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# **Learning with Knowledge**

Viktor Schlegel

### What about knowledge?

So far, we have regarded learning in a pure inductive setting. We pursued the following question:

"Assuming nothing and looking at these examples, what general rules can we learn?"

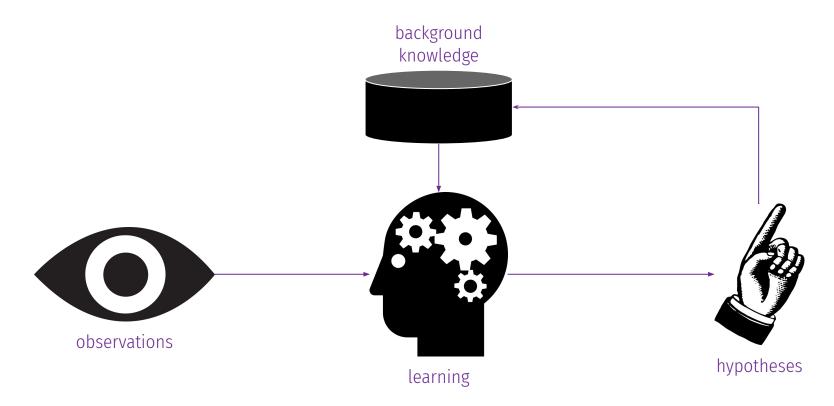
This is not how we (humans) learn!

When learning new things, we already possess some prior knowledge, which we can make use of in order to learn better and faster.



# Learning, idealised

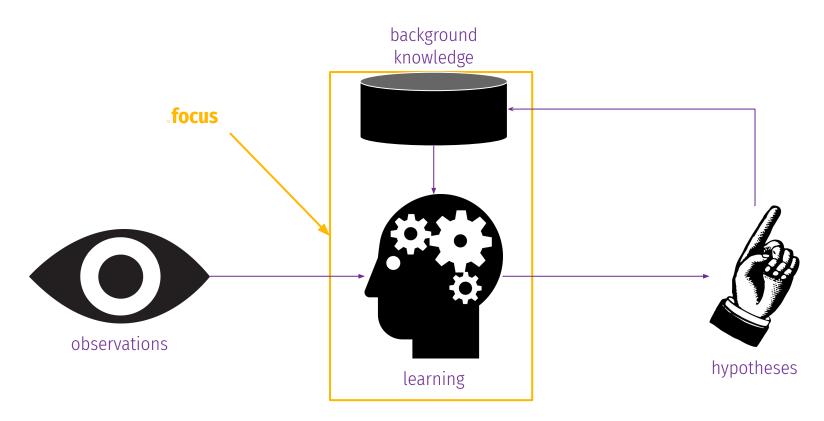
An AI agent should be able to use prior knowledge as well.





# Learning, idealised

An AI agent should be able to use prior knowledge as well.





### **Using knowledge**

When learning how to solve quadratic equations in high school, you might have encountered an example like this:

$$2x^2 + 8x + 16$$

$$= 2(x^2 + 4x + 8)$$

$$= 2((x + 2)^2 + 4)$$

Which, when substituting the numbers for variables results in

$$ax^2+bx+c=a(x+rac{b}{2a})^2+c-rac{b^2}{4a}$$

And finding x in f(x) = 0 results in

$$x_{1,2}=rac{-b\pm\sqrt{b^2+4ac}}{2a}$$



### **Using knowledge**

$$x_{1,2}=rac{-b\pm\sqrt{b^2+4ac}}{2a}$$

From there on, to solve for x you would use this formula, that you learned from the example, without deriving it every time.

You didn't learn anything "new" in the process, you just deduced a shortcut using your existing knowledge (of arithmetics).



### **Explanation based learning**

We use the term "explanation" quite liberally here, referring to a proof. We will look at explanations more formally later.

The AI equivalent for this process is called **Explanation based learning** (EBL).

