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CL 249 Computational Methods Lab

Assignment 7: Differential Equation (IVP)

Submitted By :

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Q. Solve ODE'S

$$\frac{dy}{dx} = -2y + 4e^{-x} \quad \text{and} \quad \frac{dz}{dx} = \frac{-yz^2}{3}$$

where $y = 2$ and $z = 4$ at $x = 0$

Solve using Euler's explicit technique to obtain values of y and z from $x=0$ to $x=4$. Also, find the converged/correct solution after plotting

(i) y vs x (ii) z vs x for various values of h.

METHOD

Euler's Explicit technique.

Consider a differential eq^n

$$\frac{dy}{dx} = f(x, y)$$

For a given point x_0, let $y = y_0$

~~Using~~ we can write, $x_i = x_0 + ih$

$y_i = y(x = x_i)$

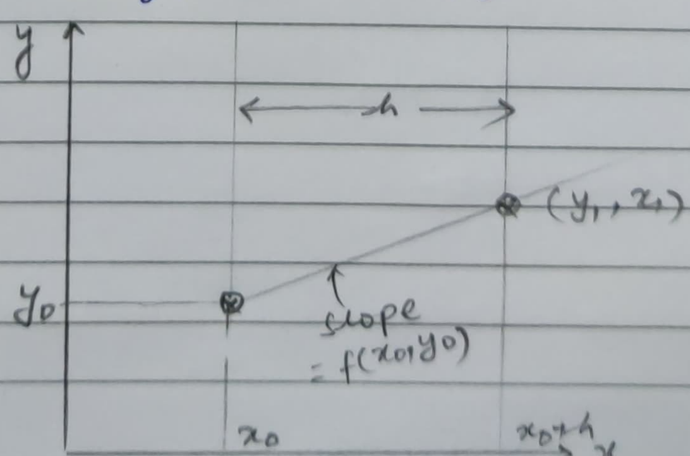
for a given step-size h.

Now, using Taylor's expansion,

$$y(x_0 + h) = y(x_0) + h y'(x_0) + \left[\frac{h^2}{2!} y''(x_0) + \dots \right. \text{, error}$$

$$\Rightarrow y_{i+1} = y_i + h\, y'(x_i)$$

$$\Rightarrow y_{i+1} = y_i + h f(x_i, y_i)$$



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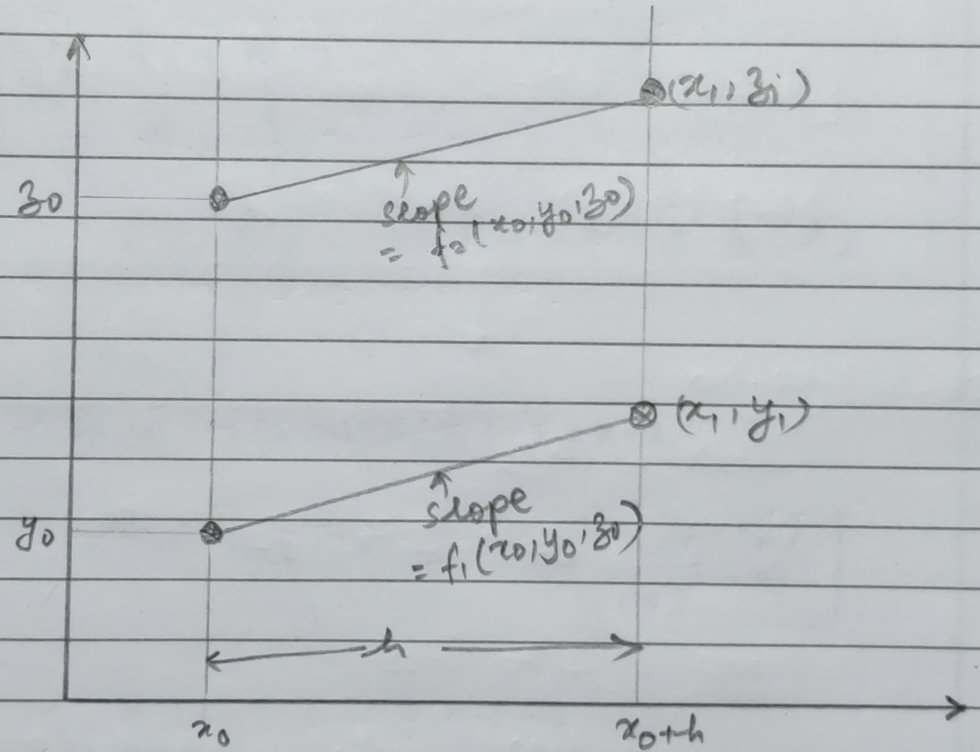
$\therefore$ for simultaneous differential eq^n, like

$$\frac{dy}{dx} = f_1(x, y, z) \quad \& \quad \frac{dz}{dx} = f_2(x, y, z)$$

$$y_{i+1} = y_i + h f_1(x_i, y_i, z_i)$$

$$z_{i+1} = z_i + h f_2(x_i, y_i, z_i)$$



PSEUDOCODE

main.m

```
h = 4 , x0 = 0 , y1 = 2 , y2 = 4
h1 = 0   h2 = 0
[yi , zi] = ODE_solver(x0, y1, y2, h);

plot (x, yi)   plot (x, zi)

loop i = 1 to n
    h = h/2

    [yj , zj] = ODE_solver(x0, y1, y2, h)

  err1 = 0
 loop k = 1 to length(yj)  k = k+2

    err_temp1 = | (yj(k) - yi((k+1)/2)) / yj(k) |

    if (err_temp1 > err1)
        err1 = err_temp1 .
  end loop
 err2 = 0
 loop l = 1 to length (zj)  l = l+2

    err_temp2 = | (zj(l) - zi((l+1)/2)) / zj(l) |

    if (err_temp2 > err2)
        err2 = err_temp2

 end loop
```

```
if(err1 > 1e-4)
    plot(x, yj), h1 = h
if(err2 > 1e-4)
    plot (x, zj), h2 = h

    yi = yj , zi = zj
end loop


h1 = h1/2 , h2 = h2/2

h = min(h1, h2)

[Y, Z] = ODE-solver(x0, y1, y2, h)

plot(x, y), plot(x, z)
```

ODE-Solver.m

```
iter = 4/h,
y(1) = y1 , z1 = y2 , x = x0

loop i = 1 to iter

    y(i+1) = y(i) + h* dy(x, y(i))
    z(i+1) = z(i) + h* dz(y(i), z(i))

    x = x+h

end loop
```

where dy and dz calculate diff value $\frac{dy}{dx}$ & $\frac{dz}{dx}$

Comments & Remarks

The error term in Euler's explicit method is given by:

$$\text{error} = \frac{h^2}{2} y''(\xi_p)$$

So, decreasing h, leads to better accuracy.

Plus, if we decrease h by $1/2$, the number of data points increase constantly.

It can be ensured that

$$\max \left| \frac{y(x_i, h_j) - y(x_i, h_{j+1})}{y(x_i, h_{j+1})} \right| < \epsilon$$

for common points in consecutive Sol^n.

This ensures converged solution.