AI1103-Assignment 6

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Download all python codes from

https://github.com/ayushjha2612/AI11003/tree/main/Assignment6/Codes

and latex-tikz codes from

https://github.com/ayushjha2612/AI11003/tree/main/Assignment6

GATE 2021(ST) Q.22 (Statistics section)

Let X be a random variable having probability density function

$$f(x) = \begin{cases} \frac{3}{13}(1-x)(9-x) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Then $\frac{4}{3}E[X(X^2 - 15X + 27)]$ equals — (round of to two decimal places).

Answer

8.67

Solution

Let X be the random variable. To find

$$\frac{4}{3}E[X(X^2 - 15X + 27)] \tag{0.0.1}$$

Let,

$$g(X) = X(X^2 - 15X + 27) \tag{0.0.2}$$
$$= X^3 - 15X^2 + 27X \tag{0.0.3}$$

Then for random variable X we have that,

$$E[g(X)] = \int_{-\infty}^{\infty} g(x)f(x)\,dx \tag{0.0.4}$$

The probability distribution of X is,

$$f(x) = \begin{cases} \frac{3}{13}(1-x)(9-x) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases} \tag{0.0.5}$$

Using 0.0.5 we have,

$$E[g(X)] = 0 + \int_0^1 g(x)f(x)\,dx + 0 \tag{0.0.6}$$
$$= \int_0^1 (x^3 - 15x^2 + 27x)[\frac{3}{13}(1-x)(9-x)]\,dx \tag{0.0.7}$$

Using Integration by substitution let,

$$t = x^3 - 15x^2 + 27x$$
$$dt = 3x^2 - 30x + 27$$
$$= 3(1-x)(9-x)$$

The corresponding limits are,

$$\text{For } x = 0 \implies t = 0^3 - 15 \times 0^2 + 27 \times 0 = 0 \tag{0.0.8}$$
$$\text{For } x = 1 \implies t = 1^3 - 15 \times 1^2 + 27 \times 1 = 13 \tag{0.0.9}$$

Therefore we have,

$$E[g(X)] = \frac{1}{13}\int_0^{13} t\,dt \tag{0.0.10}$$
$$= \frac{1}{13} \times \left(\frac{t^2}{2}\right)\Big|_0^{13} \tag{0.0.11}$$
$$= \frac{1}{13} \times \frac{13^2}{2} \tag{0.0.12}$$
$$= \frac{13}{2} \tag{0.0.13}$$

Thus,

$$\frac{4}{3}E[g(X)] = \frac{4}{3} \times \frac{13}{2} \tag{0.0.14}$$
$$= \frac{26}{3} \tag{0.0.15}$$
$$= 8.67 \text{ (rounded off)} \tag{0.0.16}$$

Therefore,

$$\frac{4}{3}E[X(X^2 - 15X + 27)] = 8.67 \tag{0.0.17}$$

The plot for PDF of X can be observed at figure 0.

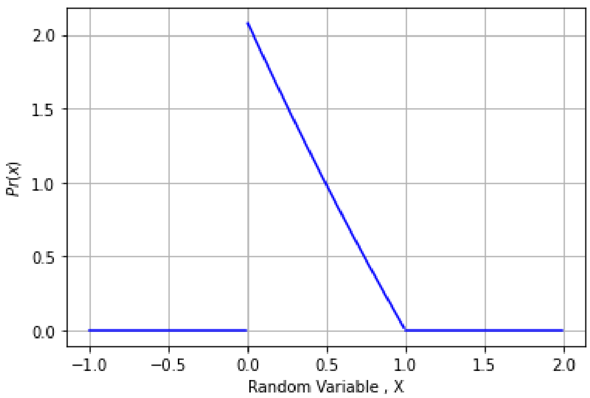

Fig. 0: The PDF of X