Assignment 10

***Abstract*—This document contains Solution of Problem.**

Download latex-tikz codes from

https://github.com/ayushkesh/Matrix−Theory−EE5609/tree/master/A10

1 Problem

Find all solutions to the system of equations

$$\begin{aligned}(1-i)\,x_1 - ix_2 &= 0\\ 2x_1 + (1-i)\,x_2 &= 0\end{aligned} \tag{1.0.1}$$

2 Solution

System of Linear Equations (1.0.1) can be expressed in matrix form as,

$$\mathbf{A}\mathbf{x} = 0 \tag{2.0.1}$$

$$\begin{pmatrix}1-i & -i\\ 2 & 1-i\end{pmatrix}\mathbf{x} = 0 \tag{2.0.2}$$

By row reduction ,

$$\begin{pmatrix}1-i & -i\\ 2 & 1-i\end{pmatrix} \xleftrightarrow[R_1\leftarrow R_1/2]{R_1\leftrightarrow R_2} \begin{pmatrix}1 & \frac{1-i}{2}\\ 1-i & -i\end{pmatrix} \tag{2.0.3}$$

$$\xleftrightarrow{R_2\leftarrow R_2-(1-i)R_1} \begin{pmatrix}1 & \frac{1-i}{2}\\ 0 & 0\end{pmatrix} \tag{2.0.4}$$

$$\begin{pmatrix}1 & \frac{1-i}{2}\end{pmatrix}\mathbf{x} = 0 \tag{2.0.5}$$

$$\begin{pmatrix}1 & \frac{1-i}{2}\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} = 0 \tag{2.0.6}$$

$$x_1 = -\frac{1-i}{2}x_2 \tag{2.0.7}$$

$$\implies \mathbf{x} = \begin{pmatrix}-\frac{1-i}{2}x_2\\ x_2\end{pmatrix} \tag{2.0.8}$$

Note: This is nothing but **Rouché–Capelli theorem**, If $\text{rank}(\mathbf{A}) = 1$ and is less than the number of variables. The system is **consistent** and there is an **infinite** number of solutions.