Date ___ Page No. ___

Q6. Given data :-

x	1	2	3	4	5	6	7	8	9
y	1	1.5	2	3	4	5	8	10	13

For linear regression model.

$$y = a_0 + a_1 x$$

$$A = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = (X^T X)^{-1} X^T Y$$

where $X = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \\ 1 & 6 \\ 1 & 7 \\ 1 & 8 \\ 1 & 9 \end{bmatrix}$ $Y = \begin{bmatrix} 1 \\ 1.5 \\ 2 \\ 3 \\ 4 \\ 5 \\ 8 \\ 10 \\ 13 \end{bmatrix}$

$$(X^T X)^{-1} = \begin{bmatrix} 0.5278 & -0.0833 \\ -0.0833 & 0.0167 \end{bmatrix}$$

This gives us $a_0 = -2.0139$ $a_1 = 1.4583$.

Now, $y_{AVG} = \frac{\sum y_i}{n} = \frac{47.5}{9} = 5.277$

$\Rightarrow S_T = \sum (y_i - y_{AVG})^2 = 139.5556.$

Date / /
Page No.

y_{linear} at given x points

$$= [-0.5556 \quad 0.9028 \quad 2.3611 \quad 3.8194 \quad 5.2778 \quad 6.7361 \quad 8.1944 \quad 9.6528 \quad 11.1111]$$

$$S_R = \sum (y_i - y_{linear})^2 = 11.9514$$

$$\Rightarrow r^2 = \frac{S_T - S_R}{S_T} = 0.914361.$$

Similarly, for parabolic regression $\Rightarrow$

$$y = a_0 + a_1 x + a_2 x^2$$

$$\underbrace{\begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \\ \vdots & \vdots & \vdots \end{bmatrix}}_{X} \underbrace{\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}}_{A} = \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \end{bmatrix}}_{Y}$$

where

$$X = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \\ 1 & 5 & 25 \\ 1 & 6 & 36 \\ 1 & 7 & 49 \\ 1 & 8 & 64 \\ 1 & 9 & 81 \end{bmatrix}$$

Y is same as before.

Date / /
Page No.

$$\Rightarrow A = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = (X^T X)^{-1} X^T Y.$$

where $(X^T X)^{-1} = \begin{bmatrix} 1.6190 & -0.6786 & 0.0595 \\ -0.6786 & 0.3413 & -0.0325 \\ 0.0595 & -0.0325 & 0.0032 \end{bmatrix}$

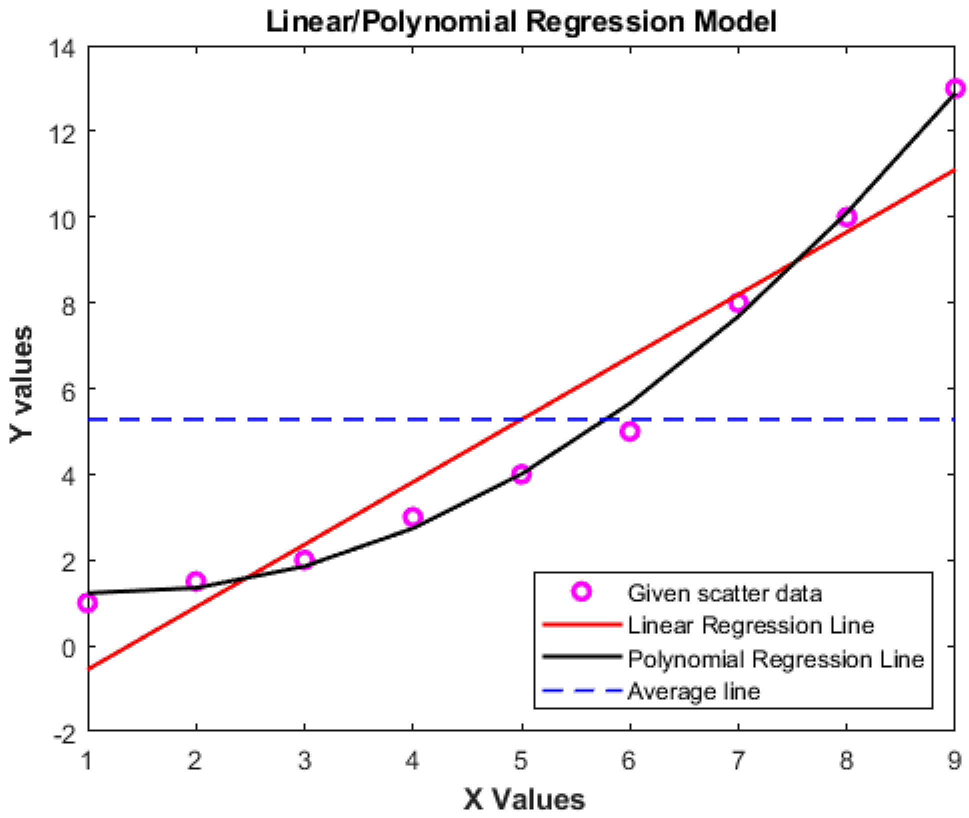
$a_0 = 1.4881 \quad a_1 = -0.4518 \quad a_2 = 0.1910$

