SAP ID - 60004200132
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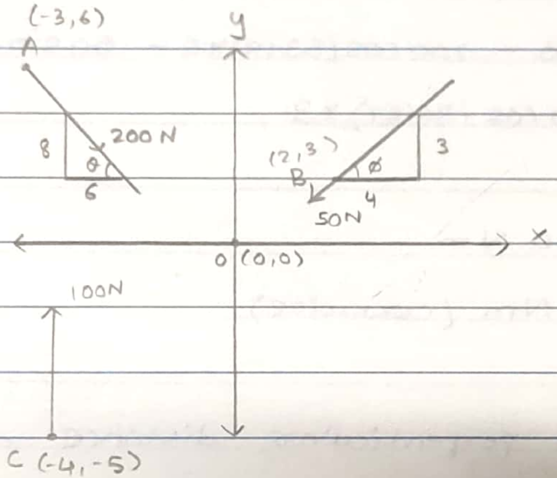
Div - J

Engeneering Mechanics

Term Test 1 Assignment.

1) Find the resultant of the following non-concurrent force system.



$\tan\theta = \frac{8}{6}$, $\tan\phi = \frac{3}{4}$

$\therefore \theta = \tan^{-1}\left(\frac{8}{6}\right) = 53.13^\circ$ $\qquad \phi = \tan^{-1}\left(\frac{3}{4}\right) = 36.87^\circ$

$\therefore R_x = \Sigma F_x = 200\cos\theta - 50\cos\phi$

$= 200\cos(53.13^\circ) - 50\cos(36.87^\circ)$

$R_x = \Sigma F_x = 80N\ (\rightarrow)$

$R_y = \Sigma F_y = 100 - 200\sin\theta - 50\sin\phi$

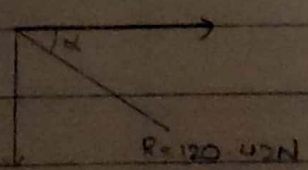
$= 100 - 200\sin(53.13^\circ) - 50\sin(36.87^\circ)$

$= -90N$

$\therefore R_y = \Sigma F_y = 90N\ (\downarrow)$

$\therefore R = \sqrt{R_x^2 + R_y^2} = \sqrt{80^2 + 90^2}$

$\therefore R = 120.42N$, $\alpha = \tan^{-1}\left(\frac{90}{80}\right) = 48.36$



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$\therefore M_{(0,0)} = -100 \times 4 - 200\cos\theta \times 6 - 50\sin\phi \times 2 + 200\sin\theta \times 3$
$+ 50\cos\phi \times 3$

$= -400 - 200\cos(53.13) \times 6 - 50\sin(36.87) \times 2 + 200\sin(53.13) \times 3$
$+ 50\cos(36.87) \times 3$

$= -580$ Nm

$\therefore M_{(0,0)} = 580$ Nm (clockwise)

Let d be the perpendicular distance of resultant from (0,0)

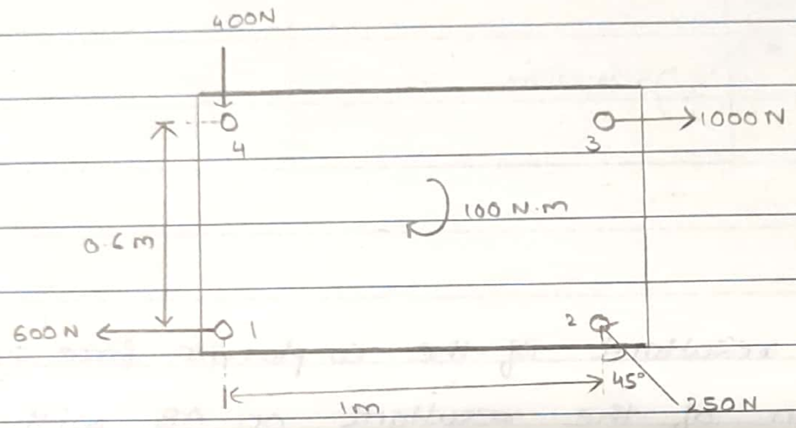
$\therefore M(0,0) = 580 = R \times d$

$\therefore 580 = 120.42\, d$

$\therefore d = 4.816$ m.

