Name : Ayush Jain
SAP ID: 60004200132
Branch: Computer Engineering
Div : J1

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Term Test 2

Engineering Physics - 2

1.) Given:

$\vec{A} = 3\vec{i} + 2xy\vec{j} + 50xz^2\vec{k}$

To find: Divergence, curl

Solution:

(i) Divergence $= \vec{\nabla} \cdot \vec{A}$

$$= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot \left(3\hat{i} + 2xy\hat{j} + 50xz^2\hat{k}\right)$$

$$= \left(\frac{\partial}{\partial x} \cdot 3\right) + \left(\frac{\partial}{\partial y} \cdot 2xy\right) + \left(\frac{\partial}{\partial z} 50xz^2\right)$$

$$= 0 + 2x + 100xz$$

$\therefore$ $\boxed{\text{Divergence} = 2x + 100xz \text{ [Scalar quantity]}}$

(ii) Curl $= \vec{\nabla} \times \vec{A}$

$$= \left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right) \times \left(3\vec{i} + 2xy\vec{j} + 50xz^2\vec{k}\right)$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3 & 2xy & 50xz^2 \end{vmatrix} = \left(\frac{\partial}{\partial y} 50xz^2 - \frac{\partial}{\partial z} 2xy\right)\hat{i}$$

$$- \left(\frac{\partial}{\partial x} \cdot 50xz^2 - \frac{\partial}{\partial z} \cdot 3\right)\hat{j}$$

$$+ \left(\frac{\partial}{\partial x}(2xy) - \frac{\partial}{\partial y}(3)\right)\hat{k}$$

$= 0\hat{i} - (50z^2)\hat{j} + 2y\,\hat{k}$

$= -50z^2\hat{j} + 2y\hat{k}$ [Vector quantity]

$\therefore \boxed{\text{Curl} = -50z^2\hat{j} + 2y\hat{k}}$

2) Given :

$V = 5x^2yz + 2y^2z^2$

$P = (3,1,2)$

To find: Volume charge density = ?

Solution: $\rho_v = \vec{\nabla} \cdot \vec{v}$

$\therefore \rho_v = \dfrac{\partial V_x}{\partial x} + \dfrac{\partial V_y}{\partial y} + \dfrac{\partial V_z}{\partial z}$

$= 10xyz + 5x^2z + 4yz^2 + 5x^2y + 4y^2z$

$= 10(3)(1)(2) + 5(3)^2(2) + 4(1)(2)^2 + 5(3)^2(1) + 4(1)^2(2)$

$= 60 + 90 + 16 + 45 + 8$

$= 219\ C/m^3$

$\therefore$ $\boxed{\text{The volume charge density at point P is } 219\ C/m^3}$

Q. 3) Given:

$\vec{E} = 4x\,\hat{i}$

$\vec{H} = 7z\,\hat{k}$

i) $\vec{\nabla}\cdot\vec{D} = 0$ . . First equation

$$\vec{\nabla}\cdot\vec{D} = \epsilon_0 \vec{\nabla}\cdot\vec{E} = \epsilon_0\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot 4x\,\hat{i}$$

$$= 4\epsilon_0 \neq 0$$

Hence it is not satisfied.

ii) Second equation: $\vec{\nabla}\cdot\vec{B} = 0$

$$\vec{\nabla}\cdot\vec{B} = \mu_0\vec{\nabla}\cdot\vec{H} = \mu_0\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot(7z\,\hat{k})$$

$$= 7\mu_0 \neq 0$$

Hence not satisfied.

iii) Third equation:

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

LHS: $\vec{\nabla}\times\vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 4x & 0 & 0 \end{vmatrix} = 0$

$$RHS = -\frac{\partial \vec{B}}{\partial t} = \mu_0 \frac{\partial \vec{H}}{\partial t} = -\mu_0 \frac{\partial}{\partial t}(7z\hat{k}) = 0$$

$\therefore$ LHS = RHS

Hence, third equation is satisfied.

iv) Fourth equation $\longrightarrow$ $\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$

$$LHS = \vec{\nabla} \times \vec{H} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & 7z \end{vmatrix} = 0$$

$$RHS = \frac{\partial \vec{D}}{\partial t} = \epsilon_0 \cdot \frac{\partial \vec{E}}{\partial t} = \epsilon_0 \frac{\partial}{\partial t}(4x\hat{i}) = 0$$

$\therefore$ LHS = RHS

Hence, 4^{th} equation is satisfied.

$\therefore$ It does not satisfy Maxwell's equation