Assignment-5

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***Abstract*—This document contains solution of Problem Ramsey**(4.1.4)

Download latex-tikz codes from

https://github.com/ayushkesh/Matrix−Theory−EE5609/tree/master/A5

1 Question

Find the equation of the circle that passes through the points $\begin{pmatrix} 2a \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 2b \end{pmatrix}$ and $\begin{pmatrix} a+b \\ a+b \end{pmatrix}$.

2 Solution

The equation of circle can be expressed as

$$\mathbf{x}^T\mathbf{x} - 2\mathbf{c}^T\mathbf{x} + f = 0 \tag{2.0.1}$$

$\mathbf{c}$ is the centre and substituting the points in the equation of circle we get

$$2\begin{pmatrix} 2a & 0 \end{pmatrix}\mathbf{c} - f = 4a^2 \tag{2.0.2}$$

$$2\begin{pmatrix} 0 & 2b \end{pmatrix}\mathbf{c} - f = 4b^2 \tag{2.0.3}$$

$$2\begin{pmatrix} a+b & a+b \end{pmatrix}\mathbf{c} - f = 2(a+b)^2 \tag{2.0.4}$$

which can be expressed in matrix form

$$\begin{pmatrix} 4a & 0 & -1 \\ 0 & 4b & -1 \\ 2(a+b) & 2(a+b) & -1 \end{pmatrix}\begin{pmatrix} \mathbf{c} \\ f \end{pmatrix} = \begin{pmatrix} 4a^2 \\ 4b^2 \\ 2(a+b)^2 \end{pmatrix} \tag{2.0.5}$$

Row reducing the augmented matrix

$$\begin{pmatrix} 4a & 0 & -1 & 4a^2 \\ 0 & 4b & -1 & 4b^2 \\ 2(a+b) & 2(a+b) & -1 & 2(a+b)^2 \end{pmatrix} \tag{2.0.6}$$

$$\xleftrightarrow[R_3 \leftarrow R_3 - 2(a+b)R_1]{R_1 \leftarrow \frac{R_1}{4a}} \begin{pmatrix} 1 & 0 & -\frac{1}{4a} & a \\ 0 & 4b & -1 & 4b^2 \\ 0 & 2(a+b) & \frac{-a+b}{2a} & 2b(a+b) \end{pmatrix} \tag{2.0.7}$$

$$\xleftrightarrow[R_2 \leftarrow \frac{R_2}{4b}]{R_3 \leftarrow R_3 - 2(a+b)R_2} \begin{pmatrix} 1 & 0 & -\frac{1}{4a} & a \\ 0 & 1 & -\frac{1}{4b} & b \\ 0 & 0 & \frac{a}{2b} + \frac{b}{2a} & 0 \end{pmatrix} \tag{2.0.8}$$

$$\xleftrightarrow{R_3 \leftarrow \frac{R_3}{\frac{a}{2b} + \frac{b}{2a}}} \begin{pmatrix} 1 & 0 & -\frac{1}{4a} & a \\ 0 & 1 & -\frac{1}{4b} & b \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{2.0.9}$$

$$\xleftrightarrow[R_1 \leftarrow R_1 - (-\frac{1}{4a})R_3]{R_2 \leftarrow R_2 - (-\frac{1}{4b})R_3} \begin{pmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{2.0.10}$$

$$\mathbf{c} = \begin{pmatrix} a \\ b \end{pmatrix} \tag{2.0.11}$$

$$f = 0 \tag{2.0.12}$$

$$r = \sqrt{\|\mathbf{c}\|^2 - f} = \sqrt{(a^2 + b^2)} \tag{2.0.13}$$

The required equation of circle is

$$\mathbf{x}^T\mathbf{x} - 2\begin{pmatrix} a & b \end{pmatrix}\mathbf{x} = 0 \tag{2.0.14}$$

Python Code to verify your result, Assuming value of a=1 and b=1,

https://github.com/ayushkesh/Matrix−Theory−EE5609/tree/master/A5.py

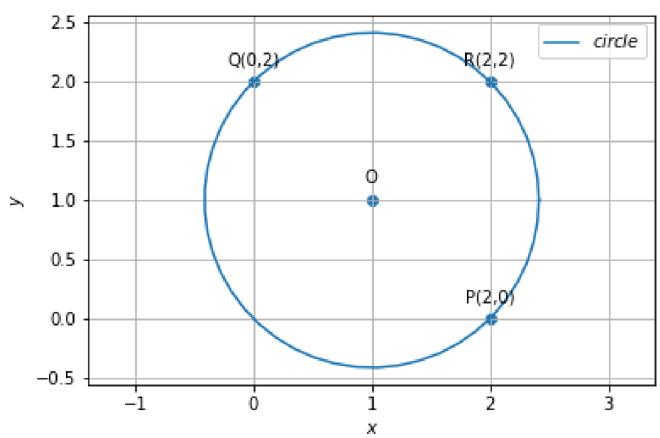

Fig. 0: Circle passing through point P and Q and R