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Discrete Structures - TT1

1) Translate each of the statements into symbols using quantifiers, variable and symbols.

→ Let $P(x)$: x can speak Tamil and
$Q(x)$: x knows the language C++

a) There is a student who can speak Tamil and knows C++

→ $\exists x \, [P(x) \wedge Q(x)]$

b) There is a student who can speak Tamil but does not know C++.

→ $\exists x \, [P(x) \wedge \sim Q(x)]$

c) Every student either speak Tamil or knows C++.

→ $\forall x \, [P(x) \vee Q(x)]$

d) No student can speak Tamil or knows C++.

→ $\forall x \, [\sim(P(x) \vee Q(x))]$

$\therefore \forall x \, [\sim P(x) \wedge \sim Q(x)]$

→ 2) Total number of students = $n(S)$ = 200

No. of students who took Marathi = $n(A) = 98$

No. of students who took Hindi = $n(B) = 75$

No. of students who took Sanskrit = $n(C) = 70$

No. of students who took Marathi and Hindi = $(A \cap B) = 35$

No. of students who took Sanskrit and Marathi = $(A \cap C) = 42$

No. of students who took Sanskrit and Hindi = $(B \cap C) = 40$

No. of students who took all courses = $(A \cap B \cap C) = 25$

No. of Students who took:

(i) Marathi, Hindi or Sanskrit:

$\therefore$ ~~(AUBUC) = A + B + C +~~

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

$$= 98 + 75 + 70 - 35 - 42 - 40 + 25$$

$$= \underline{\underline{151}}$$

(ii) None of these languages:

$$n(S) - n(A \cup B \cup C) = 200 - 151$$

$$= \underline{\underline{49}}$$

(iii) Marathi or Hindi but not in Sanskrit $\Rightarrow$

$\therefore [n(A) \cup n(B)] - [n(A) \cap n(B)] - [n(B) \cap n(C)] + n(A \cap B \cap C)$

$\therefore [n(A) + n(B) - \{n(A) \cap n(B)\}] - [n(A) \cap n(C)] - [n(B) \cap n(C)] + n(A \cap B \cap C)$

$\therefore 98 + 75 - 35 - 42 - 40 + 25$

$\therefore \underline{\underline{81}}$

(iv) Exactly one language

$\therefore [n(A) \cup n(B) \cup n(C)] - [n(A) \cap n(B)] - [n(A) \cap n(C)] - [n(B) \cap n(C)]$
$+ 2\, n(A \cap B \cap C)$

$\therefore 151 - 40 - 35 - 42 + 2 \times 25$

$\therefore \underline{\underline{84}}$

(v) exactly two language

$\therefore n(A \cap B) + n(C \cap B) + n(A \cap C) - 3\, n(A \cap B \cap C)$

$\therefore 35 + 40 + 42 - 3(25)$

$\therefore \underline{\underline{42}}$

(vi) only Sanskrit

$\therefore$ $n(C) - n(B\cap C) - n(A\cap C) + n[A\cap B\cap C]$

$70 - 40 - 42 + 25$

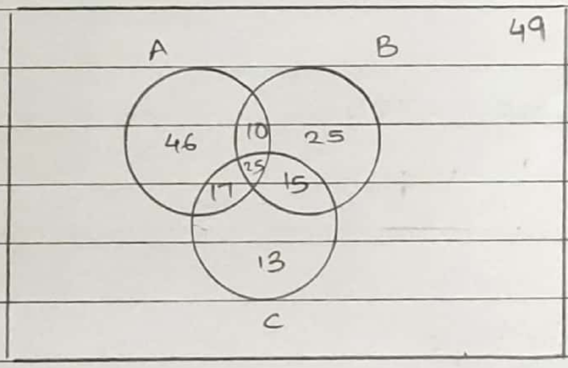
$\therefore$ $\underline{\underline{13}}$

(vii) at least two languages

$\therefore$ $n(A\cap C) + n(A\cap B) + n(B\cap C) - 2\,n(A\cap B\cap C)$

$\therefore$ $40 + 35 + 42 - 2(25)$

$\therefore$ $\underline{\underline{67}}$



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Q. 3 $A = \{1, 2, 3, 4\}$

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$W_0 = \begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$P_1(1,1)$ $P_2(2,1)$

$Q_1(1,1)$ $Q_2(1,4)$

$\therefore (1,1)$ $(1,4)$ $(2,1)$ $(2,4)$

$$W_1 = \begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$P_1(2,2)$

$Q_1(2,1)$, $Q_2(2,2)$, $Q_3(2,4)$

$\therefore (2,1)$, $(2,2)$, $(2,4)$

$$W_2 = \begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Since all column elements are 0. P will be null no (P, Q) pair will be formed.

$$W_3 = W_2 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 0 & 0 & 1 \\ 2 & 1 & 1 & 0 & 1 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 1 \end{array}$$

$P_1 = (1,4)$, $P_2 = (2,4)$, $P_3 = (3,4)$, $P_4 = (4,4)$

$Q_1 = (4,4)$

$\therefore (1,4), (2,4), (3,4), (4,4)$

$$W_4 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 0 & 0 & 1 \\ 2 & 1 & 1 & 0 & 1 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 1 \end{array}$$

