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Div - B/B1

Discrete Structure

Tutorial 3

Solutions :

1> $A = \{1, 2, 3, 4, 5, 6\}$

$\therefore R = \{(i, j) : |i - j| = 2\}$

$\therefore R = \{(1,3), (2,4), (3,5), (4,6), (3,1), (4,2), (5,3), (6,4)\}$

R is not reflexive as $(1,1), (2,2), (3,3), (4,4), (5,3), (6,6) \notin R$

R is irreflexive as there are no self loops present in R.

R is symmetric as mirror images of each pair is there.

$\{(1,3) \rightarrow (3,1)\}, \{(2,4) \rightarrow (4,2)\}, \{(3,5) \rightarrow (5,3)\}, \{(4,6) \rightarrow (6,4)\}$

R is not transitive as $(1,3) \in R$ and $(3,1) \in R$ but $(1,1) \notin R$

2> $R = \{(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)\}$

$$W_0 = M_R = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since there are no '1's in column 1, no new '1's will be added in W_0, Hence $W_0 = W_1$

$$\therefore W_1 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$P : (1,2)$

$q_1 : (2,3)$, $q_2 : (2,4)$

add $(1,3), (1,4)$ in W_1

$\therefore W_2 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $P_1 : (1,3)$ $P_2 : (2,3)$

$q : (3,4)$

add $(1,4)$, $(2,4)$ in W_2

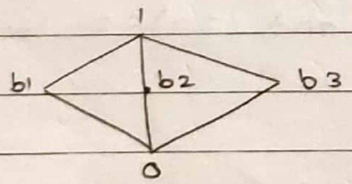
$\therefore W_3 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Since there are no '1's in row 4, no new '1's will be added in W_3. Hence $W_3 = W_4$

$\therefore$ Transitive closure of R is:

$\{(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)\}$

3) (a)



Consider three element b_1, b_2, b_3

$b_2 \vee b_3 = 1$

$b_1 \wedge (b_2 \vee b_3) = b_1 \wedge 1 = b_1$

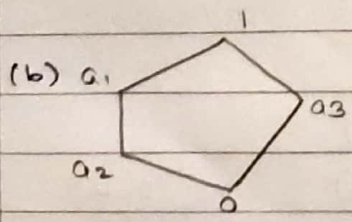
$b_1 \wedge b_2 = 0$ and $b_1 \wedge b_3 = 0$

$\therefore (b_1 \wedge b_2) \vee (b_1 \wedge b_3) = 0 \vee 0 = 0$

Since, $b_1 \wedge (b_2 \vee b_3) \neq (b_1 \wedge b_2) \vee (b_1 \wedge b_3)$

$\therefore$ Lattice is not distributive.

(b)



Consider three elements a_1, a_2, a_3

$a_2 \vee a_3 = 1$

$a_1 \wedge (a_2 \vee a_3) = a_1 \wedge 1 = a_1$

$\therefore \quad a_1 \wedge a_2 = a_2 \qquad \text{and} \qquad a_1 \wedge a_3 = 0$

$\therefore \quad (a_1 \wedge a_2) \vee (a_1 \wedge a_3) = a_2 \vee 0 = a_2$

Since, $a_1 \wedge (a_2 \vee a_3) \neq (a_1 \wedge a_2) \vee (a_1 \wedge a_3)$

$\therefore$ lattice is not distributive.

4) $R = \{(1,1), (2,1), (2,2), (3,1), (3,3), (3,4), (4,4)\}$

R is reflexive as $\{(1,1), (2,2), (3,3), (4,4)\} \in R$

R is antisymmetric as $\{(1,2), (1,3), (4,3)\} \notin R$

R is transitive as

$(2,1)$ and $(1,1) \longrightarrow (2,1) \in R$

$(2,2)$ and $(2,1) \longrightarrow (2,1) \in R$

$(3,1)$ and $(1,1) \longrightarrow (3,1) \in R$

$(3,3)$ and $(3,1) \longrightarrow (3,1) \in R$

$(3,3)$ and $(3,4) \longrightarrow (3,4) \in R$

$(3,4)$ and $(4,4) \longrightarrow (3,4) \in R$

$\therefore$ R is a partial ordering set as it is reflexive, antisymmetric and transitive.

Hasse diagram :