Assignment 12

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Download latex-tikz codes from

https://github.com/ayushkesh

1 Problem

$$\mathbf{M} = \begin{pmatrix} 2 & 0 & 3 & 2 & 0 & -2 \\ 0 & 1 & 0 & -1 & 3 & 4 \\ 0 & 0 & 1 & 0 & 4 & 4 \\ 1 & 1 & 1 & 0 & 1 & 1 \end{pmatrix} \tag{1.0.1}$$

$$\mathbf{b}_1 = \begin{pmatrix} 5 \\ 1 \\ 1 \\ 4 \end{pmatrix}, \mathbf{b}_2 = \begin{pmatrix} 5 \\ 1 \\ 3 \\ 3 \end{pmatrix}. \tag{1.0.2}$$

Then which of the following are true?

1) both systems $\mathbf{Mx} = \mathbf{b}_1$ and $\mathbf{Mx} = \mathbf{b}_2$ are inconsistent.
2) both systems $\mathbf{Mx} = \mathbf{b}_1$ and $\mathbf{Mx} = \mathbf{b}_2$ are consistent.
3) the system $\mathbf{Mx} = \mathbf{b}_1 - \mathbf{b}_2$ is consistent.
4) the system $\mathbf{Mx} = \mathbf{b}_1 - \mathbf{b}_2$ is inconsistent.

2 Explanation

See Table 4

Given	$$\mathbf{M} = \begin{pmatrix} 2 & 0 & 3 & 2 & 0 & -2 \\ 0 & 1 & 0 & -1 & 3 & 4 \\ 0 & 0 & 1 & 0 & 4 & 4 \\ 1 & 1 & 1 & 0 & 1 & 1 \end{pmatrix}, \mathbf{b_1} = \begin{pmatrix} 5 \\ 1 \\ 1 \\ 4 \end{pmatrix}, \mathbf{b_2} = \begin{pmatrix} 5 \\ 1 \\ 3 \\ 3 \end{pmatrix} \tag{2.0.1}$$
Solution	Solving for $\mathbf{Mx} = \mathbf{b_1}$, Row Reducing the augmented matrix $$\begin{pmatrix} 2 & 0 & 3 & 2 & 0 & -2 & 5 \\ 0 & 1 & 0 & -1 & 3 & 4 & 1 \\ 0 & 0 & 1 & 0 & 4 & 4 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 4 \end{pmatrix} \tag{2.0.2}$$ $$\xleftrightarrow[R_4 \leftarrow R_4 - 2R_2]{R_4 \leftarrow 2R_4 - R_1} \begin{pmatrix} 2 & 0 & 3 & 2 & 0 & -2 & 5 \\ 0 & 1 & 0 & -1 & 3 & 4 & 1 \\ 0 & 0 & 1 & 0 & 4 & 4 & 1 \\ 0 & 0 & -1 & 0 & -4 & -4 & 1 \end{pmatrix} \tag{2.0.3}$$ $$\xleftrightarrow{R_4 \leftarrow R_4 + R_3} \begin{pmatrix} 2 & 0 & 3 & 2 & 0 & -2 & 5 \\ 0 & 1 & 0 & -1 & 3 & 4 & 1 \\ 0 & 0 & 1 & 0 & 4 & 4 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix} \tag{2.0.4}$$ $$\implies Rank\left(M\right) = 3, Rank\left(\mathbf{M}\vert\mathbf{b_1}\right) = 4 \tag{2.0.5}$$ $$\implies Rank\left(M\right) \neq Rank\left(\mathbf{M}\vert\mathbf{b_1}\right) \tag{2.0.6}$$
	Solving for $\mathbf{Mx} = \mathbf{b_2}$, Row Reducing the augmented matrix $$\begin{pmatrix} 2 & 0 & 3 & 2 & 0 & -2 & 5 \\ 0 & 1 & 0 & -1 & 3 & 4 & 1 \\ 0 & 0 & 1 & 0 & 4 & 4 & 3 \\ 1 & 1 & 1 & 0 & 1 & 1 & 3 \end{pmatrix} \tag{2.0.7}$$ $$\xleftrightarrow[R_4 \leftarrow R_4 + 2R_2]{R_4 \leftarrow 2R_4 - R_1} \begin{pmatrix} 2 & 0 & 3 & 2 & 0 & -2 & 5 \\ 0 & 1 & 0 & -1 & 3 & 4 & 1 \\ 0 & 0 & 1 & 0 & 4 & 4 & 3 \\ 0 & 0 & -1 & 0 & -4 & -4 & -1 \end{pmatrix} \tag{2.0.8}$$ $$\xleftrightarrow{R_4 \leftarrow R_4 + R_3} \begin{pmatrix} 2 & 0 & 3 & 2 & 0 & -2 & 5 \\ 0 & 1 & 0 & -1 & 3 & 4 & 1 \\ 0 & 0 & 1 & 0 & 4 & 4 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix} \tag{2.0.9}$$ $$\implies Rank\left(M\right) = 3, Rank\left(\mathbf{M}\vert\mathbf{b_2}\right) = 4 \tag{2.0.10}$$ $$\implies Rank\left(M\right) \neq Rank\left(\mathbf{M}\vert\mathbf{b_2}\right) \tag{2.0.11}$$
	Solving for $\mathbf{Mx} = \left(\mathbf{b_1} - \mathbf{b_2}\right)$, Row Reducing the augmented matrix

