HW 4 com org

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1 Simplify the following expressions using Boolean algebraic laws. Give each step of your simplification and denote which laws you're using for each step. Do not skip or combine steps!

- a) $A \cdot (\overline{A} + B \cdot B) + \overline{(B + A)} \cdot (\overline{A} + B)$
  1. $A \cdot (\overline{A} + B \cdot B) + (\overline{B} \cdot \overline{A}) \cdot (\overline{A} + B)$ DeMorgan's
  2. $A \cdot (\overline{A} + B) + (\overline{B} \cdot \overline{A}) \cdot (\overline{A} + B)$ Idempotent
  3. $(\overline{A} + B) \cdot (A + (\overline{B} \cdot \overline{A}))$ Distributive
  4. $(\overline{A} \cdot A) + (\overline{A} \cdot (\overline{B} \cdot \overline{A})) + (B \cdot A) + (B \cdot (\overline{B} \cdot \overline{A})$ Distributive
  5. $0 + (\overline{A} \cdot \overline{A} \cdot \overline{B}) + (B \cdot A) + (0 \cdot \overline{A})$ Inverse
  6. $(\overline{A} \cdot \overline{B}) + (B \cdot A) + (0 \cdot \overline{A})$ Idempotent
  7. $(\overline{A} \cdot \overline{B}) + (B \cdot A) + 0$ Law of zeroes
  8. Simplified: $(\overline{A} \cdot \overline{B}) + (B \cdot A)$

- b) $\overline{C * B} + (A * B * C) + \overline{A + C + \overline{B}}$
  1. $\overline{C} + \overline{B} + (A * B * C) + \overline{A + C + \overline{B}}$ DeMorgan's
  2. $\overline{C} + \overline{B} + (A * B * C) + \overline{A} * \overline{C} * \overline{\overline{B}}$ DeMorgan's
  3. $\overline{C} + \overline{B} + (A * B * C) + \overline{A} * \overline{C} * B$ double not cancel out
  4. $\overline{C} + \overline{B} + (A * B * C)$ Absorption law $B(AC + \overline{AC})$
  5. $\overline{C} + \overline{B} + (A * B)$ Absorption law $\overline{C} + ABC$
  6. $\overline{C} + \overline{B} + (A)$ Absorption law $\overline{B} + AB$
  7. Simplified: $\overline{C} + \overline{B} + A$

- c) $(A + B) * (\overline{A} + C) * (\overline{C} + B)$
  1. $(A + B) * (\overline{A} * \overline{C} + \overline{A} * B + C * \overline{C} + C * B)$ distributive
  2. $(A + B) * (\overline{A} * \overline{C} + \overline{A} * B + 0 + C * B)$ inverse
  3. $(A * \overline{A} * \overline{C}) + (\overline{A} * B * A) + (C * B * A) + (\overline{A} * \overline{C} * B) + (\overline{A} *$

$B * B) + (C * B * B)$ distributive
4. $(0*\overline{C})+(0*B)+(C*B*A)+(\overline{A}*\overline{C}*B)+(\overline{A}*B*B)+(C*B*B)$ inverse
5. $0+0+(C*B*A)+(\overline{A}*\overline{C}*B)+(\overline{A}*B*B)+(C*B*B)$ law of zeroes
6. $(C*B*A)+(\overline{A}*\overline{C}*B)+(\overline{A}*B)+(C*B)$ Idempotent
7. $(A*B*C)+(\overline{A}*B)+(C*B)$ Absorption law
8. Simplified form: $\overline{A}*B+C*B$ Absorption law

2 Find all solutions of the following Boolean equations without using the truth tables:

- a) $(\overline{A}+C)*(\overline{B}+D+A)*(D+A*\overline{C})*(\overline{D}+A)=1$
1. $(D+A+\overline{C})(\overline{D}+A)(\overline{A}*\overline{B})+(D+A+\overline{C})(\overline{D}+A)(\overline{A}*D)+(D+A+\overline{C})(\overline{D}+A)(\overline{A}*\overline{A})+(\overline{B}+D+A)(D+A+\overline{C})(\overline{D}+A)*C$ Distribution
2. $(D+A+\overline{C})(\overline{D}+A)(\overline{A}*\overline{B})+(D+A+\overline{C})(\overline{D}+A)(\overline{A}*D)+0+(\overline{B}+D+A)(D+A+\overline{C})(\overline{D}+A)*C$ Inverse
3. $(\overline{D}+A)(\overline{A}*\overline{B}*D)+(\overline{D}+A)(\overline{A}*\overline{B}*A)+(\overline{D}+A)(\overline{A}*\overline{B}*\overline{C})+(D+A+\overline{C})(\overline{D}+A)(\overline{A}*\overline{D})+(\overline{B}+D+A)(D+A+\overline{C})(\overline{D}+A)(C)$ Distribution
4. $(\overline{D}+A)(\overline{A}*\overline{B}*D)+0+(\overline{D}+A)(\overline{A}*\overline{B}*\overline{C})+(D+A+\overline{C})(\overline{D}+A)(\overline{A}*\overline{D})+(\overline{B}+D+A)(D+A+\overline{C})(\overline{D}+A)(C)$ Inverse
5. $(\overline{A}*\overline{B}*D*\overline{D})+(\overline{A}*\overline{B}*D*A)+(\overline{D}+A)(\overline{A}*\overline{B}*\overline{C})+(D+A+\overline{C})(\overline{D}+A)(\overline{A}*D)+(\overline{B}+D+A)(D+A+\overline{C})(\overline{D}+A)(C)$ Distribution
6. $0+0+(\overline{D}+A)(\overline{A}*\overline{B}*\overline{C})+(D+A+\overline{C})(\overline{D}+A)(\overline{A}*D)+(\overline{B}+D+A)(D+A+\overline{C})(\overline{D}+A)(C)$ Inverse
7. $(\overline{A}*\overline{B}*\overline{C}*\overline{D})+(\overline{A}*\overline{B}*\overline{C}*A)+(D+A+\overline{C})(\overline{D}+A)(\overline{A}*D)+(\overline{B}+D+A)(D+A+\overline{C})(\overline{D}+A)(C)$ Distribution
8. $(\overline{A}*\overline{B}*\overline{C}*\overline{D})+0+(D+A+\overline{C})(\overline{D}+A)(\overline{A}*D)+(\overline{B}+D+A)(D+A+\overline{C})(\overline{D}+A)(C)$ Inverse
9. $(\overline{A}*\overline{B}*\overline{C}*\overline{D})+(C*\overline{B}*A)+(\overline{D}+A)(C*D*D)+(\overline{D}+A)(C*D*A)+(\overline{D}+A)(C*D*\overline{C})+(D+A+\overline{C})(\overline{D}+A)(C*A)$ Distribution
10. $(\overline{A}*\overline{B}*\overline{C}*\overline{D})+(C*\overline{B}*A)+(\overline{D}+A)(C*D)+(\overline{D}+A)(C*D*A)+(\overline{D}+A)(C*D*\overline{C})+(D+A+\overline{C})(\overline{D}+A)(C*A)$ Idempotent

