Matrix Theory Assignment 2

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Abstract—**This document contains the solution to problem No.3.10.52**

1 Problem Statement

Examine the consistency of the system of given Equation.

$$x + 2y = 2 \tag{1.0.1}$$
$$2x + 3y = 3 \tag{1.0.2}$$

2 Solution

The given system of equations can be written in the form of **AX=B** where,

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} \tag{2.0.1}$$
$$X = \begin{pmatrix} x \\ y \end{pmatrix} \tag{2.0.2}$$
$$B = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{2.0.3}$$
$$\implies AX = B \tag{2.0.4}$$
$$\implies A^{-1}AX = A^{-1}B \tag{2.0.5}$$
$$\because A^{-1}A = I \tag{2.0.6}$$
$$\implies X = A^{-1}B \tag{2.0.7}$$

Therefore if A^{-1} exists, we will have a unique solution for these linear equations. In order to use elementary row operation we may write **A= IA.**

$$\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{A} \tag{2.0.8}$$

Applying R2↔R2-2R1

$$\begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \mathbf{A} \tag{2.0.9}$$

Applying R1↔R1+2R2

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ -2 & 1 \end{pmatrix} \mathbf{A} \tag{2.0.10}$$

Applying R1↔ (-1)×$R2$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ 2 & -1 \end{pmatrix} \mathbf{A} \tag{2.0.11}$$
$$\mathbf{A}^{-1} = \begin{pmatrix} -3 & 2 \\ 2 & -1 \end{pmatrix} \tag{2.0.12}$$

A^{-1} exists, we will have a **unique solution** for these linear equation So the System of equation is **Consistent**.

Python Code:

```
https://github.com/ayushkesh/Matrix−Theory−
    EE5609/blob/master/A2/codes/A2.ipynb
```

Latex codes:

```
https://github.com/ayushkesh/Matrix−Theory−
    EE5609/blob/master/A2/latex/A2.tex
```