A^2+A+I$$

$$= \begin{bmatrix} 5 & 4 & 4 \\ 0 & 1 & 0 \\ 4 & 4 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The given matrix $= \begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}$

3) Is the following matrices are diagonalizable ? If so find the diagonal and modal matrices in each case

(i) $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. (ii) $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$

→ (i) Character equation $|A - \lambda I| = 0$

$$\lambda^3 - (-1)\lambda^2 + [-12 - 3 - 6]\lambda - 45 = 0$$
$$\lambda^3 + \lambda^2 - 21\lambda - 45 = 0$$

$$\lambda = 5, -3, -3$$

For $\lambda = -3$,

$(A + 3\lambda)X_1 = 0$

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix} X_1 = 0$$

$R_3 = R_3 + R_1$

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ 0 & 0 & 0 \end{bmatrix} X_1 = 0$$

$R_2 = R_2 - 2R_1$

$$\begin{bmatrix} 1 & 2 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} X_1 = 0$$

$$x_1 + 2x_2 - 3x_3 = 0$$
$$x_1 = -2x_2 + 3x_3$$

$$X_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2x_2 + 3x_3 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$$

Algebraic multiplicity of $\lambda = -3$ is 2 and there exists two linearly independent eigen vectors corresponding to $\lambda_1 = -3$, thus geometric multiplicity is 2

$\therefore$ AM = GM, for $\lambda = -3$

For $\lambda = 5$

$$(A - 5I)X_2 = 0$$

$$\begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix} X_2 = 0$$

$$\xrightarrow{R_3 + R_1} \begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -8 & 0 & -8 \end{bmatrix} X_2 = 0$$

$$\xrightarrow{R_2 + 2R_1} \begin{bmatrix} -7 & 2 & -3 \\ -12 & 0 & -12 \\ -8 & 0 & -8 \end{bmatrix} X_2 = 0$$

$$\xrightarrow{R_3 - 8R_2/12} \begin{bmatrix} -7 & 2 & -3 \\ -12 & 0 & -12 \\ 0 & 0 & 0 \end{bmatrix} X_2 = 0$$

$\therefore -12x_1 - 12x_3 = 0$ $\qquad$ $-7x_1 + 2x_2 - 3x_3 = 0$

$\underline{x_1 = -x_3}$ $\qquad$ $2x_1 + 4x_3 = 0$

$\underline{\underline{x_2 = -2x_3}}$

$$\therefore X_3 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -x_3 \\ -2x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$$

For $\lambda = 5$, the AM is 1 and GM = 1

$\therefore$ AM = GM

$\therefore$ A is a diagonisable matrix.

Now, $M = [x_1 \quad x_2 \quad x_3]$

$$M = \begin{bmatrix} -2 & 3 & -1 \\ 1 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix}$$

$$M^{-1} = -\frac{1}{8}\begin{bmatrix} 2 & -4 & -6 \\ -1 & -2 & -5 \\ 1 & 2 & -3 \end{bmatrix}$$

$D = M^{-1}AM$

$$= -\frac{1}{8}\begin{bmatrix} 2 & -4 & -6 \\ -1 & -2 & -5 \\ 1 & 2 & -3 \end{bmatrix}\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}\begin{bmatrix} -2 & 3 & -1 \\ 1 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\therefore D = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

→ (ii) Characteristic equation

$|A - \lambda I| = 0$

$\lambda^3 - (3+5+3)\lambda^2 + (14+8+14)\lambda - 36 = 0$

$\therefore \lambda^3 - 11\lambda^2 + 36\lambda - 36 = 0$

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

$\lambda = \underline{\underline{6, 3, 2}}$

$\because$ The eigen values are distinct, the given matrix A is diagonisable.

For $\lambda_1 = 2$,

$(A - 2I) X_1 = 0$

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 1 \end{bmatrix} X_1 = 0 \xrightarrow{R_3 - R_1} \begin{bmatrix} 1 & -1 & 1 \\ -1 & 3 & -1 \\ 0 & 0 & 0 \end{bmatrix} X_1 = 0$$

$\therefore x_1 - x_2 + x_3 = 0$

$\Rightarrow x_2 = x_1 + x_3$

$-x_1 + 3x_2 - x_3 = 0$

$\Rightarrow 3x_2 = x_1 + x_3$

$\Rightarrow x_2 = 0$

$x_1 + x_3 = 0$

$x_1 = -x_3$

$$\therefore X_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -x_3 \\ 0 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

For $\lambda_2 = 3$, $(A - 3I) X_2 = 0$

$$\begin{bmatrix} 0 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 0 \end{bmatrix} X_2 = 0$$

$$\xrightarrow{R_3 + R_2} \begin{bmatrix} 0 & -1 & 1 \\ -1 & 2 & -1 \\ 0 & 1 & -1 \end{bmatrix} X_2 = 0$$

$$\xrightarrow{R_3+R_1} \begin{bmatrix} 0 & -1 & 1 \\ -1 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix} X_2 = 0$$