11. $(\overline{A}*\overline{B}*\overline{C}*\overline{D})+(C*\overline{B}*A)+(\overline{D}+A)(C*D)+0+(\overline{D}+A)(C*D*\overline{C})+(D+A+\overline{C})(\overline{D}+A)(C*A)$ Inverse
12. $(\overline{A}*\overline{B}*\overline{C}*\overline{D})+(C*\overline{B}*A)+(\overline{D}+A)(C*D)+(D+A+\overline{C})(\overline{D}+A)(C*A)$ Absorption law
13. $(\overline{A}*\overline{B}*\overline{C}*\overline{D})+(C*\overline{B}*A)+(C*D*A)+(\overline{D}+A)(C*A*D)+(\overline{D}+A)(C*A*A)+(\overline{D}+A)(C*A*\overline{C})$ Distribution
14. $(\overline{A}*\overline{B}*\overline{C}*\overline{D})+(C*\overline{B}*A)+(C*D*A)+(\overline{D}+A)(C*A*D)+(\overline{D}+A)(C*A)+(\overline{D}+A)(C*A*\overline{C})$ Idempotent
15. $(\overline{A}*\overline{B}*\overline{C}*\overline{D})+(C*\overline{B}*A)+(C*D*A)+(\overline{D}+A)(C*A*D)+(\overline{D}+A)(C*A)+0$ Inverse
16. $(\overline{A}*\overline{B}*\overline{C}*\overline{D})+(C*\overline{B}*A)+(C*D*A)+(\overline{D}+A)(C*A)$ Absorption
17. $(\overline{A}*\overline{B}*\overline{C}*\overline{D})+(C*\overline{B}*A)+(C*D*A)+(C*A*\overline{D})+(C*A*A)$ Distribution
18. $(\overline{A}*\overline{B}*\overline{C}*\overline{D})+(C*\overline{B}*A)+(C*D*A)+(C*A*\overline{D})+(C*A)$ Idempotent
19. $(\overline{A}*\overline{B}*\overline{C}*\overline{D})+(C*D*A)+(C*A*\overline{D})+(C*A)$ Absorption law
20. $(\overline{A}*\overline{B}*\overline{C}*\overline{D})+(C*A*\overline{D})+(C*A)$ Absorption law
21. $(\overline{A}*\overline{B}*\overline{C}*\overline{D})+(C*A)$ Absorption law
22. Simplified form: $(\overline{A}*\overline{B}*\overline{C}*\overline{D})+(C*A)$

For $(\overline{A}*\overline{B}*\overline{C}*\overline{D})+(C*A)=1$ the different possibilities for A, B, C, and D are
A = 0, B = 0, C = 0, D = 0
A = 1, B = 0, C = 1, D = 0
A = 1, B = 0, C = 1, D = 1
A = 1, B = 1, C = 1, D = 0
A = 1, B = 1, C = 1, D = 1

- b) $(((\overline{K}*L*N)*(L*M))+((\overline{K}+L+N)*(K*\overline{L}*M)))*(\overline{K}+\overline{N})=1$
  1. $(\overline{K}+\overline{N})*\overline{K}*L*N*(L+M)+(\overline{K}+\overline{N})*(\overline{K}+L+N)*K*\overline{L}*\overline{M}=1$ Distribution
  2. $\overline{K}*L*N*(L+M)*\overline{K}+\overline{K}*L*N*(L+M)*\overline{N}*(\overline{K}+\overline{N})*(\overline{K}+L+N)*K*\overline{L}*\overline{M}=1$ Distribution
  3. $\overline{K}*L*N*(L+M)+\overline{K}*L*N*(L+M)*\overline{N}*(\overline{K}+\overline{N})*(\overline{K}+L+N)*K*\overline{L}*\overline{M}=1$ Idempotent

4. $\overline{K} * L * N * (L + M) + 0 + (\overline{K} + \overline{N}) * (\overline{K} + L + N) * K * \overline{L} * \overline{M} = 1$ Inverse
5. $\overline{K} * L * N * L + \overline{K} * L * N * M + (\overline{K} + \overline{N}) * (\overline{K} + L + N) * K * \overline{L} * \overline{M} = 1$ Distributive
6. $\overline{K} * L * N + \overline{K} * L * N * M + (\overline{K} + \overline{N}) * (\overline{K} + L + N) * K * \overline{L} * \overline{M} = 1$ Idempotent
7. $\overline{K} * L * N + (\overline{K} + \overline{N}) * (\overline{K} + L + N) * K * \overline{L} * \overline{M} = 1$ Absorption
8. $\overline{K} * L * N + (\overline{K} + L + N) * K * \overline{L} * \overline{M} * \overline{K} + (\overline{K} + L + N) * K * \overline{L} * \overline{M} * \overline{N} = 1$ Distribution
9. $\overline{K} * L * N + 0 + 0 = 1$ Inverse
10. Simplified: $\overline{K} * L * N = 1$

