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CL249 COMPUTATIONAL METHODS LAB

ASSIGNMENT 3 : LU DECOMPOSITION

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Q. Write a program based on the LU decomposition method to find the inverse of a given A matrix.

In main code, read A matrix from data file. Call code to find L and U matrix using Gauss Elimination method. Write function which takes L, U, B as input to solve

$$Ax = b$$

and returns x vector.

Method : LU decomposition

Using Gauss - Elimination method, we represent A matrix as

$$A = LU$$

S.t.

L is a lower triangular matrix with diagonal elements = 1.

U is an upper triangular matrix

Now, we know that for any inversible matrix A,

$$A A^{-1} = I$$

Hence, we apply LU decomposition separately for every column of I matric i.e.

$$A x_i = b_i$$

where b_i is i^{th} column of I and
x_i is i^{th} column of A^{-1}.

$$\Rightarrow \quad L U x_i = b_i$$

$$U x_i = d_i \quad \text{(back substitution)}$$

$$L d_i = b_i \quad \text{(forward substitution)}$$

PSEUDOCODE

main.m

Take matrix A as input

Calculate size of matrix A (size_A)

Feed A into LU_calc f^n to get the matrix L and U and count operations.

↓

LU_calc.m

Initialize L matrix to I of size_A.
Initialize counter to 0

Loop i: 1 to size_A

Ensure pivoting and max. diagonal element s.t.

$a_{ii} > \forall \; a_{ji}$ where $j = i+1$ to size_A

$a_{ii} \neq 0$

Loop j: i+1 to size_A

$$factor(j,i) = \frac{A(j,i)}{A(i,i)}$$

$$L(j,i) = factor(j,i)$$

$$A(j,k) = A(j,k) - factor(j,i) \cdot A(i,k)$$

$(k = 1$ to size_A$)$

Update counter

U = A

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main.m

loop i = 1 to Size_A

b is i^{th} column of I

feed L, U, B to inverse_calc

to get x(i^{th} col) and counter.

inverse_calc.m

Initialize d column vector to ZERO of length Size_A.

Initialize counter = 0

loop i : 1 to size_A

$$d(i) = B(i) - \sum_{j=1}^{i-1} L(i,j) \cdot d(j)$$

update counter.

Initialize x column vector to ZERO

loop i : Size_A to 1 :

$$x(i) = \frac{d(i) - \sum_{j=i+1}^{size\_A} U(i,j) \cdot x(j)}{U(i,i)}$$

update counter

main.m

Print x

Print counter

Comments and Remarks.

Gauss elimination method requires 2570 operation per b vector.

Hence, total operations for GEM $= 2570 \times 15$
$= 38,550$

LU decomposition method requires ~ 8870 operations

Hence, when iterations for b vector is comparable to size of A matrix,

LU decomposition is better than Gauss Elimination.