An Improved Pareto Genetic Algorithm for Multi-Objective TSP

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ABSTRACT

Multi-object traveling salesman problem (MOTSP) is a typical multi-object optimization problem. It requires to select a best route and make a balance between cost assignment and distance assignment of the route, the less cost of the whole travel and to satisfy the stipulate is the guide line. This paper gives the non-domination of genetic algorithm, and shows a simple model to put out the method that using multi-object genetic algorithm to solve the TSP. The algorithm use integer coding method, create an initial population that satisfies the basic qualification; calculate the two objective-value: distance and cost; then rank the chromosomes with Pareto function according to the objective-value; and use tournament selection to select the better chromosomes to form a series of parents, through multi-objective greedy crossover, and then use transposition mutation algorithm; we can get a new population that forms of new individuals based on genetic-searching function, and get the approximately best solution at last. The computing results of real examples of the MOTSP demonstrates that the approximate global optimal solution of the problem can be quickly obtained, and the solution with high accuracy.

1 Preface

Genetic algorithm is a kind of adaptive global searching probabilistic optimization algorithm, which simulates the process of organisms' inheritance and evolution. It has been widely applied to the areas of combinatorial optimization, pattern recognition, neural network and economic forecasts. In recent years, the potential of solving route optimization problems made a greater importance. The basic idea of genetic algorithm derived from inheritance and evolution. It mainly based on mechanism of biological evolution and genetics principle, in accordance with natural selection and survival of the fittest principle, using simple coding technology and reproduction mechanism and simulated natural evolutionary process of group's survival of the fittest to solve complex problems.

Using randomization techniques, genetic algorithm regards the individuals in group as operation objects and to achieve efficient search on coding space. Coding and forming initial population, determining the fitness function, designing the genetic algorithm and selecting control parameters are compositions of genetic algorithm's core contents.

Genetic algorithm has characteristics as follows: its searching progress don't react directly on variables, but react on the coded individual in the parameters set; the searching process is a process of iterative from one group of solutions to another, calculating a lot of individuals at the same tine so as to reduced the possibility of falling into the partial optimal solution and be easy to parallelization processing; using probabilistic rules to guide the searching direction instead of determinate ones; there is not any special requirement to the searching space and it need no more information such as derivative, gradient and the like, so it has broader applicable scope.

Traveling salesman problem is a typical combinatorial optimization problem, it has been determined to be a NP-hard problem, which can be described as: suppose that there are some cities numbered 1,2, ,n, the distance between two stochastic cities i and j is d[i][j]($i \neq j$), the problem is finding a shortest close route passing through all the cities and once for each city.

Based on this, Multi-object traveling salesman problem (MOTSP) adds a cost matrix and another object, which is less cost. So MOTSP is a typical multi-object optimization problem. It requires to select a best route and make a balance with cost assignment and distance assignment of the route, the less cost of the whole travel and to satisfy the stipulate is the guide line. Aim at this problem, this paper takes genetic algorithm as a foundation, designs a genetic optimization algorithm. The computing results of real examples of MOTSP demonstrates that the approximate global optimal

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solution of the problem can be quickly obtained, and the solution with high accuracy.

2 Some Concepts

Consider the multi-object optimization problem that has equality constraints and inequality constraints as follows:

The feasible region is:

$D=\{X\in E^n \mid g(X)\geq 0, h(X)=0\}, X=(x_1,x_2,\cdots,x_n)$.

Def 1: If there is an $X^* \in D$ (D is the feasible region), it makes any $X\in D$ meet the qualification $f_i(X^*)\leq f_i(X)$ (where $i=1,2,\cdots,m$), then X^* is the optimal solution of the multi-objective optimization problem.

Def 2: Supposes that $X_1, X_2 \in D$, to any i, $1\leq i\leq m$, if the qualification $f_i(X_1)\leq f_i(X_2)$ is tenable and there is a i causes $f_i(X_1)\leq f_i(X_2)$, then we can call X_1 dominant X_2.

Def 3:If there is a feasible solution X_p which dose not been dominated by any $X\in D$, then we call X_p Pareto optimal solution (or satisfactory solution).

All the set composed by Pareto optimal solution is called Pareto optimal solution set.

3. The mathematical model of MOTSP

Suppose that there are some cities numbered 1,2, , *n*, the distance between two stochastic cities *i* and *j* is dis[i][j](i≠j), and the cost between two stochastic cities is cos[i][j](i≠j), the mathematical model of MOTSP can be expressed as follows:

$$\min\begin{cases} f1(x)=\sum\limits_{i,j\in x} dis[i][j] \\ f2(x)=\sum\limits_{i,j\in x} \cos[i][j] \end{cases}$$

Where x is a route permutation of n numbers. So function $f1$ is the summation of the distance from one city to the next whose sequence follows the sequence in x and function $f2$ is the summation of the cost from one city to the next.