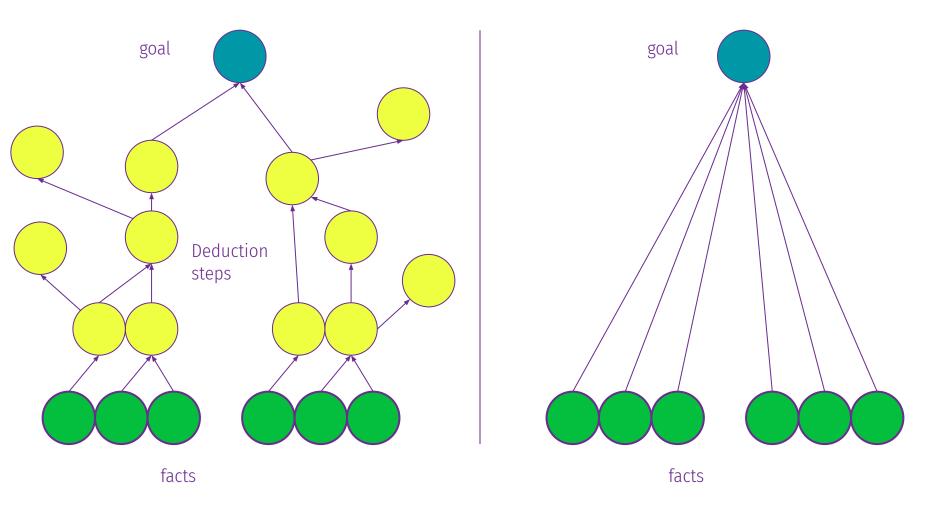
EBL aims to extract **general rules** from detailed examples, in order to apply them to a class of similar problems.

These general rules speed up the search for a proof for a problem where those rules apply.

 $Hypothesis \land Descriptions \models Classifications \ Background \models Hypothesis$ 



# **Explanation based learning**





### **EBL** example

Consider the following prolog KB (excerpt) that describes some simple arithmetic rules.

We will derive the proof for the

simplification of

```
1 \times (0 + y)
```



#### **Proof SLD**

```
[trace] ?- simplify(1 x (0 p y), X).
                                                                             Call: (10) simplify(1 x (0 p y), _29874) ? creep
                                                                            Call: (11) rewrite(1 x (0 p y), _30326) ? creep
                                                                             Exit: (11) rewrite(1 x (0 p y), 0 p y) ? creep
                                                                             Call: (11) simplify(0 p y, _29874) ? creep
                                                                             Call: (12) rewrite(0 p y, _30458) ? creep
                                                                             Exit: (12) rewrite(0 p y, y) ? creep
                                                                             Call: (12) simplify(y, _29874) ? creep
                                                                             Call: (13) rewrite(y, _30590) ? creep
                                                                             Fail: (13) rewrite(y, _30634) ? creep
                                                                            Redo: (12) simplify(y, _29874) ? creep
                                                                             Call: (13) primitive(y) ? creep
                                                                             Call: (14) variable(y) ? creep
                                                                             Exit: (14) variable(y) ? creep
                                                                             Exit: (13) primitive(y) ? creep
                 simplify(1 \times (0 p y), W)
                                                                             Exit: (12) simplify(y, y) ? creep
                                                                            Exit: (11) simplify(0 p y, y) ? creep
                                                                             Exit: (10) simplify(1 x (0 p y), y) ? creep
                                                      simplify((0 p y), W)
   rewrite(1 x (0 p y), V)
          Done | V := 0 p y
                                 rewrite((0 p y), V2)
                                                                                   simplify(y, W)
                                       Done | V2 := y
                                                                                                  W := v
simplify(U,W) :- rewrite(U,V) , simplify(V,W).
simplify(U,U) :- primitive(U).
                                                                                     primitive(y)
primitive(U) :- variable(U).
primitive(U) :- nr(U).
rewrite(1 x U, U).
rewrite(0 p U, U).
                                                                                     variable(y)
                                                                                                           MANCHESTER
variable(y).
                                                            10
                                                                                                          The University of Manchester
                                                                                          Done
```

#### **Generalised SLD**

rewrite(0 p U, U).

variable(y).

```
simplify(X x (Y p Z), W)
                                              simplify((Y p Z), W)
    rewrite(X \times (Y p Z), V)
         Done | V := Y p Z
               X := 1
                             rewrite((Y p Z), V2)
                                                                       simplify(Z, W)
                                  Done | V2 := Z
                                                                                   W := Z
                                        Y := 0
                                                                        primitive(Z)
                                                                        variable(Z)
simplify(U,W) :- rewrite(U,V) , simplify(V,W).
simplify(U,U) :- primitive(U).
primitive(U) :- variable(U).
                                                                            Done
primitive(U) :- nr(U).
rewrite(1 \times U, U).
```



#### **Generalised SLD**

rewrite(0 p U, U).

variable(y).

```
simplify(X x (Y p Z), W)
                                              simplify((Y p Z), W)
    rewrite(X \times (Y p Z), V)
         Done | V := Y p Z
               X := 1
                             rewrite((Y p Z), V2)
                                                                       simplify(Z, W)
                                  Done | V2 := Z
                                                                                   W := Z
                                        Y := 0
                                                                        primitive(Z)
                                                                        variable(Z)
simplify(U,W) :- rewrite(U,V) , simplify(V,W).
simplify(U,U) :- primitive(U).
primitive(U) :- variable(U).
                                                                            Done
primitive(U) :- nr(U).
rewrite(1 \times U, U).
```



