

Matrix Theory Assignment 2

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Abstract—This document contains the solution to problem No.3.10.52

1 PROBLEM STATEMENT

Examine the consistency of the system of given Equation.

$$x + 2y = 2 \quad (1.0.1)$$

$$2x + 3y = 3 \quad (1.0.2)$$

2 SOLUTION

The given system of equations can be written in the form of $\mathbf{AX}=\mathbf{B}$ where,

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} \quad (2.0.1)$$

$$X = \begin{pmatrix} x \\ y \end{pmatrix} \quad (2.0.2)$$

$$B = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad (2.0.3)$$

$$\Rightarrow AX = B \quad (2.0.4)$$

$$\Rightarrow A^{-1}AX = A^{-1}B \quad (2.0.5)$$

$$\because A^{-1}A = I \quad (2.0.6)$$

$$\Rightarrow X = A^{-1}B \quad (2.0.7)$$

Therefore if A^{-1} exists, we will have a unique solution for these linear equations. In order to use elementary row operation we may write $\mathbf{A}=\mathbf{IA}$.

$$\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{A} \quad (2.0.8)$$

Applying $R_2 \rightarrow R_2 - 2R_1$

$$\begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \mathbf{A} \quad (2.0.9)$$

Applying $R_1 \rightarrow R_1 + 2R_2$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ -2 & 1 \end{pmatrix} \mathbf{A} \quad (2.0.10)$$

Applying $R_1 \rightarrow (-1) \times R_2$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ 2 & -1 \end{pmatrix} \mathbf{A} \quad (2.0.11)$$

$$\mathbf{A}^{-1} = \begin{pmatrix} -3 & 2 \\ 2 & -1 \end{pmatrix} \quad (2.0.12)$$

A^{-1} exists, we will have a **unique solution** for these linear equation So the System of equation is **Consistent**.

Python Code:

<https://github.com/ayushkesh/Matrix-Theory-EE5609/blob/master/A2/codes/A2.ipynb>

Latex codes:

<https://github.com/ayushkesh/Matrix-Theory-EE5609/blob/master/A2/latex/A2.tex>