4. Algorithm description

4.1 Coding

In this algorithm, chromosome is composed by a series of integer queue, namely, the coding method is based on route expression. The gene code in each chromosome can not be appeared twice, in order to satisfy the restriction that one city should be passes and be passed only once. For example, there are 10 cities, then an individual $x=[3,4,8,5,9,6,2,7,1,10]$ is the route 3-4-8-5-9-6-7-1-10.

We use a system function "randperm(n)" in Matlab R2006a to create a random sequence which expresses a route passing all the cities, the "n" in the function expresses there are n cities in all.

4.2 Fitness function

The genetic algorithm does not use more extraneous information in the evolution search; but take the fitness function as the basis only, and do searching process by each individual's fitness. Therefore the fitness function can affect directly whether the global optimal solution of the problem can be quickly obtained or be with high accuracy. Here, we use the Pareto sort algorithm, namely, determine the fitness of each individual by non-dominate classification idea.

Description of Pareto sort algorithm:

① k = 1；

② All the individuals uncontrolled in population P compose set A, and note all the individuals serial number use k;

③ Delete the individuals in A from P;

④ If P is null, the algorithm end, distribute fitness according to its k rank;

⑤ let k = k + 1, go to ②.

There, k is the rank we set for the individuals in the population. Then decide the individuals' fitness according the rank. The higher the rank is(i.e. the smaller the k is) the bigger its fitness is, the stronger survivability is, and it have bigger probability to give the next generation as hereditary feature.

Then we calculate the fitness use k. Because we will think a lot of the relative value of fitness in our next tournament selection, we just record the fitness as $\frac{1}{k}$.

4.3 Selection

Selection is a progress selecting the individual, which has the high fitness from the current population to produce the mating pool. Its purpose is selecting better individuals to reproduce the next generation as parents. The standard to judge if an individual is fine or not is their respective fitness. Obviously this operation works by dint of the Darwin survival of the fittest evolution principle, namely the high fitness the individual has, the more opportunity it will obtain. There are many ways to do this operation.

In this paper, we use tournament selection and do the operation based on fitness. Determine that the size of tournament N is 2, that is to say, choose two individuals randomly to be compared every time. Suppose that there are two individuals A and B, if the fitness of A is bigger than B, then A is selected to the mating pool, equally, if the fitness of B is bigger than A, then B is selected to the mating pool, if the two individuals' fitness is the same, the computer will give a random number n between 0 and 1, if $0 < n \le 0.5$, we choose individual A, if $0.5 < n \le 1$, we choose individual B. Each individual's selecting probability matches its fitness. That is to say, the high fitness the individual has, the more opportunity it will obtain.
Suppose the population size is M, repeat the progress above M times. We can get M individuals in mating pool.

4.4 Crossover

The crossover operation is a progress in which we choose two individuals randomly according to a certain probability and exchanges the two-parent individual's partial structures randomly to produce two new individuals. The two new individuals produced by crossover operation contain both the parent individuals' genetics, but is different from their parents. Therefore, the crossover operation can enhance the genetic algorithm's searching ability, under the suitable choice strategy, it also can enhance the convergence rate of searching the global optimal solution.
In this paper, we use greedy crossover operator, which can optimize the fitness effectively. The greedy crossover operator used in MOTSP is different from that used in TSP. It not only can get new individuals with high fitness, but also make the individuals dispersed, so it can avoid converging prematurely in a way.
Choose a pair of parents
$p1(m11, m12, m13, \cdots m1n)$,
$p2(m21, m22, m23, \cdots m2n)$ from the mating pool. The greedy crossover operator that produces two new individuals $p1'$ and $p2'$ can be described as follows:

① Determine a city d as the start point of crossover, set d as the first genetic point.

② Find the right cities $dr1$ and $dr2$ from $p1$ and $p2$, and calculate the two distance values of $(d, dr1)$ and $(d, dr2)$, $j1$ and $j2$.

□ If $j1 < j2$, put $dr1$ as the second point of $p1'$, delete d from $p1$ and $p2$, $d = dr1$, go to ⑤.

④ If $j1 > j2$, put $dr2$ as the second point of $p1'$, delete d from $p1$ and $p2$, $d = dr2$.

□ If the number of elements in $p1$ and $p2$ is 1, progress end. Else, go to □.

