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CSE-H

# ASSIGNMENT-5

1) Eigen values of A are 0, -1, 2

i) determinant of A = |A| = product of eigen values

$= 0\cdot(-1)\cdot 2$

$= 0$

ii) sum of elements of principal diagonal matrix

= sum of eigen values

$= 0+(-1)+2$

$= 1$

iii) eigen of values of $A^2$ = (eigen values of A)$^2$

$= (0,-1,2)^2$

$= 0, 1, 4$

2) Because the eigenvalues of a triangular (upper or lower) matrix or diagonal matrix are same as its element in the main diagonal.

Hence eigen values of A $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 3 & 4 & 5 \end{bmatrix}$ are 1, 3, 5

eigen values of $A^2$ = (eigen values of A)$^2$

$= (1)^2, (3)^2, (5)^2$

$= 1, 9, 25$

3) Let $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

sum of eigen values = sum of main diagonal elements
$= 1+2+3$
$= 6$

Product of eigen values $= |A|$
$= 1(6-0) - 6(3-0) + 1(0-0)$
$= 6 - 18$
$= -12$

4) Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$A - \lambda I = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{bmatrix}$

$= (1-\lambda)^2 - 1$
$= \cancel{1} + \lambda^2 - 2\lambda - \cancel{1}$
$= \lambda^2 - 2\lambda$

characteristic equation
$|A - \lambda I| = 0$
$\lambda^2 - 2\lambda = 0$
$\lambda(\lambda - 2) = 0$
$\lambda = 0$ $\quad \lambda - 2 = 0 \Rightarrow \lambda = 2$

Eigen values are $\lambda = 0, 2$

5) Let $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{bmatrix}$$

$$= -(2+\lambda)[\lambda^2-\lambda-12] - 2[-2\lambda-6] - 3[-4+1(1-\lambda)]$$

$$= -(2+\lambda)[\lambda^2-\lambda-12] - 2[-2\lambda-6] - 3[-4+1-\lambda]$$

$$= -(2+\lambda)[\lambda^2-\lambda-12] + 4\lambda+6+9+3\lambda$$

$$= -\lambda^3-\lambda^2+21\lambda+45$$

character equation

$$-\lambda^3-\lambda^2+21\lambda+45=0$$

for $\lambda=-3$

$$27-9-63+45 = 72-72 = 0$$

$$(\lambda+3)(\lambda^2-2\lambda-15)=0$$

$\lambda=-3$ $\quad$ $\lambda^2-2\lambda-15=0$

$$\lambda^2-5\lambda+3\lambda-15=0$$

$$\lambda(\lambda-5)+3(\lambda-5)=0$$

$$(\lambda+3)(\lambda-5)=0$$

$$\lambda=-3, \quad \lambda=5$$

Eigen Values are $\lambda = -3, -3, 5$

At $\lambda = -3$

$$A - \lambda I\Big|_{\lambda=-3} = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix}$$

~~$R_2 \to R_2 + R_1$~~ ~~$R_3 \to R_3 + R_1$~~

$$A - 3I = 0 \Rightarrow \begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & 6 \\ -1 & -2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 2 & 4 & 6 & 0 \\ -1 & -2 & 3 & 0 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 + R_1 \end{array}$$

$$\begin{bmatrix} 1 & 2 & -3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x_1 + 2x_2 - 3x_3 = 0$$

$$x_1 = -2x_2 + 3x_3$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0x_1 + -2x_2 + 3x_3 \\ 0x_1 + x_2 + 0 \cdot x_3 \\ 0x_1 + 0 \cdot x_2 + x_3 \end{bmatrix} = x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$$

Assume $x_2 = 1$, $x_3 = 1$

$$X_1 = k_1 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + k_2 \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$$

