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Maths - Tutorial 2

1) In a bolt factory machines A, B and C manufactures resp 25%, 35% and 40% of the total of their total output 5, 4 and 2 percent are defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine B.

2) A random variable x has the following probability distribution

x	0	1	2	3	4	5	6	7
$P(X=x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Find: 1) k 2) $P\left(\frac{1.5 < x < 4.5}{x > 2}\right)$

3) The smallest value of λ for which $P(x \le \lambda) > 1/2$

3) A function is defined as:

$$f(x) = \begin{cases} 0 & , \text{for } x < 2 \\ \frac{2x+3}{18} & , \text{for } 2 \le x \le 4 \\ 0 & , \text{for } x > 4 \end{cases}$$

Show that $f(x)$ is a p.d.f and find the probability that $2 < x < 3$

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4) For the distribution function given below, find p.d.f

$$F(x) = \begin{cases} 0 & , x < 0 \\ 1 - e^{-x/4} & , x \geq 0 \end{cases}$$

Also find probabilities : $P(X \leq 4)$, $P(X \geq 8)$, $P(4 \leq x \leq 8)$

Solutions :

→ 1) Let E_1, E_2, E_3 denotes the events that a bolt is selected at random is manufactured by machine A, B, C respectively

let X denotes that bolt is defective.

$\therefore P(E_2/x) = ?$

$P(E_1) = \frac{25}{100} = 0.25$, $P(X/E_1) = \frac{5}{100} = 0.05$

$P(E_2) = \frac{35}{100} = 0.35$, $P(X/E_2) = \frac{4}{100} = 0.04$

$P(E_3) = \frac{40}{100} = 0.4$, $P(X/E_3) = \frac{2}{100} = 0.02$

$\therefore$ By Bayes Theorem,

$$P(E_2/x) = \frac{P(E_2) \cdot P(X/E_2)}{\sum_{i=1}^{n} P(E_i) \cdot P(H/E_i)}$$

$\therefore P(E_2/X) = \dfrac{(0.35)\cdot(0.04)}{(0.25)(0.05)+(0.35)(0.04)+(0.4)(0.02)}$

$P(E_2/X) = \dfrac{0.014}{0.0345} = 0.41$

$\therefore \boxed{P(E_2/X) = 0.41}$

$\therefore$ Probability that the defective ball was manufactured by machine B is 0.41

→ 2) Since, $\Sigma P(x_i) = 1$, we get

$0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$\therefore 10k^2 + 9k = 1$

$\therefore 10k^2 + 10k - k - 1 = 0$

$10k(k+1) - 1(k+1) = 0$

$k = \frac{1}{10}$, $k = -1$

k cannot be -1

$\therefore \boxed{k = \frac{1}{10}}$

$\therefore$ Probability distribution of X is :

X :	0	1	2	3	4	5	6	7
P(X=0) :	0	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{1}{100}$	$\frac{2}{100}$	$\frac{17}{100}$

Now, we know that $P(A/B) = \frac{P(A \cap B)}{P(B)}$

$$\therefore P\left(\frac{1.5 < x < 4.5}{x > 2}\right) = \frac{P[(1.5 < x < 4.5) \cap (x > 2)]}{P(x > 2)}$$

$$= \frac{P(2 < x < 4.5)}{P(x > 2)}$$

$$= \frac{P(x = 3, 4)}{P(x = 3, 4, 5, 6, 7)} = \frac{5/10}{70/100} = \frac{5}{7}$$

$$\therefore \boxed{P\left(\frac{1.5 < x < 4.5}{x > 2}\right) = \frac{5}{7}}$$

Now, from the table we find that

$$P(x \leq 3) = P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3)$$

$$= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} = \frac{5}{10} = \frac{1}{2}$$

Hence, $P(x \leq 4) = \frac{8}{10} > \frac{1}{2}$

Hence, $\boxed{\lambda = 4}$

→ 3) $f(x) = \begin{cases} 0 & , \text{ for } x < 2 \\ \frac{2x+3}{18} & , \text{ for } 2 \le x \le 4 \\ 0 & , \text{ for } x > 4 \end{cases}$

Since, $2 \le x \le 4$, $f(x) \ge 0$ for all x

NOW, $\int_2^4 \frac{2x+3}{18} = \frac{1}{18}\left[\frac{2x^2}{2} + 3x\right]_2^4$

$= \frac{1}{18}\left[x^2 + 3x\right]_2^4$

$= \frac{1}{18}\left[16 + 12 - 4 - 6\right] = \frac{18}{18}$

$\therefore \int_2^4 \frac{2x+3}{18} = 1$

Since, $f(x) \ge 0$ and $\int_a^b f(x)\,dx = 1$, $f(x)$ is a probability density function

NOW,

$P(2 < x < 3) = \int_2^3 \frac{2x+3}{18} = \frac{1}{18}\left[x^2 + 3x\right]_2^3$

$= \frac{1}{18}\left[9 + 9 - 4 - 6\right] = \frac{8}{18}$

$= \frac{4}{9}$

$\therefore \boxed{P(2 < x < 3) = \frac{4}{9}}$

→ 4) $F(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-x/4}, & x \geq 0 \end{cases}$

F(x) satisfies all the condition of a distribution function.
If f(x) is corresponding probability density function

$$\boxed{f(x) = F'(x) = \begin{cases} \frac{e^{-x/4}}{4}, & x \geq 0 \\ 0, & x < 0 \end{cases}}$$

Now, we have to verify that $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\therefore \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} f(x)\,dx = 0 + \int_{0}^{\infty} \frac{e^{-x/4}}{4}\,dx$$

$$= \frac{1}{4}\left[\frac{e^{-x/4}}{-1/4}\right]_0^{\infty} = -\left[e^{-x/4}\right]_0^{\infty}$$

$$= -[0-1] = 1$$

(i) $P(x \leq 4) = F(4) = 1 - e^{-4/4} = 1 - e^{-1} = 1 - \frac{1}{e} = \frac{e-1}{e}$

$$\therefore \boxed{P(x \leq 4) = \frac{e-1}{e}}$$

(2) $P(x \geq 8) = 1 - P(x \leq 8) = 1 - F(8) = 1 - [1 - e^{-2}] = e^{-2}$

$$\therefore \boxed{P(x \geq 8) = \frac{1}{e^2}}$$

(3) $P(4 \leq x \leq 8) = F(8) - F(4) = (1 - e^{-2}) - (1 - e^{-1}) = e^{-1} - e^{-2}$

$$\therefore \boxed{P(4 \leq x \leq 8) = \frac{e-1}{e^2}}$$