SAP ID - 60004200132
Name - Ayush Jain
Div - J1

DATE:

28/06/21

Engineering Mathematics

Tutorial 2: Double Integration.

1) Evaluate $\iint_R x^2\,dx\,dy$ where R is the region in the first quadrant bounded by the hyperbola $xy=16$ and the lines $y=x$, $y=0$ and $x=8$

2) Evaluate $\iint r e^{-r^2/a^2} \cos\theta \sin\theta \, d\theta\, dr$ over the upper half of the circle $r = 2a\cos\theta$.

3) Change the order of integration and evaluate

$$\int_0^2 \int_{\sqrt{2x}}^{2} \frac{y^2}{\sqrt{y^4-4x^2}}\,dx\,dy$$

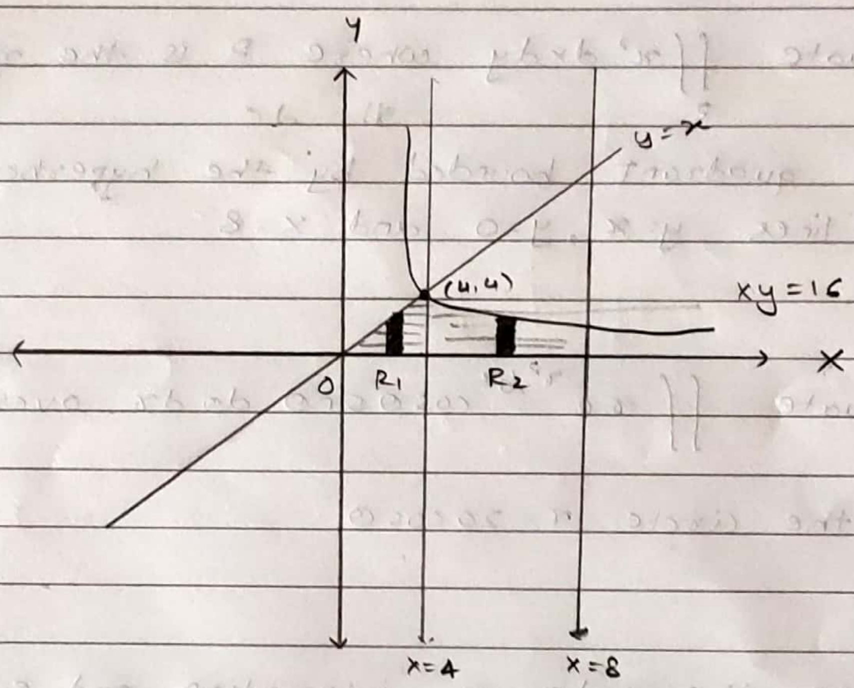
4) Change to polar coordinates and evaluate $\iint_R \frac{1}{\sqrt{xy}}\,dx\,dy$ where R is the region of integration bounded by $x^2+y^2-x=0$ and $y \geq 0$

Scanned with CamScanner

DATE:

Solutions:

1)



$$\therefore \iint_R x^2\,dx\,dy = \iint_{R_1} x^2\,dx\,dy + \iint_{R_2} x^2\,dx\,dy$$

$$\therefore I = \int_{x=0}^{4}\left(\int_{y=0}^{x} x^2\,dy\right)dx + \int_{x=4}^{8}\left(\int_{y=0}^{16/x} x^2\,dy\right)dx$$