~~$\therefore$ Coordinates of R are $(d\sin\alpha, d\cos\alpha)$~~
~~$(3.599, 3.1999)$~~

2) Four forces and a couple are applied to a rectangular plate as shown in Fig. Replace the system of forces and the couple by an equivalent force-couple system at bolt 1.



$R_x = \Sigma F_x = 1000 - 600 - 250\sin 45$

$\therefore R_x = 223.22\,N\ (\rightarrow)$

$R_y = \Sigma F_y = -400 + 250\cos 45$

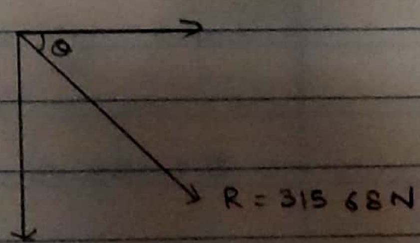
$= -223.22\ N$

$\therefore R_y = 223.22\,N\ (\downarrow)$

$\therefore R = \sqrt{R_x^2 + R_y^2} = \sqrt{(223.22)^2 + (-223.22)^2}$

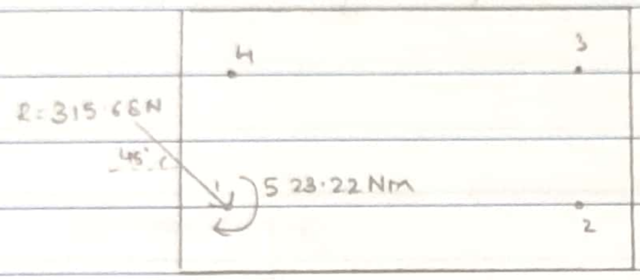
$R = 315.68\ N$

$\therefore \theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}(1) = 45°$

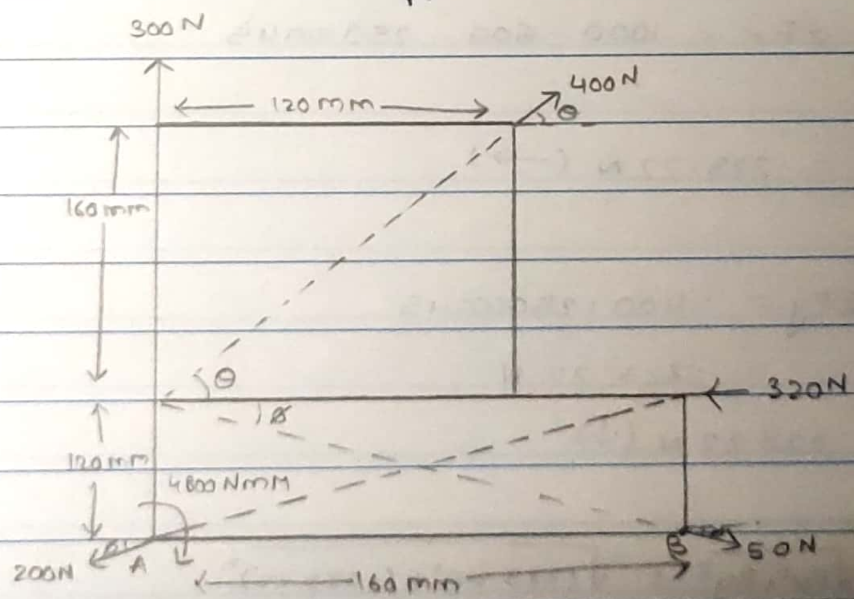


$I = -1000 \times 0.6 + 250\cos 45 \times 1 ~~-1000\times 1~~ -100$

$I = -523.22$ NM $= 523.22$ NM (clockwise)



3) Find the resultant of the co-planar force system. locate the position of the resultant on AB with due consideration to the applied moment.