#### **New rule**

$$simplify(X \times (Y p Z), W)$$

#### **Generalised SLD**

variable(y).

```
simplify(1 \times (0 p Z), Z) :- primitive(Z)
                simplify(X x (Y p Z), W)
                                               simplify((Y p Z), W)
    rewrite(X \times (Y p Z), V)
         Done | V := Y p Z
                X := 1
                             rewrite((Y p Z), V2)
                                                                       simplify(Z, W)
                                  Done | V2 := Z
                                                                                   W := Z
                                         Y := 0
                                                                         primitive(Z)
                                                                         variable(Z)
simplify(U,W) :- rewrite(U,V) , simplify(V,W).
simplify(U,U) :- primitive(U).
primitive(U) :- variable(U).
                                                                             Done
primitive(U) :- nr(U).
rewrite(1 \times U, U).
rewrite(0 p U, U).
```

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#### **Generalised SLD**

variable(y).

```
simplify(X x (Y p Z), W)
                                                    simplify(1 x (Y p Z), W) :- simplify(Y p Z, W)
                                              simplify((Y p Z), W)
    rewrite(X \times (Y p Z), V)
         Done | V := Y p Z
               X := 1
                             rewrite((Y p Z), V2)
                                                                      simplify(Z, W)
                                  Done | V2 := Z
                                                                                  W := Z
                                        Y := 0
                                                                        primitive(Z)
                                                                        variable(Z)
simplify(U,W) :- rewrite(U,V) , simplify(V,W).
simplify(U,U) :- primitive(U).
primitive(U) :- variable(U).
                                                                            Done
primitive(U) :- nr(U).
rewrite(1 \times U, U).
rewrite(0 p U, U).
```

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### **EBL Algorithm Sketch**

#### Given:

- Domain Theory (rules)
- Situation Description (facts)
- $\cdot$  Example (statement to derive with rules & facts)  $1 imes (0+y) \Leftrightarrow y$
- "Operationality criterion"

#### Result:

Other terms that can appear in the generalised rule

 Rule consistent with the example but is more general that the proof used to prove example



simplify(U,W) :- rewrite(U,V), simplify(V,W).

simplify(U,U) :- primitive(U).
primitive(U) :- variable(U).

variable(y).

### **EBL Algorithm Sketch**

- 1. Construct proof for the example using background knowledge
- 2. Construct a generalised proof, substituting the literals in the proof goal with variables, keep track of instantiated variables
- 3. Construct a new rule from generalised tree: body are leaves of the proof tree and head is the root, substitute variables for their instantiations
- 4. Drop conditions from the body that are true, regardless of the values of the variables in the head



### How is that learning?

Did we learn anything new?

Didn't we just deduce the rule from what was from our knowledge?

In a perfect world, the deductive system would already know everything in its knowledge base closure.

In the real world, there are constraints, such as time, memory, etc.



### **EBL to speed up inference**

```
[trace] ?- simplify(1 x (0 p y), X).
   Call: (10) simplify(1 x (0 p y), _29874) ? creep
  Call: (11) rewrite(1 x (0 p y), _30326) ? creep
   Exit: (11) rewrite(1 x (0 p y), 0 p y) ? creep
   Call: (11) simplify(0 p y, _29874) ? creep
  Call: (12) rewrite(0 p y, _30458) ? creep
   Exit: (12) rewrite(0 p y, y) ? creep
   Call: (12) simplify(y, _29874) ? creep
   Call: (13) rewrite(y, _30590) ? creep
   Fail: (13) rewrite(y, _30634) ? creep
  Redo: (12) simplify(y, _29874) ? creep
   Call: (13) primitive(y) ? creep
   Call: (14) variable(y) ? creep
   Exit: (14) variable(y) ? creep
  Exit: (13) primitive(y) ? creep
   Exit: (12) simplify(y, y) ? creep
   Exit: (11) simplify(0 p y, y) ? creep
  Exit: (10) simplify(1 x (0 p y), y) ? creep
 = V .
```

```
[trace] ?- simplify(1 x (0 p y), X).
    Call: (10) simplify(1 x (0 p y), _15692) ? creep
    Call: (11) variable(y) ? creep
    Exit: (11) variable(y) ? creep
    Exit: (10) simplify(1 x (0 p y), y) ? creep
X = y .
```