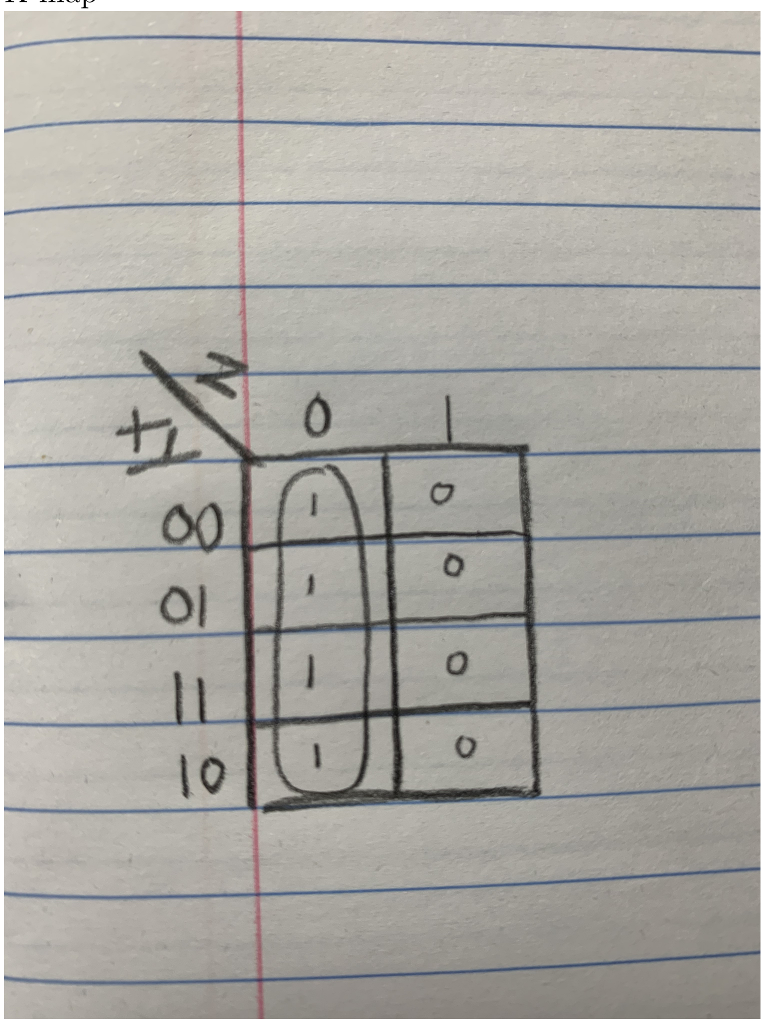
For this expression to be true or 1, the possible values of K, L, M, and N are
K = 0, L = 1, N = 1, M = 0
K = 0, L = 1, N = 1, M = 1
The value of M does not matter since its not included in the simplified expression, for the expression !K & L & N to be true all 3 components must be true, which is why the two solutions above work.

3 Simplify the following expression by first constructing a truth table, using that truth table to construct a K-map, and then using that K-map to simplify.

- $Q = (\overline{X} * \overline{Y} * Z) + (X * Y * \overline{Z}) + (\overline{X} + Y + \overline{Z}) + (X + \overline{Y} + \overline{Z})$