$\tan\theta = \frac{160}{120}$

$\therefore \theta = 53.13°$

$\tan\phi = \frac{120}{160}$

$\therefore \phi = 36.87°$

$R_x = \Sigma F_x = 400\cos\theta - 320 + 50\cos\phi - 200\cos\phi$

$= 400\cos(53.13) - 320 + 50\cos(36.87) - 200\cos(36.87)$

$= -200\text{ N}$

$\therefore R_x = 200\text{ N } (\leftarrow)$

$R_y = \Sigma F_y = 300 + 400\sin\theta - 50\sin\phi - 200\sin\phi$

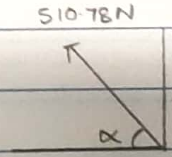
$= 300 + 400\sin(53.13) - 50\sin(36.87) - 200\sin(36.87)$

$= 470\text{ N}$

$\therefore R_y = 470\text{N } (\uparrow)$

$\therefore R = \sqrt{R_x^2 + R_y^2} = \sqrt{(200)^2 + (470)^2}$

$R = 510.78\text{ N}$



$\alpha = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{470}{200}\right) = 66.95^\circ$

Moment about A, $M_A = 400\cos\theta \times 280 + 50\sin\phi \times 160 - 400\sin\theta \times 120$

$-320 \times 120 + 4800$

$= 400\cos 53.13 \times 280 + 50\sin(36.87) \times 160 - 400\sin(53.13) \times 120$

$-320 \times 120 + 4800$

$= 0\text{ Nmm}$

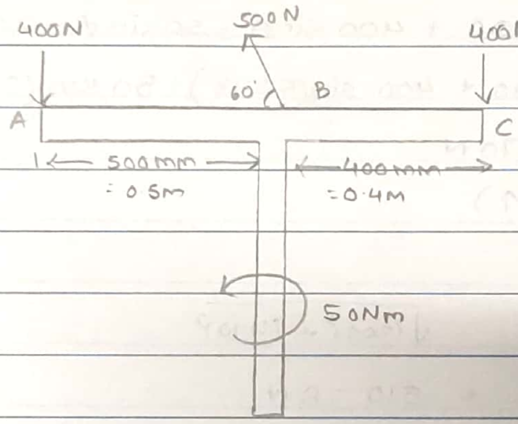
$\therefore$ using Varignon's theorem, $R \times d = M$

$d = 0\text{ mm}$

$\therefore$ x-intercept $= 0\text{ mm}$

$\therefore$ Resultant cuts x-axis at $x = 0$.

4) A bracket is subjected to a coplanar force system. Determine the magnitude and line of action from A of the single resultant of the system. If the resultant is to pass through the point B, what should be the magnitude and direction of couple?



$R_x = \Sigma F_x = -500 \cos 60$

$= -250$ N

$\therefore R_x = 250$ N $(\leftarrow)$

$R_y = \Sigma F_y = -400 - 400 + 500 \sin 60$

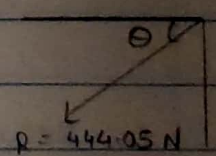
$= -366.99$ N

$\therefore R_y = 366.99$ N $(\downarrow)$

$\therefore R = \sqrt{R_x^2 + R_y^2} = \sqrt{(250)^2 + (366.99)^2}$

$\therefore R = 444.05$ N.

$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{366.99}{250}\right) = 55.74°$



$MA = -400 \times 0.9 + 50 + 500 \sin 60 \times 0.5$

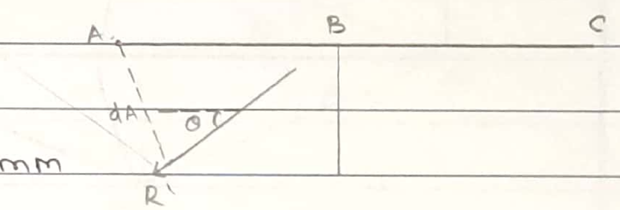
$MA = -93.49$ Nm

$\therefore MA = 93.49$ Nm (clockwise)

