Assignment-5

Ayush Kumar

***Abstract*—This document contains solution of Problem Ramsey**(4.1.4)

Download latex-tikz codes from

https://github.com/ayushkesh/Matrix−Theory−EE5609/tree/master/A5

1 Question

Find the equation of the circle that passes through the points $\begin{pmatrix}2a\\0\end{pmatrix}$, $\begin{pmatrix}0\\2b\end{pmatrix}$ and $\begin{pmatrix}a+b\\a+b\end{pmatrix}$.

2 Solution

The equation of circle can be expressed as

$$\mathbf{x}^T\mathbf{x} - 2\mathbf{c}^T\mathbf{x} + f = 0 \tag{2.0.1}$$

$\mathbf{c}$ is the centre and substituting the points in the equation of circle we get

$$2\begin{pmatrix}2a & 0\end{pmatrix}\mathbf{c} - f = 4a^2 \tag{2.0.2}$$

$$2\begin{pmatrix}0 & 2b\end{pmatrix}\mathbf{c} - f = 4b^2 \tag{2.0.3}$$

$$2\begin{pmatrix}a+b & a+b\end{pmatrix}\mathbf{c} - f = 2(a+b)^2 \tag{2.0.4}$$

which can be expressed in matrix form

$$\begin{pmatrix}4a & 0 & -1\\0 & 4b & -1\\2(a+b) & 2(a+b) & -1\end{pmatrix}\begin{pmatrix}\mathbf{c}\\f\end{pmatrix} = \begin{pmatrix}4a^2\\4b^2\\2(a+b)^2\end{pmatrix} \tag{2.0.5}$$

Solve by using Cramer's Rule

$$\Delta = \begin{vmatrix}4a & 0 & -1\\0 & 4b & -1\\2(a+b) & 2(a+b) & -1\end{vmatrix} \tag{2.0.6}$$

$$\xleftrightarrow[R_3\leftarrow R_3-\left(\frac{a+b}{2b}\right)R_2]{R_3\leftarrow R_3-\left(\frac{a+b}{2a}\right)R_1} \begin{vmatrix}4a & 0 & -1\\0 & 4b & -1\\0 & 0 & \frac{a^2+b^2}{2ab}\end{vmatrix} \tag{2.0.7}$$

$$\implies 16ab\left(\frac{a^2+b^2}{2ab}\right) = 8a^2+8b^2 \tag{2.0.8}$$

$$\Delta_1 = \begin{vmatrix}4a^2 & 0 & -1\\4b^2 & 4b & -1\\2(a+b)^2 & 2(a+b) & -1\end{vmatrix} \tag{2.0.9}$$

$$\xleftrightarrow[R_3\leftarrow R_3-\left(\frac{(a+b)^2}{2a^2}\right)R_1]{R_2\leftarrow R_2-\left(\frac{b^2}{a^2}\right)R_1} \begin{vmatrix}4a^2 & 0 & -1\\0 & 4b & \frac{-a^2+b^2}{a^2}\\0 & 2a+2b & \frac{-a^2+b^2+2ab}{2a^2}\end{vmatrix} \tag{2.0.10}$$

$$\xleftrightarrow{R_3\leftarrow R_3-\left(\frac{a+b}{2b}\right)R_2} \begin{vmatrix}4a^2 & 0 & -1\\0 & 4b & \frac{-a^2+b^2}{a^2}\\0 & 0 & \frac{a^2+b^2}{2ab}\end{vmatrix} \tag{2.0.11}$$

$$\implies 4a^2(4b)\left(\frac{a^2+b^2}{2ab}\right) = 8a^3+8ab^2 \tag{2.0.12}$$

$$\Delta_2 = \begin{vmatrix}4a & 4a^2 & -1\\0 & 4b^2 & -1\\2a+2b & 2a^2+2b^2+4ab & -1\end{vmatrix} \tag{2.0.13}$$

$$\xleftrightarrow[R_3\leftarrow R_3-\left(\frac{a+b}{2b}\right)R_2]{R_3\leftarrow R_3-\left(\frac{a+b}{2a}\right)R_1} \begin{vmatrix}4a & 4a^2 & -1\\0 & 4b^2 & -1\\0 & 0 & \frac{a^2+b^2}{2ab}\end{vmatrix} \tag{2.0.14}$$

$$\implies 4a\left(4b^2\right)\left(\frac{a^2+b^2}{2ab}\right) = 8b^3+8a^2b \tag{2.0.15}$$

$$\Delta_3 = \begin{vmatrix}4a & 0 & 4a^2\\0 & 4b & 4b^2\\2(a+b) & 2(a+b) & 2(a+b)^2\end{vmatrix} \tag{2.0.16}$$

$$\xleftrightarrow[R_3\leftarrow R_3-\left(\frac{a+b}{2b}\right)R_2]{R_3\leftarrow R_3-\left(\frac{a+b}{2a}\right)R_1} \begin{vmatrix}4a & 0 & 4a^2\\0 & 4b & 4b^2\\0 & 0 & 0\end{vmatrix} \implies 0 \tag{2.0.17}$$

Using (2.0.8),(2.0.12),(2.0.15) and (2.0.17) we get,

$$\mathbf{c_1} = \frac{\Delta_1}{\Delta} = \frac{8a^3 + 8ab^2}{8a^2 + 8b^2} = a\left(\frac{8a^2 + 8b^2}{8a^2 + 8b^2}\right) = a \tag{2.0.18}$$

$$\mathbf{c_2} = \frac{\Delta_2}{\Delta} = \frac{8b^3 + 8a^2 b}{8a^2 + 8b^2} = b\left(\frac{8b^2 + 8a^2}{8b^2 + 8a^2}\right) = b \tag{2.0.19}$$

$$f = \frac{\Delta_3}{\Delta} = \frac{0}{8a^2 + 8b^2} = 0 \tag{2.0.20}$$

$$\implies \mathbf{c} = \begin{pmatrix} a \\ b \end{pmatrix} \tag{2.0.21}$$

$$\implies f = 0 \tag{2.0.22}$$

$$r = \sqrt{\|\mathbf{c}\|^2 - f} = \sqrt{(a^2 + b^2)} \tag{2.0.23}$$

The required equation of circle is

$$\mathbf{x}^T \mathbf{x} - 2\begin{pmatrix} a & b \end{pmatrix} \mathbf{x} = 0 \tag{2.0.24}$$

Python Code to verify your result.

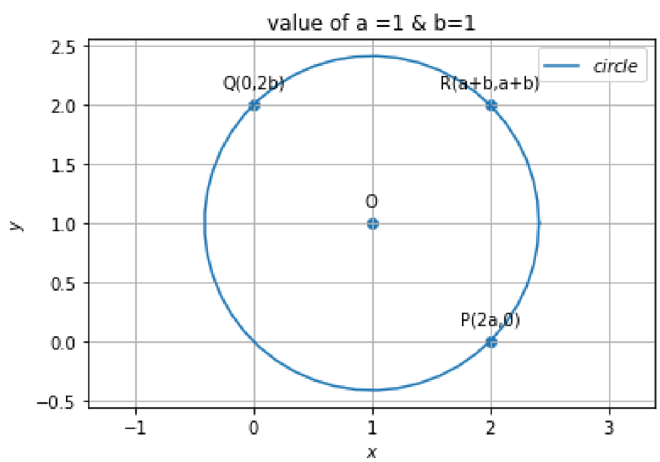

Fig. 0: Circle passing through point P and Q and R

https://github.com/ayushkesh/Matrix−Theory−EE5609/tree/master/A5.py