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FLAT - Tutorial 7

1) Design a Turing machine that computes a function $f(m,n) = m+n$ i.e. addition of two integers.

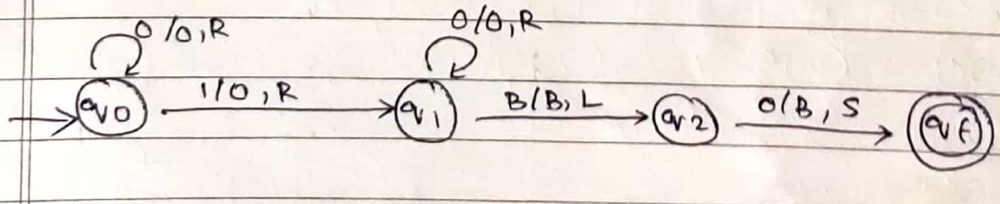
→ Let M be turing machine
$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$

Γ = Tape alphabet = $(0, 1, B)$
B = Blank Symbol
Q = Finite set of states. = (q_0, q_1, q_2, q_f)
Σ = Input alphabet = $(0, 1)$
δ = Transition function
q_0 = Initial states
F = Final set of states.

Logic:

For 0 replace with 0 move right.
For 1 replace with 0 move right.
When Blank is encounter move left and replace rightmost 0 by Blank.

Transition diagram



Transition table:

Q \ Γ	0	1	B
q_0	$(q_0, 0/0, R)$	$(q_1, 1/0, R)$	–
q_1	$(q_1, 0/0, R)$	–	$(q_2, B/B, L)$
q_2	$(q_f, 0/B, S)$	–	–
q_f	–	–	–

Simulation

$m = 2, n = 2$

B 0 0 1 0 0 B

→ 1) B 0 0 1 0 0 B
↑ q_0

2) B 0 0 1 0 0 B
↑ q_0

3) B 0 0 1 0 0 B
↑ q_0

4) B 0 0 0 0 0 B
↑ q_1

5) B 0 0 0 0 0 B
↑ q_1

6) B 0 0 0 0 0 B
↑ q_1

7) B 0 0 0 0 0 B
↑ q_2

8) B 0 0 0 0 B B
↑ q_f – string accepted.

2) Design a Turing machine for set of string with equal number of 0's and 1's over $\{0, 1\}^*$

→ let m be turing machine

$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$

Γ = Tape ~~sym~~ alphabet = (0, 1, *, B)

B = Blank symbol

Q = Finite set of states = ($q_0, q_1, q_2, q_3, q_4, q_f$)

Σ = Input alphabet = (0, 1)

δ = Transition function

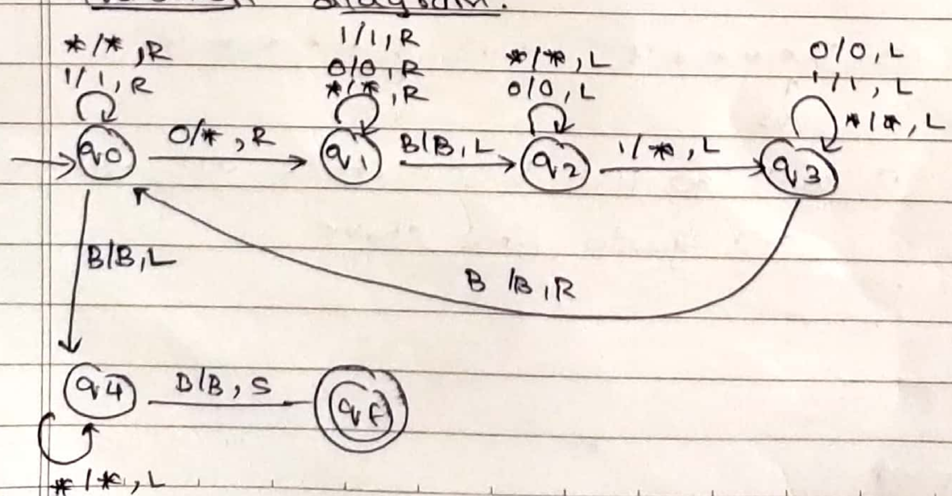
q_0 = Initial state

F = Final set of states.

<u>logic</u>:

Replace 0 with * move right till blank encouter then move left and replace 1 with * and move left. Repeat till all '0' and '1' are replace by *

<u>Transition diagram</u>:



Q \ Γ	0	1	*	B
q0	(q1, 0/*, R)	(q0, 1/1, R)	(q0, */*, R)	(q4, B/B, L)
q1	(q1, 0/0, R)	(q1, 1/1, R)	(q1, */*, R)	(q2, B/B, L)
q2	(q2, 0/0, L)	(q3, 1/*, L)	(q2, */*, L)	–
q3	(q3, 0/0, L)	(q3, 1/1, L)	(q3, */*, L)	(q0, B/B, R)
q4	–	–	(q4, */*, L)	(qf, B/B, S)
qf	–	–	–	–

Simulation

(1) B 0101 B ↑ q0

(2) B * 101 B ↑ q1

(3) B * 101 B ↑ q1

(4) B * 101 B ↑ q1

(5) B * 101 B ↑ q1

(6) B * 101 B ↑ q2

(7) B * 10 * B ↑ q3

(8) B * 10 * B ↑ q3

(9) B * 10 * B ↑ q3

(10) B * 10 * B ↑ q3

(11) B * 10 * B ↑ q3 Steps will be repeated till are become *

(12) B * * * * B ↑ q3

(13) B * * * * B ↑ q3

(14) B * * * * B ↑ q3

(15) B * * * * B ↑ q0 it will go till blank check all ~~and~~ are * and go to final state.