At $\lambda = 5$

$$A - \lambda I\Big|_{\lambda=5} = \begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix}$$

$$A \quad \begin{bmatrix} -7 & 2 & 3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Augemented matrix $\longrightarrow$

$$\begin{bmatrix} -7 & 2 & 3 & 0 \\ 2 & -4 & -6 & 0 \\ -1 & -2 & -5 & 0 \end{bmatrix} \qquad R_3 \longleftrightarrow -R_1$$

$$\begin{bmatrix} 1 & 2 & 5 & 0 \\ 2 & -4 & -6 & 0 \\ -7 & 2 & -3 & 0 \end{bmatrix} \qquad \begin{array}{l} R_2 \longrightarrow R_2 - 2R_1 \\ R_3 \longrightarrow R_3 + 7R_1 \end{array}$$

$$\begin{bmatrix} 1 & 2 & 5 & 0 \\ 0 & -8 & -16 & 0 \\ 0 & 16 & 32 & 0 \end{bmatrix} \qquad R_3 \longrightarrow R_3 + 2R_2$$

$$\begin{bmatrix} 1 & 2 & 5 & 0 \\ 0 & -8 & -16 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$-8x_2 - 16x_3 = 0$$
$$-x_2 - 2x_3 = 0$$
$$x_2 = -2x_3$$

$$x_1 + 2x_2 + 5x_3 = 0$$
$$x_1 - 4x_3 + 5x_3 = 0$$
$$x_1 + x_3 = 0$$
$$x_1 = -x_3$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -x_3 \\ -2x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$$

Eigen vectors are $X_1 = k_1 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + k_2 \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$

$$X_2 = k_3 \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

8) $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{bmatrix}$$

$|A - \lambda I| = 0$

$= (8-\lambda)[21 - 7\lambda - 3\lambda + \lambda^2 - 16] + 6[-18 + 6\lambda + 8]$
$\quad + 2(+24 - 14 + 2\lambda)$

$= (8-\lambda)(\lambda^2 - 10\lambda + 5) + 36\lambda - 60 - 4\lambda + 20$

$= 8\lambda^2 - 80\lambda + \cancel{40} - \lambda^3 + 10\lambda^2 - 5\lambda + 40\lambda - \cancel{40}$

$= -\lambda^3 + 18\lambda^2 - 75\lambda$

$\Rightarrow -\lambda^3 + 18\lambda^2 - 75\lambda = 0$

$\lambda^3 - 18\lambda^2 + 75\lambda = 0$

$\lambda(\lambda^2 - 18\lambda + 75) = 0$

$\lambda = 0 \qquad \lambda^2 - 18\lambda + 75 = 0$

$\lambda^2 - (15+3)\lambda + 75 = 0$

$(\lambda - 3)(\lambda - 15) = 0$

$\lambda = 3, 15$

(λ) Eigen values are = 0, 3, 15

At λ = 0

$$A-\lambda I\Big|_{\lambda=0} = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 8 \end{bmatrix}$$

$$C = \left[\begin{array}{ccc|c} 8 & -6 & 2 & 0 \\ -6 & 7 & -4 & 0 \\ 2 & -4 & 8 & 0 \end{array}\right] \qquad R_3 \leftrightarrow R_1,\quad R_1 \rightarrow R_1/2$$

$$= \left[\begin{array}{ccc|c} 1 & -2 & 3/2 & 0 \\ -6 & 7 & -4 & 0 \\ 8 & -6 & 2 & 0 \end{array}\right] \qquad R_2 \rightarrow R_2 + 6R_1,\quad R_3 \rightarrow R_3 - 8R_1$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3/2 & 0 \\ 0 & -5 & 5 & 0 \\ 0 & 10 & 10 & 0 \end{array}\right] \qquad R_3 \rightarrow R_3 + 2R_2$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3/2 & 0 \\ 0 & -5 & 5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$-5x_2 + 5x_3 = 0$$
$$-\cancel{5}x_2 = -\cancel{5}x_3$$
$$x_2 = x_3$$

$$x_1 - 2x_2 + 3/2\, x_3 = 0$$
$$x_1 - 2x_3 + \frac{3}{2}x_3 = 0$$
$$x_1 - \frac{4x_3 + 3x_3}{2} = 0$$
$$x_1 - \frac{x_3}{2} = 0$$
$$x_1 = \frac{x_3}{2} = \frac{1}{2}x_3$$

$$X_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}x_3 \\ x_2 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 1/2 \\ 1 \\ 1 \end{bmatrix}$$