$$S_R = \sum (y_i - y_{poly})^2 = 0.7132$$

$$\Rightarrow r^2 = \frac{S_T - S_R}{S_T} = 0.9949$$

```
clc;
clear all;
clear;

x=1:9;
y= [1 1.5 2 3 4 5 8 10 13];

[a0, a1]= linear_regression_model(x,y);

[b0, b1, b2]= polynomial_regression_model(x,y);

y_poly= b0 + b1*x + b2*x.^2;

y_avg= (sum(y)/length(y))*ones([1,length(y)]);
y_linear = a0 + a1*x;

St= sum((y-y_avg).^2);
Sr_linear= sum((y-y_linear).^2);
Sr_poly= sum((y-y_poly).^2);

fprintf('Coefficient of Determination (Linear): %f\n',(St-Sr_linear)/
St);
fprintf('Coefficient of Determination (Polynomial): %f\n',(St-
Sr_poly)/St);

figure(1);
plot(x,y,'om','Linewidth',2);
hold on;
plot(x, y_linear, 'r','Linewidth',1.5);
hold on;
plot(x, y_poly, 'k','Linewidth',1.5);
hold on;
plot(x, y_avg, '--b','Linewidth',1.2);
hold on;
title('Linear/Polynomial Regression Model');
legend('Given scatter data','Linear Regression Line','Polynomial
 Regression Line','Average line','Location','southeast');
xlabel('\bf X Values');
ylabel('\bf Y values');
```

```
Coefficient of Determination (Linear): 0.914361
Coefficient of Determination (Polynomial): 0.994889
```



Published with MATLAB® R2021a

```
function [a0,a1]= linear_regression_model(x,y)
    % XA = Y
    n = length(x);
    X = zeros(n,2);

    X(:,1)= 1;
    X(:,2)= x;

    Y= y';

    % X'XA = X'Y
    X_trans= X';

    % A = inv(X'X)X'Y
    A = inv(X_trans*X)*X_trans*Y;
    a0 = A(1);
    a1 = A(2);
end
```

Published with MATLAB® R2021a

```matlab
function [b0,b1,b2]= polynomial_regression_model(x,y)
    % XA = Y
    n = length(x);
    X = zeros(n,3);

    X(:,1)= 1;
    X(:,2)= x;
    X(:,3)= x.^2;

    Y= y';

    % X'XA = X'Y
    X_trans= X';

    % A = inv(X'X)X'Y
    A = inv(X_trans*X)*X_trans*Y;
    b0 = A(1);
    b1 = A(2);
    b2= A(3);
end
```

Published with MATLAB® R2021a