Matrix Theory Assignment 1

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Abstract—**This document contains the solution to problem No.66 from Lines and Planes**

1 Problem Statement

If $\mathbf{a} = \begin{pmatrix} 5 \\ -1 \\ -3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 3 \\ -5 \end{pmatrix}$, then show that the vectors $\mathbf{a}+\mathbf{b}$ ans $\mathbf{a}-\mathbf{b}$ are perpendicular.

2 Theory

For two lines having direction vectors $\mathbf{A}$ and $\mathbf{B}$ respectively, they will be perpendicular if the scalar product of the two direction vector is 0,

$$\mathbf{AB} = 0 \tag{2.0.1}$$

Scalar product of two vectors, $\mathbf{A} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$ is defined by

$$\mathbf{AB} = \mathbf{A^TB} = \begin{pmatrix} x_1 & y_1 & z_1 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} \tag{2.0.2}$$

$$\implies x_1x_2 + y_1y_2 + z_1z_2 \tag{2.0.3}$$

3 Solution

Let $\mathbf{a} = \begin{pmatrix} 5 \\ -1 \\ -3 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 1 \\ 3 \\ -5 \end{pmatrix}$, and $\mathbf{A}=\mathbf{a}+\mathbf{b}$ and $\mathbf{B}=\mathbf{a}-\mathbf{b}$. To check if the two lines are perpendicular, we perform scalar product of the two direction vectors $\mathbf{A}$ and $\mathbf{B}$ using equation 2.0.2 as follows

$$\mathbf{AB} = \mathbf{A^TB} \tag{3.0.1}$$

$$\mathbf{AB} = (\mathbf{a}+\mathbf{b})^T(\mathbf{a}-\mathbf{b}) \tag{3.0.2}$$

NOTE : The transpose of a sum is the sum of transposes so

$$(\mathbf{a}+\mathbf{b})^T = \left(\mathbf{a}^T + \mathbf{b}^T\right) \tag{3.0.3}$$

$$\mathbf{AB} = \left(\mathbf{a}^T + \mathbf{b}^T\right)(\mathbf{a}-\mathbf{b}) \tag{3.0.4}$$

$$\mathbf{a}^T(\mathbf{a}-\mathbf{b}) + \mathbf{b}^T(\mathbf{a}-\mathbf{b}) \tag{3.0.5}$$

$$\implies \mathbf{a}^T\mathbf{a} - \mathbf{a}^T\mathbf{b} + \mathbf{b}^T\mathbf{a} - \mathbf{b}^T\mathbf{b} \tag{3.0.6}$$

$$\because \mathbf{a}^T\mathbf{a} = ||\mathbf{a}||^2 \tag{3.0.7}$$

$$\because \mathbf{b}^T\mathbf{b} = ||\mathbf{b}||^2 \tag{3.0.8}$$

$$\because \mathbf{a}^T\mathbf{b} = \mathbf{b}^T\mathbf{a} \tag{3.0.9}$$

Using (3.0.7), (3.0.8) and (3.0.9)

$$\mathbf{AB} = ||\mathbf{a}||^2 - \mathbf{a}^T\mathbf{b} + \mathbf{a}^T\mathbf{b} - ||\mathbf{b}||^T \tag{3.0.10}$$

$$||\mathbf{a}||^2 = 5^2 + (-1)^2 + (-3)^2 = 35 \tag{3.0.11}$$

$$||\mathbf{b}||^2 = 1^2 + (3)^2 + (-5)^2 = 35 \tag{3.0.12}$$

$$\tag{3.0.13}$$

$$\mathbf{AB} = ||\mathbf{a}||^2 - ||\mathbf{b}||^2 \tag{3.0.14}$$

Using (3.0.11) and (3.0.12)

$$\implies \mathbf{AB} = 35 - 35 = 0 \tag{3.0.15}$$

Thus the direction vectors of the two lines satisfies the equation 2.0.1, hence proved that the lines are **perpendicular**.

Python Code:

https://github.com/ayushkesh/MatrixTheoryEE5609 /blob/master/A1/codes/A1 code.py

Latex codes:

https://github.com/ayushkesh/Matrix−Theory−EE5609/tree/master/A1/latex/A1.tex