Matrix Theory Assignment 1

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Abstract—**This document contains the solution to problem No.66 from Lines and Planes**

1 Problem Statement

If $\mathbf{a} = \begin{pmatrix} 5 \\ -1 \\ -3 \end{pmatrix}$ and $\mathbf{b}= \begin{pmatrix} 1 \\ 3 \\ -5 \end{pmatrix}$, then show that the vectors $\mathbf{a}+\mathbf{b}$ ans $\mathbf{a}-\mathbf{b}$ are perpendicular.

2 Theory

For two lines having direction vectors **A** and **B** respectively, they will be perpendicular if the scalar product of the two direction vector is 0,

$$\mathbf{AB} = 0 \tag{2.0.1}$$

Where scalar product of two vectors, $\mathbf{A}= \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ and $\mathbf{B}= \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$ is defined by

$$\mathbf{AB} = \mathbf{A^T B} = \begin{pmatrix} x_1 & y_1 & z_1 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = x_1x_2 + y_1y_2 + z_1z_2 \tag{2.0.2}$$

3 Solution

Let $\mathbf{a}=\begin{pmatrix} 5 \\ -1 \\ -3 \end{pmatrix}$, $\mathbf{b}=\begin{pmatrix} 1 \\ 3 \\ -5 \end{pmatrix}$, and $\mathbf{A}=\mathbf{a}+\mathbf{b}$ and $\mathbf{B}=\mathbf{a}-\mathbf{b}$.

$$\mathbf{A} = \mathbf{a} + \mathbf{b} \tag{3.0.1}$$

$$\implies \mathbf{A} = \begin{pmatrix} 5 \\ -1 \\ -3 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \\ -5 \end{pmatrix} \tag{3.0.2}$$

$$\implies \mathbf{A} = \begin{pmatrix} 6 \\ 2 \\ -8 \end{pmatrix} \tag{3.0.3}$$

$$\mathbf{B} = \mathbf{a} - \mathbf{b} \tag{3.0.4}$$

$$\implies \mathbf{B} = \begin{pmatrix} 5 \\ -1 \\ -3 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ -5 \end{pmatrix} \tag{3.0.5}$$

$$\implies \mathbf{B} = \begin{pmatrix} 4 \\ -4 \\ 2 \end{pmatrix} \tag{3.0.6}$$

To check if the two lines are perpendicular, we perform scalar product of the two direction vectors **A** and **B** using equation 2.0.2 as follows

$$\mathbf{AB} = \mathbf{A^T B} \tag{3.0.7}$$

$$\mathbf{AB} = (\mathbf{a}+\mathbf{b})^T (\mathbf{a}-\mathbf{b}) \tag{3.0.8}$$

$$\mathbf{AB} = \left(\begin{pmatrix} 5 \\ -1 \\ -3 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \\ -5 \end{pmatrix} \right)^T \left(\begin{pmatrix} 5 \\ -1 \\ -3 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \\ -5 \end{pmatrix} \right) \tag{3.0.9}$$

$$\mathbf{AB} = \begin{pmatrix} 6 \\ 2 \\ -8 \end{pmatrix}^T \begin{pmatrix} 4 \\ -4 \\ -2 \end{pmatrix} \tag{3.0.10}$$

$$\implies \mathbf{AB} = \begin{pmatrix} 6 & 2 & -8 \end{pmatrix} \begin{pmatrix} 4 \\ -4 \\ -2 \end{pmatrix} \tag{3.0.11}$$

$$\implies \mathbf{AB} = 24 - 8 - 16 \tag{3.0.12}$$

$$\implies \mathbf{AB} = 0 \tag{3.0.13}$$

Thus the direction vectors of the two lines satisfies the equation 2.0.1, hence proved that the lines are **perpendicular**.

Python Code:

https://github.com/ayushkesh/MatrixTheoryEE5609
/blob/master/A1/codes/A1 code.py

Latex codes:

https://github.com/ayushkesh/Matrix−Theory−
EE5609/tree/master/A1/latex/A1.tex