Name - Ayush Jain
SAP ID - 60004200132
Div - J1

DATE:

26/07/21

Engineering Maths

Tutorial 5 : Differential equation of First order and first degree.

1) Solve $x \sin x \, dy + [y(x\cos x - \sin x) - 2] \, dx = 0$

2) Solve $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$

3) Solve $y(\sin(xy) + xy\cos(xy))dx + x(xy\cos(xy) - \sin(xy))dy = 0$

4) Solve $(3xy - 2ay^2)dx + (x^2 - 2axy)dy = 0$

5) Solve $\frac{dy}{dx} - xy = y^2 e^{-\left(\frac{x^2}{2}\right)} \log x$

6) Solve $\tan y \frac{dy}{dx} + \tan x = \cos y \cdot \cos^3 x$.

DATE:

Solutions :

1) $x\sin x\, dy + [y(x\cos x - \sin x) - 2]\, dx = 0$ — (1)

It is of the form $M dx + N dy = 0$

where $M = xy\cos x - y\sin x - 2$

$N = x\sin x$

Now, $\frac{\partial M}{\partial y} = x\cos x - \sin x$, $\frac{\partial N}{\partial x} = x\cos x + \sin x$

$\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

$$\therefore \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{-2\sin x}{x\sin x} = -\frac{2}{x} = f(x)$$

$$\therefore I.F = e^{\int -\frac{2}{x}dx} = e^{-2\log x} = \frac{1}{x^2}$$

Multiplying I.F with equation (1)

$$\left(\frac{y\cos x}{x} - \frac{y\sin x}{x^2} - \frac{2}{x^2}\right)dx + \frac{\sin x}{x}dy = 0 \quad — (2)$$

$$\frac{\partial M}{\partial y} = \frac{\cos x}{x} - \frac{\sin x}{x^2} = \frac{\partial N}{\partial x} = \frac{\cos x}{x} - \frac{\sin x}{x^2}$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$\therefore$ equation (2) is an exact differential equation.

Scanned with CamScanner

DATE:

It's general equation is given by;

$$\int N\,dy + \int \left(M - \left(\frac{\partial}{\partial x}\int N\,dy\right)\right)dx = c$$

$$\int \frac{\sin x}{x}\,dy + \int \frac{-2}{x^2}\,dx = c$$

$$\frac{y\sin x}{x} - 2\left(\frac{-1}{x}\right) = c$$

$$\therefore \boxed{\frac{y\sin x}{x} + \frac{2}{x} = c}$$

2> $(xy^3+y)\,dx + 2(x^2y^2 + x + y^4)\,dy = 0$ — (i)

It is of the form $M\,dx + N\,dy = 0$
where $M = xy^3 + y$
$N = 2x^2y^2 + 2x + 2y^4$

$$\frac{\partial M}{\partial y} = 3xy^2 + 1 \neq \frac{\partial N}{\partial x} = 4xy^2 + 2$$

and
$$\frac{-\frac{\partial M}{\partial y} + \frac{\partial N}{\partial x}}{M} = \frac{-(3xy^2+1) + 4xy^2 + 2}{xy^3 + y}$$

$$= \frac{xy^2+1}{y(xy^2+1)} = \frac{1}{y} = f(y)$$

$$\therefore I.F = e^{\int \frac{1}{y}dy} = e^{\log y} = y$$

DATE:

Multiplying equation (i) by I.F , we get

$(xy^4+y^2)\,dx + (2x^2y^3+2xy+2y^5)\,dy = 0$ — (2)

$\therefore \frac{\partial M}{\partial y} = x\cdot 4y^3 + 2y = \frac{\partial N}{\partial x} = 2y^3\cdot 2x + 2y$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\therefore$ equation (2) is an exact differential equation.

It's general equation is given by,

$\int M\,dx + \int \left(N - \frac{\partial}{\partial y}\left(\int M\,dx\right)\right) dy = c$

$\int (xy^4+y^2)\,dx + \int 2y^5\,dy = c$

$y^4\cdot\frac{x^2}{2} + xy^2 + \frac{\cancel{2}y^6}{\cancel{6}\,3} = c$

$\boxed{3x^2y^4 + 6xy^2 + 2y^6 = C}$

3> $\underbrace{y(\sin(xy) + xy\cos(xy))}_{M}dx + \underbrace{x(xy\cos(xy) - \sin(xy))}_{N}dy = 0$ — (i)

It is of the form $M\,dx + N\,dy = 0$

$\therefore \frac{\partial M}{\partial y} = \sin xy + xy\cos(xy) + 2xy\cos xy - x^2y^2\sin xy$

$\therefore \frac{\partial N}{\partial x} = 2xy\cos xy - x^2y^2\sin xy - \sin xy - xy\cos xy$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \Rightarrow$ (i) is not exact differential equation.

DATE:

$$\therefore \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \frac{-2\sin xy - 2xy\cos xy}{y(\sin xy + xy\cos xy)}$$

$$= -\frac{2}{y} = f(y)$$

$$IF = e^{\int f(y)dy} = e^{\int -\frac{2}{y} dy} = e^{-2\log y} = \frac{1}{y^2}$$

Multiplying I.F with (i),

$$\left(\frac{\sin xy}{y} + x\cos xy\right)dx + \left(\frac{x^2\cos xy}{y} - \frac{x\sin xy}{y^2}\right)dy = 0 \quad \text{— (2)}$$

(2) is of form $M dx + N dy = 0$

$$\frac{\partial M}{\partial y} = \frac{yx\cos xy - \sin xy}{y^2} - x^2\sin xy$$

$$\therefore \frac{\partial N}{\partial x} = \frac{2x\cos xy}{y} - \frac{x^2 y\sin xy}{y} - \frac{\sin xy}{y^2} - \frac{xy\cos xy}{y^2}$$

$$= \frac{x\cos yx}{y} - \frac{x^2\sin xy}{y} - \frac{\sin xy}{y^2}$$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \implies$ (2) is exact diffraction equation.