Assume $x_3 = 2$

$$X_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

For $\lambda = 3$

$$A - \lambda I\big|_{\lambda=3} = \begin{bmatrix} 5 & -6 & 2 \\ -6 & 4 & 4 \\ 2 & -4 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 5 & -6 & 2 & 0 \\ -6 & 4 & 4 & 0 \\ 2 & -4 & 0 & 0 \end{bmatrix} \quad \begin{matrix} R_3 \longleftrightarrow R_1 \\ R_1 \rightarrow R_1/2 \end{matrix}$$

$$\begin{bmatrix} 1 & -2 & 0 & 0 \\ -6 & 4 & 4 & 0 \\ 5 & -6 & 2 & 0 \end{bmatrix} \quad \begin{matrix} R_2 \rightarrow R_2 + 6R_1 \\ R_3 \rightarrow R_3 - 5R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & -2 & 0 & 0 \\ 0 & -8 & -4 & 0 \\ 0 & 4 & 2 & 0 \end{bmatrix} \quad R_3 \rightarrow R_3 + \frac{R_2}{2}$$

$$\begin{bmatrix} 1 & -2 & 0 & 0 \\ 0 & -8 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$-8x_2 - 4x_3 = 0$$
$$-8x_2 = 4x_3$$
$$x_2 = -\frac{1}{2}x_3$$

$$x_1 - 2x_2 = 0 \Rightarrow \boxed{x_1 = 2x_2} \qquad \boxed{x_3 = -2x_2}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2x_2 \\ x_2 \\ -2x_2 \end{bmatrix} = x_2 \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

Assume $x_2 = 1$

$$X = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

for $\lambda = 15$

$$A - \lambda I \Big|_{\lambda=15} = \begin{bmatrix} -7 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{bmatrix}$$

$$C = \begin{bmatrix} -7 & -6 & 2 & 0 \\ -6 & -8 & -4 & 0 \\ 2 & -4 & -12 & 0 \end{bmatrix} \quad \begin{matrix} R_3 \leftrightarrow R_1 \\ R_1 \rightarrow R_1/2 \end{matrix}$$

$$= \begin{bmatrix} 1 & -2 & -6 & 0 \\ -6 & -8 & -4 & 0 \\ -7 & -6 & 2 & 0 \end{bmatrix} \quad \begin{matrix} R_2 \rightarrow R_2 + 6R_1 \\ R_3 \rightarrow R_3 + 7R_1 \end{matrix}$$

$$= \begin{bmatrix} 1 & -2 & -6 & 0 \\ 0 & -20 & -40 & 0 \\ 0 & 0 & -40 & 0 \end{bmatrix}$$

$$-20x_2 - 40x_3 = 0$$
$$-20x_2 = 40x_3$$
$$\boxed{x_2 = -2x_3}$$

$$x_1 - 2x_2 - 6x_3 = 0$$
$$x_1 + 4x_3 - 6x_3 = 0$$
$$x_1 - 2x_3 = 0$$
$$x_1 = 2x_3$$

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$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2x_3 \\ -2x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

Assume $x_3 = 1$

$$X = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

Eigen vectors are $\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

8) A, B are symmetric

$A = A^T$

$B = B^T$

$C = A + B$

$C^T = (A+B)^T = A^T + B^T$

$= A + B = C$

$(A+B)^T = (A+B)$

$\Rightarrow$ A+B is symmetric.