X	Y	Z	Q
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

K-map

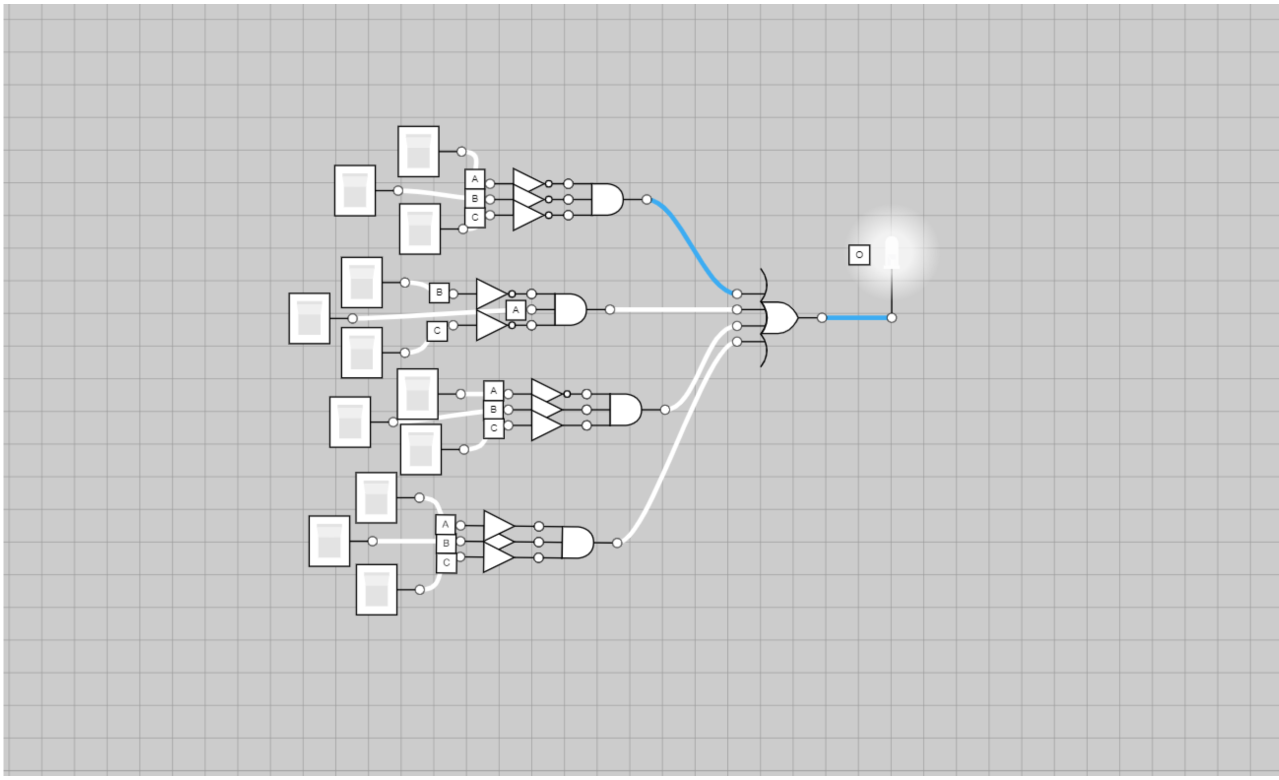


Simplified expression from k-map: $Q = \overline{Z}$

4 Convert the following truth table into its sum of products representation:

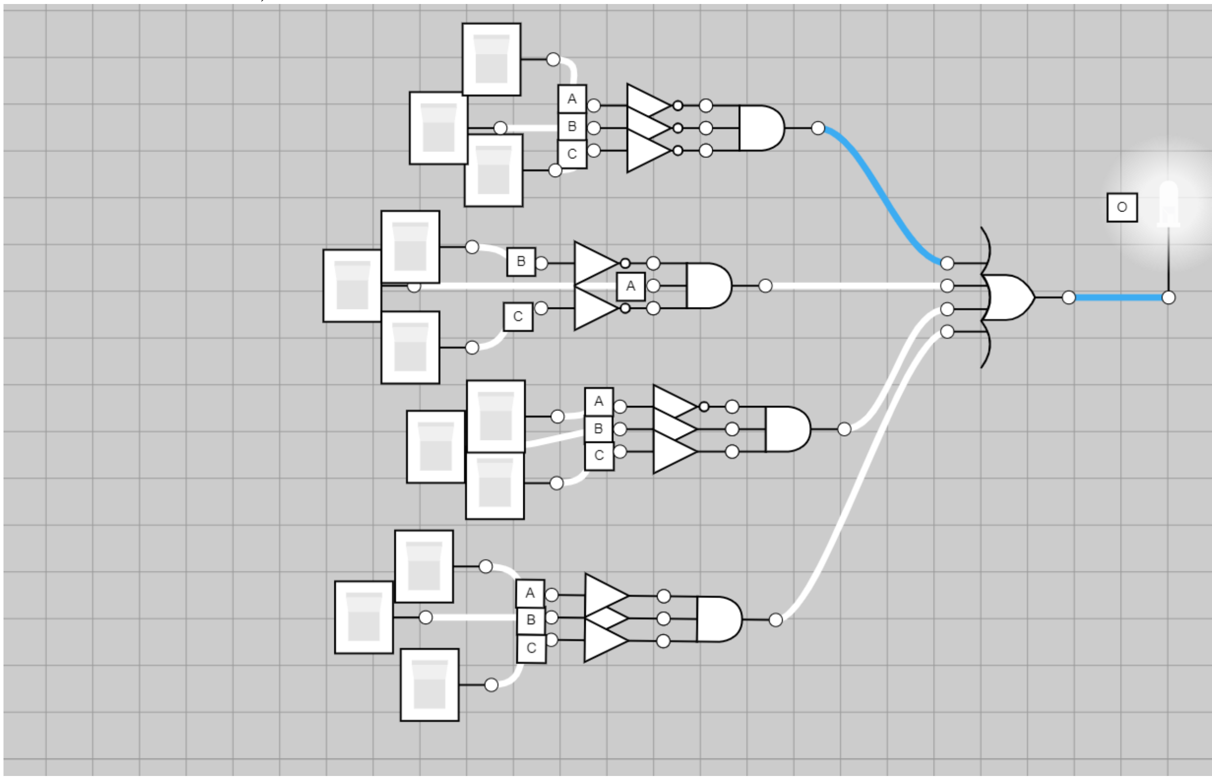
$O = (\overline{A} * \overline{B} * \overline{C}) + (\overline{A} * B * \overline{C}) + (\overline{A} * B * C) + (A * B * C)$

5 Draw a logical circuit diagram that represents the above sum of products expression using OpenCircuits (https://opencircuits.io/). Clearly label all inputs/outputs and all components. Make sure you connect appropriate input components (e.g., buttons, switches, clocks, etc.) and output components (e.g., LEDs, displays, etc.) to facilitate testing of your circuit.

