Assignment 13

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Download latex-tikz codes from

https://github.com/ayushkesh

1 Problem

Let R[x] denote the vector space of all real polynomial. Let $\mathbf{D}$:$\mathbf{R}[\mathbf{x}] \to \mathbf{R}[\mathbf{x}]$ denote the map $\mathbf{D}f = \frac{df}{dx}, \forall f$ then,

1) $\mathbf{D}$ is one-one.
2) $\mathbf{D}$ is onto.
3) There exist $E : \mathbf{R}[\mathbf{x}] \to \mathbf{R}[\mathbf{x}]$ so that $D(E(f)) = f, \forall f$.
4) There exist $E : \mathbf{R}[\mathbf{x}] \to \mathbf{R}[\mathbf{x}]$ so that $E(D(f)) = f, \forall f$.

2 Explanation

See Table 4

Given	Let $\mathbf{D}$:$\mathbf{R}[\mathbf{x}] \to \mathbf{R}[\mathbf{x}]$ denote the map $\mathbf{D}f = \frac{df}{dx}, \forall f$
Statement 1	D is one-one.
	$$\mathbf{D} \text{ is not one-one because} \quad (2.0.1)$$ $$f_1 \neq f_2 \quad (2.0.2)$$ $$\implies Df_1 = Df_2. \quad (2.0.3)$$ $$\text{Take } f_1\text{=x then } f_2\text{= x+1} \quad (2.0.4)$$ **False Statement**
Statement 2	D is onto.
	$$\text{D is not onto because} \quad (2.0.5)$$ $$f(x) = \begin{cases} 1 & x \in \mathbb{Q} \\ 0 & x \in \mathbb{Q}' \end{cases} \quad (2.0.6)$$ **False Statement**
Statement 3	There exist E :R[x] $\to \mathbf{R}[\mathbf{x}]$ so that $D(E(f)) = f, \forall f$.
	$$\text{Not True because,} \quad (2.0.7)$$ $$\text{Every integrable operator is not differentiable.} \quad (2.0.8)$$ **False Statement**
Statement 4	There exist $E : \mathbf{R}[\mathbf{x}] \to \mathbf{R}[\mathbf{x}]$ so that $E(D(f)) = f, \forall f$.
	$$\exists \text{ an integrable operator such that } E(D(f)) = f, \forall f \quad (2.0.9)$$ **True Statement**

TABLE 4: Explanation