

14/01/2022

## Discrete Structures

### Term-Test 2

Solutions:

→ 3) Suppose  $P(n) = 2 + 5 + 8 + \dots + (3n-1) = \frac{n(3n+1)}{2}$

Now, let us check for  $n=1$ ,

$$P(1) = \frac{1(4)}{2} = 2$$

∴  $P(n)$  is true for  $n=1$ .

Now, let  $P(n)$  is true for  $n=k$ , then we have to prove that  $P(k+1)$  is true.

$$P(k) = 2 + 5 + 8 + 11 + \dots + (3k-1) = \frac{1}{2} k(3k+1) \quad \text{--- (i)}$$

Therefore,

$$2 + 5 + 8 + 11 + \dots + (3k-1) + (3k+2)$$

Then, substituting the value of  $P(k)$  from equation (i),

$$= \frac{1}{2} \times k(3k+1) + (3k+2) \text{ by using equation (i)}$$

$$= \left[ \frac{3k^2 + k + 2(3k+2)}{2} \right]$$

$$= \frac{3k^2 + k + 6k + 4}{2}$$

$$= \frac{3k^2 + 7k + 4}{2}$$

$$= \frac{3k^2 + 4k + 3k + 4}{2}$$

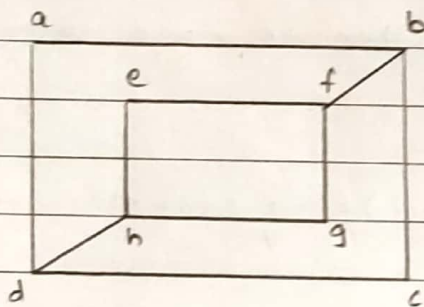
$$= \frac{3k(k+1) + 4(k+1)}{2}$$

$$= \frac{(k+1)(3k+4)}{2}$$

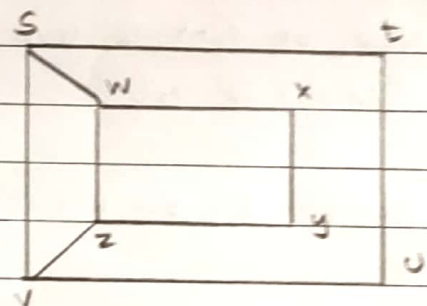
$\therefore P(n)$  is true for  $n = k+1$

Thus,  $P(n)$  is true for all  $n \in \mathbb{N}$

→ 2)



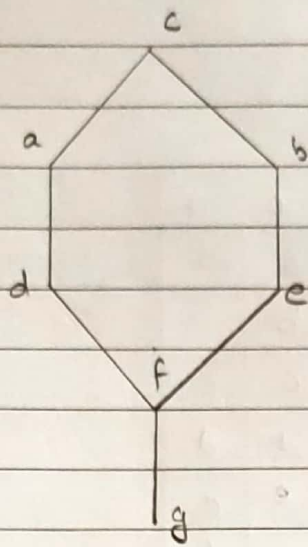
G



H

We first note the following:

- 1) Both the graphs have the same number of vertices viz. 8 and the same number of edges 10.
- 2) In G there are four vertices with degree 3 and in H also there are four vertices with degree 3.
- 3) But adjacency is not preserved in the two graphs. In G vertex with 3 edges is adjacent to only one vertex with 3 edges (f and b or d and h). But in H a vertex with 3 edges is adjacent to two vertices with edges 3 (s to w and v, w to s and u and so on). Thus adjacency is not preserved.
- 4) Hence, G and H are not isomorphic.



GLB

v	a	b	c	d	e	f	g
a	a	f	a	d	f	f	g
b	f	b	b	f	e	f	g
c	a	b	c	d	e	f	g
d	d	f	d	d	f	f	g
e	f	e	e	f	e	f	g
f	f	f	f	f	f	f	g
g	g	g	g	g	g	g	g

~~GLB~~



LUB

$\wedge$	a	b	c	d	e	f	g
a	a	c	c	a	c	a	a
b	c	b	c	c	b	b	b
c	c	c	c	c	c	c	c
d	a	c	c	d	c	d	d
e	c	b	c	c	e	e	e
f	a	b	c	d	e	f	f
g	a	b	c	d	e	f	g

Yes, it is lattice as every pair of elements has a GLB and a LUB.