

27/04/22

Maths - Tutorial 3

- 1) The following data shows ages of husband and wife for 6 couples

Age of husband (yrs)	28	30	27	26	32	29
Age of wife (yrs)	24	25	24	20	28	26

Calculate correlation coefficient between age of husband and wife and comment.

- 2) The following values were calculated on the basis of 8 paired observations on (x, y) . $\sum x = 49$, $\sum y = 73$, $\sum xy = 417$, $\sum x^2 = 359$, $\sum y^2 = 695$. It was later found that our above calculations were done on two pairs of observations wrongly copied as $(8, 7)$ and $(6, 8)$ while the correct values were $(7, 8)$ and $(6, 6)$. Find correct value of correlation equation

- 3) Find the spearman's rank correlation coefficient between height and weight from the following data.

Height (in inches)	48	40	45	50	55	55	55	50	50	40
Weight (in lbs)	11	13	14	16	16	15	15	14	13	13

- 4) The following r ranks given by 2 judges given to 8 participants in beauty contest. calculate spearman's correlation coefficient and comment.

Rank by judge 1	1	7	6	8	4	5	2	3
Rank by judge 2	8	3	4	5	1	6	2	7

Solution:

→ 1) Let the age of husband and wife be x and y respectively.

$$\therefore \text{The mean of age of husbands} = \bar{x} = \frac{28+30+27+26+32+29}{6}$$

$$\therefore \bar{x} = 28.67$$

$$\text{The mean of age of wife} = \bar{y} = \frac{24+25+24+20+28+26}{6}$$

$$= 24.5$$

X	Y	$X = x_i - \bar{x}$	$Y = y_i - \bar{y}$	x^2	y^2	xy
28	24	-0.67	-0.5	0.4489	0.25	0.335
30	25	1.33	0.5	1.7689	0.25	0.665
27	24	-1.67	-0.5	2.7889	0.25	0.835
26	20	-2.67	-4.5	7.1289	20.25	12.015
32	28	3.33	3.5	11.0889	12.25	11.655
29	26	0.33	1.5	0.1089	2.25	0.495
172	147	0	0	23.3334	35.5	26

$$\therefore \text{Correlation coefficient } (r) = \frac{\sum xy}{\sqrt{(\sum x^2)(\sum y^2)}} = \frac{26}{\sqrt{(23.3334)(35.5)}}$$

$$= \frac{26}{28.7808}$$

$$\text{Correlation coefficient } (r) = 0.9034$$

There is a strong positive correlation between husband's age and wife's age.

$$\begin{aligned} \rightarrow 2) \quad \sum x &= 49 & \sum y &= 73 \\ \sum x_c &= 49 - 8 - 6 + 7 + 6 & \sum y_c &= 73 - 70 - 8 + 8 + 6 \\ \sum x_c &= 48 & \sum y_c &= 72 \\ \sum xy &= 417 \\ \sum (xy)_c &= 417 - 8(7) - 6(8) + 7(8) + 6(6) \\ \sum (xy)_c &= 405 \end{aligned}$$

$$\sum x^2 = 359$$

$$\sum x_c^2 = 359 - (8)^2 - (6)^2 + (7)^2 + (6)^2$$

$$\sum x_c^2 = 344$$

$$\sum y^2 = 695$$

$$\sum y_c^2 = 695 - (8)^2 - (7)^2 + (8)^2 + (6)^2$$

$$\sum y_c^2 = 682$$

Karl Pearson's coefficient of correlation,

$$r = \frac{\sum x_c y_c - N \bar{x} \bar{y}}{\sqrt{(\sum x_c^2 - N \bar{x}^2)(\sum y_c^2 - N \bar{y}^2)}}$$

$$r = \frac{405 - 8(6)(9)}{\sqrt{(344 - 8(6)^2)(682 - 8(9)^2)}}$$

$$r = \frac{-27}{\sqrt{(56)(34)}}$$

$$r = -0.6187 \quad (\text{negative correlation})$$

→ 3) let X be the height (in inches) and Y be the weight (in lbs)

X	Rank (x_i)	Y	Rank (y_i)	$d_i = (x_i - y_i)$	d_i^2
48	4	11	1	3	9
40	1.5	13	3	-1.5	2.25
45	3	14	5.5	-2.5	6.25
50	6	16	9.5	-3.5	12.25
55	9	16	9.5	-0.5	0.25
55	9	15	7.5	1.5	2.25
55	9	15	7.5	1.5	2.25
50	6	14	5.5	0.5	0.25
50	6	13	3	3	9
40	1.5	13	3	-1.5	2.25

Here, $N = 10$, $\sum d_i^2 = 46.25$, $m_1 = 2$, $m_2 = 3$, $m_3 = 3$, $m_4 = 3$,
 $m_5 = 2$, $m_6 = 2$, $m_7 = 2$

\therefore Spearman's rank correlation coefficient (R) =
$$1 - \frac{6 \left[\sum d_i^2 + \frac{1}{12} (m_1^3 - m_1) + \frac{1}{12} (m_2^3 - m_2) + \dots + \frac{1}{12} (m_7^3 - m_7) \right]}{N^3 - N}$$

$$\therefore R = 1 - \frac{6(54.25)}{990}$$

$$R = 1 - 0.3288$$

$$R = 0.6712$$

→ 8.4)	Rank by Judge 1 (x_i)	Rank by Judge 2 (y_i)	$d = (x_i - y_i)$	d_i^2
	1	8	-7	49
	7	3	4	16
	6	4	2	4
	8	5	3	9
	4	1	3	9
	5	6	-1	1
	2	2	0	0
	3	7	-4	16

Here, $N = 8$, $\sum d_i^2 = 104$

\therefore Spearman's rank correlation coefficient (r) $= 1 - \frac{6 \sum d^2}{n(n^2 - 1)}$

$$r = 1 - \frac{6(104)}{8(63)}$$

$$r = -0.2381 \text{ (negative correlation)}$$

The correlation is poor and one value cannot be estimated from other.