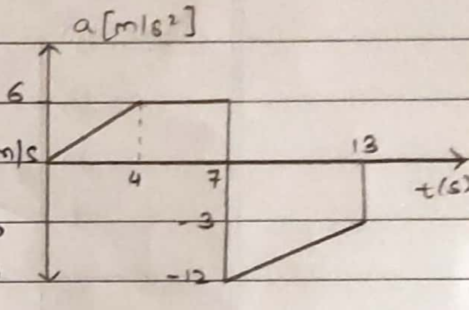
SAP ID - 60004200132
Name - Ayush Jain
Div - J

Engineering Mechanics

Kinematics of Particles



4.1.18) The a-t curve for a particle having rectilinear motion is shown at $t=0$, $v=8\,m/s$ and the particle is 60 m to the left of the origin of displacement. Draw v-t and s-t curves specifying the values at $t=4$, 7 and 13 secs.

→ At $t=0$, $v=8\,m/s$, $x=-60\,m$.

For a-t graph,

$\Delta V = $ (Area under a-t graph)

$\therefore V_{t=4} - V_{t=0} = \frac{1}{2} \times 4 \times 6$

$V_{t=4} - 8 = 12$

$\boxed{V_{t=4} = 20\ m/s}$ —

$V_{12} - V_6 = 6 \times 3$

$V_{12} - 20 = 18$

$\boxed{V_{12} = 38\ m/s}$

$V_{13} - V_7 = -\left[6 \times 3 + \frac{1}{2} \times 6 \times 9\right]$

$V_{13} - 38 = -[18 + 27]$

$\boxed{V_{13} = -7\ m/s}$

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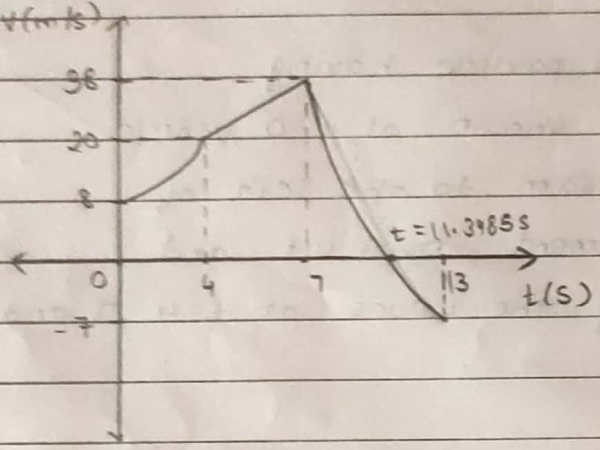
v-t graph.



$X_{t=0} = -60\,m$.

$\therefore X_4 - X_0 = \left(4 \times 8 + \frac{1}{3} \times 4 \times 12\right)$

$X_4 + 60 = 48$

$X_4 = -12\,m$.

$X_7 - X_4 = 20 \times 3 + \frac{1}{2} \times 3 \times 18$

$X_7 + 12 = 87$

$X_7 = 75\,m$

$\cancel{X_{13}} \cancel{- X_7} \rightarrow \quad v = \frac{3t^2}{4} - \frac{90t}{4} + \frac{635}{4}$

$\frac{dx}{dt} = \frac{3t^2}{4} - \frac{90t}{4} + \frac{635}{4}$

$\int_{t=7}^{t=11.3485} dx = \int \left(\frac{3t^2}{4} - \frac{90t}{4} + \frac{635}{4}\right)$

$\therefore$ ~~ds~~

~~$\Delta S = \Delta x = -6.343\,m$~~ $\quad \Delta x = 72.343\,m$

~~$\therefore S = X_{13} = 141\,m$~~ $\quad \therefore X$ at $t = 11.3485\,s$ is $147.343\,m$

