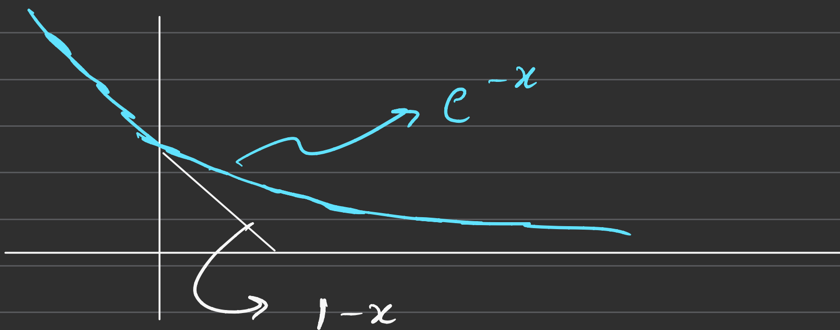
Q8

$$P\left(\bigcap_{i=1}^{n} A_i^c\right) = \prod_{i=1}^{n} P(A_i^c)$$

$$= \prod_{i=1}^{n} \left(1 - P(A_i)\right)$$



$\because P(\xi) \longrightarrow$ always from 0 to 1

$\Rightarrow \quad 1 - P(A_i) \le e^{-P(A_i)}$

$$\boxed{P\left(\bigcap_{i=1}^{n} A_i^c\right) \le e^{-\sum_{i=1}^{n} P(A_i)}}$$

Q7

(a) P(r times without returning to originator)

$$= \left(\frac{n-1}{n}\right) \cdot \left(\frac{n-1}{n}\right) \cdots\cdots \left(\frac{n-1}{n}\right)$$

$$= \left(\frac{n-1}{n}\right)^{r}$$

(b) $P = \frac{n}{n} \cdot \frac{n-1}{n} \cdot \frac{n-2}{n} \cdots \frac{n-r+1}{n}$

$$\boxed{P = \prod_{i=0}^{r-1} \frac{n-i}{n}}$$

(a) $P = \underbrace{\left(\frac{n}{n+1}\right)\left(\frac{n}{n+1}\right) \cdots\cdots \left(\frac{n}{n+1}\right)}_{r \text{ times}}$

$$\boxed{P = \left(\frac{n}{n+1}\right)^{r}}$$

Q10

$$\because E(X) = \int x f_X(x)\,dx = \int_\Omega X(\omega)\,dP(\omega)$$

$$X(\omega) = \int_0^\infty \mathbb{I}_{[0, X(\omega)]}(x)\,dx$$

$$E(X) = \int_\Omega \int_0^\infty \mathbb{I}_{[0, X(\omega)]}(x)\,dx\,dP(\omega)$$

$$= \int_0^\infty \underbrace{\int_\Omega \mathbb{I}_{[0, X(\omega)]}(x)\,dP(\omega)}_{1 - F(x)}\,dx$$

$$\boxed{E(X) = \int_0^\infty (1 - F(x))\,dx}$$

Hence proved.

Q11

$$E(e^{ux}) = \int_{-\infty}^{\infty} e^{ux} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\frac{2u\sigma^2 x - (x^2+\mu^2-2\mu x)}{2\sigma^2}} dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left[-\frac{(x^2-(2\mu+2u\sigma^2)x+\mu^2)}{2\sigma^2}\right] dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{\sigma^2\mu^2}{2\sigma^2}} \int_{-\infty}^{\infty} \exp\left[-\frac{[x-(\mu+u\sigma^2)]^2}{2\sigma^2}\right] \underbrace{dx}_{\sqrt{2}\sigma\, dy}$$

$$\frac{x-(\mu+u\sigma^2)}{\sqrt{2}\sigma} = y$$

$$= e^{u\mu} \cdot \exp\left(\frac{1}{2}u^2\sigma^2\right) = e^{u\mu+\frac{1}{2}u^2\sigma^2}$$

(b) By the above proof,

$\phi(x) = e^{ux}$

$$\underbrace{E[\phi(X)]}_{e^{u\mu+\frac{1}{2}u^2\sigma^2}} \geq \underbrace{\phi(E(X))}_{e^{u\mu}}$$

$E(x) = \mu$

as $e^{\frac{1}{2}u^2\sigma^2} \geq 1$