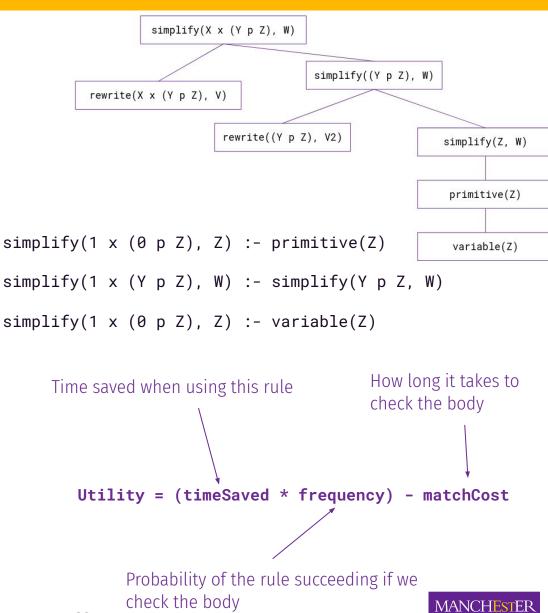
```
simplify(1 \times (0 p Z), Z) :- variable(Z)
```



#### Which rules to add?

If we add too many rules to our KB, we increase the search space, perhaps unnecessarily.

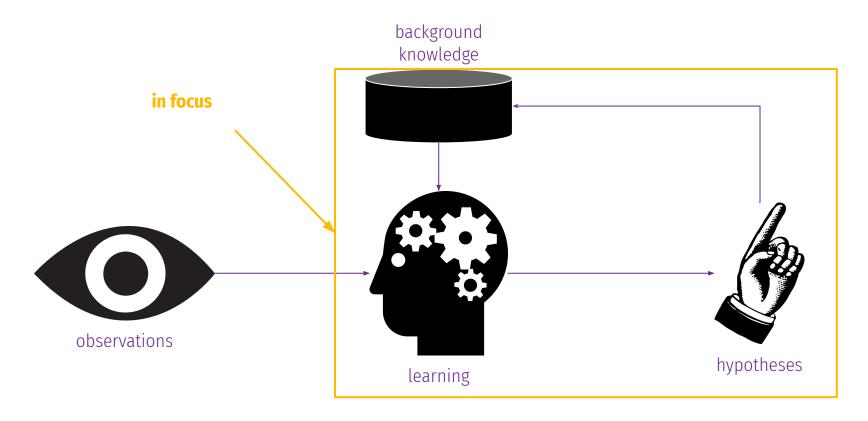
We would like to know the "utility" of a rule, before adding it.



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# Learning, idealised

An AI agent should be able to use prior knowledge as well.





### **Relevance based learning**

#### Consider another example:

When you need to predict, whether a block will float in water, you ask for the material and not necessarily for its size or its finish.

The reason for this is because you **know** that whether an object will float depends on its density (which in turn is dictated by the object's material).



#### **Determinations**

In other words, knowing the material, the "floatiness" of a block can be fully determined.

This is called a **functional dependency** or **determination**.

Material(x, m) > Floats(x)



### **Determining the hypothesis space**

Determinations provide a **sufficient set of attributes** that need to be considered when constructing hypotheses.

This is useful to reduce the dimensionality of the hypothesis space.

- Speed up learning (smaller search space)
- · Learn from fewer examples
- · In spirit, this is similar to **feature selection techniques**

```
Hypothesis \land Descriptions \models Classification \\ Background \land Descriptions \land Classifications \models Hypothesis
```



### **Learning determinations**

In practice, for a learning problem we might not know the determinations. Therefore, we want to **learn** them from observations.

A determination  $\bigwedge_{i=0}^n A_i \succ C$  is **consistent** with a set of examples,

if all examples with the same values for all  $A_i$  have the same value for C .

A consistent determination of size *n* is **minimal** if there is no consistent determination of smaller size.



#### **Minimal Consistent Determinations**

```
def is_consistent(attributes, examples):
                                                 map from attribute values to class of
  h := dict()
                                                  example
  for e in examples:
      if any(class of e =! c for c in h[e[attributes]):
         return false
                                                 Check if there are any examples with the
     h[e[attributes]] = class of e
                                                 same attribute combination but different
  return true
                                                 classifications
def minimal_consistent_determinations(attributes,
example):
                                                 —— All possible subsets of attributes
   for subset in sorted(powerset(attributes), key=len):
    if is_consistent(subset, examples):
        return subset
```

 $\Rightarrow$  Runtime  $\mathcal{O}(|examples|^n)$  in theory, in practice, in many domains n is sufficiently small.



### **Takeaways**

- · We can exploit prior knowledge to improve the learning
- Explanation based learning learns general rules from specific observations by using deductive reasoning
- Relevance based learning relies upon/learns functional dependencies to reduce the size of the hypothesis space
- · We still didn't learn anything "complicated!"