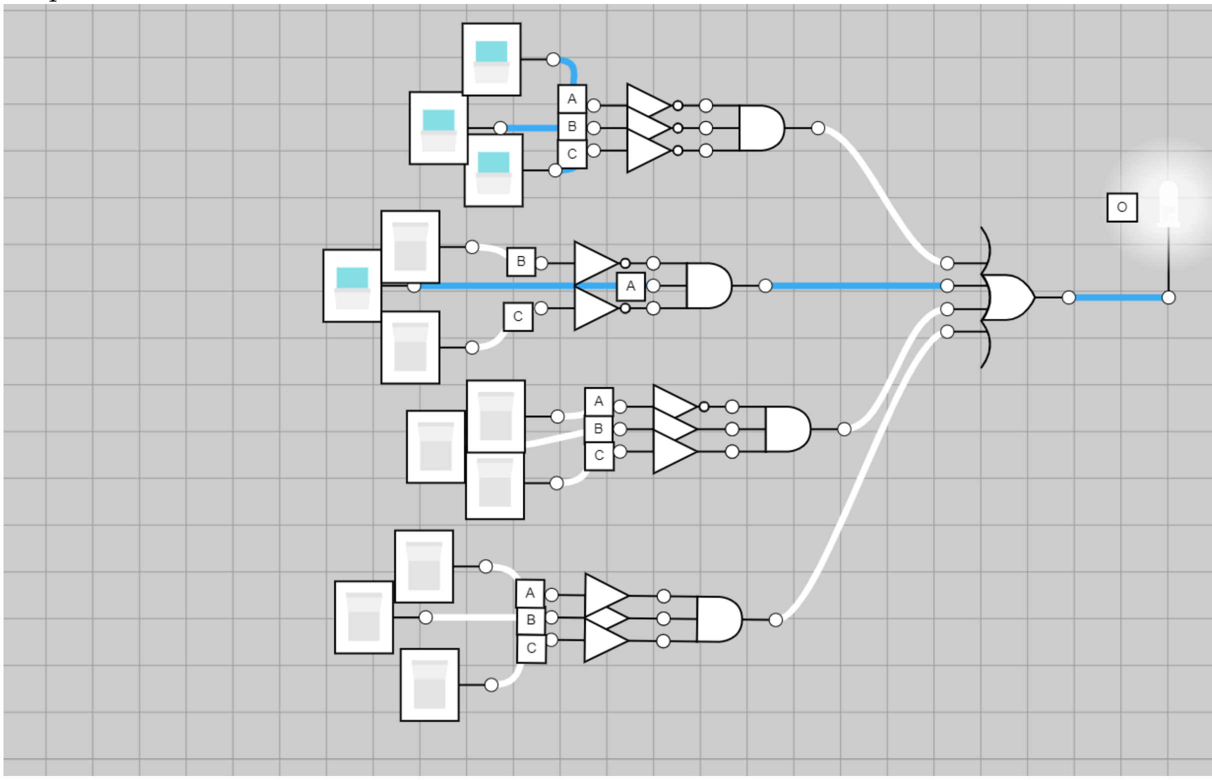


6 Test you circuit by supplying appropriate inputs and observing the expected values of the output. Explain why your set of tests is sufficient to prove that your logical circuit does in fact implement the required Boolean function. For each test, provide a picture (snapshot) of your circuit. Insert all such pictures in the hw4.pdf PDF file. You can download pictures (PNG, JPEG, or PDF) of your circuit diagram using OpenCircuits' "Export Image" feature.

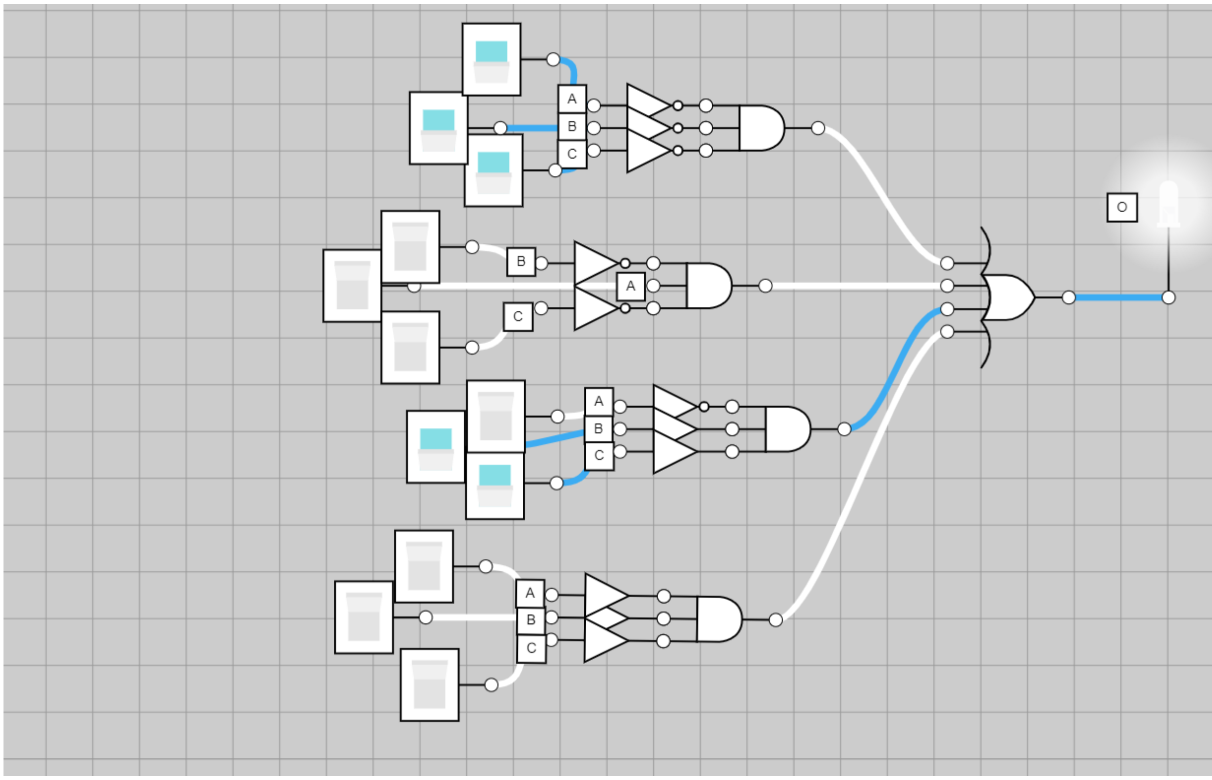
- Test 1, all false inputs for first gate, shows true since first switch equates to true when all inputs are false for A, B, and C (not gates for all inputs)



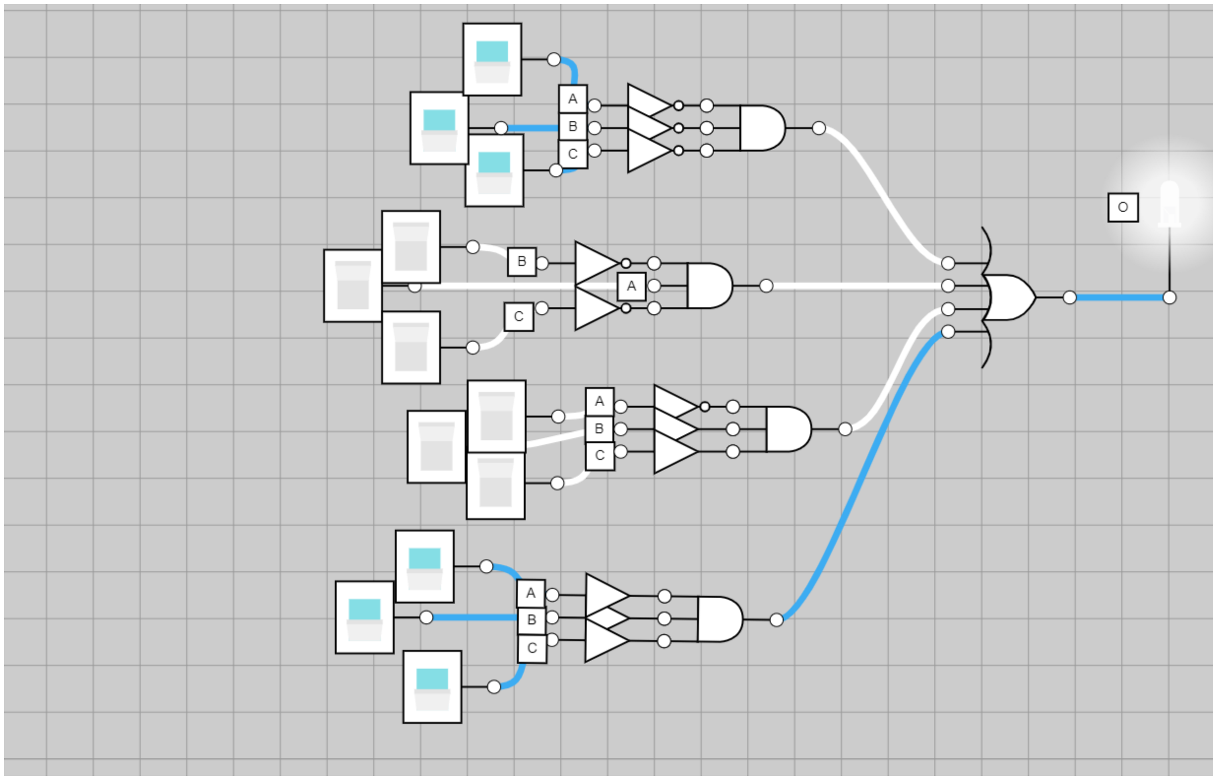
- Test 2, true input for B in second gate, and false for A and C, outputs True which matches the truth table