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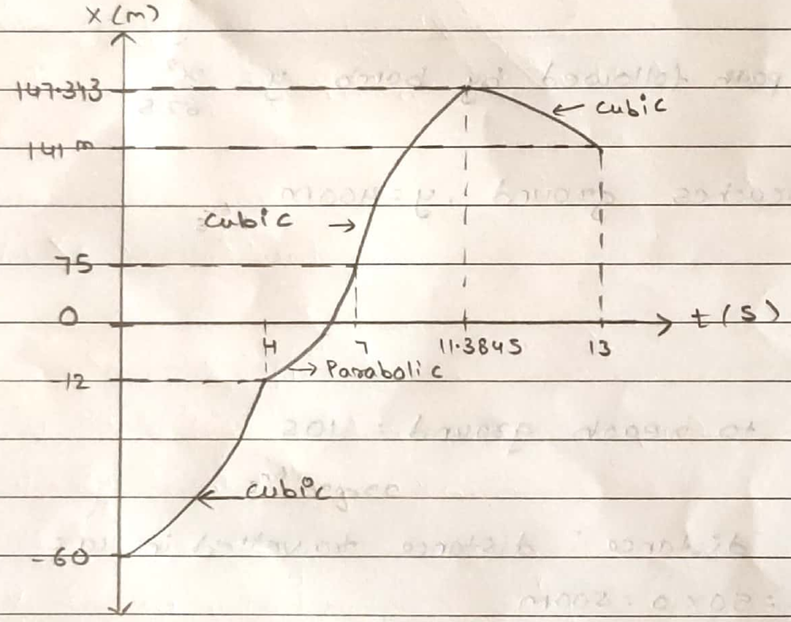
$$\left(dx\right)_{t=11.3845}^{t=13} = \left[\frac{t^3}{4} - \frac{45}{4}t^2 + \frac{635}{4}t + c\right]_{11.3485}^{13}$$

$\therefore \Delta x = -6.343\ m$

$\therefore x$ at 13 s is 141 m



4.1.22) A bomb thrown from a plane flying at height 400m moves along the path vector $r = (50t)\hat{i} + (4t^2)\hat{j}$ m. The origin is taken as the point from where, the bomb is released and the +ve y axis is taken as pointing downwards. Find, i) Equation of path followed by bomb. 2) Time taken to reach the ground. 3) Horizontal distance travelled by bomb. 4) Displacement, velocity and acceleration at $t = 5$ s. 5) Tangential and normal component of acceleration at $t = 5$ sec.

→ i) height = 400m

$r = 50t\,\hat{i} + 4t^2\,\hat{j}$

$\therefore x = 50t$, $y = 4t^2$

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$\therefore t = \frac{x}{50}$

$\therefore y = 4\left(\frac{x^2}{2500}\right)$

$\therefore y = \frac{x^2}{625}$

$\therefore$ Equation of path followed by bomb $y = \frac{x^2}{625}$

ii) When bomb reaches ground, $y = 400m$

$y = 4t^2$

$400 = 4t^2$

$\therefore t = 10s$

$\therefore$ Time taken to reach ground $= 10s$

iii) Total horizontal distance = distance travelled in 10 s

$\therefore x = 50t = 50 \times 10 = 500m$

iv) Position at $t = 5$ sec $= [50t, 4t^2]$

$[50 \times 5, 4 \times 25] = [250, 100]$

displacement $= \sqrt{(250)^2 + (100)^2} = 269.258m$

$\therefore \boxed{S = 269.258m}$

Velocity $(v) = \frac{dr}{dt} = 50\hat{i} + 8t\hat{j}$

$\therefore$ Velocity at $t = 5$ sec $= \sqrt{50^2 + (8 \times 5)^2}$

$= \sqrt{50^2 + 40^2} = 64.031 m/s$ $\quad \boxed{\therefore V = 64.031 m/s}$

$a = \frac{dv}{dt} = 0\hat{i} + 8\hat{j}$

$\therefore \boxed{a = 8 m/s\ (\downarrow)}$

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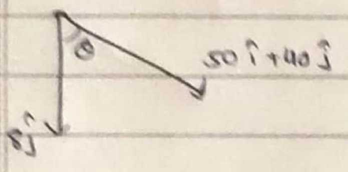
(v)