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# **Learning relations**

Viktor Schlegel

#### **What about relations?**

So far, we have regarded learning as a classification problem.

$$\forall x: Goal(x) \Leftrightarrow Attributes(x)$$

This is a restricted view of learning.

Specifically we have only one variable in the head and the body of the learned rule.



### **Representing relations**

Let's consider learning a relation like the following:

$$Grandparent(x,y) \Leftrightarrow [\exists z \ Parent(x,z) \land Parent(z,y)].$$

How can we represent some examples for this relation using the attribute-based notation?



### **Representing relations**

First problem: how do we represent the target relation to be learned as an attribute?

· Represent pairs as single objects, i.e

```
Grandparent(<Elsa, Viktor>)
Grandparent(<Robert, Andy>)
```

Second problem: how do we then represent the relations between elements of the pair?

→ Introduce new attributes

```
SecondElementIsMale(<Elsa, Viktor>)
FirstElementIsFatherOfElena(<Robert, Andy>)
....
```



### **Representing relations II**

Let's substitute the question for an easier one:

Is X the grandparent of Y  $\rightarrow$  Has X a grandchild?

Parent (X)	Child (Y)	HasGrandChild
Elena	Olga obvi01	ustyemissing
Olga	Sophie Something is obvious	False

So far, we have only considered algorithms that process example independently. This is not given in this example!



### **Representing relations III**

Let's manually flatten out the relation.

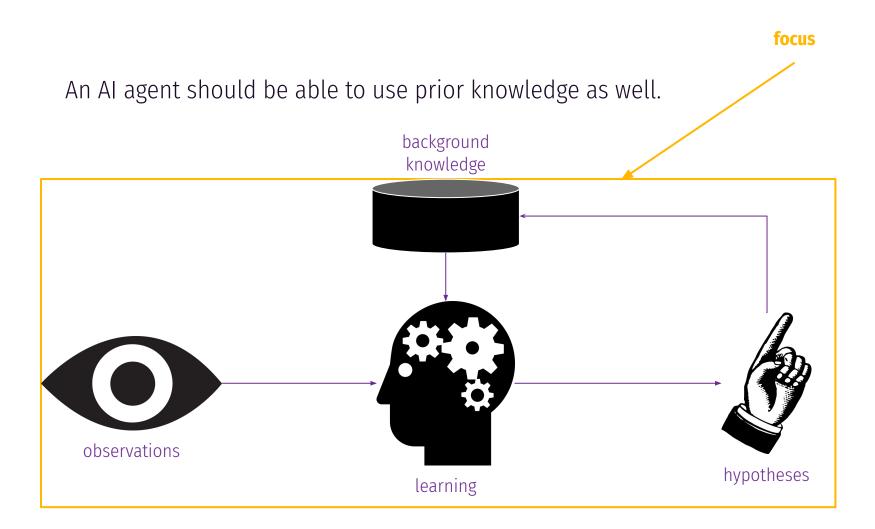
Parent (X)	Child (Y)	ChildOfChild	HasGrandChild
Elena	Olga	Sophie	True
Olga	Sophie	NULL	False

Then the search for a hypothesis becomes trivial.

We will see later how this type of transformation can be achieved automatically.



# Learning, idealised





### **Learning relations**

#### Given:

- Knowledge base
- Target predicate with arity
- Positive examples of target relation
- Negative examples of target relation

#### Goal:

Learn target relation (disjunction of horn clauses)



### **FOIL** algorithm sketch

- Start with most general clause Target relation as head, empty body (true)
- specialise until it covers some positive examples and no negative examples
- Remove positive examples covered by hypothesis
- Repeat from the beginning until no positive examples left
- Return disjunction of clauses as hypothesis

Learn one rule

Sequential covering



# **FOIL Algorithm**

```
def foil(pos_ex, neg_ex, predicates, target):
 clauses := set()
 while pos_examples not empty:
     clause := new_clause(pos_ex, neg_ex, predicates, target)
     pos_examples = [e for e in pos_examples if not clause |= e]
     clauses.push(clause)
  return disjunction of clauses as hypothesis
def new_clause(pos_ex, neg_ex, predicates, target):
 clause = (target, []) # (head, body)
 while neg_ex not empty:
     candidates := generate_candidates(clause)
     lit := argmax_c(foil\_gain(pos\_ex, neg\_ex, c) for c in candidates)
     clause[1].push(new_literal)
     update pos_ex, neg_ex with all bindings for new variables in lit
  return clause
```

Similar to decision tree learning, we first look at all possible ways to specialise a clause.