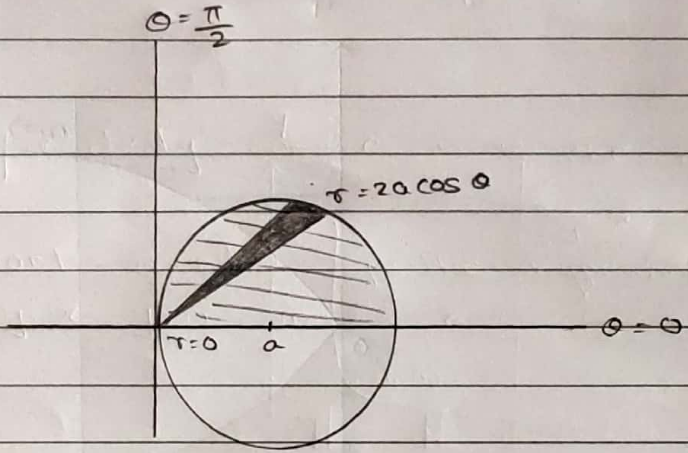
$$= \int_{x=0}^{4} x^2\,[y]_0^x\,dx + \int_{x=4}^{8} x^2\,[y]_0^{\frac{16}{x}}\,dx$$

$$= \int_{x=0}^{4} x^3\,dx + \int_{x=4}^{8} 16x\,dx$$

$$= \left[\frac{x^4}{4}\right]_0^4 + \left[8x^2\right]_4^8 = 64 + 8(64-16) = 448$$

DATE:

2)



$$I = \iint_R r e^{-r^2/a^2} \cos\theta \sin\theta \, d\theta \, dr$$

$$= \int_{\theta=0}^{\pi/2} \left(\int_{r=0}^{2a\cos\theta} r e^{-r^2/a^2} \cos\theta \sin\theta \, dr \right) d\theta$$

$$= \int_{\theta=0}^{\pi/2} \left[-\frac{a^2}{2} e^{-r^2/a^2} \cos\theta \sin\theta \right]_0^{2a\cos\theta} d\theta$$

$$= \int_0^{\pi/2} \left[\frac{a^2}{2} - \frac{a^2}{2} e^{-4\cos^2\theta} \right] \cos\theta \sin\theta \, d\theta$$

$$= \left[\frac{a^2}{4} \sin^2\theta - \frac{a^2}{16} e^{-4\cos^2\theta} \right]_0^{\frac{\pi}{2}}$$

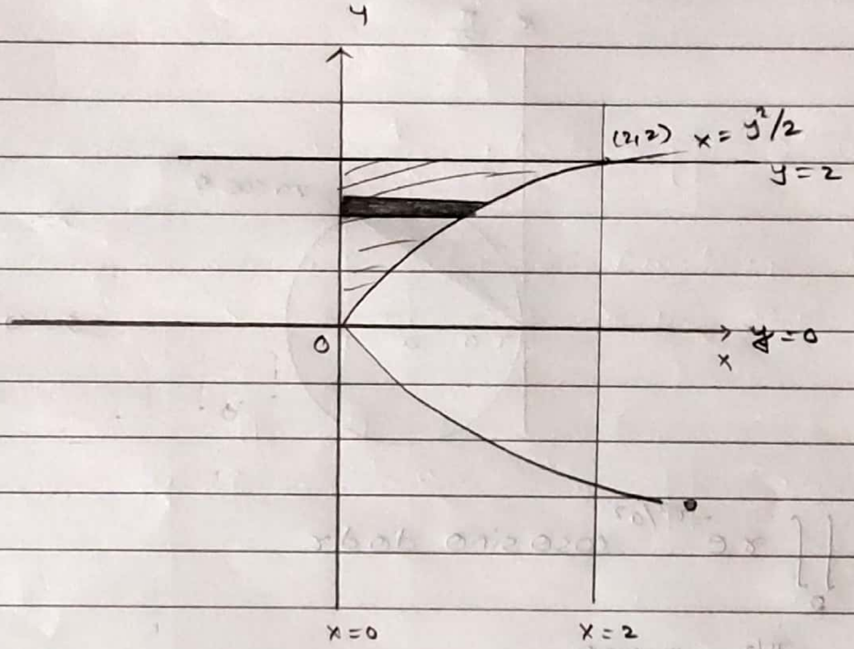
$$= \left[\frac{a^2}{4} - 0 \right] - \left[\frac{a^2}{16} - \frac{a^2}{16} e^{-4} \right]$$

$$= \frac{a^2}{16} \left[4 - 1 + e^{-4} \right]$$

$$I = \frac{a^2}{16} \left[3 + e^{-4} \right]$$

DATE:

3)



$y = \sqrt{2x}$ $\therefore$ $y^2 = 2x$ is a parabola with vertex at origin

$y = 2$ is a line parallel to x-axis.