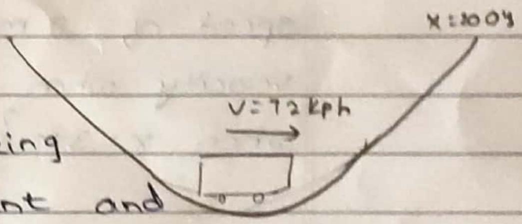
$$\cos\theta = \frac{(8\hat{j})\cdot[50\hat{i}+40\hat{j}]}{|8\hat{j}||50\hat{i}+40\hat{j}|}$$

$$\theta = \cos^{-1}\left[\frac{8\times 40 + 0\times 50}{8\times 64.031}\right]$$

$$\theta = 51.34°$$

$$\therefore \boxed{\begin{array}{l} a_T = a\cos\theta = 4.998\ m/s^2 \\ a_c = a\sin\theta = 6.247\ m/s^2 \end{array}}$$

4.1.24) A car travels along a vertical curve on a road. The equation of curve being $x^2 = 200y$. The speed of car is constant and equal to 72 Kmph. Determine the acceleration of car when it is at the dippest point on the curve.



→ Equation ⟹ $x^2 = 200y$

$$\therefore 2x = 200\frac{dy}{dx}$$

$$\therefore \frac{dy}{dx} = \frac{x}{100}$$

$$\frac{d^2y}{dx^2} = \frac{1}{100}$$

$$\rho\ (\text{radius of curvature}) = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{|d^2y/dx^2|}$$

$$= \frac{\left[1+\left(\frac{0}{100}\right)^2\right]^{3/2}}{1/100}$$

$$\rho = 100\ m$$

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$V = 72 \, km/h \quad = 72 \times \frac{5}{18} = 20 \, m/s$

$\therefore V = constant, \quad a_T = 0$

$a_c = \frac{v^2}{s} = \frac{(20)^2}{100} = 4 \, m/s^2$

$\therefore$ Acceleration of car at dippest point is $4 \, m/s^2$

4.1.25) A point moves along the path $y = \left(\frac{x^2}{3}\right)$ with a constant speed of 8 m/s. What are the x and y components of velocity when $x = 3m$? What is the acceleration of point when $x = 3m$?

→ $y = \frac{x^2}{3}$

~~$\frac{dy}{dx} = \frac{2x}{3} = \tan\theta$~~

Differentiating wrt t,

$\frac{dy}{dt} = \frac{2x}{3} \frac{dx}{dt}$

$V_y = \frac{2x}{3} V_x$

For $x = 3$,

$V_y = 2 V_x$

Now,

$\sqrt{V_x^2 + V_y^2} = 8$

$\therefore V_x^2 + 4V_x^2 = 64$

$V_x^2 = \frac{64}{5}$

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$V_x = \frac{8}{\sqrt{5}} = 3.578 \, m/s$

$V_y = 2V_x = 7.155 \, m/s$.

