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AOA

Assignment 2

Q.1 Explain Karatsuba Algorithm using divide and conquer.

→ If we have two n-digit number x and y
Assume B is the base of m and $m<n$ (for instance $m=\frac{n}{2}$)
x and y can be represented as x_1, x_2 and y_1, y_2
i.e. $x = x_1 * B^m + x_2$
$y = y_1 * B^m + y_2$

∴ Product xy can be written as
$$xy = (x_1 * B^m + x_2)(y * B^m + y_2)$$
$$xy = x_1y_1B^{2m} + x_1y_2B^m + x_2y_1B^m + x_2y_2$$
There are 4 subproblems,
x_1y_1 ; x_1y_2 ; x_2y_1 and x_2y_2
We reduce them into 3 subproblems,
let $a = x_1y_1$, $c = x_2y_2$, $b = x_2y_1 + x_1y_2$
$\therefore xy = aB^{2m} + bB^m + c$
$\therefore b = (x_1+x_2)(y_1+y_2) - a - c$ (B^m is usually taken as 10^m)

For eg:
Multiplication of 47×78
$\therefore x = 47$ $y = 78$
$x = 4*10+7$ $y = 7*10+8$
$x_1 = 4$, $x_2 = 7$, $y_1 = 7$, $y_2 = 8$
$a = x_1 * y_1 = 4\times7 = 28$, $c = x_2 * y_2 = 7\times8 = 56$
$b = (x_1+x_2)(y_1+y_2) - a - c = (11)(15) - 28 - 56 = 81$
$\therefore xy = a(10)^2 + b(10) + c$
$= 28(100) + 81(10) + 56$
$= 2800 + 810 + 56$
$\therefore xy = 3666$

Q. 2 Use simplex method to solve LPP $z = x_1 - 3x_2 + 2x_3$

subject to $3x_1 - x_2 + 2x_3 <= 7$

$-2x_1 + 4x_2 <= 12$

$-4x_1 + 3x_2 + 8x_3 <= 10$

$x_1, x_2, x_3 >= 0$

→ Express the problem in standard form.

Maximize $z' \Rightarrow z + x_1 - 3x_2 + 2x_3 + 0s_1 + 0s_2 + 0s_3 = 0$

subject to $3x_1 - x_2 + 2x_3 + s_1 + 0s_2 + 0s_3 = 7$

$-2x_1 + 4x_2 + 0x_3 + 0s_1 - s_2 + 0s_3 = 12$

$-4x_1 + 3x_2 + 8x_3 + 0s_1 + 0s_2 - s_3 = 10$

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

Simplex Table:

Iteration no.	Basic ~~Iteration~~ variables	Coefficient of x_1	x_2	x_3	s_1	s_2	s_3	RHS solution	Ratio
0	z	1	-3	2	0	0	0	0	
s_2 leaves	s_1	3	-1	2	1	0	0	7	$7/-1 = -7$
x_2 enters	s_2	-2	4	0	0	-1	0	12	$12/-4 = -3$
	s_3	-4	3	8	0	0	-1	10	$10/3 = 3.33$

Iteration no.	Basic variables	Coefficients of x_1	x_2	x_3	s_1	s_2	s_3	RHS solution	Ratio
1	z	$-1/2$	0	2	0	0	0	0	
s_1 leaves	s_1	$5/2$	0	3	1	$1/4$	0	10	$10 \times 2/5 = 4$
x_1 enters	x_2	$-1/2$	1	0	0	$1/4$	0	3	$3 \times -2 = -6$
		$-5/2$	0	8	0	$-3/4$	1	1	$1 \times -2/5 = -2/5$

Iteration no.	Basic values	Coefficient of						RHS Solution	Ratio
		x_1	x_2	x_3	s_1	s_2	s_3		
2	z_1	0	0	18/5	1/5	4/5	0	11	
	x_1	1	0	6/5	2/5	1/10	0	4	
	x_2	0	1	3/5	1/5	3/10	0	5	
	s_3	0	0	11	1	-1/2	1	11	

$\therefore x_1 = 4, \; x_2 = 5, \; x_3 = 0$

$\therefore z_{min} = -z'_{max} = \underline{\underline{-11}}$

Q. 3 Write short note on Job Sequencing with Deadlines. Explain with the help of an example.

→ The objective is to find a sequence of jobs, which is completed within their deadlines and gives maximum profits.
Consider a set of 5 jobs. We have to find a sequence of jobs which will be completed within their deadlines to yeild maximum profits.

Job	J1	J2	J3	J4	J5
Profit	60	100	20	40	20
Deadline	2	1	3	2	1

1) Sort all jobs in decreasing order of profit.

Job	J1	J2	J3	J4	J5
Profit	100	60	40	20	20
Deadline	1	2	2	3	1

2) For each job, find a slot i such that slot is empty and i ≤ deadline and i is greatest. Put the job in this slot and this slot is filled. If no such i exist then ignore the job.

0–1	1–2	2–3
J2		
J2	J1	
J2	J1	J3

→ First we select J2 as it can be completed within deadline.

→ Next J1 is selected.

→ J4 cannot be selected as its deadline is over, so we select J3 as it can be completed within deadine.

→ J5 is discarded as it cannot be completed before deadline.

∴ Total profit = 100 + 60 + 20 = 180
(J2, J1, J3)

The sequence is J2, J1, J3.

Q.4) Perform Analysis of Insertion sort and Selection sort.