9) let $B = A - A^T$

$B^T = (A - A^T)^T = A^T - (A^T)^T$

$= A^T - A$

$= -(A - A^T)$

$B^T = -B$

B is skew symmetric

OR

$A - A^T$ is skew symmetric.

10) $A = \begin{bmatrix} 4 & 2 & -3 \\ 1 & 3 & 6 \\ -5 & 0 & 7 \end{bmatrix}$ . $A^T \begin{bmatrix} 4 & 1 & -5 \\ 2 & 3 & 0 \\ -3 & 6 & 7 \end{bmatrix}$

$A + A^T = \begin{bmatrix} 8 & 3 & -8 \\ 5 & 6 & 6 \\ -8 & 6 & 14 \end{bmatrix}$

$A - A^T = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -6 \\ -2 & 6 & 0 \end{bmatrix}$

$P = \frac{1}{2}(A + A^T) = \frac{1}{2}\begin{bmatrix} 8 & 3 & -8 \\ 5 & 6 & 6 \\ -8 & 6 & 14 \end{bmatrix}$

$Q = \frac{1}{2}(A - A^T)$

$= \frac{1}{2}\begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -6 \\ -2 & 6 & 0 \end{bmatrix}$ . $Q^T = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 6 \\ 2 & -6 & 0 \end{bmatrix}$

$P^T = \begin{bmatrix} 8 & 5 & -8 \\ 3 & 6 & 6 \\ -8 & 6 & 14 \end{bmatrix}$

$Q^T = Q$

Q is skew symmetric

$P = P^T$

P is symmetric

$P + Q = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T) = A$

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13) $\alpha = (a_1, a_2)$
$\beta = (b_1, b_2) \in V_2(f)$

$(\alpha, \beta) = (a_1 b_1 - a_2 b_1 - a_1 b_2 + 4 a_2 b_2)$

If inner product space then
$\cdot \langle \alpha a_1\, b_1 \rangle - 2 a_2 \cdot b_1 \rangle - 1 \alpha a_1 - b_2 \rangle + 4 \alpha a_2 b_2 \rangle$

Inner product
$(\alpha \cdot \beta) = (a_1, a_2) \cdot (b_1, b_2)$
$= a_1 b_1 + a_1 b_2 + a_2 b_1 + a_2 b_2$
$= \alpha a_1 \cdot b_1 \rangle + 2 a_1 \cdot b_2 \rangle + 2 a_2 b_1 \rangle + \alpha a_2 \cdot b_2 \rangle$

Hence it is not inner product.

14) i) $(0, \beta) = 0$ of $\beta$ in $V$
first element is 0. Hence in vector inner product they multiply with other. They become 0.
$(0, \beta) = 0$

(ii) If $(\alpha, \beta) = 0$ of $\beta \in V$ Then $\alpha = 0$
yes if $(\alpha, \beta) = 0$
$\beta$ is constant
Then $\alpha$ must be zero.

15) $\beta_1 = (1, 0, 1)$ $\beta_2 = (1, 0, -1)$ $\beta_3 = (0, 3, 4)$

$y_1 = \dfrac{x_1}{|x_1|} = \dfrac{(1,0,1)}{\sqrt{1^2+0^2+1^2}} = \dfrac{(1,0,1)}{\sqrt{2}}$

$= \left(\dfrac{1}{\sqrt{2}}, 0, \dfrac{1}{\sqrt{2}}\right)$

$$y_2 = \frac{z_2}{\|z_2\|}$$

$$z_2 = x_2 - \langle x_2 \cdot y_1 \rangle \cdot y_1$$

$$= (1,0,-1) - \left[(1,0,-1)\left[\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right]\right] y_1$$

$$= (1,0,-1) - \cancel{0 \cdot y_1} = (1,0,-1)$$

$$y_2 = \frac{(1,0,-1)}{\sqrt{1^2+0^2+(-1)^2}} = \frac{(1,0,-1)}{\sqrt{2}}$$

$$= \left(\frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}}\right)$$

$$z_3 = x_3 - \langle x_3 \cdot y_1 \rangle y_1 - \langle x_3 \cdot y_2 \rangle y_2$$

