Assignment-5

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***Abstract*—This document contains solution of Problem Ramsey**(4.1.4)

Download latex-tikz codes from

https://github.com/ayushkesh/Matrix−Theory−EE5609/tree/master/A5

1 Question

Find the equation of the circle that passes through the points $\begin{pmatrix} 2a \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 2b \end{pmatrix}$ and $\begin{pmatrix} a+b \\ a+b \end{pmatrix}$.

2 Solution

The equation of circle can be expressed as

$$\mathbf{x}^T\mathbf{x} - 2\mathbf{c}^T\mathbf{x} + f = 0 \tag{2.0.1}$$

$\mathbf{c}$ is the centre and substituting the points in the equation of circle we get

$$2\begin{pmatrix} 2a & 0 \end{pmatrix}\mathbf{c} - f = 4a^2 \tag{2.0.2}$$

$$2\begin{pmatrix} 0 & 2b \end{pmatrix}\mathbf{c} - f = 4b^2 \tag{2.0.3}$$

$$2\begin{pmatrix} a+b & a+b \end{pmatrix}\mathbf{c} - f = 2(a+b)^2 \tag{2.0.4}$$

which can be expressed in matrix form

$$\begin{pmatrix} 4a & 0 & -1 \\ 0 & 4b & -1 \\ 2(a+b) & 2(a+b) & -1 \end{pmatrix}\begin{pmatrix} \mathbf{c} \\ f \end{pmatrix} = \begin{pmatrix} 4a^2 \\ 4b^2 \\ 2(a+b)^2 \end{pmatrix} \tag{2.0.5}$$

Solve by using Cramer's Rule

$$\Delta = \begin{vmatrix} 4a & 0 & -1 \\ 0 & 4b & -1 \\ 2(a+b) & 2(a+b) & -1 \end{vmatrix} \tag{2.0.6}$$

$$\xleftrightarrow[R_2 \leftarrow R_2 - R_3]{R_1 \leftarrow R_1 - R_2} \begin{vmatrix} 4a & -4b & 0 \\ -2(a+b) & -2(a-b) & 0 \\ 2(a+b) & 2(a+b) & -1 \end{vmatrix} \tag{2.0.7}$$

Expanding along third column we get,

$$-1\left\{(4a)(-2)(a-b) - (-4b)(-2)(a+b)\right\} \tag{2.0.8}$$

$$8\left(a^2 - ab + ab + b^2\right) \implies 8a^2 + 8b^2 \tag{2.0.9}$$

$$\Delta_1 = \begin{vmatrix} 4a^2 & 0 & -1 \\ 4b^2 & 4b & -1 \\ 2(a+b)^2 & 2(a+b) & -1 \end{vmatrix} \tag{2.0.10}$$

$$\xleftrightarrow[R_2 \leftarrow R_2 - R_3]{R_1 \leftarrow R_1 - R_2} \begin{vmatrix} 4a^2 - 4b^2 & -4b & 0 \\ -2a^2 + 2b^2 - 4ab & -2a + 2b & 0 \\ 2(a+b)^2 & 2(a+b) & -1 \end{vmatrix} \tag{2.0.11}$$

Expanding Along the third Column we get,

$$-1\left\{\left(4a^2 - 4b^2\right)(-2a + 2b) - (-4b)\left(-2a^2 + 2b^2 - 4ab\right)\right\} \tag{2.0.12}$$

$$\left(8a^3 - 8a^2b - 8ab^2 + 8b^3\right) + \left(8a^2b - 8b^3 + 16ab^2\right) \tag{2.0.13}$$

$$\implies 8a^3 - 8ab^2 + 16ab^2 = 8a^3 + 8ab^2 \tag{2.0.14}$$

$$\Delta_2 = \begin{vmatrix} 4a & 4a^2 & -1 \\ 0 & 4b^2 & -1 \\ 2a + 2b & 2a^2 + 2b^2 + 4ab & -1 \end{vmatrix} \tag{2.0.15}$$

$$\xleftrightarrow[R_2 \leftarrow R_2 - R_3]{R_1 \leftarrow R_1 - R_2} \begin{vmatrix} 4a & 4a^2 - 4b^2 & 0 \\ -2(a+b) & -2a^2 + 2b^2 - 4ab & 0 \\ 2(a+b) & 2a^2 + 2b^2 + 4ab & -1 \end{vmatrix} \tag{2.0.16}$$