$\therefore MA = R \times dA$

$93.49 = 444.05\ dA$

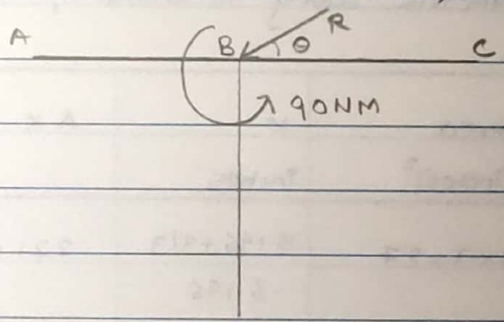
$\therefore dA = 0.21$ m $= 210$ mm



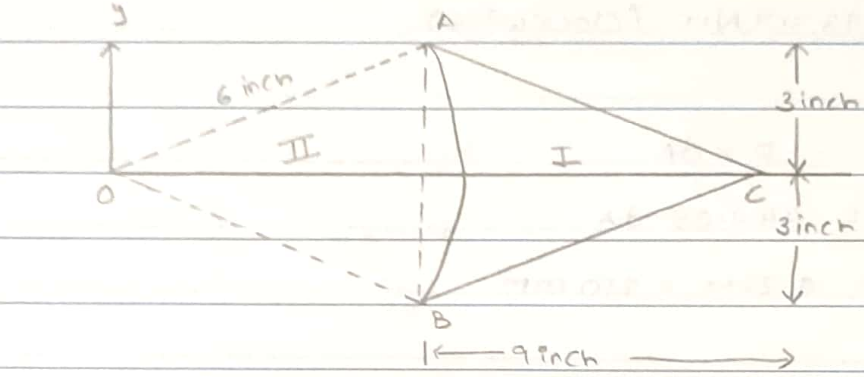
$MB = 400 \times 0.5 + 50 - 400 \times 0.4$

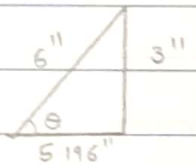
$= 90$ Nm (anticlockwise)

$\therefore$ Resultant with moment for B,



5) Locate the centroid of the plane area shown.

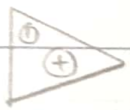
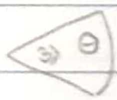




$\sin 30 = \frac{3}{6} = \frac{1}{2}$

$\therefore \theta = 30^\circ = 30 \times \frac{\pi}{180} = \frac{\pi}{6}$ radians $= \alpha$

Since, it is symmetric along x-axis, $\bar{y} = 0$.

Component	Area (inches)^2	x Inches	A x
	$\frac{1}{2} \times 6 \times 9 = 27$	$5.196 + 9/3 = 8.196$	221.292
	$\frac{1}{2} \times 6 \times 5.196 = 15.588$	$5.196 - 5.196/3 = 3.464$	53.997
	$-r^2\alpha = -6^2 \times \pi/6 = -6\pi = -18.84$	$\frac{2}{3}\frac{r\sin\alpha}{\alpha} = 3.822$	-72.006
	$\Sigma A = 23.748$		$\Sigma Ax = 203.283$

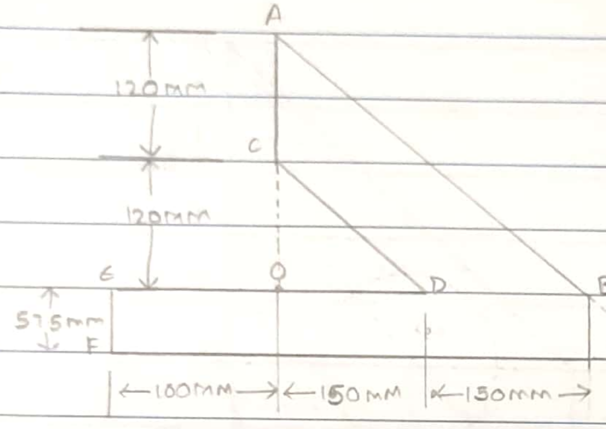
$\therefore \bar{x} = \frac{\Sigma Ax}{\Sigma A}$

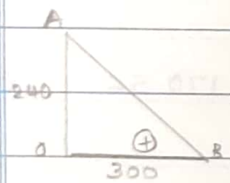
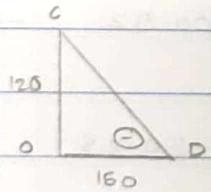
$\therefore \bar{x} = \frac{203.283}{23.748}$

$\therefore \bar{x} = 8.56''$

$\therefore$ Centroid of the composite figure is $(8.56, 0)$ inches.