General solution : $\int M\,dx + \int \left(N - \frac{\partial}{\partial y}\int M\,dx\right)dy = c$

$$\therefore \int \left(\frac{\sin xy}{y} + x\cos xy\right)dx + \int 0\,dy = c$$

$$\therefore \int \frac{\sin xy}{y}\,dx + \int \frac{x}{4}\cos xy\,dx = c$$

$$\therefore \int \frac{\sin xy}{y}\,dx + x\int \cos xy\,dy - \int\left(\int \cos xy\,dx\right)dx = c$$

DATE:

$$\int \frac{\sin xy}{y}\,dx + \frac{x \sin xy}{y} - \int \frac{\sin xy}{y}\,dx = c$$

$$\therefore \frac{x \sin xy}{y} = c$$

$$\therefore \boxed{x \sin(xy) = cy}$$

4) $(3xy - 2ay^2)\,dx + (x^2 - 2axy)\,dy = 0$ — (1)

Eq (1) is of form $M\,dx + N\,dy = 0$

where $M = 3xy - 2ay^2$

$N = x^2 - 2axy$

$$\frac{\partial M}{\partial y} = 3x - 4ay \neq \frac{\partial N}{\partial x} = 2x - 2ay$$

Eq (1) is not exact differential equation.

And $$\frac{\frac{dM}{dy} - \frac{dN}{dx}}{N} = \frac{x - 2ay}{x^2 - 2axy} = \frac{1}{x} = f(x)$$

$$\therefore IF = e^{\int \frac{1}{x} dx} = x$$

Multiplying IF with eq (1)

$$\therefore \underbrace{(3x^2y - 2axy^2)}_{M}\,dx + \underbrace{(x^3 - 2ax^2y)}_{N}\,dy = 0$$

DATE:

$$\frac{\partial M}{\partial y} = 3x^2 - 4axy = \frac{\partial N}{\partial x}$$

$\therefore$ Eq (2) is an exact differential equation.

General solution is given by:

$$\int M\,dx + \int \left(N - \frac{\partial}{\partial y}\left(\int M\,dx\right)\right)dy = c$$

$$\therefore \int (3x^2y - 2axy^2)\,dx + \int 0\,dy = c$$

$$\therefore \frac{3x^3y}{3} - \frac{2ax^2y^2}{2} = c$$

$$\therefore x^3y - ax^2y^2 = c$$

$$\therefore \boxed{x^2y(x-ay)=c}$$

5) $$\frac{dy}{dx} - xy = y^2 \cdot e^{-\left(\frac{x^2}{2}\right)} \log x$$

$$\frac{1}{y^2}\frac{dy}{dx} - \frac{x}{y} = e^{\left(-\frac{x^2}{2}\right)} \log x \quad \text{— (i)}$$

Putting $-\frac{1}{y} = u$

$$\therefore \frac{1}{y^2}\frac{dy}{dx} = \frac{du}{dx}$$

$\therefore$ Eq (i) becomes,

$$\frac{du}{dx} + xu = e^{-x^2/2}\log x$$

This is a linear differential equation of form $\frac{dy}{dx} + Py = Q$

DATE:

$\therefore$ IF $= e^{\int x\,dx} = e^{\frac{x^2}{2}}$

The solution is

$$ue^{\frac{x^2}{2}} = \int e^{x^2/2} \cdot e^{-x^2/2} \log x \,dx + C$$

$$ue^{\frac{x^2}{2}} = \int \log x \,dx + C$$

$$ue^{x^2/2} = x\log x - \int x \cdot \frac{1}{x}\,dx + C$$

$$ue^{x^2/2} = x\log x - x + C$$

But $u = -\frac{1}{y}$

$$\therefore -\frac{e^{\frac{x^2}{2}}}{y} = x(\log x - 1) + C$$

$$\therefore -\frac{e^{\frac{x^2}{2}}}{y} = x\log x - x + C$$

$$\therefore \boxed{\frac{1}{y}e^{x^2/2} + x\log x - x = C}$$

DATE:

6) $\tan y \frac{dy}{dx} + \tan x = \cos y \cdot \cos^3 x$.

$\sec y \tan y \frac{dy}{dx} + \tan x \sec y = \cos^3 x$

Putting $\sec y = u$

$\therefore \sec y \cdot \tan y \cdot \frac{dy}{dx} = \frac{du}{dx}$

$\therefore$ Equation reduce to,

$\frac{du}{dx} + \tan x \cdot u = \cos^3 x$

It is linear equation of the form $\frac{dy}{dx} + Py = Q$

$IF = e^{\int \tan x \, dx} = e^{\log \sec x} = \sec x$

The solution is,

$u \sec x = \int \sec x \cdot \cos^3 x \, dx + c$

$u \sec x = \int \cos^2 x \, dx + c$

$u \sec x = \int \frac{1+\cos 2x}{2} \, dx + c$

$u \sec x = \frac{x}{2} + \frac{\sin 2x}{4} + c$

But $u = \sec y$

$\therefore$ $\boxed{\sec y \cdot \sec x = \frac{x}{2} + \frac{\sin 2x}{4} + c}$