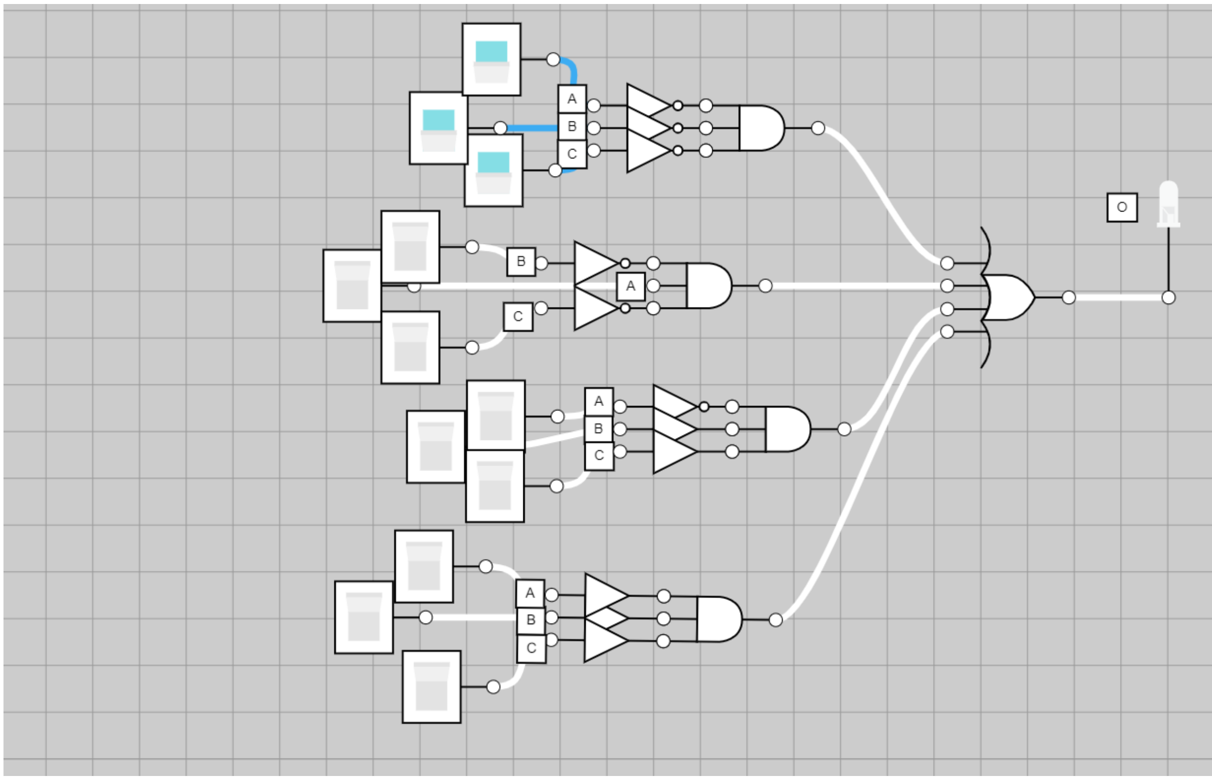
- Test 3, True inputs for B and C in gate 3, false input for A, outputs true which matches the truth table



- Test 4, True inputs for A, B, and C in gate 4, outputs true which matches truth table

