AI1103-Assignment 5

Name : Ayush Jha
Roll Number: CS20BTECH11006

Download all python codes from

https://github.com/ayushjha2612/AI11003/tree/main/Assignment5/Codes

and latex-tikz codes from

https://github.com/ayushjha2612/AI11003/tree/main/Assignment5

1 GATE 2020 XE-A Q.11

Players A and B take turns to throw a fair dice with six faces. If A is the first player to throw, then the probability of B being the first one to get a six is — (round of to two decimal places).

2 Answer

0.45

3 Solution

Let the random variable X represent which player gets six first. That is $X = 0$ when A gets a six first and $X = 1$ when B gets six first.
Let another random variable Y represent getting a six on the dice. $Y = 1$ for six and $Y = 0$ for any other number.
Let N be the number of turns until we get a six.

$$\Pr(Y = 0) = \frac{5}{6} \tag{3.0.1}$$

$$\Pr(Y = 1) = \frac{1}{6} \tag{3.0.2}$$

The event success is when B gets a six for first time and failure is when neither A nor B gets six. Let p denote probability of success

$$p = \Pr(Y = 1) \tag{3.0.3}$$

$$\Pr(Y = 0) = 1 - p \tag{3.0.4}$$

$$p = \frac{1}{6} \tag{3.0.5}$$

To get $X = 1$ in N turns we have to get $N-1$ failures for B and N failures for A and finally one success for B. Therefore the geometric distribution is,

$$f(N) = (1-p)^{n-1} \times p \times (1-p)^n \tag{3.0.6}$$

$$= (1-p)^{2n-1} \times p \tag{3.0.7}$$

$$= \left(\frac{5}{6}\right)^{2n-1} \times \frac{1}{6} \tag{3.0.8}$$

The result has been summarized in table 0.

No. of turns	Probability
1	$5^1/6^2$
2	$5^3/6^4$
⋮	⋮
n	$5^{2n-1}/6^{2n}$
⋮	⋮

TABLE 0: Summary of turns

Thus the total probability is sum of these individual probabilities i.e.

$$\Pr(X = 1) = \sum_{N=1}^{\infty} f(N) \tag{3.0.9}$$

$$= \frac{5}{6^2} + \frac{5^3}{6^4} + \ldots + \frac{5^{2n-1}}{6^{2n}} + \ldots \tag{3.0.10}$$

$$= \frac{5}{6^2} \times \left(1 + \frac{5^2}{6^2} + \frac{5^4}{6^4} + \ldots\right) \tag{3.0.11}$$

By Using sum of infinite GP we have,

$$\Pr(X = 1) = \frac{5}{6^2} \times \left(\frac{1}{1 - \frac{25}{36}}\right) \tag{3.0.12}$$

$$= \frac{5}{36} \times \frac{36}{11} \tag{3.0.13}$$

$$= \frac{5}{11} = 0.45 \tag{3.0.14}$$

Therefore the probability of B being the first one to get a six is 0.45.

The theory vs Simulation plot can be seen at figure 0 and the geometric distribution plot can be seen at figure 0.

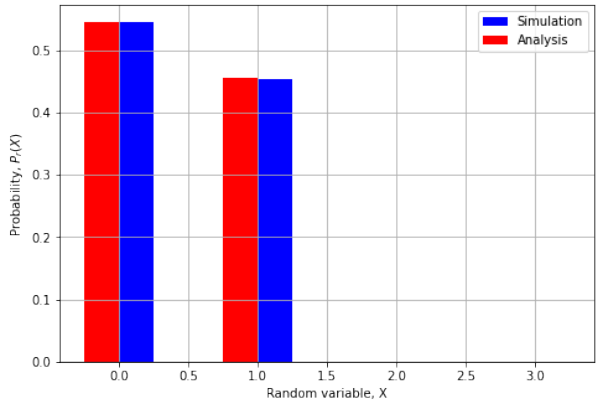

Fig. 0: Probability distribution of X

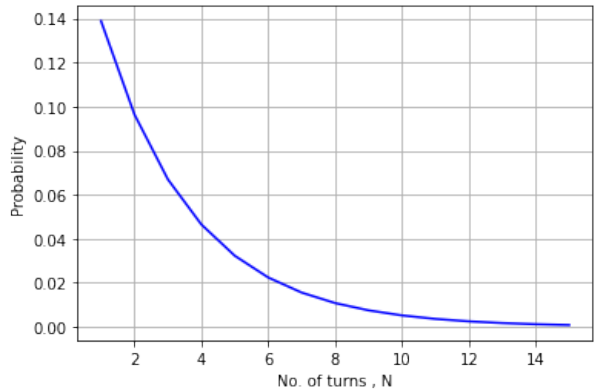

Fig. 0: Geometric distribution