Matrix Theory Assignment 1

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Abstract—**This document contains the solution to problem No.66 from Lines and Planes**

1 Problem Statement

If $\mathbf{a} = \begin{pmatrix} 5 \\ -1 \\ -3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 3 \\ -5 \end{pmatrix}$, then show that the vectors $\mathbf{a}+\mathbf{b}$ ans $\mathbf{a}-\mathbf{b}$ are perpendicular.

2 Solution

To check if the two lines are perpendicular, we perform scalar product of the two direction vectors

$$\mathbf{A}^{\mathbf{T}}\mathbf{B} = 0 \tag{2.0.1}$$

$$\mathbf{A}^T\mathbf{B} = (\mathbf{a}+\mathbf{b})^T(\mathbf{a}-\mathbf{b}) \tag{2.0.2}$$

The transpose of a sum is the sum of transposes so,

$$(\mathbf{a}+\mathbf{b})^T = \left(\mathbf{a}^T + \mathbf{b}^T\right) \tag{2.0.3}$$

$$\mathbf{A}^T\mathbf{B} = \left(\mathbf{a}^T + \mathbf{b}^T\right)(\mathbf{a}-\mathbf{b}) \tag{2.0.4}$$

$$\mathbf{a}^T(\mathbf{a}-\mathbf{b}) + \mathbf{b}^T(\mathbf{a}-\mathbf{b}) \tag{2.0.5}$$

$$\implies \mathbf{a}^T\mathbf{a} - \mathbf{a}^T\mathbf{b} + \mathbf{b}^T\mathbf{a} - \mathbf{b}^T\mathbf{b} \tag{2.0.6}$$

$$\because \mathbf{a}^T\mathbf{a} = ||\mathbf{a}||^2 \tag{2.0.7}$$

$$\because \mathbf{b}^T\mathbf{b} = ||\mathbf{b}||^2 \tag{2.0.8}$$

$$\because \mathbf{a}^T\mathbf{b} = \mathbf{b}^T\mathbf{a} \tag{2.0.9}$$

Using (2.0.7), (2.0.8) and (2.0.9)

$$\mathbf{A}^T\mathbf{B} = ||\mathbf{a}||^2 - \mathbf{a}^T\mathbf{b} + \mathbf{a}^T\mathbf{b} - ||\mathbf{b}||^T \tag{2.0.10}$$

$$||\mathbf{a}||^2 = 5^2 + (-1)^2 + (-3)^2 = 35 \tag{2.0.11}$$

$$||\mathbf{b}||^2 = 1^2 + (3)^2 + (-5)^2 = 35 \tag{2.0.12}$$

$$\mathbf{A}^T\mathbf{B} = ||\mathbf{a}||^2 - ||\mathbf{b}||^2 \tag{2.0.13}$$

Using (2.0.11) and (2.0.12)

$$\implies \mathbf{A}^T\mathbf{B} = 35 - 35 = 0 \tag{2.0.14}$$

Thus the direction vectors of the two lines satisfies the equation 2.0.1, hence proved that the lines are **perpendicular**.

Python Code:

https://github.com/ayushkesh/MatrixTheoryEE5609/blob/master/A1/codes/A1 code.py

Latex codes:

https://github.com/ayushkesh/Matrix−Theory−EE5609/tree/master/A1/latex/A1.tex