$\therefore$ By changing the order of integration,

$$R = \left\{ (x, y) \in \mathbb{R}^2 \;\middle|\; \begin{array}{l} 0 \le y \le 2 \\ 0 \le x \le y^2/2 \end{array} \right\}$$

$$\therefore I = \int_{y=0}^{2} \left(\int_{x=0}^{\frac{y^2}{2}} \frac{y^2}{\sqrt{y^4 - 4x^2}} \, dx \right) dy$$

$$= \int_{y=0}^{2} \frac{y^2}{2} \left[\sin^{-1}\left(\frac{2x}{y^2}\right) \right]_0^{y^2/2} dy$$

DATE:

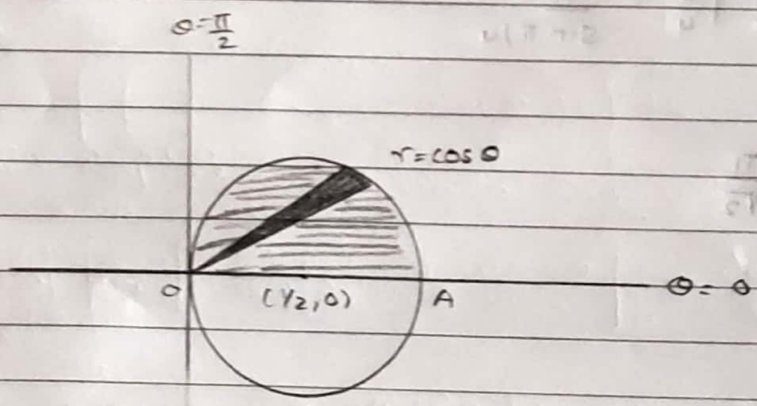
$$= \int_{y=0}^{2} \frac{y^2}{2}\left(\sin^{-1}(1) - \sin^{-1}(0)\right) dy$$

$$= \frac{1}{2}\int_{y=0}^{2} y^2 \cdot \frac{\pi}{2}\, dy$$

$$= \frac{\pi}{4}\left[\frac{y^3}{3}\right]_0^2$$

$$I = \frac{2\pi}{3}$$

4)



Change to polar coordinates,

$x = r\cos\theta$, $dx\,dy = J\,dr\,d\theta$

$y = r\sin\theta$ $\quad = r\,dr\,d\theta$

$$x^2 + y^2 - x = 0$$

$$\therefore r^2 = r\cos\theta$$

$$\therefore r = \cos\theta.$$

$$\therefore I = \int_{\theta=0}^{\pi/2}\left(\int_{r=0}^{\cos\theta} \frac{1}{r\sqrt{\sin\theta\cos\theta}}\, dr\right) d\theta$$

Scanned with CamScanner

DATE:

$$= \int_0^{\pi/2} \left[\frac{r}{\sqrt{\sin\theta \cos\theta}} \right]_0^{\cos\theta} d\theta$$

$$= \int_0^{\pi/2} (\cos\theta)^{1/2} (\sin\theta)^{-1/2} \, d\theta$$

$$= \frac{1}{2} B\left[\frac{1}{4}, \frac{3}{4}\right]$$

$$= \frac{1}{2} \frac{\Gamma 1/4 \; \Gamma 3/4}{\Gamma 1}$$

But $\Gamma\frac{1}{4} \Gamma\frac{3}{4} = \frac{\pi}{\sin \pi/4} = \sqrt{2}\,\pi$

$$\therefore \; I = \frac{\pi}{\sqrt{2}}$$