AI1103-Assignment 5

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Download all python codes from

```
https://github.com/ayushjha2612/AI11003/tree/main
    /Assignment5/Codes
```

and latex-tikz codes from

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https://github.com/ayushjha2612/AI11003/tree/main
    /Assignment5
```

1 GATE 2020 XE-A Q.11

Players A and B take turns to throw a fair dice with six faces. If A is the first player to throw, then the probability of B being the first one to get a six is — (round of to two decimal places).

2 Answer

0.45

3 Solution

Let the random variable X represent which player gets six first. That is $X = 0$ when A gets a six first and $X = 1$ when B gets six first.
Let another random variable Y represent getting a six on the dice. $Y = 1$ for six and $Y = 0$ for any other number.

$$\Pr(Y = 0) = \frac{5}{6} \tag{3.0.1}$$
$$\Pr(Y = 1) = \frac{1}{6} \tag{3.0.2}$$

Case 1 :
A does not get six and B does in the first turn i.e.

$$X = 1 \tag{3.0.3}$$
$$\Pr(X = 1) = \Pr_A(Y = 0) \times \Pr_B(Y = 1) \tag{3.0.4}$$
$$= \frac{5}{36} \tag{3.0.5}$$

Case 2 :
A does not get six and B does in the second turn i.e.

$$\Pr(X = 1) = \Pr(Y = 0)^3 \times \Pr(Y = 1) \tag{3.0.6}$$
$$= \frac{5^3}{36^2} \tag{3.0.7}$$

Case 3 :
A does not get six and B does in the third turn i.e.

$$\Pr(X = 1) = \Pr(Y = 0)^5 \times \Pr(Y = 1) \tag{3.0.8}$$
$$= \frac{5^5}{36^3} \tag{3.0.9}$$

And so on ...
The further cases have been summarized in table 0

Case	No. of turns	Probability
4	4	$5^7/36^4$
5	5	$5^9/36^5$
$\vdots$	$\vdots$	$\vdots$
n	n	$5^{2n-1}/6^{2n}$
$\vdots$	$\vdots$	$\vdots$

TABLE 0: Generalization of cases

Thus the total probability is sum of these individual probabilities i.e.

$$\Pr(X = 0) = \frac{5}{6^2} + \frac{5^3}{6^4} + \ldots + \frac{5^{2n-1}}{6^{2n}} + \ldots \tag{3.0.10}$$
$$= \frac{5}{6^2} \times \left(1 + \frac{5^2}{6^2} + \frac{5^4}{6^4} + \ldots\right) \tag{3.0.11}$$

By Using sum of infinite GP we have,

$$\Pr(X = 0) = \frac{5}{6^2} \times \left(\frac{1}{1 - \frac{25}{36}}\right) \tag{3.0.12}$$
$$= \frac{5}{36} \times \frac{36}{11} \tag{3.0.13}$$
$$= \frac{5}{11} = 0.45 \tag{3.0.14}$$

Therefore the probability of B being the first one to get a six is 0.45.
The theory vs Simulation plot can be seen at figure 0.

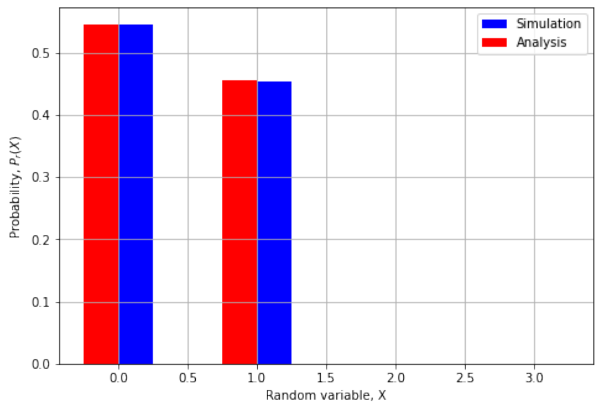

Fig. 0: Probability distribution of X