Assignment - 2.1

Date: ___/___/___

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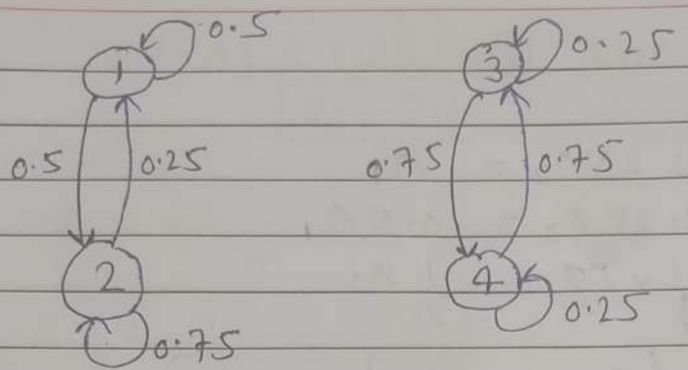
Subject

(1)



(1) (a) Transition matrix Q for this chain

$$Q = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0.5 & 0.5 & 0 & 0 \\ 0.25 & 0.75 & 0 & 0 \\ 0 & 0 & 0.25 & 0.75 \\ 0 & 0 & 0.75 & 0.25 \end{bmatrix} \end{array}$$

(b) Recurrent state = $\{1, 2, 3, 4\}$

Transient state = No transient state.

(c) Stationary distribution for the chain.

$\pi = (\pi_1, \pi_2, \pi_3, \pi_4)$

$$\pi Q = \pi$$

$$[\pi_1, \pi_2, \pi_3, \pi_4] \begin{bmatrix} .5 & .5 & 0 & 0 \\ .25 & .75 & 0 & 0 \\ 0 & 0 & .25 & .75 \\ 0 & 0 & .75 & .25 \end{bmatrix} = [\pi_1, \pi_2, \pi_3, \pi_4]$$

$\pi_1 + \pi_2 + \pi_3 + \pi_4 = 1$

$0.5\pi_1 + 0.25\pi_2 = \pi_1$ — (1)

$0.5\pi_1 + 0.75\pi_2 = \pi_2$ — (2)

$0.25\pi_3 + 0.75\pi_4 = \pi_3$ — (3)

$0.75\pi_3 + 0.25\pi_4 = \pi_4$ — (4)

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using ① and ②

$0.5\pi_1 + 0.25\pi_2 = \pi_1$

$0.5\pi_1 + 0.75\pi_2 = \pi_2$

$0.5\pi_1 + 0.25\pi_2 = \pi_1$

$0.25\pi_2 = 0.5\pi_1$

$\pi_2 = 2\pi_1$

put in eq ②

$0.5\pi_1 + 0.75(2\pi_1) = 2\pi_1$

$2\pi_1 = 2\pi_1$ (satisfy)

so $\pi_2 = 2\pi_1$ $\quad \pi_1 + \pi_2 + \pi_3 + \pi_4 = 1$

$\pi_3 = \pi_4 = 0$

so, $\left(\frac{1}{3}, \frac{2}{3}, 0, 0\right)$

using ③ and ④

$0.25\pi_3 + 0.75\pi_4 = \pi_3$

$0.75\pi_3 + 0.25\pi_4 = \pi_4$

$0.25\pi_3 + 0.75\pi_4 = \pi_3$

$0.75\pi_4 = 0.75\pi_3$

$\boxed{\pi_4 = \pi_3}$ put in eq ④

$0.75\pi_3 + 0.25\pi_3 = \pi_3$

$\boxed{\pi_3 = \pi_3}$ (satisfied)

so, $\left(0, 0, \frac{1}{2}, \frac{1}{2}\right)$

so, $\left(\frac{1}{3}, \frac{2}{3}, 0, 0\right)$, $\left(0, 0, \frac{1}{2}, \frac{1}{2}\right)$ are two

different stationary distributions for the chain.

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(2) The transition matrix.

$$\begin{array}{c|cc} & W & L \\ \hline W & 0.8 & 0.2 \\ L & 0.3 & 0.7 \end{array}$$

(a) stationary distribution

$$\pi = [\pi_W \ \pi_L]$$

$$[\pi_W \ \pi_L]\begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix} = [\pi_W \ \pi_L]$$

$$0.8\pi_W + 0.3\pi_L = \pi_W \quad - (1)$$
$$0.2\pi_W + 0.7\pi_L = \pi_L \quad - (2)$$
$$\pi_W + \pi_L = 1 \quad - (3)$$

from (1)

$$0.8\pi_W + 0.3\pi_L = \pi_W$$
$$\boxed{0.3\pi_L = 0.2\pi_W} \text{ put in (2)}$$

$$0.2\pi_W + 0.7\frac{(0.2\pi_W)}{0.3} = \frac{0.2}{0.3}\pi_W$$

$$\pi_W[0.2 + 0.4667] = 0.6667\pi_W$$

$$\boxed{\pi_W(0.667) = 0.6667\pi_W}$$

satisfied

so from (3)

$$\pi_W + \pi_L = 1$$
$$\pi_W + \frac{0.2}{0.3}\pi_W = 1$$

$$1.667\pi_W = 1$$
$$\pi_W = \frac{1}{1.6667} \simeq 0.600024$$

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(a) In long run team win 0.6 [approximately]
60% of the games win

(b) Team wins dinner probability 0.7
Team loses dinner probability 0.2

$\pi_W = 0.6 \quad \pi_L = 0.4$

long run proportion of dinner is.

$$= 0.7 \times 0.6 + 0.2 \times 0.4$$
$$= 0.42 + 0.08$$
$$= 0.5$$

50% proportion of games result in a team dinner.

