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Engineering Mathematics

Term - Test 1 Assignment.

1.) $A = \begin{bmatrix} 1 & 5 & 7 \\ -1 & -2 & -4 \\ 8 & 2 & 13 \end{bmatrix}$

$\therefore A^T = \begin{bmatrix} 1 & -1 & 8 \\ 5 & -2 & 2 \\ 7 & -4 & 13 \end{bmatrix}$

$\therefore A = \frac{1}{2}\left[(A+A^T) + (A-A^T)\right]$

$\frac{1}{2}(A+A^T) = \frac{1}{2}\left(\begin{bmatrix} 1 & 5 & 7 \\ -1 & -2 & -4 \\ 8 & 2 & 13 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 8 \\ 5 & -2 & 2 \\ 7 & -4 & 13 \end{bmatrix}\right)$

$= \frac{1}{2}\begin{bmatrix} 2 & 4 & 15 \\ 4 & -4 & -2 \\ 15 & -2 & 26 \end{bmatrix}$

$\frac{1}{2}(A+A^T) = \begin{bmatrix} 1 & 2 & 15/2 \\ 2 & -2 & -1 \\ 15/2 & -1 & 13 \end{bmatrix}$

$\frac{1}{2}(A-A^T) = \frac{1}{2}\left(\begin{bmatrix} 1 & 5 & 7 \\ -1 & -2 & -4 \\ 8 & 2 & 13 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 8 \\ 5 & -2 & 2 \\ 7 & -4 & 13 \end{bmatrix}\right)$

$$= \frac{1}{2}\begin{bmatrix} 0 & 6 & -1 \\ -6 & 0 & -6 \\ 1 & 6 & 0 \end{bmatrix}$$

$$\frac{1}{2}(A - A^T) = \begin{bmatrix} 0 & 3 & -1/2 \\ -3 & 0 & -3 \\ 1/2 & 3 & 0 \end{bmatrix}$$

$$\therefore \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T) = \begin{bmatrix} 1 & 2 & 15/2 \\ 2 & -2 & -1 \\ 15/2 & -1 & 13 \end{bmatrix} + \begin{bmatrix} 0 & 3 & -1/2 \\ -3 & 0 & -3 \\ 1/2 & 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 5 & 7 \\ -1 & -2 & -4 \\ 8 & 2 & 13 \end{bmatrix} = A$$

Let $P = \frac{1}{2}(A + A^T) = \begin{bmatrix} 1 & 2 & 15/2 \\ 2 & -2 & -1 \\ 15/2 & -1 & 13 \end{bmatrix}$

$$\therefore P^T = \begin{bmatrix} 1 & 2 & 15/2 \\ 2 & -2 & -1 \\ 15/2 & -1 & 13 \end{bmatrix}$$

$\because P = P^T$

$\therefore$ P is a symmetric matrix.

NOW,

$$Q = \frac{1}{2}(A - A^T) = \begin{bmatrix} 0 & 3 & -1/2 \\ -3 & 0 & -3 \\ 1/2 & 3 & 0 \end{bmatrix}$$

$$\therefore Q^T = \begin{bmatrix} 0 & -3 & 1/2 \\ 3 & 0 & 3 \\ -1/2 & -3 & 0 \end{bmatrix}$$

$$= -\begin{bmatrix} 0 & 3 & -1/2 \\ -3 & 0 & -3 \\ 1/2 & 3 & 0 \end{bmatrix} = -Q$$

$$\because Q^T = -Q$$

$\therefore$ Q is a skew-~~Hermition~~ symmetric matrix.

Hence, the matrix can be expressed in the form of sum of symmetric and skew-symmetric matrix.

2) $A = \frac{1}{3}\begin{bmatrix} -2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{bmatrix}$

$$\therefore A' = \frac{1}{3}\begin{bmatrix} -2 & 2 & 1 \\ 1 & 2 & -2 \\ 2 & 1 & 2 \end{bmatrix}$$

$$AA^T = \frac{1}{9}\begin{bmatrix} -2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{bmatrix}\begin{bmatrix} -2 & 2 & 1 \\ 1 & 2 & -2 \\ 2 & 1 & 2 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Now,

$$A^TA = \frac{1}{9}\begin{bmatrix} -2 & 2 & 1 \\ 1 & 2 & -2 \\ 2 & 1 & 2 \end{bmatrix}\begin{bmatrix} -2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Since, $AA^T = A^TA = I$

$\therefore$ A is orthogonal matrix

$$\therefore A^{-1} = A^{T} = \frac{1}{3}\begin{bmatrix} -2 & 2 & 1 \\ 1 & 2 & -2 \\ 2 & 1 & 2 \end{bmatrix}$$

