

Q5. Given $y = a x e^{bx}$

$$\Rightarrow \log y = \log a + \log x + bx$$

$$\Rightarrow \log \left(\frac{y}{x} \right) = \log a + bx$$

$$\Rightarrow Y = A_0 + A_1 X$$

where $Y = \log \left(\frac{y}{x} \right)$ $X = x$.

Given data :-

x	0.1	0.2	0.4	0.6	0.9	1.3	1.5	1.7	1.8
y	0.75	1.25	1.45	1.25	0.85	0.55	0.35	0.28	0.18

The set of linear eqⁿ can be written as:

$$\begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_3 \\ 1 & x_4 \\ 1 & x_5 \\ 1 & x_6 \\ 1 & x_7 \\ 1 & x_8 \\ 1 & x_9 \end{bmatrix} \begin{bmatrix} A_0 \\ A_1 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \end{bmatrix}$$

$$\Rightarrow X A = Y$$

where $X = \begin{bmatrix} 1 & 0.1 \\ 1 & 0.2 \\ 1 & 0.4 \\ 1 & 0.6 \\ 1 & 0.9 \\ 1 & 1.3 \\ 1 & 1.5 \\ 1 & 1.7 \\ 1 & 1.8 \end{bmatrix}$ $Y = \log\left(\frac{Y}{X}\right) = \begin{bmatrix} 2.6149 \\ 1.8326 \\ 1.2879 \\ 0.7340 \\ -0.0572 \\ -0.8602 \\ -1.4553 \\ -1.8036 \\ -2.3026 \end{bmatrix}$

$$\Rightarrow XA = Y$$

$$X^T X A = X^T Y$$

$$A = (X^T X)^{-1} X^T Y.$$

where $(X^T X)^{-1} = \begin{bmatrix} 0.3718 & -0.2760 \\ -0.2760 & 0.2922 \end{bmatrix}$

$$\Rightarrow \text{This gives } a_0 = 2.2682 \quad a_1 = -2.4733$$

$$\text{Now, } a_0 = \ln a \Rightarrow a = e^{a_0} = 9.6618$$

$$b = a_1 = -2.4733$$

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clc;
clear all;
clear;

% y = axexp(bx) => ln(y/x) = ln(a) + bx

x= [0.1 0.2 0.4 0.6 0.9 1.3 1.5 1.7 1.8];
y= [0.75 1.25 1.45 1.25 0.85 0.55 0.35 0.28 0.18];

Y = log(y./x);
X = x;

[a0, a1]= linear_regression_model(X,Y);
a = exp(a0);
b = a1;

y_avg= (sum(y)/length(y))*ones([1,length(y)]);
y_poly= a.*x.*exp(b.*x);
St= sum((y-y_avg).^2);
Sr_poly= sum((y-y_poly).^2);
fprintf('Coefficient of Determination (Polynomial): %f\n',(St-
Sr_poly)/St);

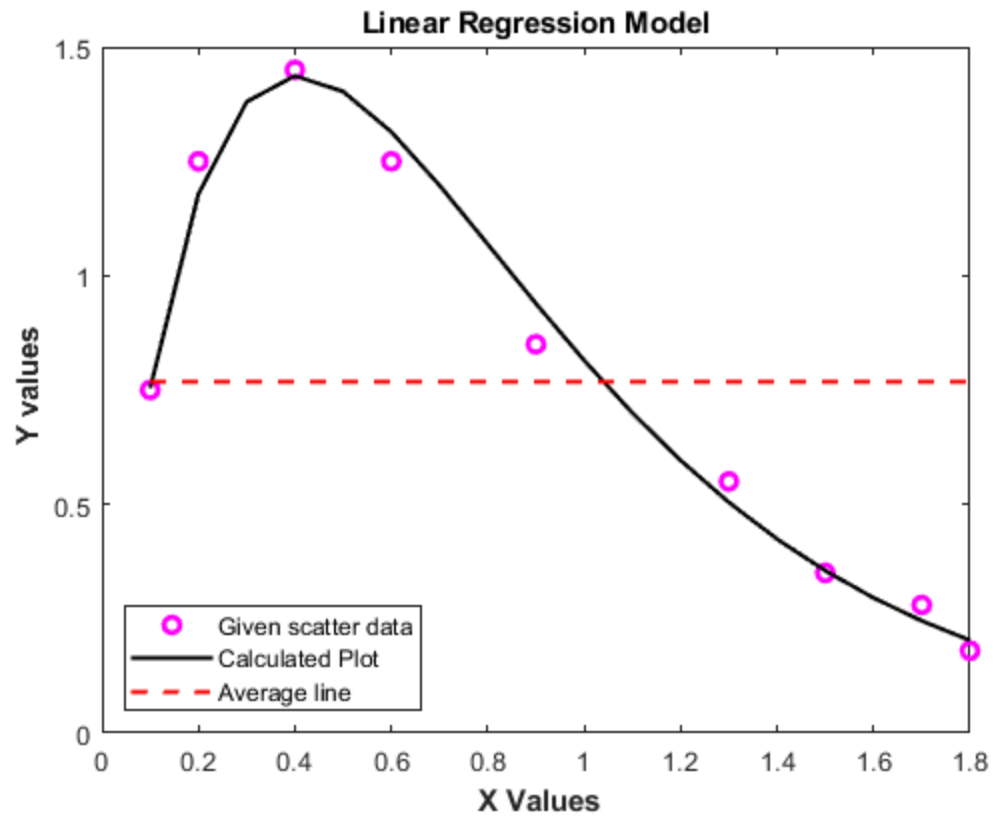
x_grid=0.1:0.1:1.8;
y_calc= a.*x_grid.*exp(b.*x_grid); % for higher resolution

y_avg= (sum(y)/length(y))*ones([1,length(y)]);

figure(1);
plot(x,y,'om','Linewidth',2);
hold on;
plot(x_grid, y_calc, 'k','Linewidth',1.5);
hold on;
plot(x, y_avg, '--r','Linewidth',1.5);
hold on;
title('Linear Regression Model');
legend('Given scatter data','Calculated Plot','Average
line','Location','southwest');
xlabel('\bf X Values');
ylabel('\bf Y values');

Coefficient of Determination (Polynomial): 0.987833

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function [a0,a1]= linear_regression_model(x,y)
    % XA = Y
    n = length(x);
    X = zeros(n,2);

    X(:,1)= 1;
    X(:,2)= x;

    Y= y';

    % X'XA = X'Y
    X_trans= X';

    % A = inv(X'X)X'Y
    A = inv(X_trans*X)*X_trans*Y;
    a0 = A(1);
    a1 = A(2);
end
```

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