Transitive closure of R is :

$$\boxed{\{(1,1), (1,4), (2,1), (2,2), (2,4), (3,4), (4,4)\}}$$

Q. 4>

a> $\sim [P \rightarrow \sim (P \vee q)]$

$= \sim [\sim P \vee \sim (P \vee q)]$ ... $[P \rightarrow q \equiv \sim P \vee q]$

$= \sim(\sim P) \wedge \sim(\sim(P \vee q))$ ... [DeMorgan's Law]

$= P \wedge (P \vee q)$

$= P$ ... [Absorbtive law]

b) $\left[(p \leftrightarrow q) \rightarrow \sim(r \rightarrow \sim p)\right] \vee (r \rightarrow \sim q)$

$= \left[\left[(p \rightarrow q) \wedge (q \rightarrow p)\right] \rightarrow \sim(r \rightarrow \sim p)\right] \vee (r \rightarrow \sim q)$ .... $[p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)]$

$= \left[((\sim p \vee q) \wedge (\sim q \vee p)) \rightarrow \sim(\sim r \vee \sim p)\right] \vee (\sim r \vee \sim q)$ .... $[p \rightarrow q \equiv \sim p \vee q]$

$= \left[\sim((\sim p \vee q) \wedge (\sim q \vee p)) \vee \sim(\sim r \vee \sim p)\right] \vee (\sim r \vee \sim q)$ .... $[p \rightarrow q \equiv \sim p \vee q]$

$= (p \wedge \sim q) \vee (q \wedge \sim p) \vee (r \wedge p) \vee \sim r \vee \sim q$ ... [DeMorgan's law]

$= (\sim q \wedge p) \vee \left[(\sim p \wedge q) \vee \sim q\right] \vee \left[(r \wedge p) \vee \sim r\right]$

$= (\sim q \wedge p) \vee \left[(\sim p \vee \sim q) \wedge (q \vee \sim q)\right] \vee \left[(r \vee \sim r) \wedge (p \vee \sim r)\right]$
... [Distributive law]

$= (\sim q \wedge p) \vee \left[(\sim p \vee \sim q) \wedge T\right] \vee \left[(p \vee \sim r) \wedge T\right]$ ... (Complement law)

$= (\sim q \wedge p) \vee (\sim p \vee \sim q) \vee (p \vee \sim r)$ ... [Identity law]

$= p \vee \sim r \vee \sim q \vee \left[\sim p \vee (\sim q \wedge p)\right]$

$= p \vee \sim r \vee \sim q \vee \left[(\sim p \vee \sim q) \wedge (\sim p \vee p)\right]$ ... [Distributive law]

$= p \vee \sim r \vee \sim q \vee \left[(\sim p \vee \sim q) \wedge T\right]$ ... [Complement law]

$= \sim r \vee [\sim q \vee p] \vee [\sim p \vee \sim q]$ ... [Identity law]

$= \sim r \vee [\sim q \vee (p \vee \sim p)]$

$= \sim r \vee \sim q \vee T$ ... [Complement law]

$= T$

Q. 5) $A = \{a, b, c, d\}$

$R = \{(a,a), (a,c), (c,b), (c,d), (d,b)\}$

$S = \{(b,a), (c,c), (c,d), (d,a)\}$

(a)

RoS (R follows S)	RoS
$(b,a) \rightarrow (a,a)$	(b,a)
$(b,a) \rightarrow (a,c)$	(b,c)
$(c,c) \rightarrow (c,b)$	(c,b)
$(c,c) \rightarrow (c,d)$	(c,d)
$(c,d) \rightarrow (d,b)$	(c,b)
$(d,a) \rightarrow (a,a)$	(d,a)
$(d,a) \rightarrow (a,c)$	(d,c)

$\boxed{\therefore RoS = \{(b,a), (b,c), (c,b), (c,d), \cancel{(c,b)}, (d,a), (d,c)\}}$

(b)

SoR (S follows R)	SoR
$(a,a) \rightarrow \times$	
$(a,c) \rightarrow (c,c)$	(a,c)
$(a,c) \rightarrow (c,d)$	(a,d)
$(c,b) \rightarrow (b,a)$	(c,a)
$(c,d) \rightarrow (d,a)$	(c,a)
$(d,b) \rightarrow (b,a)$	(d,a)

$\boxed{\therefore SoR = \{(a,c), (a,d), (c,a), (d,a)\}}$

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c) RoR (R follows R)

			RoR
(a,a)	$\longrightarrow$	(a,a)	(a,a)
(a,a)	$\longrightarrow$	(a,c)	(a,c)
(a,c)	$\longrightarrow$	(c,b)	(a,b)
(a,c)	$\longrightarrow$	(c,d)	(a,d)
(c,b)	$\longrightarrow$	X	
(c,d)	$\longrightarrow$	(d,b)	(c,b)
(d,b)	$\longrightarrow$	X	

$\therefore$ $\boxed{RoR = \{(a,a), (a,c), (a,b), (a,d), (c,b)\}}$

d) SoS (S follows S)

			SoS
(b,a)	$\longrightarrow$	x	
(c,c)	$\longrightarrow$	(c,c)	(c,c)
(c,c)	$\longrightarrow$	(c,d)	(c,d)
(c,d)	$\longrightarrow$	(d,a)	(c,a)
(d,a)	$\longrightarrow$	X	

$\therefore$ $\boxed{SoS = \{(c,c), (c,d), (c,a)\}}$