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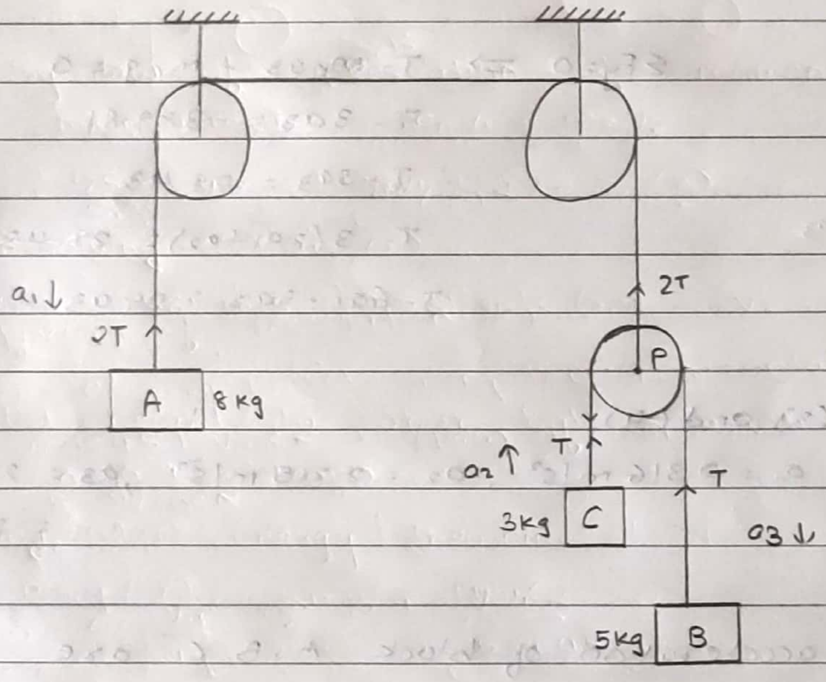
Engineering Mechanics

Assignment - 05

5.1.4) Find the acceleration for masses A, B and C when the system is released from rest.



→ Obtain kinematic relation,

$\Sigma Tx = 0$

$\pm T_1 x_1 \pm T_2 x_2 \pm T_3 x_3 = 0$

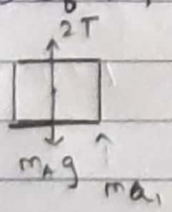
$2Tx_1 + Tx_2 - Tx_3 = 0$

Differentiate twice wrt time, we get

$2a_1 + a_2 = a_3$ — (i)

Using D'Alembert's Principle,

FBD of A,



$\Sigma F_y = 0 \Rightarrow 2T + m_A a_1 - m_A g = 0$

$\therefore 2T + 8a_1 = 8 \times 9.81$

$\therefore 2T + 8a_1 = 78.48$ — (2)

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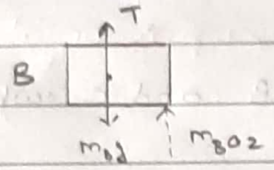
FBD of B

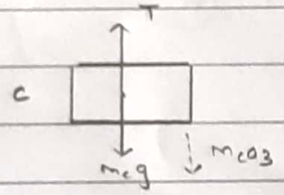


$\Sigma F_y = 0 \Rightarrow T + m_B a_2 - m_B g = 0$

$T + 5a_2 = 5 \times 9.81$

$T + 5a_2 = 49.05 \quad -(3)$

FBD of C



$\Sigma F_y = 0 \Rightarrow T - m_c a_3 - m_c g = 0$

$T - 3a_3 = 3 \times 9.81$

$\therefore T - 3a_3 = 29.43$

$T - 3(2a_1 + a_2) = 29.43$. (from i)

$\therefore T - 6a_1 - 3a_2 = 29.43 \quad -(4)$

Solving (2), (3) and (4)

