

Assignment-5

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Abstract—This document contains solution of Problem Geolin(1.12)

Download latex-tikz codes from

<https://github.com/ayushkesh/Matrix-Theory-EE5609/tree/master/A5>

1 QUESTION

Find the equation of the circle that passes through the points $\begin{pmatrix} 2a \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 2b \end{pmatrix}$ and $\begin{pmatrix} a+b \\ a+b \end{pmatrix}$.

2 SOLUTION

The equation of circle can be expressed as

$$\mathbf{x}^T \mathbf{x} - 2\mathbf{c}^T \mathbf{x} + f = 0 \quad (2.0.1)$$

\mathbf{c} is the centre and substituting the points in the equation of circle we get

$$2 \begin{pmatrix} 2a & 0 \end{pmatrix} \mathbf{c} - f = 4a^2 \quad (2.0.2)$$

$$2 \begin{pmatrix} 0 & 2b \end{pmatrix} \mathbf{c} - f = 4b^2 \quad (2.0.3)$$

$$2 \begin{pmatrix} a+b & a+b \end{pmatrix} \mathbf{c} - f = 2(a+b)^2 \quad (2.0.4)$$

which can be expressed in matrix form

$$\begin{pmatrix} 4a & 0 & -1 \\ 0 & 4b & -1 \\ 2(a+b) & 2(a+b) & -1 \end{pmatrix} \begin{pmatrix} \mathbf{c} \\ f \end{pmatrix} = \begin{pmatrix} 4a^2 \\ 4b^2 \\ 2(a+b)^2 \end{pmatrix} \quad (2.0.5)$$

Row reducing the augmented matrix

$$\begin{pmatrix} 4a & 0 & -1 & 4a^2 \\ 0 & 4b & -1 & 4b^2 \\ 2(a+b) & 2(a+b) & -1 & 2(a+b)^2 \end{pmatrix} \quad (2.0.6)$$

$$\begin{matrix} R_1 \leftarrow \frac{R_1}{4a} \\ R_3 \leftarrow R_3 - 2(a+b)R_1 \end{matrix} \rightarrow \begin{pmatrix} 1 & 0 & -\frac{1}{4a} & a \\ 0 & 4b & -1 & 4b^2 \\ 0 & 2(a+b) & \frac{-a+b}{2a} & 2b(a+b) \end{pmatrix} \quad (2.0.7)$$

$$\begin{matrix} R_3 \leftarrow R_3 - 2(a+b)R_2 \\ R_2 \leftarrow \frac{R_2}{4b} \end{matrix} \rightarrow \begin{pmatrix} 1 & 0 & -\frac{1}{4a} & a \\ 0 & 1 & -\frac{1}{4b} & b \\ 0 & 0 & \frac{a}{2b} + \frac{b}{2a} & 0 \end{pmatrix} \quad (2.0.8)$$

$$\begin{matrix} R_3 \leftarrow \frac{R_3}{\frac{a}{2b} + \frac{b}{2a}} \end{matrix} \rightarrow \begin{pmatrix} 1 & 0 & -\frac{1}{4a} & a \\ 0 & 1 & -\frac{1}{4b} & b \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad (2.0.9)$$

$$\begin{matrix} R_2 \leftarrow R_2 - (-\frac{1}{4b})R_3 \\ R_1 \leftarrow R_1 - (-\frac{1}{4a})R_3 \end{matrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad (2.0.10)$$

$$\mathbf{c} = \begin{pmatrix} a \\ b \end{pmatrix} \quad (2.0.11)$$

$$f = 0 \quad (2.0.12)$$

$$r = \sqrt{\|\mathbf{c}\|^2 - f} = \sqrt{a^2 + b^2} \quad (2.0.13)$$

The required equation of circle is

$$\mathbf{x}^T \mathbf{x} - 2 \begin{pmatrix} a & b \end{pmatrix} \mathbf{x} = 0 \quad (2.0.14)$$

Python Code to verify your result, Assuming value of a=1 and b=1,

<https://github.com/ayushkesh/Matrix-Theory-EE5609/tree/master/A5.py>

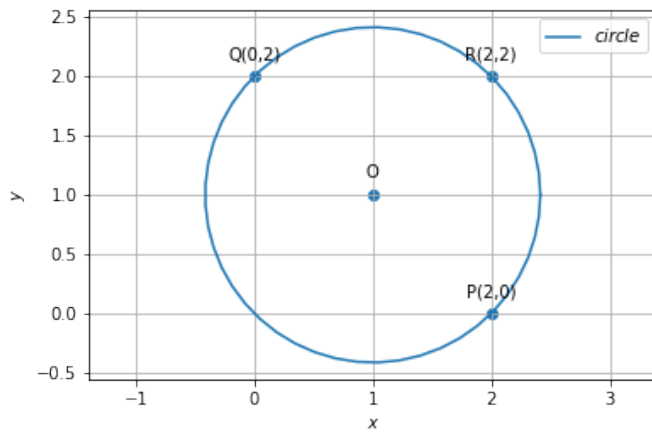


Fig. 0: Circle passing through point P and Q and R