$\frac{dy}{dx} = \frac{2x}{3}$

$\therefore \left(\frac{dy}{dx}\right)_{x=3} = 2$

$\therefore \frac{d^2y}{dx^2} = \frac{2}{3}$

$\therefore \rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2}$

$\rho = \frac{\left[1 + (2)^2\right]^{3/2}}{2/3}$

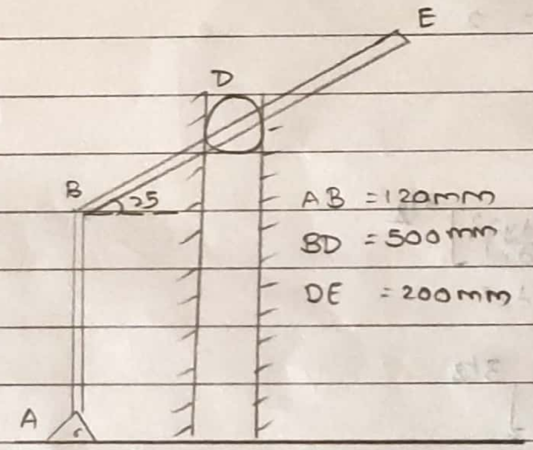
$\rho = 16.77 \, m$

$\therefore a_N = \frac{v^2}{\rho} = \frac{64}{16.77} = 3.816 \, m/s^2$

$\therefore a = a_N = 3.816 \, m/s^2$

Acceleration at $x = 3m$ is $3.816 \, m/s^2$

4.2.2) Rod BDE is guided partially by roller at D, which moves in vertical track. Knowing that at the instant shown, the angular velocity of rod AB is 5 rad/s (clockwise), find angular velocity of rod BE and velocity of point E.



$V_B = 120 \times 5$

$= 600$ mm/s

$BD = 500$ mm

$D\,I_{CR} = 500 \cos 25$

$= 453.15$ mm

$B\,I_{CR} = 500 \sin 25$

$= 211.31$ mm

$\omega_{BDE} = \frac{V_B}{B I_{CR}} = \frac{600}{211.31}$

$\omega_{BDE} = 2.839$ rad/s.

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$\angle EDI_{CR} = 180 - 25 = 155$

$ED = 200\,mm$

$DI_{CR} = 453.15$

$\therefore$ Using cosine rule for triangle EDI_{CR},

$$EI_{CR} = \sqrt{ED^2 + DI_{CR}^2 - 2(ED)(DI_{CR})\cdot \cos 155}$$

$$= \sqrt{(200)^2 + (453.15)^2 - 2(200)(453.15)\cos 155}$$

$EI_{CR} = 640.02\,mm$

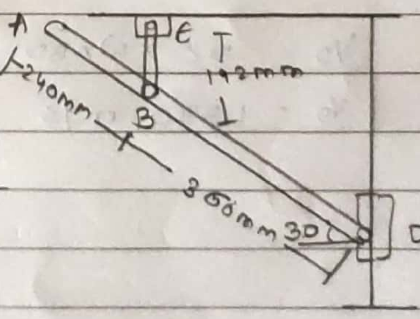
$V_E = \omega_{BDE} \times EI_{CR}$

$= 2.839 \times 640.02$

$V_E = 1817.01\,mm/s$

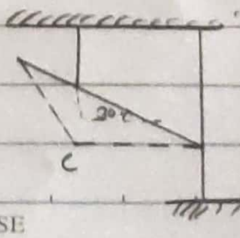
$\therefore$ Velocity at point E is $1817.01\,mm/s$.

4.2.10) Knowing that at the instant shown the angular velocity of rod BE is 4 rad/s counterclockwise, determine
(a) angular velocity of rod AD,
(b) velocity of collar D,
(c) velocity of point A.



$\rightarrow$ $DC = BD\cos 30$

$= 360\cos 30 = 311.77\,mm$



$BC = BD \sin 30$

$= 360 \sin 30 = 180$ mm

$V_B = \in B \times W_{\in B}$

$= 192 \times 4$

$V_B = 768$ mm/s $= 0.768$ m/s

$\omega_{ABD} = \frac{V_B}{BC} = \frac{0.768}{0.180} = 4.267$ rad/s

$\boxed{\omega_{ABD} = 4.267 \text{ rad/s}}$

By cosine rule,

$AC = \sqrt{AD^2 + DC^2 - 2(AD)(DC)\cos 30}$

$= \sqrt{600^2 + (311.77)^2 - 2 \times 600 \times 311.77 \cos 30}$

$= 364.97$ mm

$AC = 0.36497$ m

$V_A = AC \times \omega_{ABD}$

$V_A = 1.557$ m/s

$V_D = DC \times \omega_{ABD}$

$\boxed{V_D = 1.330 \text{ m/s}}$

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