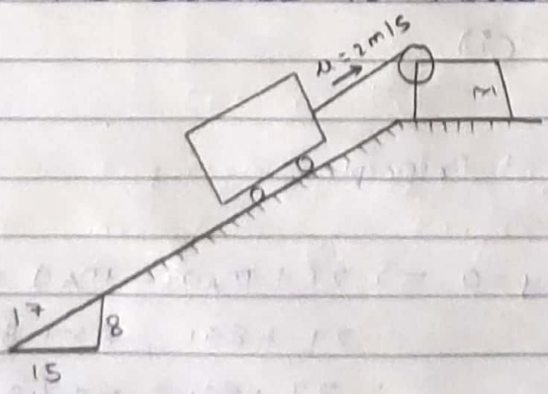
$T = 37.97\,N$, $a_1 = 0.316\, m/s^2$, $a_2 = 2.215\, m/s^2$, $a_3 = 2a_1 + a_2$

$= 2.847\, m/s^2$

$\therefore$ The values of acceleration of block A, B, C are

$a_A = 0.316\, m/s^2\ (\downarrow)$, $a_B = 2.215\, m/s^2\ (\downarrow)$, $a_C = 2.847\, m/s^2\ (\uparrow)$

5.1.7) The 400 kg mine car is hoisted up the plane by using a cable and a motor M. the force in the cable is $F = 3200\,t^2\,N$, where t is in seconds. If the car has initial velocity of 2 m/s determine the distance it moved up the plane after t = 2 sec.



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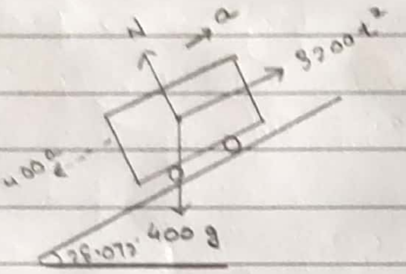
→ $\tan\theta = \frac{8}{15}$

$\therefore \theta = \tan^{-1}\left(\frac{8}{15}\right) \approx 28.072$

FBD of mine car.



$\Sigma F_x = 0 \Rightarrow 3200t^2 - 400g\sin(28.072) = 400a$

$\therefore 3200t^2 - 400(9.81)(0.47) = 400a$

$\therefore a = 8t^2 - 4.616 \quad — (i)$

We know that $a = \frac{dv}{dt}$ where v = velocity.

$\therefore$ From equation (i)

$dv = a\,dt$

Integrating both sides,

$$\int_2^v dv = \int_0^t (8t^2 - 4.616)\,dt$$

$$\therefore v - 2 = 2.667t^3 - 4.616t \Rightarrow v = \frac{ds}{dt} = 2.667t^3 - 4.616t + 2$$

$$\therefore \int_0^s ds = \int_0^2 (2.667t^3 - 4.616t + 2)\,dt$$

$$\therefore s = \left(0.667t^4 - 2.313t^2 + 2t\right)_0^2$$

$\therefore s = 5.42\,m$

$\therefore$ The mine cart moved 5.42 m up the plane after two seconds.

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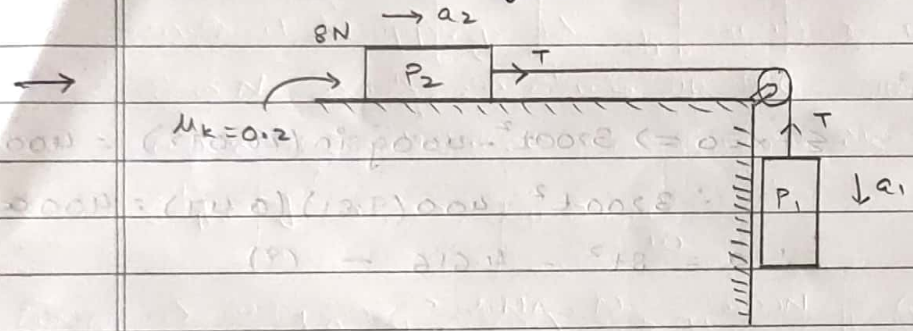
5.2.3) Block P_1 of weight 4N is connected to block P_2 of weight 8N by an inextensible string. Find the velocity of block P_1, if it falls by 0.6 m starting from rest. $\mu_k = 0.2$. Also find the tension in the string.