6) Locate the centroid of the plane area shown.



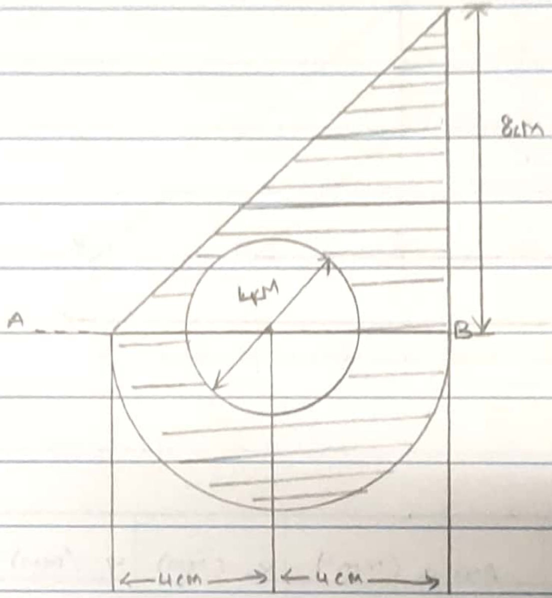
Component	Area (mm^2)	x (mm)	y (mm)	Ax (·	Ay
	$400 \times 57.5 = 23000$	100	-28.75	2300000	-661250
	$\frac{1}{2} \times 300 \times 240 = 36000$	100	80	3600000	2880000
	$-\frac{1}{2} \times 150 \times 120 = -9000$	50	40	-450000	-360000
	$\Sigma A = 50000$			$\Sigma Ax = 5450000$	$\Sigma Ay = 1858750$

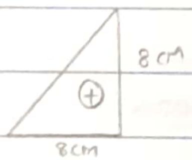
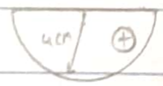
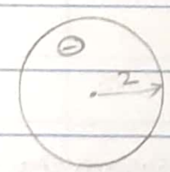
Now, $\bar{x} = \frac{\Sigma Ax}{\Sigma A} = \frac{5450000}{50000} = 109 \text{ mm}$

$\bar{y} = \frac{\Sigma Ay}{A} = \frac{1858750}{50000} = 37.175 \text{ mm}$

$\therefore$ Centroid is located at $(109, 37.175)$ mm.

7) Determine the centroid of shaded area.



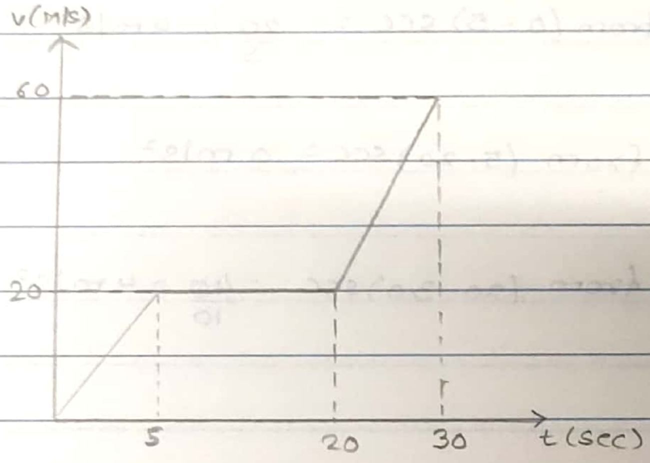
Component	Area (cm^2)	x (cm)	y (cm)	Ax	Ay
	$\frac{1}{2} \times 8 \times 8 = 32$	$8 - 8/3 = 5.33$	$8/3 = 2.67$	170.56	85.44
	$\pi(4)^2/2 = 25.13$	4	$-4\times4/3\pi = -1.69$	100.52	−42.47
	$\pi(2)^2 = -12.57$	4	0	−50.28	0
	$\Sigma A = 44.56$			$\Sigma Ax = 220.8$	$\Sigma Ay = 42.97$

$$\therefore \bar{x} = \frac{\Sigma Ax}{A} = \frac{220.8}{44.56} = 4.95 \text{ cm}$$

$$\bar{y} = \frac{\Sigma Ay}{A} = \frac{42.97}{44.56} = 0.96 \text{ cm}$$

$\therefore$ Centroid of shaded area is (4.95, 0.96) cm.

