Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

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8.1. Plot the Bode magnitude and phase plots for the following system

$$G(s) = \frac{Ks^2}{(1+0.2s)(1+0.02s)} \tag{8.1.1}$$

Also compute gain margin and phase margin .
Solution: Substituting $s = j\omega$ in (8.1.1)
Initially assuming K =1

$$G(j\omega) = \frac{(j\omega)^2}{(1+0.2j\omega)(1+0.02j\omega)} \tag{8.1.2}$$

The corner frequencies are

$$\omega c1 = 1/0.2 = 5rad/sec \tag{8.1.3}$$

$$\omega c2 = 1/0.02 = 50rad/sec \tag{8.1.4}$$

8.2. Magnitude Plot Calculation.
Solution:

$$20logG(j\omega) = 20log(j\omega)^2 - 20log(1+0.2j\omega) - 20log(1+0.02j\omega) \tag{8.2.1}$$

The various values of G($j\omega$) are below TABLE 8.2, in the increasing order of their corner frequencies also slope contributed by each term and the change in slope at the corner frequency.

TERM	**Corner Freq**	**Slope**	**Slope change**
$(j\omega)^2$	$--$	+40	$--$
$\frac{1}{1+j0.2}$	$\omega c1 = \frac{1}{0.2}$	-20	40-20=20
$\frac{1}{1+j0.02}$	$\omega c2 = \frac{1}{0.02}$	-20	20-20=0

TABLE 8.2: Magnitude

8.3. Phase Angle Calculation
Solution:

$$\phi = \angle Gj\omega = 180^\circ - tan^{-1}(0.2\omega) - tan^{-1}(0.02\omega) \tag{8.3.1}$$

The phase angle of G($j\omega$) are calculated for various value of ω TABLE 8.3.

ω	$\mathbf{tan}^{-1}(0.2\omega)$	$\mathbf{tan}^{-1}(0.02\omega)$	$\phi = \angle G(j\omega)$
0.5	5.7	0.6	174
1	11.3	1.1	168
2	21.8	2.3	156
5	45	5.7	130
10	63.4	11.3	106
50	84.3	45	50

TABLE 8.3: Phase

The magnitude and phase plot are as follows: Fig8.3
The python code to obtain the graphs:

```
codes/es17btech11002.py
```

8.4. Calculation for K
Solution: The gain crossover frequency is 2rad/sec,

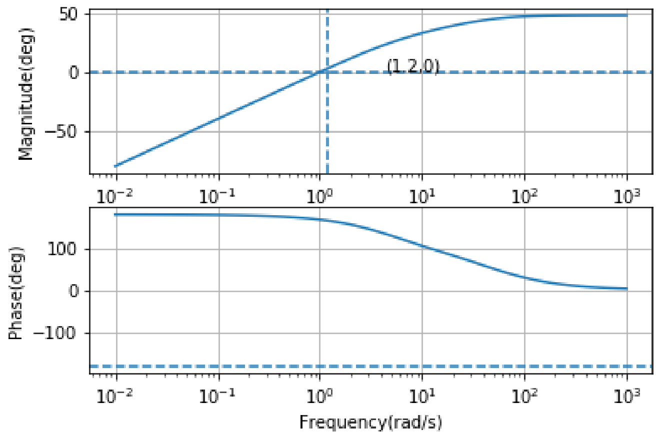

Fig. 8.3: Graphs

At $\omega = 2$, gain= 13db

$$20logK = -13db \tag{8.4.1}$$

$$LogK = -13/20 \implies K = 0.65 \tag{8.4.2}$$

8.5. Finding the Phase Margin (PM) where ω_{gc} is frequency when gain = 1. This is known as phase margin(PM).
Solution:

$$G\left(j\omega\right) = \frac{(j\omega)^2}{(1+0.2j\omega)(1+0.02j\omega)} \tag{8.5.1}$$

Solving (8.5.1) or from Fig 8.3 frequency at which gain = 1 ,is gain crossover frequency ω_{gc} .

$$\omega_{gc} = 1.2 \tag{8.5.2}$$

$$\implies PM = 344.8 \tag{8.5.3}$$

8.6. Find $-G(\jmath\omega)$ db , where ω is frequency when phase $= -180°$. This is known as *gain margin* (GM)
Solution: From Fig 8.3 ,we can say that phase never crosses $-180°$. So , the gain margin is *infinite*. Which means we can add any gain , and the equivalent closed loop system never goes unstable.

9 Phase Margin

9.1 Intoduction

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