Obtain kinematic relation,

$\Sigma Tx = 0$

$\pm T_1 x_1 \pm T_2 x_2 = 0$

$\therefore T_1 x_1 - T_2 x_2 = 0$

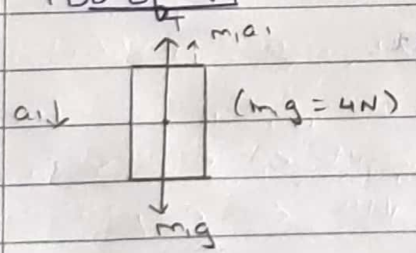
$\therefore x_1 = x_2$

Differentiating the above equation twice, we get

$a_1 = a_2$ — (i)

Using D'Alembert's Principle,

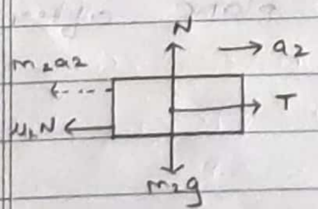
FBD of P_1



$\Sigma F_y = 0 \Rightarrow T + m_1 a_1 - m_1 g = 0$

$T + \left(\frac{4}{9.81}\right) a_1 = 4$ — (2)

FBD of P_2



$\Sigma F_y = 0 \Rightarrow N - m_2 g = 0$

$N = 8N$

$\Sigma F_x = 0 \Rightarrow T - \mu_k N - m_2 a_2 = 0$

$T - \frac{8}{9.81} a_2 = (0.2)(8)$ —(3)

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Let $a_1 = a_2 = a$ ....(from i)

$\therefore T + \frac{4}{9.81} a = 4$, $T - \frac{8}{9.81} a = 1.6$

Solving above equations we get

$\boxed{T = 3.2N}$, $a = 1.962 m/s^2$

Now, $x = 0.6 m$

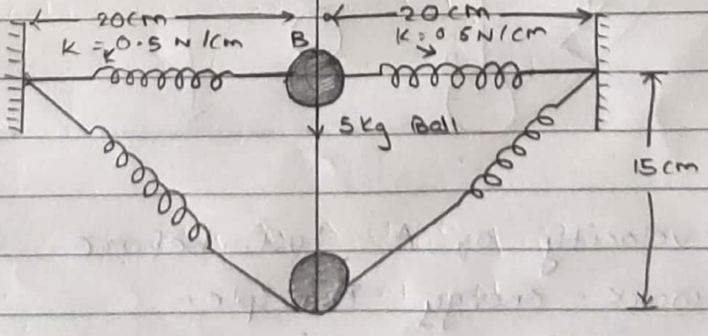
$\therefore v^2 - u^2 = 2ax$

$v^2 - 0 = 2(1.962)(0.6)$

$\therefore v = 1.534$ m/s

$\therefore$ velocity of P_1 after falling 0.6 m is 1.534 m/s and Tension in the string is 3.2 N.

5.2.6) Two springs each having a stiffness of 0.5 N/cm are connected to a ball B having a mass of 5 kg, in a horizontal position producing initial tension of 1.5N in each spring. If the ball is allowed to fall from rest, what will be its velocity after it has fallen through a height of 15 cm.



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$K = 0.5 N/cm = 50 N/m$

Let x be the initial extension in spring and 'L' be length of spring.

$x = \frac{1.5}{50} = 0.03 m$ and $L = 0.2 - 0.03 = 0.17 m$

Let x_1 be final extension in spring after ball goes down by 15 cm.

$(L + x_1)^2 = (15)^2 + (20)^2 = (25)^2$

$\therefore L + x_1 = 25 cm = 0.25 m \Rightarrow x_1 = 0.08 m$

Using work - Energy Principle,

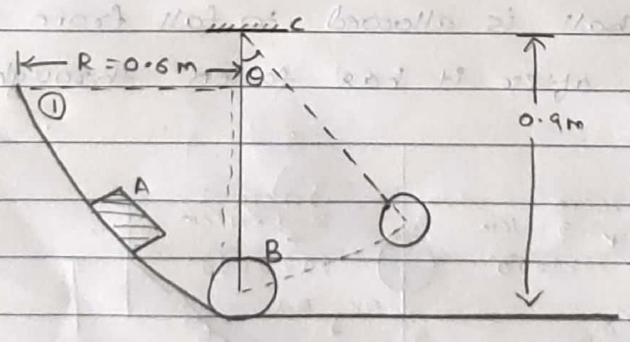
$mgh + 2 \times \frac{1}{2} K (x_1^2 - x_2^2) = \frac{1}{2} mv^2 - \frac{1}{2} mu^2$