8) The motion of a jet plane while travelling along a runway is defined by v-t curve. Construct s-t and a-t graph for the motion. The plane starts from rest.

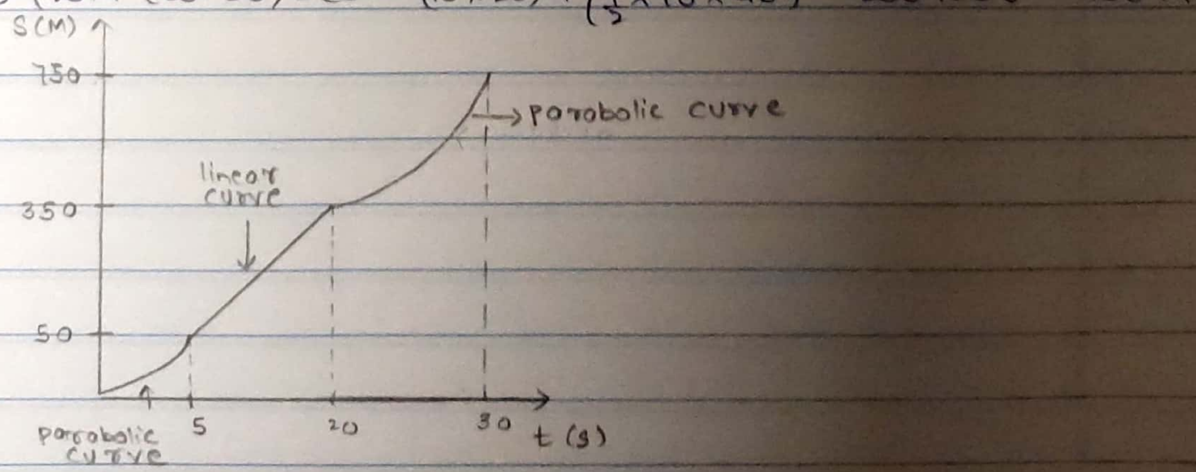


For s-t graph,

s in (0-5) sec $= \frac{1}{2} \times 5 \times 20 =$ 50 m

s from (5-20) sec $= \frac{1}{2} \times 15 \times 20 =$ 300 m

s from (20-30) sec $= (10 \times 20) + (\frac{1}{2} \times 10 \times 40) = 200 + 200 = 400$ m



For A-t graph,

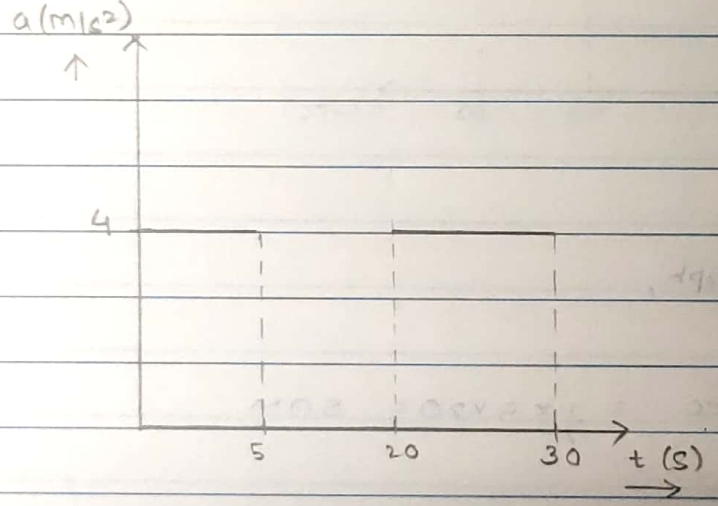
change in acceleration = slope of (v-t) curve.

