

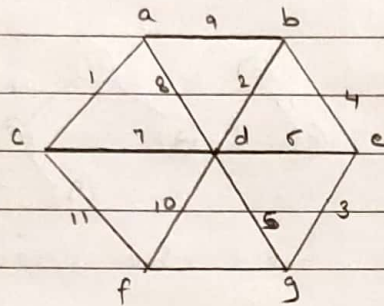
24/01/2022

Discrete Structures

Tutorial - 8

Solutions

→ 1)

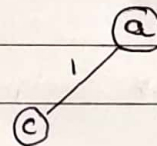


Remark

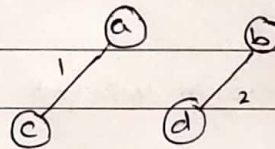
Spanning trees

1) Edge (a, c) has smallest weight.

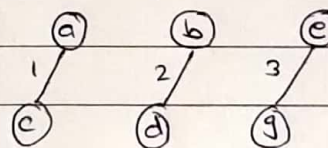
Selected (a, c)



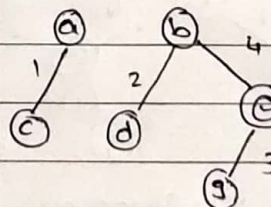
2) Edge (d, b) with weight 2



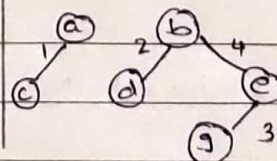
3) Edge (e, g) with weight 3



4) Edge (b, e) with weight 4



5) edge (d, g) with weight 5

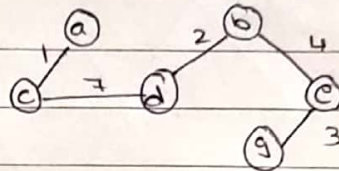


(d, g) can't be selected as it form cycle

6) edge (d,e) with weight 6

can't be selected as it forms cycle.

7) edge (d,e) with weight 7



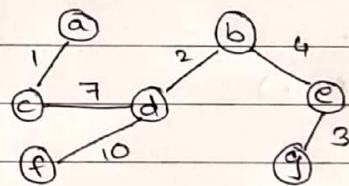
8) edge (a,d) with weight 8

can't be selected as it forms cycle.

9) edge (a,b) with weight 9

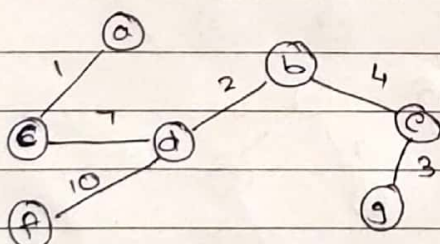
can't be selected as it forms cycle.

10) edge (f,d) with weight 10



11) edge (c,f) with weight 11

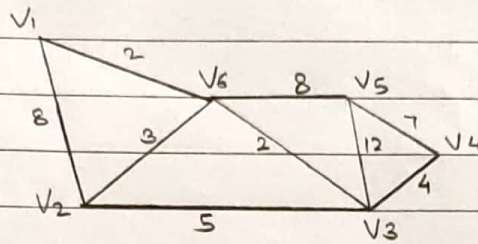
can't be selected as it forms cycle.



is minimal spanning tree.

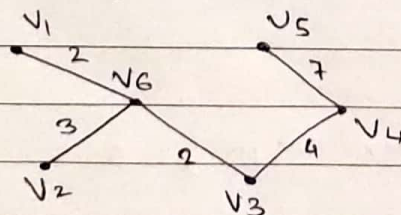
$$\text{Cost MST} = 1 + 2 + 3 + 4 + 10 + 7 = 27$$

→ 2)



edges consider	edge selected	cost	Spanning tree
1) -	-	-	V_1
2) $(V_1, V_2), (V_1, V_6)$	(V_1, V_6)	2	$V_1 \xrightarrow{2} V_6$
3) $(V_1, V_2), (V_6, V_2)$ $(V_6, V_3), (V_6, V_5)$	(V_6, V_3)	2	$V_1 \xrightarrow{2} V_6 \xrightarrow{2} V_3$
4) $(V_1, V_2), (V_6, V_2),$ $(V_6, V_5), (V_3, V_2)$ $(V_3, V_5), (V_3, V_4)$	(V_6, V_2)	3	$V_1 \xrightarrow{2} V_6 \xrightarrow{2} V_3 \xrightarrow{3} V_2$
5) $(V_1, V_2), (V_6, V_5),$ $(V_3, V_2), (V_3, V_5),$ $(V_3, V_4).$	(V_3, V_4)	4	$V_1 \xrightarrow{2} V_6 \xrightarrow{2} V_3 \xrightarrow{3} V_2 \xrightarrow{4} V_4$
6) $(V_3, V_5), (V_4, V_5)$	(V_4, V_5)	7	$V_1 \xrightarrow{2} V_6 \xrightarrow{2} V_3 \xrightarrow{3} V_2 \xrightarrow{4} V_4 \xrightarrow{7} V_5$

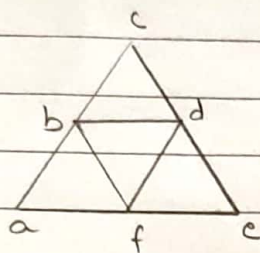
∴ Minimum Spanning tree by Prim's Algorithm



$$\text{cost MST} = 2 + 2 + 4 + 7 + 3 = 18$$

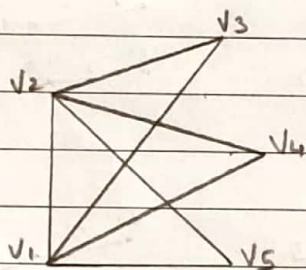
→ 3)

a)



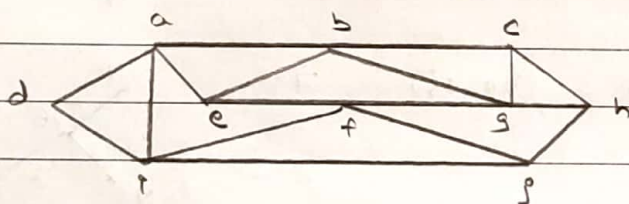
Since vertex a is of odd degree
The given graph does not contain Eulerian circuit.

b)



Since all vertices are of even degree,
Hence the graph is Eulerian graph and contain Eulerian circuit.

4)



Number of vertices, $n = 10$

$$\therefore \frac{n}{2} = 5$$

The degree of each vertex $> (\frac{n}{2}) = 5$

Hence by theorem 2, there is no Hamiltonian circuit and hence no hamiltonian region.

\therefore The above condition is sufficient but not necessary.