3) $A = \frac{1}{2}\begin{bmatrix} 1 & -i & -1+i \\ i & 1 & 1+i \\ 1+i & -1+i & 0 \end{bmatrix}$

$$\therefore A^{\Theta} = \frac{1}{2}\begin{bmatrix} 1 & -i & 1-i \\ i & 1 & -1-i \\ -1-i & 1-i & 0 \end{bmatrix}$$

$$\therefore AA^{\Theta} = \frac{1}{4}\begin{bmatrix} 1 & -i & -1+i \\ i & 1 & 1+i \\ 1+i & -1+i & 0 \end{bmatrix}\begin{bmatrix} 1 & -i & 1-i \\ i & 1 & -1-i \\ -1-i & 1-i & 0 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

NOW,

$$A^{\Theta}A = \frac{1}{4}\begin{bmatrix} 1 & -i & 1-i \\ i & 1 & -1-i \\ -1-i & 1-i & 0 \end{bmatrix}\begin{bmatrix} 1 & -i & -1+i \\ i & 1 & 1+i \\ 1+i & -1+i & 0 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Since, $AA^{\theta} = A^{\theta}A = I$

$\therefore$ A is unitary matrix.

$$\therefore A^{-1} = A^{\theta} = \frac{1}{2}\begin{bmatrix} 1 & -i & 1-i \\ i & 1 & -1-i \\ -1-i & 1-i & 0 \end{bmatrix}$$

4) $$A = \begin{bmatrix} 0 & -1 & 2 & 3 \\ 2 & 3 & 4 & 5 \\ 1 & 3 & -1 & 2 \\ 3 & 2 & 4 & 1 \end{bmatrix}$$

R_{13}

$$\sim \begin{bmatrix} 1 & 3 & -1 & 2 \\ 2 & 3 & 4 & 5 \\ 0 & -1 & 2 & 3 \\ 3 & 2 & 4 & 1 \end{bmatrix}$$

$R_2 - 2R_1$, $R_4 - 3R_1$

$$\sim \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & -3 & 6 & 1 \\ 0 & -1 & 2 & 3 \\ 0 & -7 & 7 & -5 \end{bmatrix}$$

R_{23}

$$\sim \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -3 & 6 & 1 \\ 0 & -7 & 7 & -5 \end{bmatrix}$$

$R_2(-1)$

$$\sim \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -3 & 6 & 1 \\ 0 & -7 & 7 & -5 \end{bmatrix}$$

$R_3 + 3R_2, \quad R_4 + 7R_2$

$$\sim \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & -8 \\ 0 & 0 & -7 & -26 \end{bmatrix}$$

R_{34}

$$\sim \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -7 & -26 \\ 0 & 0 & 0 & -8 \end{bmatrix}$$

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$R_3\left(-\frac{1}{7}\right)$, $R_4\left(-\frac{1}{8}\right)$

$$\sim \quad A = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 26/7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ The above matrix is in Echelon form.

Now, number of non-zero rows in matrix $A = 4$

$\therefore$ Rank of matrix $A = 4$

5) $$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

R_{12}

$$\sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

$R_4 - (R_1+R_2+R_3)$

$$\sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 - 2R_1, \quad R_3 - 3R_1$

$$\sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 - R_3$

$$\sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & -6 & -3 \\ 0 & 4 & 9 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 - 4R_2$

$$\sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & -6 & -3 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3\left(\frac{1}{11}\right)$

$$\sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & -6 & -3 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_2 + C_1,\ C_3 + 2C_1,\ C_4 + 4C_1$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -6 & -3 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_3 + 6C_2,\ C_4 + 3C_2$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_4 - \frac{2}{3}C_3,\ \cancel{C_3\left(\frac{1}{3}\right)}$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_3\left(\frac{1}{3}\right)$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right]$$

$$\therefore \begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}$$

The above matrix is in normal form.

$\therefore$ Rank of matrix $= r = 3$

6) $$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

~~Let A = PAQ~~

Let $A = I_{3\times3} \, A \, I_{3\times3}$

$$\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 + R_1$,

$$\sim \begin{bmatrix} 1 & 2 & -2 \\ 0 & 5 & -2 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_2 - 2C_1,\ C_3 + 2C_1,$

$$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & -2 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -2 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_2 + 2C_3,$

$$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

$R_2 + 2R_3,$

$$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

$\therefore \quad [I_3] = P\ A\ Q$

where, $P = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$, $Q = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$.

$\therefore$ Rank of matrix $= r = 3$

For A^{-1},

$$|A| = \begin{vmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{vmatrix} = 1(3) - 2(-1) - 2(2)$$
$$= 3 + 2 - 4 = 1$$
$$\therefore |A| \neq 0$$

$\therefore A^{-1}$ exist.

