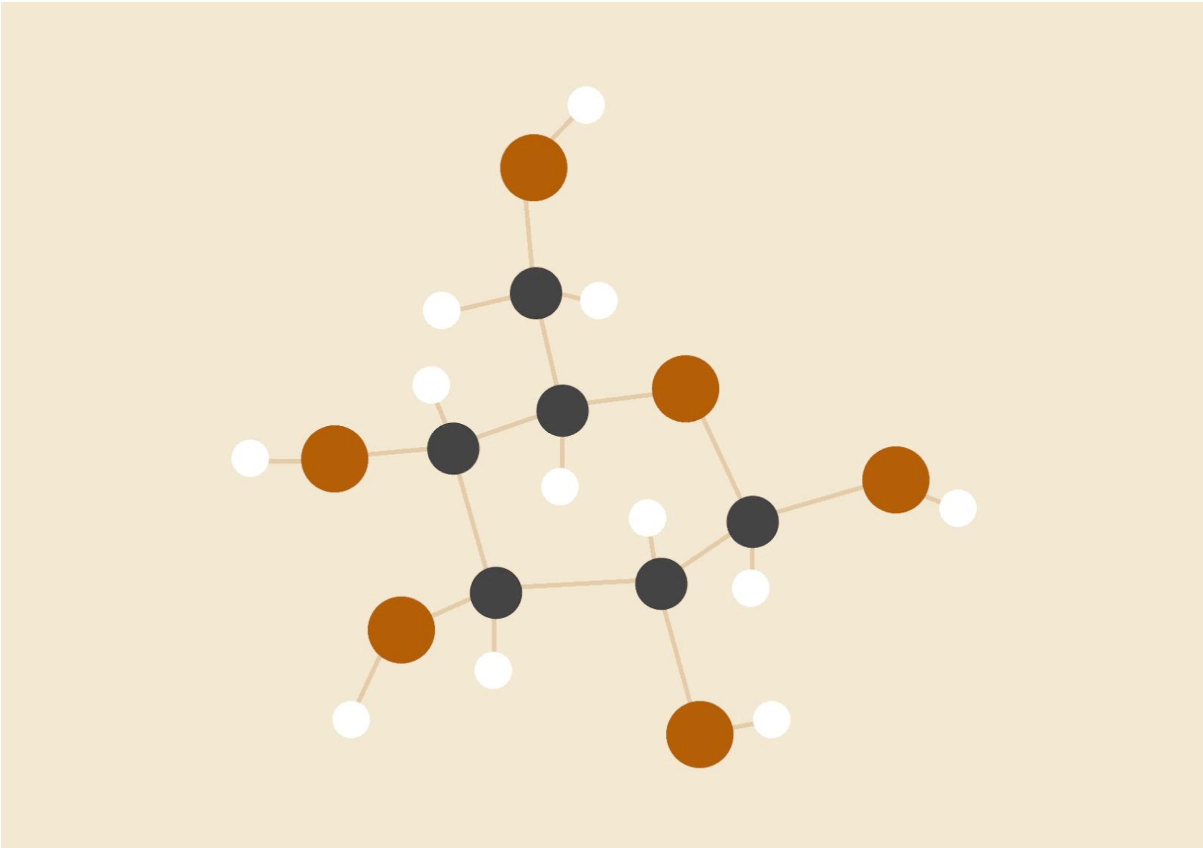
Quantitative Foundations

Project 3 - Confidence Intervals



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A. RESULTS TABLE

We define that if the fraction missed was less than the alpha = 0.25 (75% confidence interval), then we declare the corresponding function valid, otherwise invalid.

Confidence Interval generated by given functions	Confidence Interval - Valid?	Confidence Level (α)	Exact (All data sizes) / Asymptotic Confidence Interval (CI)
Function 1	**Yes**	**0.01 (99%)**	**Exact C.I. (All)**
Function 2	**Yes**	**0.01 (99%)**	**Exact C.I. (All)**
Function 3	**Yes**	**0.06 (94%)**	**Asymptotic C.I.**
Function 4	**No**	**-**	**-**
Function 5	**Yes**	**0.11 (89%)**	**Exact C.I. (All)**
Function 6	**No**	**-**	**-**
Function 7	**Yes**	**0.12 (88%)**	**Exact C.I. (All)**
Function 8	**Yes**	**0.02 (98%)**	**Exact C.I. (All)**
Function 9	**Yes**	**0.02 (98%)**	**Asymptotic C.I.**
Function 10	**Yes**	**0.05 (95%)**	**Exact C.I. (All)**

B. DATA DESIGN

We sampled data using three different distributions with their corresponding parameters:

1. Bernoulli distribution with theta = 0.5 (True Mean = 0.5)
2. Uniform distribution with a = 0, b = 1. (True Mean = 0.5)
3. Uniform distribution with a = 0, b = 0.05. (True Mean = 0.025)
4. Beta distribution with a = 1, b = 3 (True Mean = 0.25)

We used different distributions since for the confidence interval generated by the functions to be valid, it must satisfy the definition irrespective of the distribution chosen.

C. TESTING PROCEDURE

We used the data sampled from each distribution as input to each of the 10 functions and calculated the confidence interval. This process was repeated 10,000 times to generate 10,000 confidence intervals for each case.

Then we compared the fraction of the **10,000 confidence intervals** that missed the true mean for each case and compared with 4 different values of **alpha: 0.25, 0.1, 0.05, and 0.01**. We define that if the fraction missed was less than the **alpha** = **0.25 (75% confidence interval)**, then we declare the corresponding function valid, otherwise invalid.

For valid cases of confidence intervals generated by the 10 functions, we further compare the missed fractions with more concrete alpha values to determine the specific confidence level. For example, if we find that it reaches the level of 0.1 and does not reach the level of 0.05, then we compare the missed fractions with 0.06, 0.07, 0.08, 0.09, 0.1 to get the more concrete level.

Finally, for the valid cases of the confidence interval generated by the 10 functions, we compare the fraction of the 10,000 confidence intervals that missed the true mean with alpha for different values of data size: **N** = **10, 100, 1000, 10000.** If the fraction missed is more for small data sizes like 10 but less for larger data sizes like 10000, we conclude that the confidence interval generated by the function is asymptotic. On the other hand, if the fraction missed is less than alpha for all values of N, then we conclude that the confidence interval is exact (for all data sizes).

D. CONCLUSION

Hence, in this project, we generated multiple samples of confidence intervals and computed the fraction of the confidence intervals that missed to capture the true mean for different distributions, different data sizes. We then compared the fractions with different confidence levels (α) for determining the validity and nature of the confidence intervals generated by the given 10 functions. By using input from different distributions, we also find that it would have an effect on our final judgment to detect some invalid confidence intervals.