#### Possibilities:

Predicate literals

```
grandparent(X,Y) :- parent(X, V_1)
```

- a. Must be a "known" predicate from KB
- b. Arguments must be variables
- c. At least one variable must be in the rule already (head or body so far)



Similar to decision tree learning, we first look at all possible ways to specialise a clause.

#### Possibilities:

- 2. Equality grandfather(X,Y) :- father(X,  $V_1$ ), father( $V_1$ ,  $V_2$ ),  $V_2 = Y$ 
  - a. Between variables: both variables must be in the rule
  - b. Between constants: must be provided additionally



Similar to decision tree learning, we first look at all possible ways to specialise a clause.

#### Possibilities:

- 3. Negation sibling(X,Y) :- parent(X, $v_1$ ), parent(Y, $v_1$ ), X != Y
  - a. Negated predicate literals
  - b. Negated Equality (disequality)



Similar to decision tree learning, we first look at all possible ways to specialise a clause.

#### Other possibilities:

- 3. Arithmetic comparisons liquid(X,Y) :- [...] , X > 50.
  - a. Learn to split continuous values
- 4. Target predicate ancestor(X,Y) :- parent(X,Z), ancestor(Z,Y).
  - a. Additional checks to prevent infinite recursion
  - b. need to add target predicate to the KB at some point!



#### **Choose candidate**

Similar to decision tree learning FOIL chooses greedily the candidate specialisation that has the highest **information gain**.

```
def foil_gain(pos_ex, neg_ex, candidate):
  pos_ex_new, neg_ex_new = examples updated with bindings
                             for all new variables in candidate
  p_1, n_1, p_0, n_0 = len(pos_ex_new), len(neg_ex_new), len(pos_ex), len(neg_ex)
  t = len(p for p in pos_ex if p is represented by any p' in pos_ex_new)
  return |t| * (|log(p_1/(p_1+n_1))| - |log(p_0/(p_0+n_0))|)
                                                               Entropy of current hypothesis
                             Entropy of hypothesis specialised with
```

Examples covered by hypothesis and by specialised hypothesis

candidate literal



## **FOIL: example**

```
mother(anne, peter) .
mother(anne, zara) .
mother(sarah, beatrice) .
mother(sarah, eugenie) .
mother(elizabeth, anne) .
mother(elizabeth, andrew) .
father(mark, peter) .
father(mark, zara) .
father(andrew, beatrice) .
father(andrew, eugenie) .
father(philip, anne) .
father(philip, andrew) .
male(philip) .
male(mark) .
male(andrew) .
male(peter) .
```

```
female(elizabeth) .
female(anne) .
female(sarah) .
female(zara) .
female(beatrice) .
female(eugenie) .
```



# **FOIL: Generate Candidates Example**

```
clause = parent(X,Y) :- true
predicates = [Mother/2]
```

#### Possible Candidates:

```
mother(X,Y)
mother(Y,X)
```

mother(X,X)
mother(Y,Y)

 $mother(X, V_0)$ 

 $mother(Y, V_0)$ 

 $mother(V_0, X)$ 

 $mother(V_0, Y)$ 

#### Predicate literals

- a. Must be a "known" predicate from KB
- b. Arguments must be variables
- At least one variable must be in the rule already (head or body so far)

#### Impossible Candidates:

```
mother(V_0, V_0)
```

$$mother(V_0, V_1)$$

$$mother(V_1, V_0)$$



# **FOIL: Update bindings Example**

Extend example with binding based on current knowledge base

```
literal = father(X, V_0)

[...]

example_1 = {'X': 'mark', 'Y': zara}

⇒ {'X': 'mark', 'Y': 'zara', 'V_0': 'zara'}

⇒ {'X': 'mark', 'Y': 'zara', 'V_0': 'peter'}

father(andrew, beatrice) .
father(andrew, eugenie) .
father(philip, anne) .
father(philip, andrew) .

example_2 = {'X': 'elizabeth', 'Y': andrew}

[...]
```



