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Q6. Given data :-

x	1	2	3	4	5	6	7	8	9
y	1	1.5	2	3	4	5	8	10	13

For linear regression model.

$$y = a_0 + a_1 x$$

$$A = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = (X^T X)^{-1} X^T Y$$

where $X = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \\ 1 & 6 \\ 1 & 7 \\ 1 & 8 \\ 1 & 9 \end{bmatrix}$ $\quad Y = \begin{bmatrix} 1 \\ 1.5 \\ 2 \\ 3 \\ 4 \\ 5 \\ 8 \\ 10 \\ 13 \end{bmatrix}$

$$(X^T X)^{-1} = \begin{bmatrix} 0.5278 & -0.0833 \\ -0.0833 & 0.0167 \end{bmatrix}$$

This gives us $a_0 = -2.0139$ $\quad a_1 = 1.4583$.

Now, $y_{AVG} = \dfrac{\sum y_i}{n} = \dfrac{47.5}{9} = 5.277$

$\Rightarrow S_T = \sum (y_i - y_{AVG})^2 = 139.5556.$

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y_{linear} at given x points

$$= \begin{bmatrix} -0.5556 & 0.9028 & 2.3611 & 3.8194 & 5.2778 & 6.7361 \\ & & 8.1944 & 9.6528 & 11.1111 \end{bmatrix}$$

$$S_R = \sum (y_i - y_{linear})^2 = 11.9514$$

$$\Rightarrow r^2 = \frac{S_T - S_R}{S_T} = 0.914361.$$

Similarly, for parabolic regression $\Rightarrow$

$$y = a_0 + a_1 x + a_2 x^2$$

$$\underbrace{\begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \\ \vdots & \vdots & \vdots \end{bmatrix}}_{X} \underbrace{\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}}_{A} = \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \end{bmatrix}}_{Y}$$

where

$$X = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \\ 1 & 5 & 25 \\ 1 & 6 & 36 \\ 1 & 7 & 49 \\ 1 & 8 & 64 \\ 1 & 9 & 81 \end{bmatrix}$$

Y is same as before.

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$$\Rightarrow A = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = (X^T X)^{-1} X^T Y.$$

where $(X^T X)^{-1} = \begin{bmatrix} 1.6190 & -0.6786 & 0.0595 \\ -0.6786 & 0.3413 & -0.0325 \\ 0.0595 & -0.0325 & 0.0032 \end{bmatrix}$

$a_0 = 1.4881 \quad a_1 = -0.4518 \quad a_2 = 0.1910$

$$S_R = \sum (y_i - y_{poly})^2 = 0.7132$$

$$\Rightarrow r^2 = \frac{S_T - S_R}{S_T} = 0.9949$$