

Matrix Theory Assignment 2

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Abstract—This document contains the solution to problem No.3.10.52

Latex codes:

<https://github.com/ayushkesh/Matrix-Theory-EE5609/blob/master/A2/latex/A2.tex>

1 PROBLEM STATEMENT

Examine the consistency of the system of given Equation.

$$x + 2y = 2 \quad (1.0.1)$$

$$2x + 3y = 3 \quad (1.0.2)$$

2 SOLUTION

The given system of equations can be written the matrix equation form as

$$\mathbf{Ax} = \mathbf{b} \quad (2.0.1)$$

$$\Rightarrow \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad (2.0.2)$$

The augmented matrix for (2.0.2) is row reduced as follows

$$\begin{pmatrix} 1 & 2 & 2 \\ 2 & 3 & 3 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 - 2R_1} \begin{pmatrix} 1 & 2 & 2 \\ 0 & -1 & -1 \end{pmatrix} \quad (2.0.3)$$

$$\xrightarrow{R_1 \leftarrow R_1 + 2R_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & -1 \end{pmatrix} \xrightarrow{R_2 \leftarrow (-1)R_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \quad (2.0.4)$$

$$\Rightarrow \text{Rank} \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} = \text{Rank} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} = 2 \quad (2.0.5)$$

Hence, system of equations is consistent and has unique solution $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Note: If the rank of the coefficient matrix and the rank of the augmented matrix, are same then it is **Consistent** and If the rank of the coefficient matrix is less than the rank of the augmented matrix then it is **Inconsistent**.

Python Code:

<https://github.com/ayushkesh/Matrix-Theory-EE5609/blob/master/A2/codes/A2.ipynb>