# **FOIL: Gain calculation example**

```
candidate = mother(X,Y)
ex_pos = [{"X": "elizabeth", "Y": "anne"},
                                                        [ \ldots ]
          {"X": "elizabeth", "Y": "andrew"}]
ex_neg = [{"X": "anne", "Y": "eugenie"},
                                                        mother(anne, peter) .
          {"X": "beatrice", "Y": "eugenie"}]
                                                        mother(anne, zara) .
ex_pos_new = ex_pos
                                                        mother(sarah, beatrice) .
ex_neq_new = []
                                                        mother(sarah, eugenie) .
                                                        mother(elizabeth, anne) .
represented = ex_pos \Rightarrow t = 2
                                                        mother(elizabeth, andrew) .
\Rightarrow ig = 2 * (log2(2/(2+0)) - log2(2/(2+2))) = 2
                                                        [ \dots ]
candidate = mother(X, V_0)
ex_pos = [{"X": "elizabeth", "Y": "anne"}, {"X": "elizabeth", "Y": "andrew"}]
ex_neg = [{"X": "anne", "Y": "eugenie"}, {"X": "beatrice", "Y": "eugenie"}]
ex_pos_new = [{"X": "elizabeth", "Y": "anne", "V_0": "andrew"},
              {"X": "elizabeth", "Y": "anne", "V_0": "anne"},
               {"X": "elizabeth", "Y": "andrew", "V_0": "anne"},
               {"X": "elizabeth", "Y": "andrew", "V_0": "andrew"}]
ex_neg_new = [{"X": "anne", "Y": "eugenie", "V_0": "peter"},
               {"X": "anne", "Y": "eugenie", "V_0": "zara"}]
represented = ex_pos \Rightarrow t = 2
^{-0.585} ^{-9} -1 ^{-1} \Rightarrow ig = 2 * (log2(4/(4+2)) - log2(2/(2+2))) = 0.83
```

## **FOIL: Summary**

- FOIL can learn relations that simple, attribute-based learning algorithms cannot learn
- · FOIL can learn recursive relations
- · FOIL learns hypotheses as a disjunction of horn clauses
  - Each clause covers a subset of positive examples in the training set
  - · Specific to general search
- These hypotheses are generated by specialising, until they cover no negative examples
  - · Greedy, general to specific search



## **FOIL: Drawbacks and Improvements**

- · Clauses can only be "specialised, until they cover no negative examples" if there is no noise in data
  - · With noise, the refinement can take forever
  - As a solution, a stopping criterion based on the length of the current clause can be added
- · The search space when generating candidate literals can be very large
  - · Use domain theory (= background knowledge) to speed up search: EBL



#### **FOCL Sketch**

#### Given:

- Domain Theory (rules)
- Situation Description (facts)
- Example (statement to derive with rules & facts)
- · "Operationality criterion"

In addition to training set, FOCL uses an (incomplete) domain theory & operational criterion as additional inputs

- When generating candidates, FOCL first generates logically sufficient conditions for the target based on the domain theory
- Unnecessary literals are pruned
- The resulting condition is added as a conjunction to the set of all candidates



## **FOCL Example**

HasHandle

MadeOfCeramic

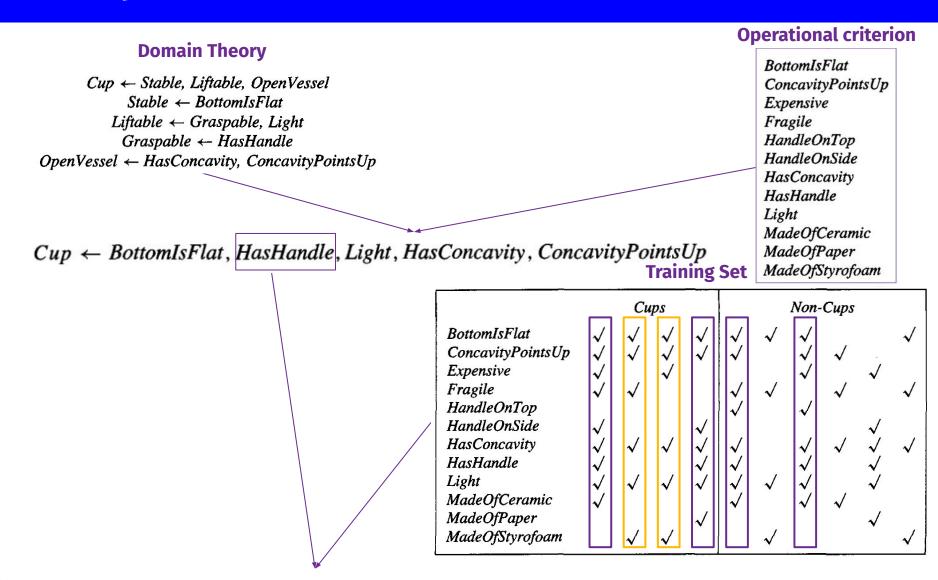
MadeOfStyrofoam

**MadeOfPaper** 

Light

```
Cup \leftarrow Stable, Liftable, OpenVessel
concavityPointsUp(object8).
                                                            Stable \leftarrow BottomIsFlat
fragile(object8).
hasConcavity(object8).
                                                         Liftable \leftarrow Graspable, Light
madeOfCeramic(object8).
                                                           Graspable \leftarrow HasHandle
\texttt{negative\_ex} = [\{\text{``X''} : \text{`object8'}\}...] \textit{OpenVessel} \leftarrow \textit{HasConcavity, ConcavityPointsUp}
                               Operational criterion
                                                                             Domain Theory
                                                                               Non-Cups
                                                   Cups
                 BottomIsFlat
                 ConcavityPointsUp
                 Expensive
                 Fragile
                 HandleOnTop
                 HandleOnSide
                 HasConcavity
```

