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CL 249 Computational Methods Lab
Assignment 6: Numerical Integration

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Q. Obtain the value of the integration:

$$I = \int_0^{30} \left(\frac{250x}{x+6}\right) \cdot e^{-\frac{x}{10}} \, dx$$

using (a) trapezoidal rule

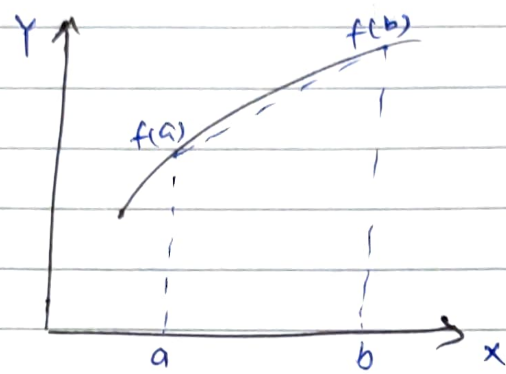
(b) Gauss-Quadrature rule.

Submit plot of of step size (h) vs integral value (I) using the two techniques.

Method

(i) Trapezoidal Rule

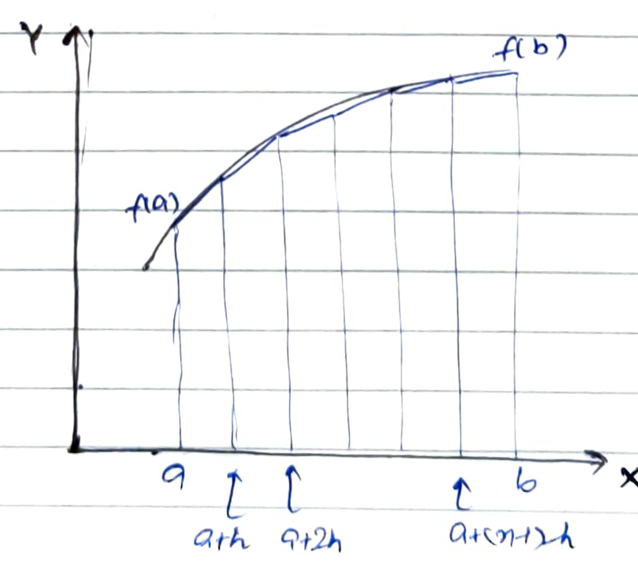
Assume a function, $y = f(x)$ as shown below:



For an approximate of $\int f(x)\,dx$, we calculate the area of the trapezium $= \left(\frac{b-a}{2}\right) \times \left(f(a) + f(b)\right)$

i.e. $\frac{(\text{height of interval})}{2} \left(f(x_i) + f(x_{i+1})\right)$

For better approximation, divide ~~interval~~ domain into equal intervals of h.



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Now, calculate area of each trapezium.

$$\text{Area} = \frac{h}{2}\Big[f(a) + f(a+h) \qquad \text{Ist Interval}$$

$$+ f(a+h) + f(a+2h) \qquad \text{IInd Interval}$$

$$+ \ldots$$

$$\vdots$$

$$+ f(a+(n-1)h) + f(b) \Big] \qquad \text{Nth Interval}$$

~~Here~~ $\Rightarrow \quad I = \frac{h}{2}\left(f(a) + 2\sum_{i=1}^{n-1} f(a+ih) + f(b) \right)$

Here, $n = \frac{(b-a)}{h}$

(ii) <u>Gauss-Quadrature Rule</u>.

For a function, $y = f(x)$

$$I = \int_a^b f(x)\,dx$$

we write it as $I = \int_{-1}^{1} g(t)\,dt$ using appropriate ~~xoy~~ $x-t$ relation.

Now, let's approximate $g(t)$ as a third degree polynomial

$$g(t) = mt^3 + nt^2 + pt + q$$

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$\therefore \int_{-1}^{1} g(t)\, dt = \frac{2n}{3} + 2q$

$\Rightarrow \frac{2n}{3} + 2q = C_0\, g(t_0) + C_1 g(t_1)$

$\because$ It's a general case, regardless of coefficient value, the answer remains same.

$\therefore$ After trying with different m, n, p, q values

$\Rightarrow C_0 = C_1 = 1 \quad t_0 = \frac{1}{\sqrt{3}} \quad t_1 = -\frac{1}{\sqrt{3}}$

After manipulations, we get

$x = \frac{(b-a)t}{2} + \frac{(b+a)}{2}$

$\Rightarrow g(t) = \frac{(b-a)}{2} f\left(\frac{(b-a)t}{2} + \frac{(a+b)}{2}\right)$

Divide ~~dom~~ limits into intervals of height h, and apply Gauss Quadrature separately, and add to get a better approximation.

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main.m

```
a=0 ;    b=30;

loop i = 1 to 300
        h(i) = (b-a)/i
End loop.

Call trapezoid_rule (h,a,b)
Call gauss-quadrature (h,a,b)
```

trapezoidal_rule.m

```
loop i = 1 to length of h

        Itemp = f(a) + f(b)
        loop j = 1 to i-1
              Itemp = Itemp + 2f(a+jh(i))
        end loop
        I(i) = Itemp . h(i)/2

end loop
```

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gauss_quadrature.m

loop $i = 1$ to length of h

$x_l = a$

$t_0 = 1/\sqrt{3}$

$t_1 = -1/\sqrt{3}$

$I_{temp} = 0$

loop $j = 1$ to i

$x_u = x_l + h(i)$

$$I_{temp} = I_{temp} + \frac{(x_u - x_l)}{2} * \left[f\left(\left(\frac{x_u - x_l}{2}\right)t_0 + \left(\frac{x_u + x_l}{2}\right)\right) + f\left(\left(\frac{x_u - x_l}{2}\right)t_1 + \left(\frac{x_u + x_l}{2}\right)\right)\right]$$

$x_l = x_u$

end loop

$I(i) = I_{temp}$

end loop

main.m

plot graph h vs I (trapezoidal)

plot graph h vs I (gauss Quadrature)

Define fn for returning function value at input x.

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Comments & Remarks.

The trapezoidal rule returns integral value which decreases as step size increases.

$$\text{Error} = \frac{-h^3}{12}\left(f''(\xi_1) + f''(\xi_2) + \cdots + f''(\xi_n)\right)$$

$$= \frac{-h^3}{12} \cdot n f''(\xi_0) = \frac{-h^3}{12} \cdot \frac{(b-a)}{h} f''(\xi_0) = \frac{-h^2}{12}(b-a) f''(\xi_0)$$

$$\Rightarrow \text{Error} \Rightarrow O(h^2)$$

The Gauss Quadrature is a cubic level approximation, and from plot, it is seen that, integral value increases as step size increases.

$$\text{Error} = O(h^5) f^{4}(\xi)$$ → for cubic expressions result are accurate.

hence, it gives better results.