Assignment 4

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Download latex-tikz codes from

https://github.com/ayushkesh/Matrix−Theory−EE5609/tree/master/A4

1 Question

Show that $\sin 30° = \frac{1}{2}$ and $\cos 30° = \frac{\sqrt{3}}{2}$.

2 Solution

Consider an equilateral $\triangle ABC$ as shown in figure: 0.1.

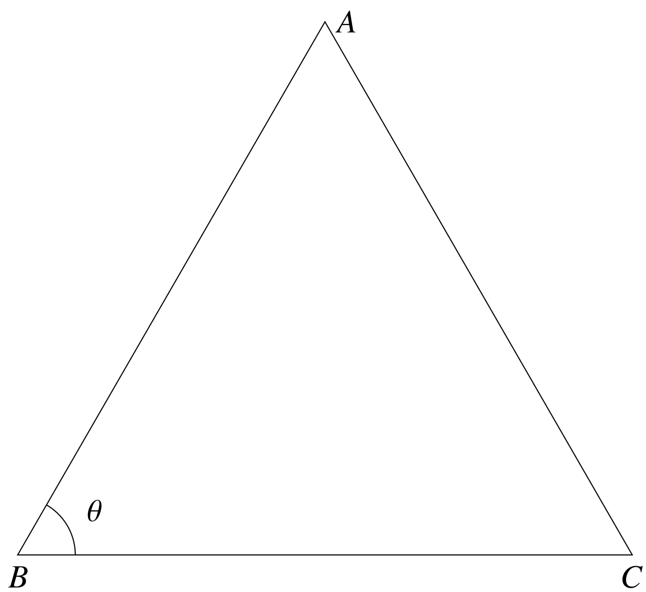

Fig. 0.1

$$\|\mathbf{A}-\mathbf{B}\| = \|\mathbf{B}-\mathbf{C}\| = \|\mathbf{A}-\mathbf{C}\| \tag{2.0.1}$$

Let $\mathbf{B} = 0$. Then substituting in (2.0.1)

$$\|\mathbf{A}\| = \|\mathbf{C}\| \tag{2.0.2}$$
$$\|\mathbf{A}\| = \|\mathbf{A}-\mathbf{C}\| \tag{2.0.3}$$

Square on both sides in (2.0.3).

$$\|\mathbf{A}\|^2 = \|\mathbf{A}-\mathbf{C}\|^2 \tag{2.0.4}$$
$$\|\mathbf{A}\|^2 = \|\mathbf{A}\|^2 + \|\mathbf{C}\|^2 - 2\mathbf{A}^T\mathbf{C} \tag{2.0.5}$$
$$\|\mathbf{A}\|^2 = \|\mathbf{A}\|^2 + \|\mathbf{A}\|^2 - 2\mathbf{A}^T\mathbf{C} \tag{2.0.6}$$
$$\implies 0 = \|\mathbf{A}\|^2 - 2\mathbf{A}^T\mathbf{C} \tag{2.0.7}$$
$$\implies 2\mathbf{A}^T\mathbf{C} = \|\mathbf{A}\|^2 \tag{2.0.8}$$
$$\implies \mathbf{A}^T\mathbf{C} = \frac{\|\mathbf{A}\|^2}{2} \tag{2.0.9}$$

let $2\theta = \angle ABC$. Taking the inner product of sides AB and BC.

$$(\mathbf{A}-\mathbf{B})^T(\mathbf{B}-\mathbf{C}) = \|\mathbf{A}-\mathbf{B}\|\,\|\mathbf{B}-\mathbf{C}\|\cos 2\theta \tag{2.0.10}$$
$$\implies \cos 2\theta = \frac{(\mathbf{A}-\mathbf{B})^T(\mathbf{B}-\mathbf{C})}{\|\mathbf{A}-\mathbf{B}\|\,\|\mathbf{B}-\mathbf{C}\|} \tag{2.0.11}$$

Substitute $\mathbf{B} = 0$ in (2.0.11)

$$\implies \cos 2\theta = \frac{\mathbf{A}^T\mathbf{C}}{\|\mathbf{A}\|\,\|\mathbf{C}\|} \tag{2.0.12}$$

Substitute (2.0.2),(2.0.9) in (2.0.12)

$$\implies \cos 2\theta = \frac{\frac{\|\mathbf{A}\|^2}{2}}{\|\mathbf{A}\|^2} \tag{2.0.13}$$
$$\implies \cos 2\theta = \frac{1}{2} \tag{2.0.14}$$

In equilateral triangle, $\angle ABC = 60°$

$$\implies \cos 2\theta = \cos 60° = \frac{1}{2} \tag{2.0.15}$$
$$\because 2\theta = 60° \implies \theta = 30° \tag{2.0.16}$$
$$\because \cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}}. \tag{2.0.17}$$

Substitute 2.0.15 and 2.0.16 in 2.0.17

$$\cos 30^\circ = \sqrt{\frac{1+\frac{1}{2}}{2}} \tag{2.0.18}$$

$$\implies \cos 30^\circ = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}. \tag{2.0.19}$$

$$\because \cos^2\theta + \sin^2\theta = 1 \tag{2.0.20}$$

$$\implies \sin 30^\circ = \sqrt{1-\cos^2 30^\circ} \tag{2.0.21}$$

$$\implies \sin 30^\circ = \frac{1}{2}. \tag{2.0.22}$$