AI1103-Assignment 3

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Download all python codes from

https://github.com/ayushjha2612/AI11003/tree/main/Assignment3/Codes

and latex-tikz codes from

https://github.com/ayushjha2612/AI11003/tree/main/Assignment3

GATE Problem 34

Let X and Y be two statistically independent random variables uniformly distributed in the range (-1, 1) and (-2, 1) respectively. Let $Z = X + Y$, then the probability that $[Z \leq -2]$ is

(A) zero (B) $\frac{1}{6}$ (C)$\frac{1}{3}$ (D)$\frac{1}{12}$

Answer

Option (D) $\frac{1}{12}$

Solution

X and Y are two independent random variables.
The range of X is $-1 \leq X \leq 1$ and the range of Y is $-2 \leq X \leq 1$.
As it is a uniform distribution we have,

$$\Pr(-1 \leq X \leq 0) = \Pr(0 \leq X \leq 1) \tag{0.0.1}$$

X is distributed in the range(-1,1) which implies that

$$\Pr(-1 \leq X \leq 0) + \Pr(0 \leq X \leq 1) = 1 \tag{0.0.2}$$

$$2 \times \Pr(-1 \leq X \leq 0) = 1 \tag{0.0.3}$$

Therefore we have,

$$\Pr(-1 \leq X \leq 0) = \frac{1}{2} \tag{0.0.4}$$

Similarly random variable Y is distributed in the range(-2 ,1)

$$\Pr(-2 \leq Y \leq -1) = \Pr(-1 \leq Y \leq 0) = \Pr(0 \leq Y \leq 1)$$

And sum of these three probabilities is 1 Therefore we have,

$$3 \times \Pr(-2 \leq Y \leq -1) = 1 \tag{0.0.5}$$

$$\Pr(-2 \leq Y \leq -1) = \frac{1}{3} \tag{0.0.6}$$

Now we have another random variable Z, which is defined as

$$Z = X + Y \tag{0.0.7}$$

We need to find $\Pr(Z \leq -2)$

$$Z \leq 2 \implies X + Y \leq 2 \tag{0.0.8}$$

When the random variable, X lies in the range (-1,0) and Y lies in the range(-2,-1) we have that Z lies in the range (-3,-1).
As X and Y are independent random variables we have that

$$\Pr(XY) = \Pr(X) \times \Pr(Y) \tag{0.0.9}$$

Therefore we have,

$$\Pr(-3 \leq Z \leq -1) = \Pr((-2 \leq Y \leq -1)(-1 \leq X \leq 0)) \tag{0.0.10}$$

$$= \frac{1}{3} \times \frac{1}{2} \tag{0.0.11}$$

$$= \frac{1}{6} \tag{0.0.12}$$

As it is a uniform distribution,

$$\Pr(-3 \leq Z \leq -2) = \Pr(-2 \leq Z \leq -1) \tag{0.0.13}$$

Which gives us that,

$$\Pr(-3 \leq Z \leq -2) = \frac{1}{12} \tag{0.0.14}$$

So $\Pr(Z \leq -2) = \frac{1}{12}$ i.e. option (D).
The theory vs simulation plot can be viewed at figure 0.

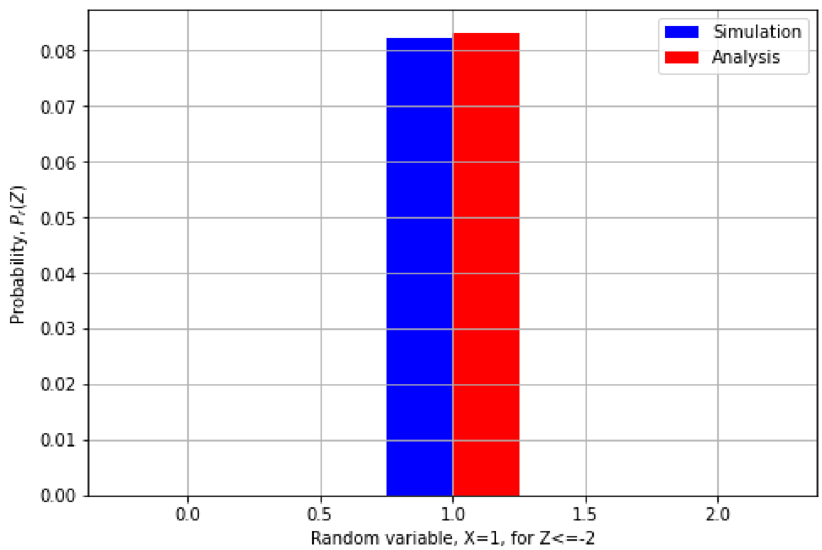

Fig. 0: Theory VS Simulation plot