Name - Ayush Jain
SAP ID - 60004200132

Tutorial 4 - Electrodynamics

Q. 7> $\bar{w} = x^2y\hat{i} + (x^2+y^2)\hat{j} + (yz+zx)\hat{k}$

The divergence of ~~vector~~ $\vec{w}$ is given by

$$\bar{\nabla}\cdot\bar{w} = \frac{\partial w_x}{\partial x} + \frac{\partial w_y}{\partial y} + \frac{\partial w_z}{\partial z}$$

$$\frac{\partial w_x}{\partial x} = 2xy, \quad \frac{\partial w_y}{\partial y} = 2y, \quad \frac{\partial w_z}{\partial z} = y+z$$

so,

$$\vec{\nabla}\cdot\vec{w} = 2xy + 2y + z + y$$

At (1,2,1)

$$\vec{\nabla}\cdot\vec{w} = 2(1)(2) + 2(2) + 1 + 2$$
$$= 11$$

Q. 9> (2,4,3)

$x = r\cos\theta, \quad y = r\sin\theta, \quad z = z$

$\therefore z = 3$

$\therefore 2 = r\cos\theta, \quad 4 = r\sin\theta$

$\therefore r = \sqrt{2^2+4^2} = 4.47 \qquad \therefore \boxed{r = 4.47}$

$\tan\theta = \frac{4}{2} = 2$

$\therefore \theta = \tan^{-1}(2)$

$\therefore \boxed{\theta = 63.43'}$

60004200132

Q. 4) Imagine the circle as a part of cylinder in cylinderical system.

$r =$ constant , $z = 0$

$dr = 0$, $dz = 0$

then $dl = r\,d\phi$

Total length i.e. circumference $= \int dl$

$$= \int_0^{2\pi} r\,d\phi$$

$$= 4 \times [\phi]_0^{2\pi}$$

$$= 4 \times 2\pi$$

$$= 25.12\,m.$$

Q. 8) Volume of cylinder

$r = 3m$, $h = 5m$

$\therefore$ Volume $(V) = \pi r^2 h = \pi (3)^2 \times 5 = 141.37\,m^3$

Q. 6) $r = 4cm$

Differential area $= ds = r\,dr\,d\theta$

$$A = \int ds = \int_0^{2\pi} \int_0^{4} r\,dr\,d\theta$$

$$= [\theta]_0^{2\pi} \left[\frac{r^2}{2}\right]_0^{4}$$

$$= 2\pi \times 16/2 = 50.24\,cm^2$$

Q.3) Given, vector $\omega = \vec{\omega} = x^2 i + x^2 y j + 24xy^2z^3 k$

To find: curl of vector ω at point $(1,3,1)$

Solution: curl of $\vec{\omega} = \vec{\nabla} \times \vec{\omega}$

$$= \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right) \times \left(x^2 i + x^2 yj + 24xy^2z^3 k\right)$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & x^2y & 24xy^2z^3 \end{vmatrix} = i\,(48\,xyz^3) - j\,(24y^2z^3) + k\,(2xy)$$

curl of ω at $(1,3,1)$ is

$$= i(48(3)) - j\,24(9) + K(2)(3)$$

$$= 144\hat{i} - 216\hat{j} + 6\hat{k}$$