Name - Ayush Jain
SAP ID - 60004200132
Div - B/B1

DIS - Tutorial - 2

1) Solution:

$[(p\vee q)\wedge\sim(\sim p\wedge(\sim q\vee\sim r))]\vee(\sim p\wedge\sim q)\vee(\sim p\wedge\sim r)$

$\equiv [(p\vee q)\wedge(p\vee(q\wedge r))]\vee\sim(p\vee q)\vee\sim(p\vee r)$ ... [De Morgan's law]

$\equiv [((p\vee q)\wedge p)\vee((p\vee q)\wedge(q\wedge r))]\vee(\sim p\wedge(\sim q\vee\sim r))$ ...(Distributive law)

$\equiv [p\vee((p\vee q)\wedge q)\wedge(p\vee q\wedge r)]\vee(\sim(p\vee(q\wedge r))$ ...(Absorption law)

$\equiv [(p\vee q)\wedge((p\vee q)\wedge r)]\vee(\sim(p\vee(q\wedge r))$ ... [Absorption law)

$\equiv ((p\vee q)\wedge r)\vee(\sim(p\vee(q\wedge r))$ ... [Distributive law]

$\equiv T$ ... [Complement law]

2) Solution:

$\sim\forall y(\exists x P(x,y)\wedge\sim\forall x Q(x,y))$

$\equiv \exists y\sim(\exists x P(x,y)\wedge\sim\forall x Q(x,y))$

$\equiv \exists y(\sim(\exists x P(x,y)\vee\sim(\sim\forall x Q(x,y))$ ... [De Morgan's law]

$\equiv \exists y(\forall x\sim P(x,y))\vee\forall x Q(x,y))$ ... [Double negation]

Q. 3) Solution

→ (i) $\sim(p \rightarrow q)$

$\equiv \sim(\sim p \vee q)$

$\equiv p \wedge \sim q$ ... [DeMorgan's law]

→ (ii) $\sim(p \vee (\sim p \wedge q))$

$\equiv \sim p \wedge (p \vee \sim q)$ ... [DeMorgan's law]

$\equiv (\sim p \wedge p) \vee (\sim p \wedge \sim q)$ ... [Distributive law]

$\equiv F \vee (\sim p \wedge \sim q)$ ... [Complement law]

$\equiv \sim p \wedge \sim q$ ... [Identity law]

4) Solutions :

$P(x, y)$ is the formula $x + y = 4$

$Q(x, y)$ is the formula "$x < y$"

a) for $x = 1$, $y = 3$

$P(1,3) = 1 + 3 = 4$

$\therefore P(x, y) = T$ and $Q(x, y) = T$ for $x = 1$ and $y = 3$

$P(x, y) \wedge Q(x, y) = T \wedge T = T$

$\sim P(x, y) \vee Q(x, y) = F \vee T = T$

$P(x, y) \rightarrow \sim Q(x, y) = T \rightarrow F = F$

$\sim P(x, y) \leftrightarrow Q(x, y) = F \leftrightarrow T = F$

(b) For $x = 1$, $y = 2$

$P(x,y) = 1+2 = 3 = F$

$Q(x,y) = T$

$\therefore P(x,y) \wedge Q(x,y) = F \wedge T = F$

$\therefore \sim P(x,y) \vee Q(x,y) = T \vee T = T$

$\therefore P(x,y) \rightarrow \sim Q(x,y) = F \rightarrow F = T$

$\therefore \sim P(x,y) \leftrightarrow Q(x,y) = T \leftrightarrow T = T$

(c) For $x = 3$, $y = 1$

$P(x,y) = 3+1 = 4 = T$, $Q(x,y) = F$

$\therefore P(x,y) \wedge Q(x,y) = T \wedge F = F$

$\sim P(x,y) \vee Q(x,y) = F \vee F = F$

$P(x,y) \rightarrow \sim Q(x,y) = T \rightarrow T = T$

$\sim P(x,y) \leftrightarrow Q(x,y) = F \leftrightarrow F = T$

(d) For $x = 2$, $y = 1$

$P(x,y) = 2+1 = 3 = F$, $Q(x,y) = F$

$\therefore P(x,y) \wedge Q(x,y) = F \wedge F = F$

$\therefore \sim P(x,y) \vee Q(x,y) = T \vee F = T$

$\therefore P(x,y) \rightarrow \sim Q(x,y) = F \rightarrow T = T$

$\therefore \sim P(x,y) \leftrightarrow Q(x,y) = T \leftrightarrow F = F$