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Discrete Structure

Tutorial 5

Solutions:

→ 1) Let the name of bit string beginning with 1 be event A and bit string with atleast 2 consecutive '0' as B. So

$$P(A) = \frac{1 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2} = \frac{1}{2}$$

$$P(A \cap B) = \frac{(1 \times 2 + 1 \times 1)}{2 \times 2 \times 2 \times 2} = \frac{3}{16}$$

Thus, $P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{3}{8}$

→ 2) Number of ways of choosing 3 bulbs out of 15 are:

$$C(15,3) = \frac{15 \times 14 \times 13}{3 \times 2 \times 1} = 5 \times 7 \times 13$$

$\therefore n(S) = 5 \times 7 \times 13$

(i) Let E_1 be the event that from selected bulbs none is defective.

$$C(10,3) = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 3 \times 4 \times 10$$

$\therefore n(E_1) = 3 \times 4 \times 10$

$$\therefore P(E_1) = \frac{n(E_1)}{n(S)} = \frac{3 \times 4 \times 10}{5 \times 7 \times 13} = \frac{24}{91}$$

(ii) Let E_2 be the event that from selected bulbs exactly one is defective.

$$C(10,2) \times C(5,1) = \frac{10 \times 9}{2} \times \frac{5}{1} = 5 \times 9 \times 5$$

$$\therefore n(E_2) = 5 \times 9 \times 5$$

$$\therefore P(E_2) = \frac{n(E_2)}{n(S)} = \frac{5 \times 9 \times 5}{5 \times 7 \times 13} = \frac{45}{91}$$

(iii) Let E_3 be the event that from selected bulbs atleast one is defective.

$$C(10,2) \times C(5,1) + C(10,1) \times C(5,2) + C(5,3)$$

$$= 5 \times 9 \times 5 + 10 \times 10 + 10$$

$$= 335$$

$$= n(E_3)$$

$$\therefore P(E_3) = \frac{n(E_3)}{n(S)} = \frac{335}{5 \times 7 \times 13} = \frac{67}{91}$$

→ 3) There are 4 different books with 3 copies each.

∴ There are total of 12 books and number of ways for arranging them is 12!

As there are 3 copies of the book present, the number of ways of arranging them is 3!

There are 4 books with 3 ~~books~~ copies, hence the number of ways of arranging them is $(3!)^4$

∴ The total number of ways of arranging 4 different books with 3 copies each is $\frac{12!}{(3!)^4}$

→ 4) $a_{n+2} - 5a_{n+1} + 6a_n = 2$

∴ Since this is a homogeneous equation, its solution consist of two parts, (i) the solution of the corresponding homogenous equation and (ii) particular solution.

We shall obtain the solution of homogeneous equation,

$$a_{n+2} - 5a_{n+1} + 6a_n = 0$$

Characteristic equation is, $r^2 - 5r + 6 = 0$

$\therefore \; r = 2, 3$

Solution of the homogeneous equation,

$$a_n^{n} = A(2)^n + B(3)^n$$

Since $F(n) =$ a constant we assume that particular solution to be constant, i.e. $a_n = c$, constant, $a_{n+1} = a_{n+2} = c$ putting these values in given recurrence relation.

$$c - 5c + 6c = 2 \implies c = 1$$

∴ The particular solution is $a_n^{(P)} = 1$

Hence, the solution of given recurrence relation is,

$$a_n = a_n^{(K)} + a_n^{(P)}$$

$$a_n = A(2^n) + B(3^n) + 1$$

when $n = 0$, $a_0 = 1$, $\implies A + B = 0$

when $n = 1$, $a_0 = -1$ $\implies 2A + 3B = -2$

By solving simultaneously these two equations, we get $A = 2$, $B = -2$

∴ The solution is $a_n = 2 \cdot 2^n + 2 \cdot 3^n + 1$

→ 5) Let $P(n)$ be the statement, $n! \geq 2^{n-1}$

(i) Basis of induction:

for $n = 1$

$P(1)$: $1! = 2^{1-1}$

$1 = 1$

Hence $P(1)$ is true

(ii) Induction step:

Assume $P(K)$ is true

$\therefore P(K) \Rightarrow K! \geq 2^{K-1}$ — (i)

$\therefore$ Prove $P(K+1)$ is true

$\therefore$ for $n = K+1$,

$R.H.S = 2^{(K+1)-1}$

$= 2^K = 2 \cdot 2^{K-1}$

$\therefore RHS \geq 2K!$ ... from (i)

$LHS = (K+1)!$

$\therefore$ ~~2K!~~ $2K!$ is smaller than $(K+1)!$

i.e. $(K+1)! \geq 2K!$

$\therefore$ It is true for $n = K+1$

$\therefore P(n) : n! \geq 2^{n-1}$ is true for $n \geq 1$

→ 6) By division algorithm every integer n can be written as $n = 10q + r$ where $0 \le r \le 9$. Since, there are 11 integers but only 10 possible values for the remainder r on division by 10.

So, there are 10 possible values for remainder r as 10 pigeonholes, $m=10$ and there are 11 integers to be chosen as 11 pigeon, $n=11$

∴ According to pigeonholes principle,

In n pigeon are assigned to m pigeonholes and $m<n$ then at least one pigeonhole contains two or more pigeons.

Hence, in the set of integers choosen, suppose it contains some x and y integer that have the same remainder on division by 10 that is, there exist a s with $0 \le s \le 9$ such that $x = 10q_1 + 5$ and $y = 10q_2 + 5$. For this, x and y we have $x - y = 10(q_1 - q_2)$. Since $(q_1 - q_2)$ is an integer, the difference of the two integers is divisible by 10.