(16) B * * * * B ↑ qf

String accepted.

3) Design a TM with no more than 3 states that accepts the language $L = (a(a+b)^*)$

→ Above language consists of string that starts with 'a' followed by 0 or more combination of 'a' and 'b'

Step 1: Let m be Turing machine.

$M = (Q, \Gamma, \Sigma, \delta, B, F, q0)$

Q = Finite set of states = $(q0, q_1, q_f)$

B = Blank Symbol

Γ = Tape alphabet = $(a, b\, B)$

Σ = Input = (a, b)

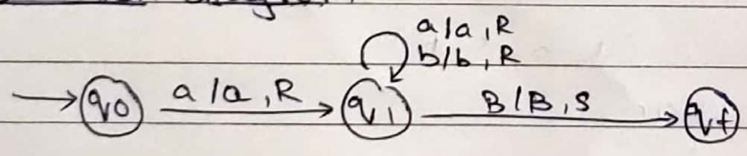
F = Final set of state = (q_f)

$q0$ = Initial state = $(q0)$

δ = Transition function

Logic: Replace a with a move right.

Transition diagram:



Transition Table

Q \ Γ	a	b	B
q0	(q1, a/a, R)	-	-
q1	(q1, a/a, R)	(q1, b/b, R)	(qf, B/B, R)
qf	-	-	-

Simulation

→ B a b b B

→ B a b b B
$\uparrow$
q_0

→ B a b b B
$\uparrow$
q_1

→ B a b b B
$\uparrow$
q_1

→ B a b b B
$\uparrow$
q_f

Machine stops at q_f.
∴ String is accepted.

4) Construct a TM which accepts the language
$L = \{a^n b^{n+1}\}$

→ Step 1: Let m be turing machine
$M = (\Gamma, \delta, F, q_0, B, Q, \Sigma)$

Γ = Tape alphabet = (a, b, x, B)
δ = Transition function
F = Final state = (q_f)
q_0 = Initial state = (q_0)
B = Blank symbol
Q = Set of states = ($q_0, q_1, q_2, q_3, q_4, q_f$)
Σ = Input = (a, b)

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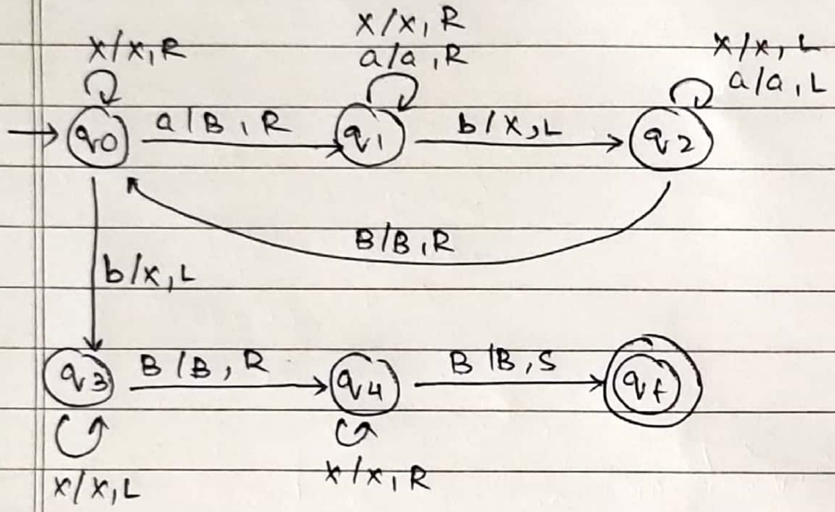
Logic:

Replace a with Blank and move right till first b.
Replace first B with x and move left till blank.

Transition diagram



Transition Table

Q \ Γ	a	b	x	B
q0	(q1, a/B, R)	(q3, b/x, L)	(q0, x/x, R)	-
q1	(q1, a/a, R)	(q2, b/x, L)	(q1, x/x, R)	-
q2	(q2, a/a, L)	-	(q2, x/x, L)	(q0, B/B, R)
q3	-	-	(q3, x/x, L)	(q4, B/B, R)
q4	-	-	(q4, x/x, R)	(qf, B/B, S)
qf	-	-	-	-

Simulation

~~B a a b b b B~~

B a b b B
$\uparrow$
q_0

$\vdash$ B B b b B
$\uparrow$
q_1

$\rightarrow$ B B X b B
$\uparrow$
q_2

$\rightarrow$ (B B X b B)
$\uparrow$
q_0

$\rightarrow$ (B B X b B)
$\uparrow$
q_0

$\rightarrow$ (B B X X B)
$\uparrow$
q_3

$\rightarrow$ (B B X X B)
$\uparrow$
q_3

$\rightarrow$ (B B X r B)
$\uparrow$
q_4

$\rightarrow$ (B B X X B)
$\uparrow$
q_4

$\rightarrow$ (B B X X B)
$\uparrow$
q_f

Machine stops at q_f.
Hence, String is accepted.