Assignment 8

***Abstract*—This document contains QR decomposition of 2**$\times$2*matrix.*

Download latex-tikz codes from

https://github.com/ayushkesh/Matrix−Theory−EE5609/tree/master/A8

Download Python code from

https://github.com/ayushkesh/Matrix−Theory−EE5609/tree/master/A8.pynb

1 Problem

Find the QR Decomposition of matrix,

$$\mathbf{A} = \begin{pmatrix} 4 & -3 \\ 6 & -2 \end{pmatrix} \tag{1.0.1}$$

2 Solution

Let c_1 and c_2 be the column vectors of given matrix $\mathbf{A}$

$$c_1 = \begin{pmatrix} 4 \\ 6 \end{pmatrix} \tag{2.0.1}$$

$$c_2 = \begin{pmatrix} -3 \\ -2 \end{pmatrix} \tag{2.0.2}$$

We can express the matrix $\mathbf{A}$ as,

$$\mathbf{A} = \mathbf{QR} \tag{2.0.3}$$

Where, $\mathbf{Q}$ is an orthogonal matrix given as,

$$\mathbf{Q} = \begin{pmatrix} \mathbf{u_1} & \mathbf{u_2} \end{pmatrix} \tag{2.0.4}$$

and $\mathbf{R}$ is an upper triangular matrix given as,

$$\mathbf{R} = \begin{pmatrix} k_1 & r_1 \\ 0 & k_2 \end{pmatrix} \tag{2.0.5}$$

Now, we can express α and β as,

$$c_1 = k_1\mathbf{u_1} \tag{2.0.6}$$

$$c_2 = r_1\mathbf{u_1} + k_2\mathbf{u_2} \tag{2.0.7}$$

$$\text{where, } k_1 = \|c_1\| = \sqrt{4^2 + (6^2} = \sqrt{52} \tag{2.0.8}$$

Solving equation (2.0.6) for $\mathbf{u_1}$,

$$\mathbf{u_1} = \frac{c_1}{k_1} = \frac{1}{\sqrt{52}}\begin{pmatrix} 4 \\ 6 \end{pmatrix} \tag{2.0.9}$$

$$\text{Now, } r_1 = \frac{\mathbf{u_1}^T c_2}{\|\mathbf{u_1}\|^2} \tag{2.0.10}$$

$$\implies \frac{\frac{1}{\sqrt{52}}\begin{pmatrix} 4 & 6 \end{pmatrix}\begin{pmatrix} -3 \\ -2 \end{pmatrix}}{1} \tag{2.0.11}$$

$$\text{Hence, } r_1 = -\frac{-24}{\sqrt{52}} \tag{2.0.12}$$

$$\mathbf{u_2} = \frac{c_2 - r_1\mathbf{u_1}}{\|c_2 - r_1\mathbf{u_1}\|} \tag{2.0.13}$$

$$\implies \frac{\begin{pmatrix} -3 \\ -2 \end{pmatrix} - \left(\frac{-24}{\sqrt{52}}\right)\left(\frac{1}{\sqrt{52}}\begin{pmatrix} 4 \\ 6 \end{pmatrix}\right)}{\left\|\begin{pmatrix} -3 \\ -2 \end{pmatrix} - \left(-\frac{24}{\sqrt{52}}\frac{1}{\sqrt{52}}\begin{pmatrix} 4 \\ 6 \end{pmatrix}\right)\right\|} \tag{2.0.14}$$

$$\implies \mathbf{u_2} = \frac{1}{\sqrt{335}}\begin{pmatrix} -15 \\ 10 \end{pmatrix} \tag{2.0.15}$$

$$\text{Now, } k_2 = u_2^T c_2 \tag{2.0.16}$$

$$\implies \frac{1}{\sqrt{335}}\begin{pmatrix} -15 & 10 \end{pmatrix}\begin{pmatrix} -3 \\ -2 \end{pmatrix} \tag{2.0.17}$$

$$\implies k_2 = \frac{25}{\sqrt{335}} \tag{2.0.18}$$

Hence substituting the values of unknown parameter from equations (2.0.8), (2.0.18), (2.0.9), (2.0.15) and (2.0.12) to equation (2.0.4) and (2.0.5) we get,

$$\mathbf{Q} = \begin{pmatrix} \frac{4}{\sqrt{52}} & \frac{-15}{\sqrt{335}} \\ \frac{6}{\sqrt{52}} & \frac{10}{\sqrt{335}} \end{pmatrix} \tag{2.0.19}$$

$$\mathbf{R} = \begin{pmatrix} \sqrt{52} & \frac{-24}{\sqrt{52}} \\ 0 & \frac{25}{\sqrt{335}} \end{pmatrix} \tag{2.0.20}$$