$\therefore -x_2 + x_3 = 0$

$\underline{\underline{x_2 = x_3}}$

$-x_1 + 2x_2 - x_3 = 0$

$-x_1 + x_3 = 0$

$\underline{\underline{x_1 = x_3}}$

$$\therefore X_2 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

For $\lambda_3 = 6$, $(A - 6I) X_3 = 0$

$$\begin{bmatrix} -3 & -1 & 1 \\ -1 & -1 & -1 \\ 1 & -1 & -3 \end{bmatrix} X_3 = 0$$

$$\xrightarrow{R_3+R_1,\ R_2 - R_3/2} \begin{bmatrix} -3 & -1 & 1 \\ 0 & 0 & 0 \\ -2 & -2 & -2 \end{bmatrix} X_3 = 0$$

$\therefore -3x_1 - x_2 + x_3 = 0$

$-3x_1 + (x_1 + x_3) + x_3 = 0$

$-2x_1 + 2x_3 = 0$

$\underline{\underline{x_1 = x_3}}$

$-2x_1 - 2x_2 - 2x_3 = 0$

$\underline{\underline{x_1 + x_3 = -x_2}}$

$\therefore x_2 = -(x_1 + x_3)$

$\underline{\underline{x_2 = -2x_3}}$

$$X_3 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ -2x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$\therefore M = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$

$$M = \begin{bmatrix} -1 & 1 & 1 \\ 0 & 1 & -2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$M^{-1} = \frac{-1}{6}\begin{bmatrix} 3 & 0 & -3 \\ -2 & -2 & -2 \\ -1 & 2 & -1 \end{bmatrix}$$

$$D = M^{-1}AM$$

$$= \frac{-1}{6}\begin{bmatrix} 3 & 0 & -3 \\ -2 & -2 & -2 \\ -1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}\begin{bmatrix} -1 & 1 & 1 \\ 0 & 1 & -2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

4) $A = \begin{bmatrix} 3 & -2 \\ 4 & -3 \end{bmatrix}$, Find $3A^{57} + 2A^{18}$

$\rightarrow$ Characteristic equation: $|A - \lambda I| = 0$

$\lambda^2 - (3-3)\lambda + (-9+8) = 0$

$\lambda^2 - 1 = 0$

$\lambda = -1, 1$

$\because$ Matrix A is diagonisable.

$\because$ Matrix A is of order 2

$\therefore \phi(A) = 3A^{57} + 2A^{18} = a_1 A + a_0 I$

$\therefore 3\lambda^{57} + 2\lambda^{18} = a_1\lambda + a_0$

For $\lambda = -1$, $-a_1 + a_0 = 3(-1)^{57} + 2(-1)^{18}$

$a_0 - a_1 = -1$ — (i)

For $\lambda = 1$, $a_1 + a_0 = 3(1)^{57} + 2(1)^{18}$

$a_0 + a_1 = 5$ — (2)

on solving (i) & (2)

$\therefore$ $\underline{\underline{a_0 = 2}}$, $\underline{\underline{a_1 = 3}}$

$\therefore 3A^{57} + 2A^{18} = 3A + 2I = 3\begin{bmatrix} 3 & -2 \\ 4 & -3 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore 3A^{57} + 2A^{18} = \underline{\underline{\begin{bmatrix} 11 & -6 \\ 12 & -7 \end{bmatrix}}}$

5) If $A = \begin{bmatrix} y & y \\ y & y \end{bmatrix}$, prove that $e^A = e^y \begin{bmatrix} \cosh y & \sinh y \\ \sinh y & \cosh y \end{bmatrix}$

$\rightarrow$ Characteristic equation: $|A - \lambda I| = 0$

$\therefore \lambda^2 - (y+y)\lambda + 0 = 0$

$\lambda^2 - 2y\lambda = 0$

$\underline{\lambda = 0, 2y}$

$\because$ Matrix A is of order 2

$\phi(A) = e^A = a_1 A + a_0 I$

$\therefore e^\lambda = a_1\lambda + a_0$

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for $d = 0$, $a_0 = 1$ — (i)

For $d = 2y$, $(2y)a_1 + a_0 = e^{2y}$

$$\therefore a_1 = \frac{e^{2y} - 1}{2y}$$

$$\therefore e^A = \left(\frac{e^{2y}-1}{2y}\right) A + I$$

$$\therefore e^A = \frac{e^{2y}-1}{2y}\begin{bmatrix} y & y \\ y & y \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{e^{2y}-1}{2} & \frac{e^{2y}-1}{2} \\ \frac{e^{2y}-1}{2} & \frac{e^{2y}-1}{2} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} e^{2y}+1/2 & e^{2y}-1/2 \\ e^{2y}-1/2 & e^{2y}+1/2 \end{bmatrix}$$

$$= \begin{bmatrix} e^y\left(\frac{e^y+e^{-y}}{2}\right) & e^y\left(\frac{e^y-e^{-y}}{2}\right) \\ e^y\left(\frac{e^y-e^{-y}}{2}\right) & e^y\left(\frac{e^y+e^{-y}}{2}\right) \end{bmatrix}$$

$$e^A = e^y \begin{bmatrix} \cosh y & \sinh y \\ \sinh y & \cosh y \end{bmatrix}$$

- Hence proved.