$$= (0,3,4) - \left[(0,3,4)\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)\right] y_1 - \left[(0,3,4)\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)\right] y_2$$

$$= (0,3,4) - \frac{4}{\sqrt{2}} y_1 - \left(-\frac{4}{\sqrt{2}}\right) y_2$$

$$(0,3,4) - (2,0,2) = (-2,0,2)$$

$$z_3 = (0,3,0)$$

$$y_3 = \frac{(0,3,0)}{3}$$

$$y_3 = (0,1,0)$$

12) $A = \begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

$A - \lambda I = \begin{bmatrix} 3-\lambda & 1 & -1 \\ -2 & 1-\lambda & 2 \\ 0 & 1 & 2-\lambda \end{bmatrix}$

$|A - \lambda I| = (3-\lambda)[(1-\lambda)(2-\lambda) - 2] - 1[(-2)(2-\lambda) - 0]$
$+ (-1)[(-2) - (1-\lambda)]$

$= -\lambda^3 + 6\lambda^2 - 13\lambda + 12$

$-\lambda^3 + 6\lambda^2 - 13\lambda + 12 = 0$

$\lambda^3 - 6\lambda^2 + 13\lambda - 12 = 0$

~~Putting~~ $\lambda = 1$

$\cancel{1 - 6 + 13 - 12}$

$\lambda^2 + 5\lambda + 6 = 0$

$(\lambda - 2)(\lambda - 3) = 0$

$\lambda = 2, 3$

for $\lambda = 1$

$\begin{bmatrix} 2 & 1 & -1 \\ -2 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix}$ $R_2 \rightarrow R_2 + R_1$

$\begin{bmatrix} 2 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ $R_3 \rightarrow R_3 - R_1$

$\begin{bmatrix} 2 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$x_3 + x_2 = 0$ $\qquad$ $2x_1 + x_2 - x_3 = 0$

$x_2 = -x_3$ $\qquad$ $x_1 = 0$

$$X_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \quad (\because \text{Assume } x_3 = 1)$$

for $\lambda = 2$

$$\begin{bmatrix} 1 & 1 & -1 \\ -2 & -1 & 2 \\ 0 & 1 & 0 \end{bmatrix} \quad R_2 \to R_2 + 2R_1$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad x_3 = 0 \qquad x_1 + x_2 - x_3 = 0$$

$x_1 = -x_2$

$$X_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

for $\lambda = 3$

$$\begin{bmatrix} 0 & 1 & -1 \\ -2 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix} \quad R_2/2 \longleftrightarrow R_1$$

$$\begin{bmatrix} -1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix} \quad R_3 \to R_3 - R_2$$

$$\begin{bmatrix} -1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix}$$

$x_2 - x_3 = 0$

$x_2 = x_3$

$-x_1 + x_2 + x_3 = 0$

$x_1 = 2x_3$

$$x_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$P = [x_1 : x_2 : x_3]$

$$= \begin{bmatrix} 0 & 1 & 2 \\ -1 & -1 & -1 \\ 1 & 0 & 1 \end{bmatrix}$$

$|P| = 4$

$A_{11} = -1 \qquad A_{21} = -1 \qquad A_{31} = 3$

$A_{12} = 2 \qquad A_{22} = -2 \qquad A_{32} = -2$

$A_{13} = 1 \qquad A_{23} = 1 \qquad A_{33} = 1$

$$P^{-1} = \frac{1}{4}\begin{bmatrix} -1 & -1 & 3 \\ 2 & -2 & -2 \\ 1 & 1 & 1 \end{bmatrix}$$

$P^{-1}AP$

$$\frac{1}{4}\begin{bmatrix} -1 & -1 & 3 \\ 2 & -2 & -2 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 \\ -1 & -1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 4 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$