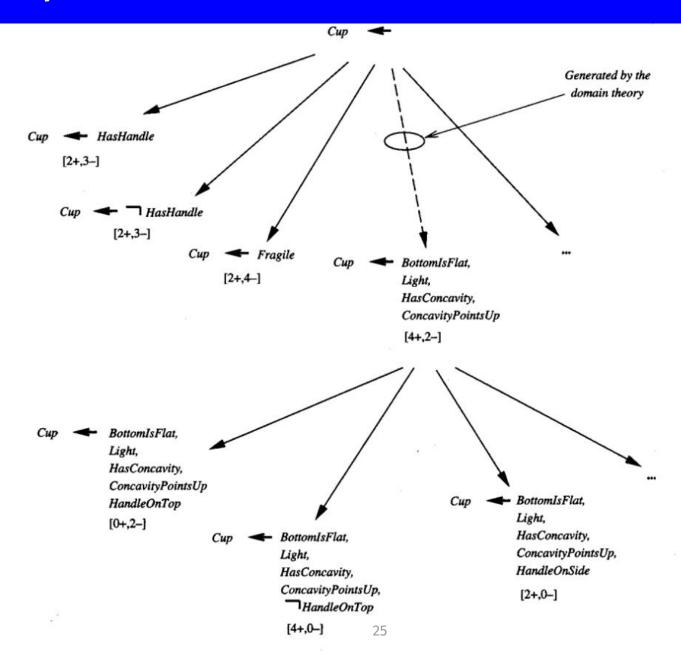
# **FOCL Example**



 $Cup \leftarrow BottomIsFlat, Light, HasConcavity, ConcavityPointsUp$ 



# **FOCL Example**



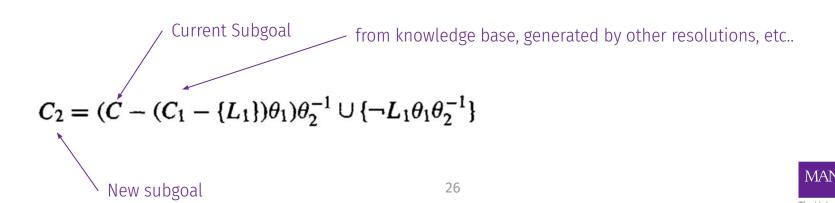


## **Induction as inverted deduction**

Induction is just inverted deduction!

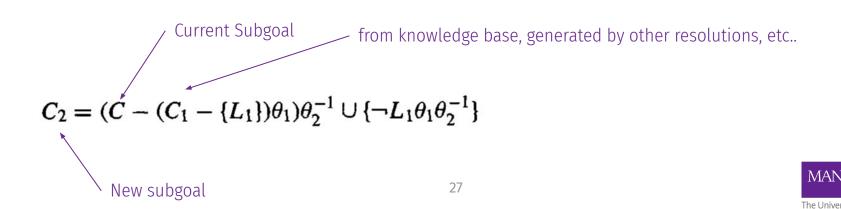
# $Hypothesis \land Descriptions \land Background \models Classifications$

Invert resolution to arrive at hypothesis starting from example observations



## **Induction as inverted deduction**

- · Inverted deduction is utilising inverted resolution and learns a hypothesis starting from single examples, "bottom up"
- · Like FOCL, can make use of domain theory to guide and speed up the search
- · Like many other algorithms we observed, unsuitable to handle noisy data



# Statistical vs Symbolic Approaches to learning & Al



#### **Statistical AI**

- · Focus on learning
- Performs well with noisy training data
- Can operate on raw input (text, pictures)
- Requires minimal human interaction

## Symbolic AI

- · Focus on inference
- Can be verified formally
- Produces results interpretable by humans
- Allows to incorporate existing knowledge intuitively



# Statistical + Symbolic Approaches to learning & Al



- → Use statistical learning approaches to learn to guide the search in large hypothesis spaces
  - E.g. in Automated Theorem Provers (ATP)
- → Use statistical learning approaches to generate symbolic representations from noisy input data
  - · Semantic Parsing
  - · Scene graph generation
- → Utilise knowledge to bootstrap statistical learning (e.g. in recommender systems)
- → Use statistical models to learn to operate over symbolic representations