$\therefore 5 \times 9.81 \times 0.15 + 1 \times 50 [(0.03)^2 - (0.08)^2] = \frac{1}{2} \times 5 V^2$

$\therefore V = \sqrt{7.083 \times 2/5}$ $\therefore$ $\boxed{V = 1.683 m/s (\downarrow)}$

$\therefore$ velocity of ball after falling 15 cm is $1.683 m/s (\downarrow)$

5.3.3)



→ Let u be velocity of A just before collision.

Using work - energy Principle.

$m_A g R = \frac{1}{2} m_A V_1^2 - \frac{1}{2} m_A V_2^2$

$m_A g R = \frac{1}{2} m_A u_1^2 \Rightarrow u_1 = \sqrt{2gR} = \sqrt{2 \times 0.6 \times 9.81} = 3.43 m/s$

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Let v_1 be final velocity of A just after impact
v_2 be velocity of B just after impact
$e = 0.9$ (given)

$\therefore v_2 - v_1 = 0.9 \times 3.43 = 3.087$ m/s — (1)

By law of conservation of momentum,

$m_A u_1 + m_B u_2 = m_A v_1 + m_B v_2$

$1.125 \times 3.43 = 1.125 v_1 + 1.8 v_2$ — (2)

On solving (1) and (2)

$v_1 = -0.581$ m/s $= 0.581$ m/s $(\leftarrow)$, $v_2 = 2.507$ m/s $(\rightarrow)$

Let height to which ball B rises be 'h'

$h = 0.9 - 0.9\cos\theta$

Using work-Energy Theorem.

$m_B g(-h) = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$v = 0$, $u = v_2$

$\therefore -1.8 \times g \times h = 0 - \frac{1}{2} \times 1.8 \times (2.507)^2$

$\therefore h = 0.32$ m

$0.32 = 0.9 - 0.9\cos\theta$

$\theta = \cos^{-1}(0.6444)$

$\therefore \boxed{\theta = 49.879^\circ}$

$\therefore$ Velocity of B immediately after impact is 2.507 m/s $(\rightarrow)$

Max angular displacement of pendulum $= 49.879^\circ$

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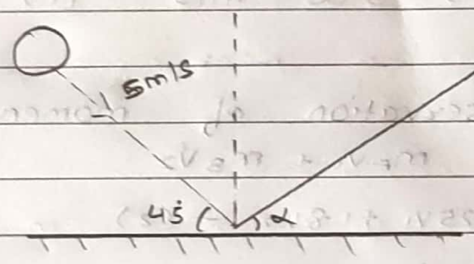
5.3.8) A billard ball moving with a velocity of 5m/s strikes a smooth horizontal plane at an angle of 45° with horizontal. If the coefficient of restitution is 0.9 what is the velocity with which ball rebounds.



→ Coefficient of restitution (e) = 0.9
since 'e' won't act along the horizontal axis.

$u\cos 45 = v\cos\alpha$ ∴ (Law of conservation of momentum)

$\frac{5}{\sqrt{2}} = v\cos\alpha$

Now, $e = \frac{v_y}{u_y} \Rightarrow 0.9 = \frac{v\sin\alpha}{5\sin 45}$

$\therefore v\sin\alpha = 0.9 \times \frac{5}{\sqrt{2}}$ — (2)

Dividing eq. (2) by eq. (i)

$\tan\alpha = 0.9$

$\alpha = \tan^{-1}(0.9)$

$\alpha = 41.99°$

$\therefore v\cos\alpha = \frac{5}{\sqrt{2}} \Rightarrow v = \frac{5}{\sqrt{2}\cos(41.99)}$

$\therefore v = 4.757$ m/s

∴ velocity with which the ball rebounds = 4.757 m/s ($\angle 41.99°$)