$\therefore$ acceleration from (0 - 5) sec = $\frac{20}{5}$ = 4 m/s²

acceleration from (5-20) sec = 0 m/s²

acceleration from (20-30) sec = $\frac{40}{10}$ = 4 m/s²

$\therefore$



9) The acceleration of particle is defined by the relation $a = 21 - 12x^2$ where a = acceleration in m/s^2 and x is meter. The particle starts with rest at $x = 0$.

Determine

a) velocity when $x = 1.5$ m

b) The position where the velocity is again zero.

c) The position where the velocity is max.

→ $a = 21 - 12x^2$

We know that, $a = v\frac{dv}{dx}$

$\therefore v\frac{dv}{dx} = 21 - 12x^2$

$\therefore v\,dv = (21 - 12x^2)\,dx$

Integrating both the sides,

$\int v\,dv = \int (21 - 12x^2)\,dx$

$\frac{v^2}{2} = 21x - 4x^3 + C$

For $x = 0$, v is 0 m/s.

$\therefore 0 = 0 + C$

$\therefore C = 0$

$\therefore \frac{v^2}{2} = 21x - 4x^3$

a) For $x = 1.5$ m,

$\frac{v^2}{2} = 21(1.5) - 4(1.5)^3$

$\therefore \frac{v^2}{2} = 18$

$\therefore v^2 = 36$

$\therefore v = 6 \, m/s.$

$\therefore$ The velocity of particle at $x = 1.5m$ is $6 \, m/s$.

b) For $v = 0$

$\therefore 21x - 4x^3 = 0$

$\therefore x(21 - 4x^2) = 0$

$\therefore x = 0 \quad , \quad x^2 = \frac{21}{4}$

$\therefore x = \pm 2.29 \, m$

c) For velocity to be maximum,

$a = \frac{dv}{dt} = 0$

$\therefore 21 - 12x^2 = 0$

$x^2 = \frac{21}{12}$

$\therefore x = 1.322 \, m$

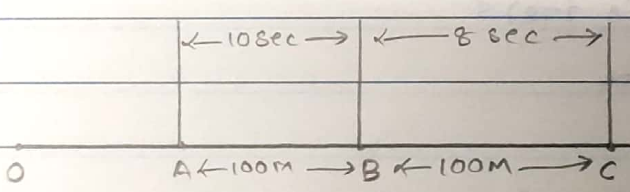
$\therefore$ The position where velocity is max is $1.322 \, m$.

10) Three vertical poles A, B and C spaced at distance of 100 M along a straight road. A car starting from rest and accelerates uniformly passes pole A and takes 10 sec to reach pole B and further 8 sec. to reach the pole C. Calculate:

a) acceleration of car.

b) velocity at A and B

c) starting position of car.



let the starting position of car be O,

Time taken by car from A to B, $t_1 = 10$ sec.

Time taken by car from B to C, $t_2 = 8$ sec.

a: acceleration of car

V_A: velocity at A

V_B: velocity at B

Now,

$$S = ut + \frac{1}{2}at^2$$

For car travelling from A to B

$$100 = V_A(10) + \frac{1}{2}a(100)$$

$$\therefore 100 = 10V_A + 50a \quad -(i)$$

For car travelling from A to C,

$$200 = 18\,V_A + \frac{1}{2} a (18)^2$$

$$\therefore 200 = 18\,V_A + 162a \quad -(2)$$

$\therefore$ solving equation (1) and (2),

$a = 0.278\ m/s^2$,

$V_A = 8.61\ m/s$

For car moving from O to A,

$$v^2 = u^2 + 2as$$

$$(8.61)^2 = 0 + 2(0.278)\,s$$

$$\therefore s = 133.33\,m$$

For car travelling from B to C,

$$100 = 8\,V_B + \frac{1}{2}(0.278)(8)^2$$

$$\therefore V_B = 11.388\ m/s.$$

$\therefore$ The acceleration of car is $0.278\ m/s^2$

The velocity of car at pole A is 8.61 m/s and at pole B is 11.388 m/s and the starting position of car is 133.33 m from pole A.