	$$\begin{pmatrix} 2 & 0 & 3 & 2 & 0 & -2 & 0 \\ 0 & 1 & 0 & -1 & 3 & 4 & 0 \\ 0 & 0 & 1 & 0 & 4 & 4 & -2 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 \end{pmatrix} \quad (2.0.12)$$ $$\xleftrightarrow[R_4 \leftarrow R_4 - 2R_2]{R_4 \leftarrow 2R_4 - R_1} \begin{pmatrix} 2 & 0 & 3 & 2 & 0 & -2 & 0 \\ 0 & 1 & 0 & -1 & 3 & 4 & 0 \\ 0 & 0 & 1 & 0 & 4 & 4 & -2 \\ 0 & 0 & -1 & 0 & -4 & -4 & 2 \end{pmatrix} \quad (2.0.13)$$ $$\xleftrightarrow{R_4 \leftarrow R_4 + R_3} \begin{pmatrix} 2 & 0 & 3 & 2 & 0 & -2 & 0 \\ 0 & 1 & 0 & -1 & 3 & 4 & 0 \\ 0 & 0 & 1 & 0 & 4 & 4 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.0.14)$$ $$\implies Rank\left(M\right) = 3, Rank\left(\mathbf{M}\middle|\left(\mathbf{b_1} - \mathbf{b_2}\right)\right) = 3 \quad (2.0.15)$$ $$\implies Rank\left(M\right) = Rank\left(\mathbf{M}\middle|\left(\mathbf{b_1} - \mathbf{b_2}\right)\right) \quad (2.0.16)$$
Statement 1	Both systems $\mathbf{Mx} = \mathbf{b_1}$ and $\mathbf{Mx} = \mathbf{b_2}$ are inconsistent
	Eq.(2.0.6) and (2.0.11) violate the condition of consistency (2.0.17) **True Statement**
Statement 2	Both systems $\mathbf{Mx} = \mathbf{b_1}$ and $\mathbf{Mx} = \mathbf{b_2}$ are consistent
	Eq.(2.0.6) and (2.0.11) violate the condition of consistency (2.0.18) **False Statement**
Statement 3	Systems $\mathbf{Mx} = \mathbf{b_1} - \mathbf{b_2}$ are consistent
	Eq.(2.0.16) satisfy the condition of consistency (2.0.19) **True Statement**
Statement 4	Systems $\mathbf{Mx} = \mathbf{b_1} - \mathbf{b_2}$ are inconsistent
	Eq.(2.0.16) satisfy the condition of consistency (2.0.20) **False Statement**

TABLE 4: Explanation