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CL249 - Computational Methods Lab
Assignment 4 : Iterative Techniques

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Q. Given a 15x15 matrix A and vector b, solve the equation :

$$Ax = b$$

using the Gauss - Seidel and Jacobi method of iterative technique. use suitable initial guess and tolerance value for converge.

Compare the no. of operations for Jacobi, Gauss Seidel and Gauss Elimination method.

Method

(i) Jacobi Method

It is an iterative method to calculate solutions of the eq^n

$$Ax = b$$

Starting with an initial guess of

$$x^0 = (0\ 0\ 0\ \cdots\ 0)^T$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ \vdots \\ b_n \end{bmatrix}$$

$$\therefore \quad x_i^{(1)} = \frac{b_i - \sum\limits_{\substack{j=1 \\ j \neq i}}^{n} a_{ij} x_j^0}{a_{ii}}$$

For general k,

$$x_i^{(k+1)} = \frac{b_i - \sum\limits_{\substack{j=1 \\ j \neq i}}^{n} a_{ij} x_j^{(k)}}{a_{ii}}$$

Check for convergence ⇒

$$\max \left| \frac{x_i^{(k+1)} - x_i^{(k)}}{x^{(k+1)}} \right| < \text{tolerance.}$$

Gauss Seidel

Gauss Seidel is similar to Jacobi method, except that we use the latest value available for x in each iteration.

i.e.

After calculating x_1^1, latest value

$$x_2^1 = \frac{b_2 - \left(a_{11}^{?} x_1^1 + \sum_{j=3}^{n} a_{2j} x_j^0 \right)}{a_{22}}$$

∴ for general k,

$$x_i^{(k+1)} = \frac{b_i - \left(\sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} + \sum_{j=i+1}^{n} a_{ij} x_j^{(k)} \right)}{a_{ii}}$$

Use same convergence criteria.

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PSEUDOCODE

main.m

```
load A matrix from file
Initialize b vector

call Jacobi_method
call Gauss_seidel method
call Gauss Elimination method.
```

Jacobi_method.m

```
sz = length of b
x0 = x1 = zero vector

operations = 0
iter = 1e4.
ensure pivoting i.e. |Aii| >= |Aji|  ∀ j > i
while (iter ≠ 0)

    loop i = 1 to sz

        temp = 0
        loop j = 1 to sz
            if (j==i) skip (continue)
            temp = temp + A(i,j)·x0(j)
        end loop
    x1(i) = (b(i) - temp)/A(i,i)

    update operations.
    end loop
```

```
% calculate max error

max_error = max | (x1(j) - x0(j)) / x1(j) |  ∀ j ∈ [1, SZ]

% convergence cond.

if (max_error < tol)
    return x1.
else
    x0 = x1.
    iter = iter - 1.

end loop (while)
```

gauss-seidel.m

```
SZ = length of b
x0 = x1 = zero vectors

op = 0
iter = 1e6
ensure pivoting i.e. |aii| ≥ |aji| ∀ j > i
while (iter ≠ 0)

    loop i = 1 to SZ
        temp = 0

        loop j = 1 to i - 1.
            temp = temp + A(i,j) . x1(j)
        end loop
```

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```
loop j = i+1 to sz
        temp = temp + A(i,j) · x0(j)
end loop.

x1(i) = (b(i) - temp) / A(i,i)
update op

end loop
~~end while (loop)~~

% calculate max error
```

$$max\_error = \max \left| \frac{x_1(j) - x_0(j)}{x_1(j)} \right| \quad \forall j \in [1, s3]$$

```
% convergence condition

if (max_error < tol)
        return x1

else
        x0 = x1
        iter = iter - 1

end loop (while)
```

Comments & Remarks.

x vector for Jacobi method does not converge.

This is because the spectral radius > 1 for given matrix. (As taught in CL244).

Answer is obtained in Gauss Seidel method, but number of operations are remarkably higher than Gauss Eliminations due to many iterations.

for $\epsilon = 1e-10$, operations = 6.06 cr

as compared to Gauss Elimination's 2570

For perfect convergence, $\epsilon = 1e-11$.

No. of operations = 7.07 cr

$A^*x = b$ for Gauss Seidel & Gauss Elimination