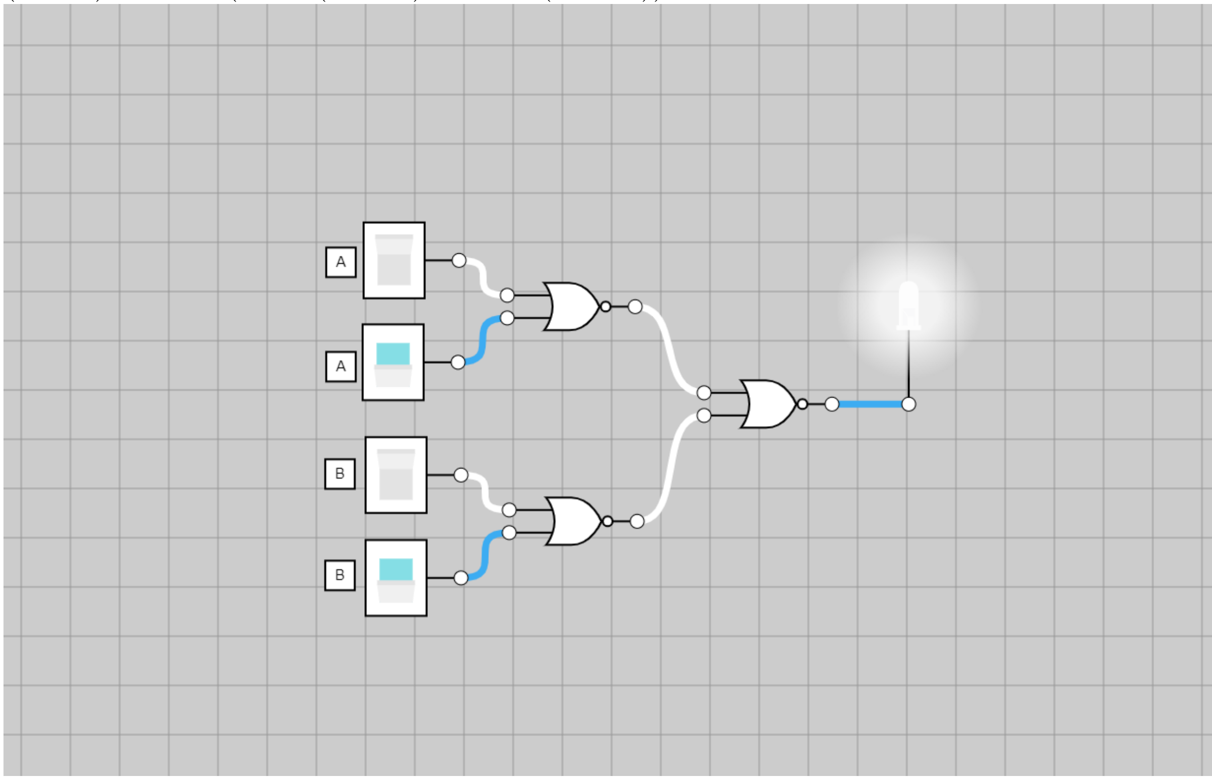


- Test 5, All false inputs for all gates, outputs false which matches the false outputs for truth table


A
B
C
B
A
C
A
B
C
A
B
C
O

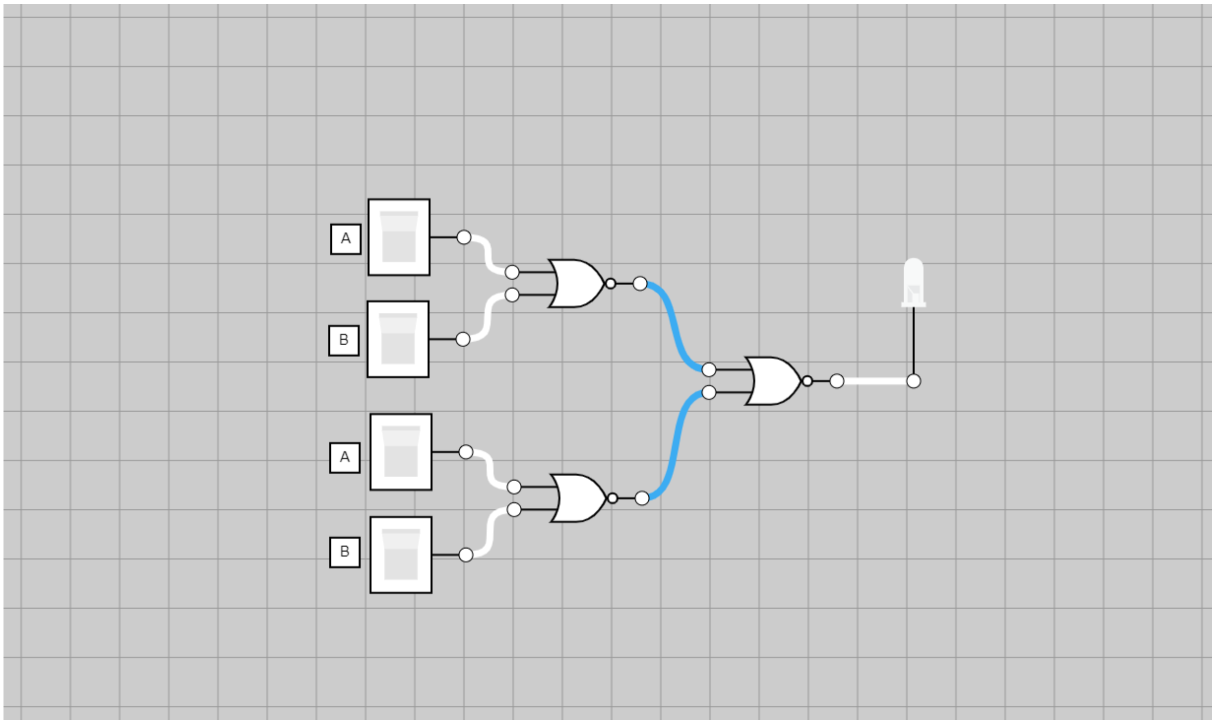
7 Given inputs A and B, show that NOR $\overline{(A+B)}$ is functionally complete by giving logical circuits equivalent to AND $(A*B)$, OR (A + B), and NOT $\overline{A}$ gates using only NOR gates in their construction.

