Q1. The value of x which satisfies the equation $\log_2(x^2-3)-\log_2(6x-10)+1=0$

Solution: $x^2-3>0, 6x-10>0 \Rightarrow x>\sqrt{3}$

Also $\log_2\left(\frac{x^2-3}{6x-10}\right)=-1 \Rightarrow \frac{x^2-3}{6x-10}=\frac{1}{2}$

$\Rightarrow \quad x^2-3=3x-5$

$\Rightarrow \quad x^2-3x+2=0 \Rightarrow x=1,2$

Thus, $\quad x=2$

Q2. Solve $\log_{10}(2^x+1)+x=\log_{10}(6)+x\log_{10}(5)$.

A. 4

B. 5

C. 2

D. 1

Solution:

The given expression is

$\log_{10}(2^x+1)+x=\log_{10}(6)+x\log_{10}(5)$

We know that $\log_m(x)+\log_m(y)=\log_m(xy)$ & $\log_m(x)+\log_m(y)=\log_m\left(\frac{x}{y}\right)$

$\Rightarrow \log_{10}(2^x+1)=x(\log_{10}(5)-\log_{10}(10))+\log_{10}(6)$

$\Rightarrow \log_{10}(2^x+1)=-\log_{10}(2)^x+\log_{10}(6)$

$\Rightarrow \log_{10}(2^x+1)+\log_{10}(2^x)=\log_{10}(6)$

$\Rightarrow \log_{10}[(2^x)(2^x+1)]=\log_{10}(6)$

Taking antilog on both sides, we get

$\Rightarrow(2^x)(2^x+1)=6$

$\Rightarrow (2^x)^2+2^x-6=0$

$\Rightarrow(2^x-2)(2^x+3)=0$

$\Rightarrow 2^x=2 \Rightarrow x=1$ Hence, $x=1$

Q3. $\log_{\frac{1}{3}}\left(x^2+2x\right)>0$**, if** x **belongs to the set**

A. $\left(-1-\sqrt{2},-1+\sqrt{2}\right)$

B. $(-\infty,\ -2)\cup(0,\ \infty)$

C. $\left(-1-\sqrt{2},-2\right)\cup\left(0,\sqrt{2}-1\right)$

D. None of these

Solution: As we know that $\log_a(b)>0$, if $0<a<1\&0<b<1$.

Given, $\log_{\frac{1}{3}}\left(x^2+2x\right)>0$

$\Rightarrow 0<x^2+2x<1$

Breaking into two cases:

Case I : $x^2+2x>0$

$\Rightarrow x(x+2)>0$

$\Rightarrow x\in(-\infty,-2)\cup(0,\infty) \quad \ldots(1)$

Case II : $x^2+2\text{x}<1$

$\Rightarrow x^2+2x-1<0$

$\Rightarrow\left(x+1+\sqrt{2}\right)\left(x+1-\sqrt{2}\right)<0$

$\Rightarrow -1-\sqrt{2}<x<-1+\sqrt{2} \quad \ldots(2)$

From equation (1) and (2),we get

$x\in\left(-1-\sqrt{2},-2\right)\cup\left(0,\sqrt{2}-1\right)$

Thus, $\left(-1-\sqrt{2},-2\right)\cup\left(0,\sqrt{2}-1\right)$ is correct option.

Q4. If $\log_{175}(5x)=\log_{343}7x$, then the value of $\log_{42}\left(x^4-2x^2+7\right)$ is equal to

A. 1

B. 2

C. 3

D. 4

Solution:

$\log_{175}5x=\log_{343}7x=k$

$\Rightarrow \frac{5}{7}=\left(\frac{175}{343}\right)^k \Rightarrow k=\frac{1}{2}\Rightarrow x=\sqrt{7}.$

Q5. Sum of all possible values of x which satisfy the equation $\log_3(x-3)=\log_9(\mathrm{x}-1)$ is:

A. 2

B. 5

C. 7

D. 10

Solution:

We have,

$\log_3(x-3)=\log_9(\mathrm{x}-1)$

$\Rightarrow \frac{\log(x-3)}{\log 3}=\frac{\log(\mathrm{x}-1)}{\log 9}$

$\Rightarrow \frac{\log(x-3)}{\log 3}=\frac{\log(\mathrm{x}-1)}{\log 3^2}$

$\Rightarrow \frac{\log(x-3)}{\log 3}=\frac{\log(\mathrm{x}-1)}{2\log 3}$

$\Rightarrow 2\log(x-3)=\log(\mathrm{x}-1)$

$\Rightarrow \log(x-3)^2=\log(\mathrm{x}-1)$

$\Rightarrow (\mathrm{x}-3)^2=(\mathrm{x}-1)$

$\Rightarrow \mathrm{x}^2-7\mathrm{x}+10=0$

$\Rightarrow \mathrm{x}=2, 5$

$\mathrm{x}=2$ is not possible as $\log_3(x-3)$ is not defined for $x=2$.

Therefore, $x=5$.

