Matrix Theory Assignment 1

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Abstract—This document contains the solution to problem No.66 from Lines and Planes

1 PROBLEM STATEMENT

If $\mathbf{a} = \begin{pmatrix} 5 \\ -1 \\ -3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 3 \\ -5 \end{pmatrix}$, then show that the vectors $\mathbf{a} + \mathbf{b}$ ans $\mathbf{a} - \mathbf{b}$ are perpendicular.

2 SOLUTION

To check if the two lines are perpendicular, we perform scalar product of the two direction vectors

$$\mathbf{A^TB} = 0 \qquad (2.0.1)$$

$$\mathbf{A}^T\mathbf{B} = (\mathbf{a} + \mathbf{b})^T (\mathbf{a} - \mathbf{b}) \qquad (2.0.2)$$

NOTE : The transpose of a sum is the sum of transposes so

$$(\mathbf{a} + \mathbf{b})^T = \left(\mathbf{a}^T + \mathbf{b}^T\right) \qquad (2.0.3)$$

$$\mathbf{A}^T\mathbf{B} = \left(\mathbf{a}^T + \mathbf{b}^T\right)(\mathbf{a} - \mathbf{b}) \qquad (2.0.4)$$

$$\mathbf{a}^T (\mathbf{a} - \mathbf{b}) + \mathbf{b}^T (\mathbf{a} - \mathbf{b}) \qquad (2.0.5)$$

$$\implies \mathbf{a}^T\mathbf{a} - \mathbf{a}^T\mathbf{b} + \mathbf{b}^T\mathbf{a} - \mathbf{b}^T\mathbf{b} \qquad (2.0.6)$$

$$\because \mathbf{a}^T\mathbf{a} = \|\mathbf{a}\|^2 \qquad (2.0.7)$$

$$\because \mathbf{b}^T\mathbf{b} = \|\mathbf{b}\|^2 \qquad (2.0.8)$$

$$\because \mathbf{a}^T\mathbf{b} = \mathbf{b}^T\mathbf{a} \qquad (2.0.9)$$

Using (2.0.7), (2.0.8) and (2.0.9)

$$\mathbf{A}^T\mathbf{B} = \|\mathbf{a}\|^2 - \mathbf{a}^T\mathbf{b} + \mathbf{a}^T\mathbf{b} - \|\mathbf{b}\|^T \qquad (2.0.10)$$

$$\|\mathbf{a}\|^2 = 5^2 + (-1)^2 + (-3)^2 = 35 \qquad (2.0.11)$$

$$\|\mathbf{b}\|^2 = 1^2 + (3)^2 + (-5)^2 = 35 \qquad (2.0.12)$$

$$\qquad (2.0.13)$$

$$\mathbf{A}^T\mathbf{B} = \|\mathbf{a}\|^2 - \|\mathbf{b}\|^2 \qquad (2.0.14)$$

Using (2.0.11) and (2.0.12)

$$\implies \mathbf{A}^T\mathbf{B} = 35 - 35 = 0 \qquad (2.0.15)$$

Thus the direction vectors of the two lines satisfies the equation 2.0.1, hence proved that the lines are **perpendicular**.

Python Code:

https://github.com/ayushkesh/MatrixTheoryEE5609 /blob/master/A1/codes/A1 code.py

Latex codes:

https://github.com/ayushkesh/Matrix−Theory− EE5609/tree/master/A1/latex/A1.tex