(c) Expected number of games the team needs to play for a dinner

Probability of a dinner in game is 0.5

expected number of game until first dinner happen $= \dfrac{1}{(\text{Dinner}_{\text{rate}})}$ = having dinner probability

$$= \frac{1}{0.5} = 2$$

So expected number of game is 2 until first dinner happen //

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(3) Cat and mouse Game -

Transition matrix for cat chain

$$\begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 0.2 & 0.8 \\ 0.8 & 0.2 \end{bmatrix} \end{array}$$

Transition matrix for mouse chain

$$\begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 0.7 & 0.3 \\ 0.6 & 0.4 \end{bmatrix} \end{array}$$

(a) stationary distribution of the cat chain

$$\pi Q = \pi$$

$$[\pi_1 \ \pi_2]\begin{bmatrix} 0.2 & 0.8 \\ 0.8 & 0.2 \end{bmatrix} = [\pi_1 \ \pi_2]$$

$$\begin{cases} 0.2\pi_1 + 0.8\pi_2 = \pi_1 & -(1) \\ 0.8\pi_1 + 0.2\pi_2 = \pi_2 & -(2) \\ \pi_1 + \pi_2 = 1 & -(3) \end{cases}$$

from (1) $0.8\pi_2 = 0.8\pi_1$

$$\boxed{\pi_1 = \pi_2}$$

also satisfied by (2) when $\pi_1 = \pi_2$

$$\pi_1 = \pi_1$$

put in eq (3)

$$\pi_1 + \pi_2 = 1$$

$$\pi_1 + \pi_1 = 1$$

$$\boxed{\pi_1 = 1/2}$$

$$\boxed{\pi_2 = 1/2}$$

The stationary distribution for cat is $(1/2, 1/2)$

$\rightarrow$ stationary distribution for mouse

$$[\theta_1 \quad \theta_2] \begin{bmatrix} 0.7 & 0.3 \\ 0.6 & 0.4 \end{bmatrix} = [\theta_1 \quad \theta_2]$$

$$0.7\theta_1 + 0.6\theta_2 = \theta_1 \quad - (4)$$
$$0.3\theta_1 + 0.4\theta_2 = \theta_2 \quad - (5)$$
$$\theta_1 + \theta_2 = 1 \quad - (6)$$

using eq. (4)

$$0.6\theta_2 = 0.3\theta_1$$
$$\frac{3}{5}\theta_2 = \frac{3}{10}\theta_1$$
$$2\theta_2 = \theta_1 \quad \text{put in } (5)$$
$$2(0.3)\theta_2 + 0.4\theta_2 = \theta_2$$
$$\boxed{\theta_2 = \theta_2} \text{ satisfied}$$

Now put in (6)

$$2\theta_2 + \theta_2 = 1$$
$$\theta_2 = 1/3 \qquad \theta_1 = 2/3$$

$(2/3, 1/3)$

The stationary distribution for mouse is $(2/3, 1/3)$

(b) There are 4 possible states -

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(1) both in room 1 (1, 1)
(2) cat in room 1 and mouse in room 2 (1, 2)
(3) Cat in room 2 and mous in room 1 (2, 1)
(4) both in room 2 (2, 2)

Z_n = current state (cat and mouse location) at time n.

Each Z_n takes one of 4 possible state.

→ for Z_n to be markov state, the transition state must depend only on last state.

Cat and mouse move independtly so, the transition probability for Z_n is the product of the individual transition probability of the cat and the mouse.

so,

$$P(Z_{n+1} = (R', L') \mid Z_n = (R, L))$$
$$= P(R_{n+1} = R' \mid R_n = R) \times P(L_{n+1} = L' \mid L_n = 2)$$

notation here R = cat
L = mouse

Also transition matrix for Z_n

$$\begin{array}{c} \\ (1,1) \\ (1,2) \\ (2,1) \\ (2,2) \end{array} \begin{array}{c} \begin{array}{cccc} (1,1) & (1,2) & (2,1) & (2,2) \end{array} \\ \begin{bmatrix} 0.14 & 0.06 & 0.56 & 0.24 \\ 0.12 & 0.08 & 0.48 & 0.32 \\ 0.56 & 0.24 & 0.14 & 0.06 \\ 0.48 & 0.32 & 0.12 & 0.08 \end{bmatrix} \end{array}$$

transition probability depends only on the current state Z_n satisfies the markov property.

$Z_0, Z_1, Z_2 \ldots$ is a markov chain because depend the transition probability for Z_{n+1} depend only on Z_n. not on any previous state.

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