On the same theory, change the right cities in step ② to left cities $dl1$ and $dl2$, compare the cost values of $(d, dl1)$ and $(d, dl2)$ step by step, then determine the new individual $p2'$.
For example, we have the distance value matrix of 5 cities as matrix 1 and cost value matrix of 5 cities as matrix 2, suppose that the parents are $p1 = (3,2,5,4,1)$ and $p2 = (4,3,5,1,2)$.

$$\begin{pmatrix} 0 & 23 & 25 & 15 & 17 \\ 23 & 0 & 36 & 9 & 18 \\ 25 & 36 & 0 & 23 & 4 \\ 15 & 9 & 23 & 0 & 16 \\ 17 & 18 & 4 & 16 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 30 & 13 & 6 & 24 \\ 30 & 0 & 18 & 8 & 19 \\ 13 & 18 & 0 & 22 & 42 \\ 6 & 8 & 22 & 0 & 21 \\ 24 & 19 & 42 & 21 & 0 \end{pmatrix}$$

$matrix\,1$ $\qquad$ $matrix\,2$

Select city 3 as the start city, follow the operator above, we can get two new individuals $p1' = (3,5,4,1,2)$ and $p2' = (3,1,4,2,5)$. The distance value of $p1'$ is 94, it's less than the distance value of $p1$, which is 110. The cost value of $p2'$ is 88, it's less than the cost value of $p2$, which is 126.

4.5 Mutation

For TSP, exchange mutation, insert mutation and inverse mutation have been known. Except inverse mutation, these mutate operator didn't consider the adjacent relation of cities. If we can't hold the intrinsic adjacent relation, we would not transmit the dominant character in the route to next population, and would not increase the searching speed.
Use a kind of greedy inverse mutation, firstly, decide a beginning city m1 randomly, then find the city m2 having the shortest distance with m1 in the cities after the nearest city on m1's right. Secondly, transpose the cities from m1 to m2 and complete the mutation. At the same time, calculate the fitness after mutation, if fitness hasn't been optimized, then keep the individual there.
The principle of inverse mutation is not difficult. For instance, there is a individual P=(2 6 5 1 3 7 4), select a mutate point 6, realize that the nearest point with 6 is 7, so take 7 as the second mutate point, transpose the cities form 5 to 7, and the individual after mutation is P'=(2 6 7 3 1 5 4).

5. Example

Multi-object traveling salesman problem requires to select a best route and make a balance with cost assignment and distance assignment of the route, the less cost of the whole travel and to satisfy the stipulate is the guide line.

We use an example from references [8] to check the algorithm in this paper. There are 6 cities in this example, the distance value matrix and the cost value matrix are shown as follows:

$$\underset{dist\ matrix}{\begin{pmatrix} 0 & 81 & 72 & 55 & 81 & 3 \\ 81 & 0 & 3 & 44 & 9 & 40 \\ 72 & 3 & 0 & 87 & 77 & 21 \\ 55 & 44 & 87 & 0 & 67 & 25 \\ 81 & 9 & 77 & 67 & 0 & 93 \\ 3 & 40 & 21 & 25 & 93 & 0 \end{pmatrix}} \underset{\cos t\ matrix}{\begin{pmatrix} 0 & 82 & 14 & 14 & 43 & 47 \\ 82 & 0 & 61 & 76 & 29 & 47 \\ 14 & 61 & 0 & 29 & 31 & 51 \\ 14 & 76 & 29 & 0 & 78 & 67 \\ 43 & 29 & 31 & 78 & 0 & 28 \\ 47 & 47 & 51 & 67 & 28 & 0 \end{pmatrix}}$$

Run the program at Matlab R2006a, set the crossover probability $p_c = 1$ and the mutation probability $p_m = 0.04$. After 5 times' iterative using 3 seconds in average, we can obtain 4 satisfactory solutions, the best routes are p1=(1,6,3,2,5,4), p2=(4,1,6,5,2,3), p3=(4,1,6,2,5,3), p4=(2,5,4,1,6,3) The distance value and the cost value of the 4 routes are (158,280), (250,208), (271,197),(209,248). Form the result of the example, we can see that compared to traditional genetic algorithms, the algorithm in this paper let solution be quickly obtained, and the solution with high accuracy. What's more, we get more satisfactory solutions than former algorithms referred in reference.

6. Conclusion

One remarkable characteristic of modern science and technology development is the intercross, inter-penetrate and inter-promote of the life sciences and the engineering science. The vigorous development of genetic algorithm has embodies the characteristic and the tendency of science development. Genetic algorithm's connotation philosophy comes from such a long and excellent evolution process in which the nature biology evaluate themselves from preliminary and simple ones, to advanced and complex ones, and even humankind. It profits from Darwin's theory of natural selection and survival of the fittest and also the nature heredity mechanism; its essence is a kind of high effective parallel global searching method for problem solution. It can in the search process the automatic gain with the accumulation related search space knowledge, and controls the search process to obtain the optimal solution auto-adapted.

The algorithm in this paper is based on multi-object genetic algorithm; operate chromosomes with greedy inverse mutation operation especially. The operation combined crossover and mutation can make ensure the ability of searching and the solutions' global convergence. The computing results of real examples demonstrates that the solution can be quickly obtained, and the solution with high accuracy.

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