- $(A*B) = NOR(NOR(A+A) + NOR(B+B))$



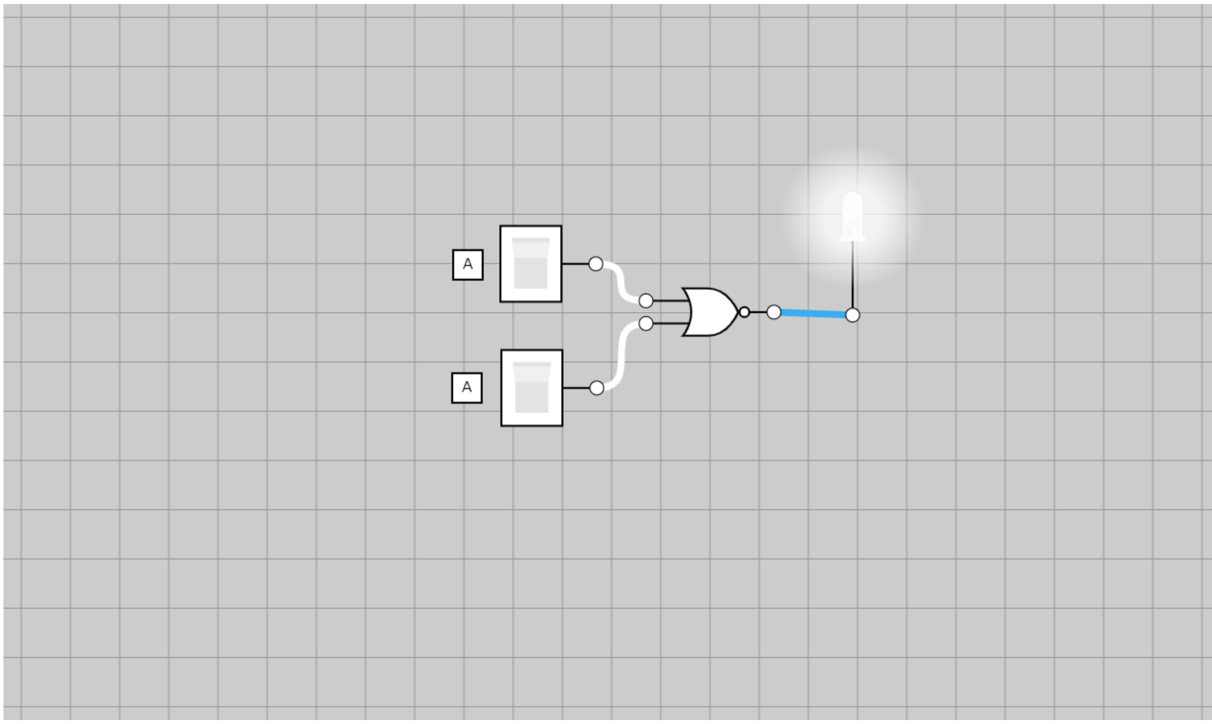
As you can see in this image the only time the output is true is when one of the A inputs is true and one of the B inputs is true, otherwise output is always false (equivalent to truth table for (A*B))

- $(A+B) = NOR(NOR(A+B) + NOR(A+B))$



For this example the only inputs in the truth table where the output is false, is when both A and B are false, as the image shows when all inputs are false output is false. When either side has one switch turned on the output is true, similar to example above but labels are now switched with one A and B on each side matches truth table for A OR B using only NOR gates

- $NOT(A) = NOR(A + A)$



AS you can see in the image the output shows true when both inputs are false, when either input is true the output is false, essentially matching the truth table for NOT(A)

8 9.13.1

- $(5ED4)_{16}$ = $(0101\ 1110\ 1101\ 0100)_2$
  $(07A4)_{16}$ = $(0000\ 0111\ 1010\ 0100)_2$
  Subtracting the two we get = $(0101\ 0111\ 0011\ 0000)_2$
  Converting to hexidecimal numbers = (0101 = 5) (0111 = 7) (0011 = 3) (0000 = 0) = $(5730)_{16}$
  $(5ED4)_{16} - (07A4)_{16} = (5730)_{16}$

9 9.13.2

- $(5ED4)_{16}$ = $(0101\ 1110\ 1101\ 0100)_2$
  $(07A4)_{16}$ = $(0000\ 0111\ 1010\ 0100)_2$

Both the numbers have a 0 as the Most Significant Bit(leftmost bit) hence they are both positive and subtraction can be carried out normally

1's compliment of 07A4 = 1111 1000 0101 1011 (inverting the bits, i.e 0 to 1 and 1 to 0)

2's compliment of 07A4 = 1111 1000 0101 1100 (adding 1 to the 1's compliment)

Answer = 0101 1110 1101 0100 + 1111 1000 0101 1100 = 0101 0111 0011 0000 (with a carry of 1)

Hence the result is 0101 0111 0011 0000 or 5730 in hexadecimal

10 9.13.6

- The given numbers 185 and 122 are unsigned 8-bit integers so we don't need to convert them.

  Difference: 185 - 122 = 63

  63 in binary is (00111111)

  This binary number is represented in 8 bits so their is no underflow or overflow, meaning neither.

11 9.13.10

- Convert first decimal to binary and then take the 2's complement and Add 1

  A=151=10010111 (binary)
  B=214=11010110 (binary)

A's 2 compliment= 0110 1001 (105)
B's 2 compliment = 00101010 (42)

subtraction on A-B = 0011 1111 (63)

Since signed 8-bit integers range is -128 127, the result using saturating arithmetic is 63.

12 9.13.11

- Convert first decimal to binary and then take the 2's complement and Add 1

  A=151=10010111 (binary)
  B=214=11010110 (binary)

  A's 2 compliment= 0110 1001 (105)
  B's 2 compliment = 00101010 (42)

  addition on A+B = 01111111 (127)

  Since signed 8-bit integers range is -128 127, the result using saturating arithmetic is 127, not 365.

13 9.13.20

- Binary representation of the hexadecimal number (0x0C000000) = 0000 1100 00000000 0000 0000 0000 0000_{two}

  32nd bit is 0, so both two's complement and unsigned integer represent same decimal number. So, for unsigned integer we get $(0*16)^7 + (12*16)^7 + (0*16)^5 + (0*16)^4 + (0*16)^3 + (0*16)^2 + (0*16)^1 + (0*16)^0 = 0 + 12*16777216 + 0 + 0 + 0 + 0 + 0 + 0 =$ 2013266529

So both the unsigned integer and two's complement are represented as 201326529 as a decimal number.

14 9.13.21

- Binary representation of the hexadecimal number (0x0C000000) = 0000 1100 00000000 0000 0000 0000 0000_{two}

  Divided into two sub fields opcode and target
  First 6 bits represent opcode remaining bits represent target
  Bit pattern divided as
  opcode: 000011
  target: 00 0000 0000 0000 0000 0000 0000

  opcode represents jal(jump and link instruction)
  Therefore the MIPS instruction Jal is used for the bit pattern 0x0C000000 is placed into the instruction register

15 Give a reason why we use two's complement representation for negative numbers in computer arithmetic. Give an example of its usage.

- We use two's complement to represent negative numbers in computer arithmetic so that we can use the same logical circuit to perform addition and subtraction. For example to add 6+3 using 5 bit two's complement

  00110 + 00011 = 01001
  Now to subtract we rewrite as 6 + (-3) instead of 6-3 to reuse addition circuit
  00110 + 11101 = 00011

  Looking at this example we can see how much simpler it is made by using the same logical circuit then making a new one to accommodate for both subtraction and addition.