Now, $I_3 = PAQ$

$AQ = P^{-1}$

Pre-multiplying by A^{-1} on both sides,

$A^{-1}AQ = A^{-1}P^{-1}$

$Q = A^{-1}P^{-1}$

$\therefore QP = A^{-1}P^{-1}P$

$\therefore QP = A^{-1}$

$\therefore A^{-1} = QP$

$$\therefore A^{-1} = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

7) Given vectors are $(1,2,4)$, $(2,-1,3)$, $(0,1,2)$, $(-3,7,2)$

$\therefore$ Let $x_1 = (1,2,4)$

$x_2 = (2,-1,3)$

$x_3 = (0,1,2)$

$x_4 = (-3,7,2)$

Now,

Consider the matrix equation,

$k_1 x_1 + k_2 x_2 + k_3 x_3 + k_4 x_4 = 0$

$k_1[1,2,4] + k_2[2,-1,3] + k_3[0,1,2] + k_4[-3,7,2] = [0,0,0]$

$k_1 + 2k_2 + 0k_3 - 3k_4 = 0$

$2k_1 - k_2 + k_3 + 7k_4 = 0$

$4k_1 + 3k_2 + 2k_3 + 2k_4 = 0$

Converting into matrix form,

$\therefore$

$$\begin{bmatrix} 1 & 2 & 0 & -3 \\ 2 & -1 & 1 & 7 \\ 4 & 3 & 2 & 2 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 - 2R_1$, $R_3 - 4R_1$

$$\sim \begin{bmatrix} 1 & 2 & 0 & -3 \\ 0 & -5 & 1 & 13 \\ 0 & -5 & 2 & 14 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 - R_2$

$$\sim \begin{bmatrix} 1 & 2 & 0 & -3 \\ 0 & -5 & 1 & 13 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore$ $k_1 + 2k_2 - 3k_4 = 0$

$-5k_2 + k_3 + 13k_4 = 0$

$k_3 + k_4 = 0$

Let $k_4 = t$,

$\therefore k_3 = -t$

$k_2 = \frac{12}{5} t$

$k_1 = \frac{-9}{5} t$

$\therefore k_1 x_1 + k_2 x_2 + k_3 x_3 + k_4 x_4 = 0$

$\therefore \frac{-9}{5} t x_1 + \frac{12}{5} t x_2 - t x_3 + t x_4 = 0$

$\therefore \frac{-9}{5} x_1 + \frac{12}{5} x_2 - x_3 + x_4 = 0$

$\therefore x_1, x_2, x_3$ and x_4 are linearly dependent.

$\therefore x_4 = \frac{9}{5} x_1 - \frac{12}{5} x_2 + x_3$

8) $x - y + z = 1$

$x + 2y + 4z = a$

$x + 4y + 6z = a^2$

$\therefore$ In matrix form,

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ a \\ a^2 \end{bmatrix}$$

$$A \quad X = B$$

∴ Augmented matrix is given by,

$$C = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 1 & 2 & 4 & a \\ 1 & 4 & 6 & a^2 \end{array}\right]$$

$R_2 - R_1$, $R_3 - R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 3 & 3 & a-1 \\ 0 & 5 & 5 & a^2-1 \end{array}\right]$$

∴ $R_2\left(\frac{1}{3}\right)$, $R_3 - \frac{5}{3}R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 1 & 1 & (a-1)/3 \\ 0 & 0 & 0 & a^2-1 - \frac{5}{3}(a-1) \end{array}\right]$$

Here, Rank of matrix A = 2

∴ As the system of equation is consistent.

$R(A) = R(C) = 2$

$\therefore (a^2-1) - \frac{5}{3}(a-1) = 0$

$3(a^2-1) - 5(a-1) = 0$

$3a^2 - 3 - 5a + 5 = 0$

$3a^2 - 5a + 2 = 0$

$(a-1)(3a-2) = 0$

$a = 1$; $a = 2/3$

$\therefore$ The system of equations is consistent for $a=1$ and $a=2/3$.

For $a=1$,

$$3y+3z=0$$

$\therefore$ Let $z=t$.

$\therefore$ $3y+3t=0$

$\therefore$ $y=-t$

$\therefore$ $x=1-2t$

$\therefore$ The solution of system of equations for $a=1$ are $x=1-2t$, $y=-t$, $z=t$

For $a=2/3$

$$3y+3z=-1/3$$

let $z=k$

$\therefore$ $3y+3k=-1/3$

$3y=-1/3-3k$

$$y=\frac{-9k-1}{9}=-k-\frac{1}{9}$$

$$x=1-k-\frac{9k-1}{9}$$

$$=\frac{8-18k}{9}=\frac{8}{9}-2k$$

$\therefore$ The solution of system of equations for $a=2/3$ are $x=\frac{8}{9}-2k$, $y=-k-\frac{1}{9}$, $z=k$

9) $x + 2y + 3z = 0$

$3x + 4y + 4z = 0$

$7x + 10y + 12z = 0$

Converting in matrix form,

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 4 \\ 7 & 10 & 12 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A \quad X = B$$

