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Discrete Structure

Tutorial - 1

1.) To prove: $A-(B\cap C) = (A-B)\cup(A-C)$

→ i) let $x = A-(B\cap C)$

$\therefore x \in A$ and $x \notin (B\cap C)$

$\therefore x \in A$ and $x \notin B$ or $x \notin C$

$\therefore x \in (A-B)$ or $x \in (A-C)$

$\therefore A-(B\cap C) \subseteq (A-B)\cup(A-C)$

$\therefore x \in A$ and $x \notin B$ and $x \notin C$

$\therefore x \in A$ and $x \notin B$ or $x \in A$ and $x \notin C$

$\therefore x \in (A-B)\cup(A-C)$

—— (1)

ii) let $x = (A-B)\cup(A-C)$

$\therefore x \in (A-B)$ or $x \in (A-C)$

$\therefore x \in A$ and $x \notin B$ and $x \notin C$

$x \in A-(B\cap C)$

$\therefore x \in A$ and $x \notin B$ or $x \in A$ and $x \notin C$

$\therefore x \in A$ and $x \notin (B\cap C)$

$\therefore (A-B)\cup(A-C) \subseteq A-(B\cap C)$ —— (2)

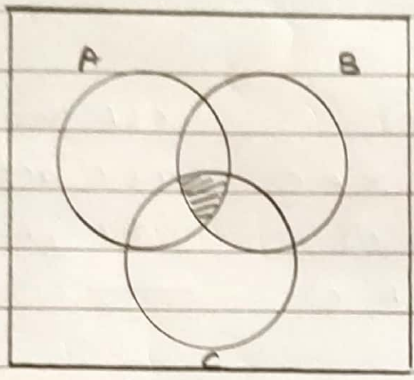
From (1) and (2), we get

$A-(B\cap C) = (A-B)\cup(A-C)$

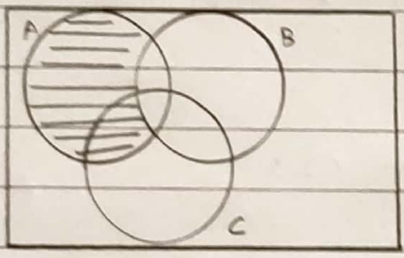
- Hence Proved.

2) TO prove: $A \cap B \cap C \neq [(A-B) \cup (A-C)]$

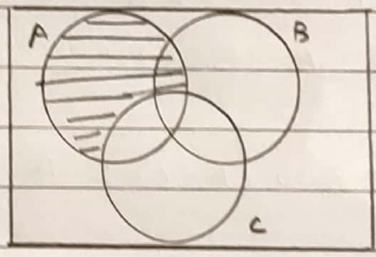
→ LHS: $A \cap B \cap C$:



$A \cap B \cap C$

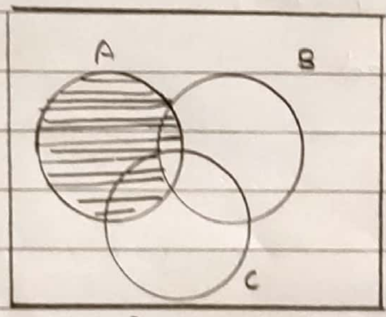


$A - B$



$A - C$

RHS: $(A-B) \cup (A-C)$



$(A-B) \cup (A-C)$

Since LHS $\neq$ RHS

$\therefore A \cap B \cap C \neq [(A-B) \cup (A-C)]$

3) let $S = \{1,2,3,4,5,6,7,8,9\}$

→ i) $\{\{1,3,5\}, \{2,6\}, \{4,8,9\}\}$

let $A_1 = \{1,3,5\}$, $A_2 = \{2,6\}$, $A_3 = \{4,8,9\}$

$\therefore A_1 \cup A_2 \cup A_3 = \{1,2,3,4,5,6,8,9\} \neq S$ — (1)

$\therefore A_1 \cap A_2 \cap A_3 = \{\} = \phi$ — (2)

Since, A_1, A_2 and A_3 are not satisfying (2) ~~both~~ ~~the~~ conditions,
It is not partition of S

→ ii) $\{\{1,3,5\}, \{2,4,6,8\}, \{5,7,9\}\}$

let $A_1 = \{1,3,5\}$, $A_2 = \{2,4,6,8\}$, $A_3 = \{5,7,9\}$

$\therefore A_1 \cup A_2 \cup A_3 = \{1,2,3,4,5,6,7,8,9\} = S$ — (1)

$\therefore A_1 \cap A_2 \cap A_3 = \{5\} \neq \phi$ — (2)

Since, A_1, A_2 and A_3 are not satisfying (2) condition,
It is not partition of S

→ (iii) $\{\{1,3,5\}, \{2,4,6,8\}, \{7,9\}\}$

let $A_1 = \{1,3,5\}$, $A_2 = \{2,4,6,8\}$, $A_3 = \{7,9\}$

$\therefore A_1 \cup A_2 \cup A_3 = \{1,2,3,4,5,6,7,8,9\} = S$ — (i)

$\therefore A_1 \cap A_2 \cap A_3 = \{\} = \phi$ — (2)

$\because$ It is satisfying both the conditions.

It is partition of ~~A~~ S.

→ iv) $\{\{S\}\}$

$\therefore$ Since S is a subset of S

$\therefore$ It is partition of S

5) let A : data structures

B : assembly languages

C : Foundation.

$|A \cup B \cup C = 119|$

$|A| = 96$

$|B| = 39$

$|C| = 53$

$|B \cap C| = 31$

$|A \cap B| = 32$

$|A \cap C| = 38$

$|A \cap B \cap C| = 22$

Acc. to defination,

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

$\therefore$ LHS $= |A \cap B \cap C| = 119$

RHS $= |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

$= 96 + 39 + 53 - 32 - 31 - 38 + 22$

$= 109$

$\therefore$ LHS $\neq$ RHS

$\therefore$ The information given is incorrect.

6) To prove: $A \cap B \cup [B \cap ((C \cap D) \cup (C \cap \bar{D}))] = B \cap (A \cup C)$

Proof:

$LHS = (A \cap B) \cup [B \cap ((C \cap D) \cup (C \cap \bar{D}))]$

$= (A \cap B) \cup [B \cap (C \cap (D \cup \bar{D}))]$ ... (Distributive law)

$= (A \cap B) \cup [B \cap (C \cap U)]$ ... $(D \cup \bar{D} = U)$

$= (A \cap B) \cup [B \cap C]$ ... $[C \cap U = C]$

$\cancel{BA} = (B \cap A) \cup (B \cap C)$ ... (Commutative law)

$= B \cap (A \cup C)$ ... (Distributive law)

$= RHS$

$\therefore LHS = RHS$

$\therefore (A \cap B) \cup [B \cap ((C \cap D) \cup (C \cap \bar{D}))] = B \cap (A \cup C)$

4) Simplify: $\overline{\overline{A \cup B} \cap C \cup \bar{B}}$

$\therefore \overline{\overline{A \cup B} \cap C \cup \bar{B}} = \overline{\bar{A} \cap \bar{B} \cap C \cup \bar{B}}$ ... (De Morgan's law)

$= \overline{\bar{A} \cap \bar{C} \cap \bar{B} \cup \bar{B}}$ ... (Commutative law)

$= \overline{\bar{A} \cap \bar{C} \cap \bar{B}}$ ... (De Morgan's law)

$= A \cup B \cup C$