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Term Test 2

Engineering Mathematics - 2

1) Solve $(2xy^4e^y + 2xy^3 + y)dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0$

→ We have, $M = 2xy^4e^y + 2xy^3 + y$

and $N = x^2y^4e^y - x^2y^3 - 3x$

$$\therefore \frac{\partial M}{\partial y} = 2x(y^4e^y + 4y^3e^y) + 6xy^2 + 1$$

and $$\frac{\partial N}{\partial x} = 2xy^4e^y - 2xy^2 - 3$$

$$\therefore \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) \Big/ M = \frac{-8xy^2 - 4 - 8xy^3e^y}{y(2xy^3e^y + 2xy^2 + 1)}$$

$$= -\frac{4}{y} \cdot \frac{(2xy^3e^y + 2xy^2 + 1)}{(2xy^3e^y + 2xy^2 + 1)}$$

$$= -\frac{4}{y} = f(y)$$

$$\therefore I.F = e^{\int -(4/y)dy} = e^{-4\log y} = e^{\log(1/y^4)} = \frac{1}{y^4}$$

Multiplying by the I.F, we get

$\left(2xe^y + \frac{2x}{y} + \frac{1}{y^3}\right)dx + \left(x^2e^y - \frac{x^2}{y^2} - \frac{3x}{y^4}\right)dy = 0$ which is exact.

$\therefore \int M\,dx = \int \left(2xe^{y} + \frac{2x}{y} + \frac{1}{y^3}\right) dx$

$= x^2 e^{y} + \frac{x^2}{y} + \frac{x}{y^3}$

$\boxed{\therefore \text{The solution is } x^2 e^{y} + \frac{x^2}{y} + \frac{x}{y^3} = c}$

Q. 2) Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2e^{x} \cos\frac{x}{2}$

→ The auxiliary equation is $D^2 - 3D + 2 = 0$

$\therefore (D-1)(D-2) = 0 \qquad \therefore D = 1, 2.$

$\therefore$ The C.F is $y = c_1 e^{x} + c_2 e^{2x}$

$P.I = 2 \cdot \frac{1}{D^2 - 3D + 2} \cdot e^{x} \cos\left(\frac{x}{2}\right)$

$= 2 \cdot e^{x} \cdot \frac{1}{(D+1)^2 - 3(D+1) + 2} \cdot \cos\left(\frac{x}{2}\right)$

$\therefore PI = 2 \cdot e^{x} \cdot \frac{1}{D^2 - D} \cos\left(\frac{x}{2}\right) = 2 \cdot e^{x} \cdot \frac{1}{(-1/4) - D} \cos\left(\frac{x}{2}\right)$

$= -8e^{x} \cdot \frac{1}{4D+1} \cos\left(\frac{x}{2}\right)$

$$PI = -8e^{x} \cdot \frac{4D-1}{16D^2-1} \cdot \cos\left(\frac{x}{2}\right)$$

$$= \frac{8}{5} e^{x}\left[-2\sin\left(\frac{x}{2}\right) - \cos\left(\frac{x}{2}\right)\right]$$

∴ The complete solution is :

$$\boxed{y = c_1 e^{x} + c_2 e^{2x} - \frac{8}{5} e^{x}\left[2\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right]}$$

Q. 3) Solve $(3x+2)^2 \frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1$

→ Put $3x+2 = v \implies x = (v-2)/3$

$$\therefore \frac{dv}{dx} = 3, \quad \frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{dx}$$

$$\therefore \frac{dy}{dx} = 3\frac{dy}{dv}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(3\frac{dy}{dv}\right)$$

$$\therefore \frac{d^2y}{dx^2} = 3\frac{d}{dv}\left(\frac{dy}{dv}\right) \cdot \frac{dv}{dx}$$

$$\frac{d^2y}{dx^2} = 9\frac{d^2y}{dv^2}$$

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Q. 3) Solve $(3x+2)^2 \frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = 3x^2+4x+1$

→ ~~Take $(3x+2) = e^z$~~ $\therefore$ ~~$z = \log(3x+2)$~~

It is legrange's linear differential equation.

let $3x+2 = e^z$ $\therefore$ $z = \log(3x+2)$

$$(3x+2)^2 \frac{d^2y}{dx^2} = (3)^2 D(D-1)y$$

$$(3x+2)\frac{dy}{dx} = 3Dy \qquad D = \frac{d}{dz}$$

$\therefore$ $9D(D-1)y + 9Dy - 36y = 0$ ⟶ Auxiliory equation.

$(9D^2 - 9D + 9D - 36)y = 0$

$\therefore$ $(D^2 - 4) = 0$

$\therefore$ $D = \pm 2$

$\therefore$ Complementary function : $y_c = c_1 e^{2z} + c_2 e^{-2z}$

$\therefore$ $f(x) = 3x^2 + 4x + 1$

$$= 3\left(\frac{e^z - 2}{3}\right)^2 + 4\left(\frac{e^z - 2}{3}\right) + 1$$

$$= 3\left(\frac{e^{2z} + 4 - 4e^z}{9}\right) + \frac{4e^z - 8}{3} + 1$$

$$= \frac{e^{2z}}{27} - \frac{1}{27}$$

$\therefore$ $y_p = \frac{1}{D^2 - 4} \cdot \frac{1}{27}(e^{2z} - 1)$

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$$= \frac{1}{27}\left[\frac{e^{2z}}{(D-2)(D+2)} - \frac{e^{0z}}{D-4}\right]$$

$$= \frac{1}{27}\left[\frac{z e^{2z}}{2D} + \frac{1}{4}\right]$$

$$= \frac{1}{27}\left[\frac{z e^{2z}}{4} + \frac{1}{4}\right]$$

$$= \frac{1}{108}\left[z e^{2z} + 1\right]$$

∴ Complete solution:

$y = y_c + y_p$

$$= c_1 e^{2z} + c_2 e^{-2z} + \frac{1}{108}(z e^{2z} + 1)$$

$$y = c_1(3x+2)^2 + c_2(3x+2)^{-2} + \frac{1}{108}\left[(3x+2)^2 \log(3x+2) + 1\right]$$