Expanding Along third column we get,

$$-1\left\{(4a)\left(-2a^2 + 2b^2 - 4ab\right) - \left(4a^2 - 4b^2\right)(-2a - 2b)\right\} \tag{2.0.17}$$

$$\left(8a^3 - 8ab^2 + 16a^2b\right) - \left(8a^3 - 8ab^2 + 8a^2b - 8b^3\right) \tag{2.0.18}$$

$$\implies 8b^3 + 8a^2b \tag{2.0.19}$$

$$\Delta_3 = \begin{vmatrix} 4a & 0 & 4a^2 \\ 0 & 4b & 4b^2 \\ 2(a+b) & 2(a+b) & 2(a+b)^2 \end{vmatrix} \quad (2.0.20)$$

$$\xleftrightarrow[R_3 \leftarrow R_3 - \left(\frac{a+b}{2b}\right)R_2]{R_3 \leftarrow R_3 - \left(\frac{a+b}{2a}\right)R_1} \begin{vmatrix} 4a & 0 & 4a^2 \\ 0 & 4b & 4b^2 \\ 0 & 0 & 0 \end{vmatrix} \implies 0 \quad (2.0.21)$$

Using (2.0.9),(2.0.14),(2.0.19) and (2.0.21) we get,

$$\mathbf{c_1} = \frac{\Delta_1}{\Delta} = \frac{8a^3 + 8ab^2}{8a^2 + 8b^2} = a\left(\frac{8a^2 + 8b^2}{8a^2 + 8b^2}\right) = a \quad (2.0.22)$$

$$\mathbf{c_2} = \frac{\Delta_2}{\Delta} = \frac{8b^3 + 8a^2b}{8a^2 + 8b^2} = b\left(\frac{8b^2 + 8a^2}{8b^2 + 8a^2}\right) = b \quad (2.0.23)$$

$$f = \frac{\Delta_3}{\Delta} = \frac{0}{8a^2 + 8b^2} = 0 \quad (2.0.24)$$

$$\implies \mathbf{c} = \begin{pmatrix} a \\ b \end{pmatrix} \quad (2.0.25)$$

$$\implies f = 0 \quad (2.0.26)$$

$$r = \sqrt{\|\mathbf{c}\|^2 - f} = \sqrt{(a^2 + b^2)} \quad (2.0.27)$$

The required equation of circle is

$$\mathbf{x}^T\mathbf{x} - 2\begin{pmatrix} a & b \end{pmatrix}\mathbf{x} = 0 \quad (2.0.28)$$

Python Code to verify your result.

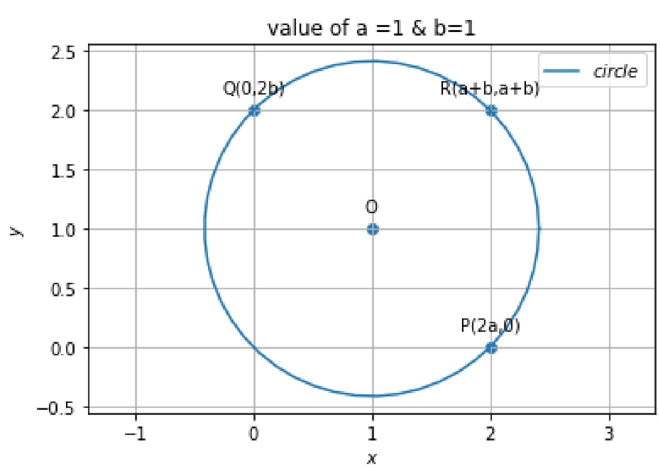

Fig. 0: Circle passing through point P and Q and R

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