→ Insertion sort :

	Cost	(Best) Time	(Worst) Time
for j ← 2 to n do	c1	n	n
key ← A[j]	c2	n − 1	n − 1
i ← j − 1	c3	n − 1	n − 1
while(i > 0 && key < A[i])	c4	n − 1	$\frac{n(n+1)}{2} - 1$
A[i+1] = A[i]	c5	0	n(n−1)/2
i--	c6	0	n(n−1)/2
A[i+1] = key	c7	n − 1	n − 1

Best case:

Total $= c_1(n) + c_2(n-1) + c_3(n-1) + c_4(n-1) + c_7(n-1)$

$= (c_1 + c_2 + c_3 + c_7)n - (c_1 + c_2 + c_3 + c_7)$

$= an + b$

$= an$

$\Rightarrow O(n)$

Worst case and average case:

Total $= c_1(n) + c_2(n-1) + c_3(n-1) + c_4\left(\frac{n(n+1)}{2} - 1\right) + c_5\left(\frac{n(n-1)}{2}\right) + c_6\left(\frac{n(n-1)}{2}\right) + c_7(n-1)$

$= an^2 + bn + c$

$\Rightarrow O(n^2)$

Selection Sort	Cost	(Best) Time	(Worst) Time
for i = 1 to n-1	c_1	n	n
imin = i	c_2	$n-1$	$n-1$
for j = i+1 to n	c_3	$\frac{n(n+1)}{2} - 1$	$\frac{n(n+1)}{2} - 1$
if (A[imin > A[j])	c_4	$n(n-1)/2$	$n(n-1)/2$
imin = j	c_5	0	$n(n-1)/2$
temp = A[imin]	c_6	$n-1$	$n-1$
A[imin] = A[i]	c_7	$n-1$	$n-1$
A[i] = temp	c_8	$n-1$	$n-1$

Best case:

Total $= c_1(n) + c_2(n-1) + c_3(n^2+n-2/2) + c_4(n^2-n/2) + (c_6 + c_7 + c_8)n-1$

$= an^2 + bn + c$

$\Rightarrow O(n^2)$

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Worst case and Average case

$$\text{Total} = c_1(n) + c_2(n-1) + c_3\left(\frac{n^2+n-2}{2}\right) + c_4\left(\frac{n^2-n}{2}\right) + (c_6 + c_7 + c_8)n - 1 + c_5\left(\frac{n^2-n}{2}\right)$$

$$= an^2 + bn + c$$

$$\Rightarrow \underline{\underline{O(n^2)}}$$

Q. 5) Explain sub of Subset Algorithm with the help of example.

→ Consider a set of non-negative integers {3, 4, 5, 2) and sum = 9.

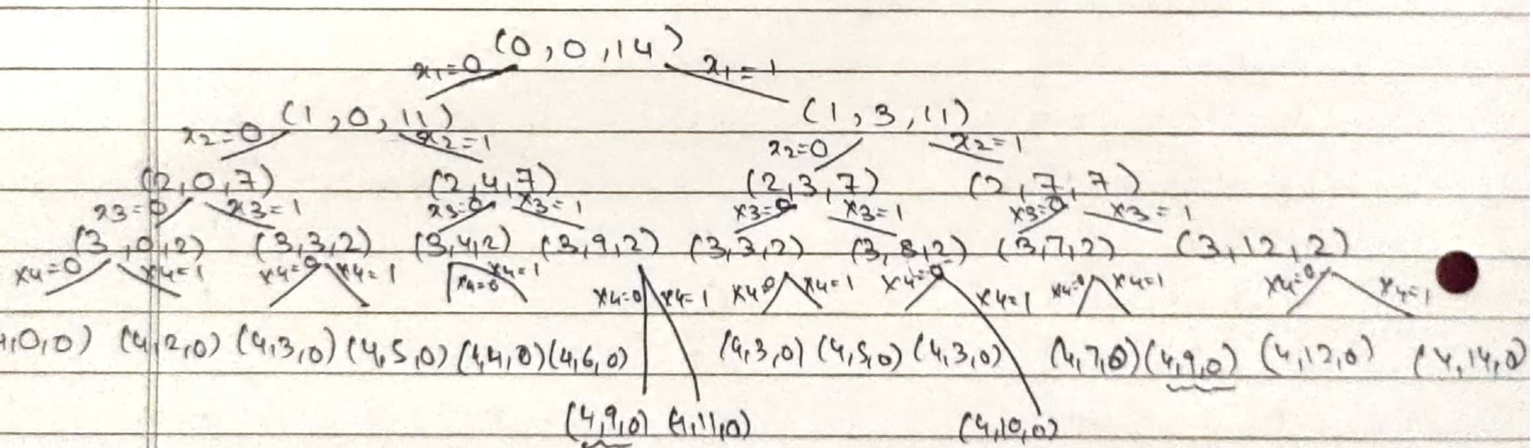
Sum of given set of integers is 14, we find state space tree.

Σw_i = sum of all set of integers

$\Sigma w_i x_i$ = required sum

k = kth element of set.

$$\boxed{k, \Sigma w_i x_i, \Sigma w_i}$$



Thus, the solutions that we got from above state space tree are:

(1) $x_1 = 0,\ x_2 = 1,\ x_3 = 1,\ x_4 = 0 \Rightarrow \{4, 5\}$

(2) $x_1 = 1,\ x_2 = 1,\ x_3 = 0,\ x_4 = 1 \Rightarrow \{3, 4, 2\}$

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