

FLAT - Tutorial 4

Q. 1) Eliminate useless productions/symbols for given CFG.

(a) $S \rightarrow AB|a$
 $A \rightarrow BC|b$
 $B \rightarrow aB|C$
 $C \rightarrow aC|B$

B, C forms a cycle and neither of them terminate. B and C terms can be removed.

$$\therefore S \rightarrow a$$
$$A \rightarrow b$$

A is not reachable and can be removed.

$$\therefore S \rightarrow a$$

(b) $S \rightarrow AB|Ca$
 $B \rightarrow BC|AB$
 $A \rightarrow a$
 $C \rightarrow aB|b$

B creates cycle and also doesn't terminate. \therefore B can be removed.

$$\therefore S \rightarrow Ca$$
$$A \rightarrow a$$
$$C \rightarrow \cancel{aB}|b$$

A is not reachable and thus can be removed.

$$\therefore S \rightarrow Ca$$
$$C \rightarrow b$$

Q.2) Express the following grammar using CNF.

$$S \rightarrow ABA$$

$$A \rightarrow aA| \epsilon$$

$$B \rightarrow bB| \epsilon$$

→ No, useless productions.

Eliminate ϵ -productions.

$$S \rightarrow ABA|BA|AB|AA|A|B$$

$$A \rightarrow aA|a$$

$$B \rightarrow bB|b$$

Eliminate unit productions.

$$S \rightarrow ABA|BA|AB|AA|aA|a|bB|b$$

$$A \rightarrow aA|a$$

$$B \rightarrow bB|b$$

Following is the productions format for CNF.

$$A \rightarrow BC|a$$

\therefore Let $C = a$, $D = b$, $E = AB$

$$\therefore S \rightarrow EA|BA|AB|AA|CA|a|DB|b$$

$$A \rightarrow CA|a$$

$$B \rightarrow DB|b$$

$$C \rightarrow a$$

$$D \rightarrow b$$

$$E \rightarrow AB$$

Thus, the above grammar satisfies CNF constraints.

Q. 3 Show following grammar is ambiguous.

→ Given grammar:

$$S \rightarrow a | a b S b | a A b$$

$$A \rightarrow b S | a A b$$

Consider the string "ababab" which can be processed by above grammar.

$$S \rightarrow a b S b$$

$$\xrightarrow{\text{LMD}} a b a A b$$

$$\xrightarrow{\text{LMD}} a b a b S b$$

$$\xrightarrow{\text{LMD}} a b a b a b$$

$$(S \rightarrow a A b)$$

$$(A \rightarrow b S)$$

$$(S \rightarrow a)$$

$$S \rightarrow a A b$$

$$\xrightarrow{\text{LMD}} a b S b \quad (A \rightarrow b S)$$

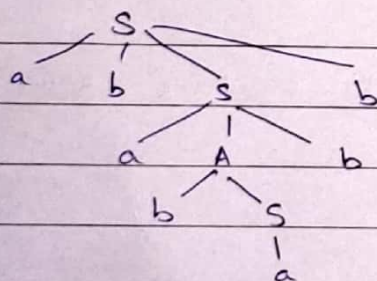
$$\xrightarrow{\text{LMD}} a b a A b \quad (S \rightarrow a A b)$$

$$\xrightarrow{\text{LMD}} a b a b S b \quad (A \rightarrow b S)$$

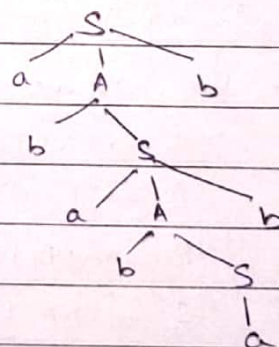
$$\xrightarrow{\text{LMD}} a b a b a b \quad (S \rightarrow a)$$

Parse tree:

For LMD (i)



For LMD (ii)



It can be observed that two different parse tree exist for a string and thus, the given grammar is ambiguous.

Q. 4) Consider the following CFG into CNF.

$$S \rightarrow AB10$$

$$A \rightarrow Bx11$$

$$B \rightarrow CD12$$

$$C \rightarrow AD10$$

$$D \rightarrow 1$$

→ There is no rule for 'x'. Thus all productions containing x can be removed.

$$\therefore S \rightarrow AB10$$

$$A \rightarrow 1$$

$$B \rightarrow CD12$$

$$C \rightarrow AD10$$

$$D \rightarrow 1$$

D can be removed, since A and D produce same string.

$$\therefore S \rightarrow AB10$$

$$A \rightarrow 1$$

$$B \rightarrow CA12$$

$$C \rightarrow AA10$$

CFG can be simplified now,

$$C \rightarrow 1A10$$

$$B \rightarrow 1AA10A12$$

$$S \rightarrow 1B10$$

$$\text{Thus, } S \rightarrow 1B10$$

$$A \rightarrow 1$$

$$B \rightarrow 1AA10A12$$

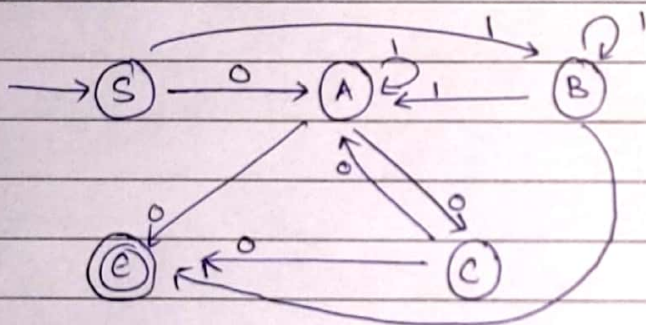
$$C \rightarrow 1A10$$

Thus, all terms are in $A \rightarrow aX$ form. Thus, it is in CNF.

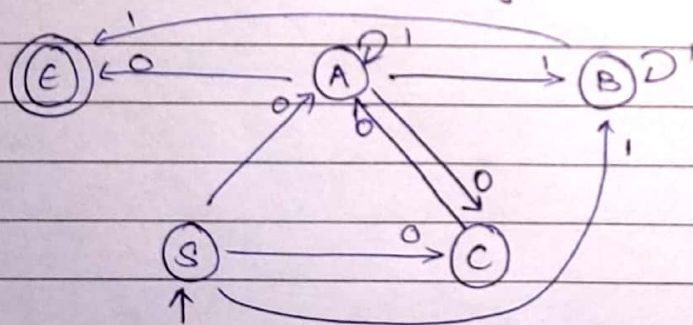
Q. 5) write an equivalent left-linear grammar for the right linear grammar, which is defined as

→ $S \rightarrow OA \mid B$
 $A \rightarrow OC \mid A \mid O$
 $B \rightarrow IB \mid A \mid I$
 $C \rightarrow O \mid OA$

Transition graph:



Replace E with S and change the direction of arrows.



$\therefore S \rightarrow AO \mid B \mid CO$
 $A \rightarrow A \mid B \mid CO \mid O$
 $B \rightarrow B \mid I$
 $C \rightarrow AO$

The above is left linear equivalent grammar for the given right linear grammar.