Augmented matrix is given by,

$$C = \left[\begin{array}{ccc:c} 1 & 2 & 3 & 0 \\ 3 & 4 & 4 & 0 \\ 7 & 10 & 12 & 0 \end{array}\right]$$

$R_2 - 3R_1$, $R_3 - 7R_1$

$$\sim \left[\begin{array}{ccc:c} 1 & 2 & 3 & 0 \\ 0 & -2 & -5 & 0 \\ 0 & -4 & -9 & 0 \end{array}\right]$$

$R_2\left(\frac{-1}{2}\right)$

$$\sim \left[\begin{array}{ccc:c} 1 & 2 & 3 & 0 \\ 0 & 1 & 5/2 & 0 \\ 0 & -4 & -9 & 0 \end{array}\right]$$

$R_3 + 4R_2$

$$C \sim \left[\begin{array}{ccc:c} 1 & 2 & 3 & 0 \\ 0 & 1 & 5/2 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$\therefore$ Number of non. zero rows in $C = 3$

$\therefore$ Rank $(C) =$ Rank$(A) = 3 =$ number of unknowns.

Hence, the system has zero solution i.e. trivial solution.

$\therefore x = y = z = 0$

10) D R _ J O H N _ I S _ T H E _ S P Y

4 18 0 10 15 8 14 0 9 19 0 20 8 5 0 19 16 25

$$A = \begin{bmatrix} 5 & 3 & 6 \\ 4 & 3 & -1 \\ 5 & 2 & 2 \end{bmatrix}_{3\times 3}$$

$\therefore$ There are 18 elements

$\therefore$ Matrix B will be of order 3×6

$$\therefore B = \begin{bmatrix} 4 & 10 & 14 & 19 & 8 & 19 \\ 18 & 15 & 0 & 0 & 5 & 16 \\ 0 & 8 & 9 & 20 & 0 & 25 \end{bmatrix}$$

NOW,

$$AB = C = \begin{bmatrix} 5 & 3 & 0 \\ 4 & 3 & -1 \\ 5 & 2 & 2 \end{bmatrix} \begin{bmatrix} 4 & 10 & 14 & 19 & 8 & 19 \\ 18 & 15 & 0 & 0 & 5 & 16 \\ 0 & 8 & 9 & 20 & 0 & 25 \end{bmatrix}$$

$$= \begin{bmatrix} 74 & 95 & 70 & 95 & 55 & 143 \\ 70 & 77 & 47 & 56 & 47 & 99 \\ 56 & 96 & 88 & 135 & 50 & 177 \end{bmatrix}$$

$\therefore$ The coded message is :

~~70, 74~~

74, 70, 56, 95, 77, 96, 70, 47, 88, 95, 56, 135, 55, 47, 50, 143, 99, 177

Since the encoded matrix A is 3x3, we begin by entering the coded message in column of matrix C in 3 rows.

$$\therefore C = \begin{bmatrix} 74 & 95 & 70 & 95 & 55 & 143 \\ 70 & 77 & 47 & 56 & 47 & 99 \\ 56 & 96 & 88 & 135 & 50 & 177 \end{bmatrix}$$

If B is the matrix containing the uncoded message then $C = AB$

$$A^{-1}C = B$$

$$\therefore B = A^{-1}C$$

$$|A| = \begin{vmatrix} 5 & 3 & 0 \\ 4 & 3 & -1 \\ 5 & 2 & 2 \end{vmatrix} = 5(6+2) - 3(8+5) + 0$$
$$= 40 - 39 = 1$$

$\because |A| \neq 0$

$\therefore A^{-1}$ exist.

$\therefore A^{-1} = \frac{1}{|A|} \text{adj } A$

$$A^{-1} = \begin{bmatrix} 8 & -6 & -3 \\ -13 & 10 & 5 \\ -7 & 5 & 3 \end{bmatrix}$$

Now,

$B = A^{-1} C$

$$= \begin{bmatrix} 8 & -6 & -3 \\ -13 & 10 & 5 \\ -7 & 5 & 3 \end{bmatrix} \begin{bmatrix} 74 & 95 & 70 & 95 & 55 & 143 \\ 70 & 77 & 47 & 56 & 47 & 99 \\ 56 & 96 & 88 & 135 & 50 & 147 \end{bmatrix}$$

$$B = \begin{bmatrix} 4 & 10 & 14 & 19 & 8 & 19 \\ 18 & 15 & 0 & 0 & 5 & 16 \\ 0 & 8 & 9 & 20 & 0 & 25 \end{bmatrix}$$

$\therefore$ Decode message is

4, 18, 0, 10, 15, 8, 14, 0, 9, 19, 0, 20, 8, 5, 0, 19, 16, 25

"DR JOHN IS THE SPY"

Hence, the message is verify by decoding.