Quick Review (Abridged)

- PIE: Way to calculate $\left|\bigcup_i A_i\right|$ for sets $A_1, \ldots, A_n$, which can overlap.

$$\left|\bigcup_i A_i\right| = \sum_i |A_i| - \sum_{i_1 \neq i_2} |A_{i_1} \cap A_{i_2}| + \sum_{i_1 \neq i_2 \neq i_3} |A_{i_1} \cap A_{i_2} \cap A_{i_3}|$$

$$- \cdots + (-1)^{n-1} \sum_{i_1 \neq i_2 \neq \cdots \neq i_n} |A_{i_1} \cap A_{i_2} \cap \cdots \cap A_{i_n}|$$

- Stars + Bars (aka Balls + Bins, ...)
  - ↳ Way to calculate how to distribute k indistinguishable objects among n distinguishable bins
  - ↳ $\Leftrightarrow$ Ways of choosing k objects from n, <u>with</u> replacement and where order <u>doesn't</u> matter
  - ↳ Consider a sequence of k "stars" and $n-1$ "bars":

$$\underset{\substack{\uparrow\\\text{Person 1}\\\text{gets 2\$}}}{**}\ |\ \underset{\substack{\uparrow\\\text{Person 2}\\\text{gets 3\$}}}{***}\ |\ * \cdots |\ \underset{\substack{\uparrow\\\text{Person } n-1\\\text{gets 2\$}}}{**}\ |\ \underset{\substack{\uparrow\\\text{Person } n\\\text{gets 3\$}}}{***}$$

  - ↳ 1-1 correspondence $\Rightarrow \binom{n+k-1}{k}$.

- Probability Formalism
  - ↳ Sample space Ω, "Probability" function $\mathbb{P} : \Omega \to \mathbb{R}$
  - ↳ Events are subsets of Ω and if $E \subseteq \Omega$, we can define $\mathbb{P}[E] := \sum_{e \in E} \mathbb{P}[e]$.