Q1. (a)

(i) Jacobi method :

$$D x^{(k+1)} = -(L+U) x^{k} + b$$

$$\therefore S = D \qquad T = -(L+U)$$

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & -2 \end{bmatrix} \qquad T = \begin{bmatrix} 0 & -1 & -6 \\ -1 & 0 & 1 \\ -4 & -2 & 0 \end{bmatrix}$$

(ii) Gauss - Siedle :

$$(L+D) x^{(k+1)} = -U x^{k} + b$$

$$\therefore S = L+D \qquad T = -U$$

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 5 & 0 \\ 4 & 2 & -2 \end{bmatrix} \qquad T = \begin{bmatrix} 0 & -1 & -6 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

(iii) Successive Over Relaxation :

$$(D+\omega L) x^{(k+1)} = \left((1-\omega) D - \omega U\right) x^{k} + \omega b \qquad \omega = 1.2$$

$$S = D + \omega L \qquad T = (1-\omega) D - \omega U$$

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 1.2 & 5 & 0 \\ 4.8 & 2.4 & -2 \end{bmatrix} \qquad T = \begin{bmatrix} 0.2 & -1.2 & -7.2 \\ 0 & 1 & 1.2 \\ 0 & 0 & -0.4 \end{bmatrix}$$

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(b) No convergence for Jacobi method within 100 iteration.

Gauss-Siedel method : $\lambda_{max} = -9.3659$

SOR method : $\lambda_{max} = -14.2084$

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Q4.

$$A_1 = \begin{bmatrix} 4 & -1 & -1 & -1 \\ -1 & 4 & -1 & -1 \\ -1 & -1 & 4 & -1 \\ -1 & -1 & -1 & 4 \end{bmatrix}$$

Since matrix A is symmetric, we can use deflation method to obtain all eigen values

i.e. iterative use of power method.

$\because$ A is symmetric, its eigen vectors are orthogonal.

i.e. $x_i^T x_j = 0$ and $x_i^T x_i = 1$

$\hookrightarrow$ euclidean normalized.

if $|\lambda_1| > |\lambda_2| > |\lambda_3| - - - > |\lambda_n|$

we first compute λ_1 using power method.

Now, $A_2 = A_1 - \lambda_1 x_1 x_1^T$

eigen values of $A_2 = 0, \lambda_2, \lambda_3, \ldots, \lambda_n$

Hence, using power method, we can calculate λ_2 and continue similarly.