Q6. $\begin{vmatrix} 5^{\sqrt{\log_5 3}} & 5^{\sqrt{\log_5 3}} & 5^{\sqrt{\log_5 3}} \\ 3^{-\log_{1/3}(4)} & (0.1)^{\log_{0.01}(4)} & 7^{\log_7(3)} \\ 7 & 3 & 5 \end{vmatrix}$ **is equal to**

A. 0

B. $5^{\sqrt{\log_5 3}}$

C. $2 \cdot 5^{\sqrt{\log_5 3}}$

D. None of these

Solution:

Let

$$D = \begin{vmatrix} 5^{\sqrt{\log_5 3}} & 5^{\sqrt{\log_5 3}} & 5^{\sqrt{\log_5 3}} \\ 3^{-\log_{1/3}(4)} & (0.1)^{\log_{0.01}(4)} & 7^{\log_7(3)} \\ 7 & 3 & 5 \end{vmatrix}$$

$C_2 \leftrightarrow C_2 - C_1$ and $C_3 \leftrightarrow C_3 - C_1$

$$\Rightarrow D = \begin{vmatrix} 5^{\sqrt{\log_5 3}} & \left[5^{\sqrt{\log_5 3}} - 5^{\sqrt{\log_5 3}}\right] & \left[5^{\sqrt{\log_5 3}} - 5^{\sqrt{\log_5 3}}\right] \\ 3^{-\log_{1/3}(4)} & \left[\left(0.1\right)^{\log_{0.01}(4)} - 3^{-\log_{1/3}(4)}\right] & \left[7^{\log_7(3)} - 3^{-\log_{1/3}(4)}\right] \\ 7 & -4 & -2 \end{vmatrix}$$

$$\Rightarrow D = \begin{vmatrix} 5^{\sqrt{\log_5 3}} & 0 & 0 \\ 3^{-\log_{1/3}(4)} & \left[\left(0.1\right)^{\log_{0.01}(4)} - 3^{-\log_{1/3}(4)}\right] & \left[7^{\log_7(3)} - 3^{-\log_{1/3}(4)}\right] \\ 7 & -4 & -2 \end{vmatrix}$$

$$\Rightarrow D = \begin{vmatrix} 5^{\sqrt{\log_5 3}} & 0 & 0 \\ 3^{-\log_{1/3}(4)} & \left[(4)^{\log_{0.01}(0.1)} - (4)^{-\log_{1/3}(3)}\right] & \left[3^{\log_7(7)} - (4)^{-\log_{1/3}(3)}\right] \\ 7 & -4 & -2 \end{vmatrix}$$

$\left[\because c^{\log_a(b)} = b^{\log_a(c)}\right]$

$$\Rightarrow D = \begin{vmatrix} 5^{\sqrt{\log_5 3}} & 0 & 0 \\ 3^{-\log_{1/3}(4)} & \left[(4)^{\frac{1}{2}} - (4)^1\right] & \left[3^1 - (4)^1\right] \\ 7 & -4 & -2 \end{vmatrix}$$

$\left[\because \log_a(b) = \frac{1}{\log_b(a)}\right]$

$$\Rightarrow D \begin{vmatrix} 5\sqrt{\log_5 3} & 0 & 0 \\ 4 & -2 & -1 \\ 7 & -4 & -2 \end{vmatrix} = 0$$

Q7. The set of all solutions of the equation $\log_3 x \log_4 x \log_5 x = \log_3 x \log_4 x + \log_4 x \log_5 x + \log_5 x \log_3 x$ **is**

A. $\{1\}$

B. $\{1, 60\}$

C. $\{1, 5, 10, 60\}$

D. $\{1, 4, 8, 60\}$

Solution: For $x = 1$, both parts of the equation vanish, consequently $x = 1$ is root of the equation. For $x \neq 1$

$$1 = \frac{1}{\log_5 x} + \frac{1}{\log_3 x} + \frac{1}{\log_4 x} = \log_x 5 + \log_x 3 + \log_x 4$$
$$= \log_x 60$$

$\Rightarrow x = 60$. Thus the required set is $\{1, 60\}$.

Q8. If $n > 1$**, the value of** $\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \ldots + \frac{1}{\log_{53} n}$ **is**

A. $\frac{1}{\log_{53!} n}$

B. 1

C. $\frac{1}{\log_{n!} 53}$

D. $\frac{1}{53}$

Solution: The given expression is equal to

$$\log_n 2+\log_n 3+\ldots+\log_n 53$$
$$=\log_n(2.3\ldots 53)=\log_n 53!=\frac{1}{\log_{53!} n}$$

Q9. The solution of the equation $4^{\log_2 \log x}=\log x-(\log x)^2+1$ is

A. $x=1$

B. $x=4$

C. $x=e$

D. $x=e^2$

Solution: $\log_2 \log x$ is meaningful if $x>1$

Since $4^{\log_2 \log x}=2^{2\log_2 \log x}=\left(2^{\log_2 \log x}\right)^2=(\log x)^2$

$\left[a^{\log_a x}=x, a>0, a\neq 1\right]$

So the given equation reduces to

$$2(\log x)^2-\log x-1=0$$
$$\Rightarrow \log x=1, \log x=-1/2. \text{ But for } x>1$$
$$\log x>0 \text{ so } \log x=1 \text{ i.e. } x=e$$

Q10. Suppose $x,\ y,\ z>0$ and distinct and $\ln x+\ln y+\ln z=0$, if the value of $x^{\frac{1}{\ln y}+\frac{1}{\ln z}}\cdot y^{\frac{1}{\ln z}+\frac{1}{\ln x}}\cdot z^{\frac{1}{\ln x}+\frac{1}{\ln y}}$ is e^{-k}, then $k=$

Solution: Let $X=x^{\frac{1}{\ln y}+\frac{1}{\ln z}}.y^{\frac{1}{\ln z}+\frac{1}{\ln x}}.z^{\frac{1}{\ln x}+\frac{1}{\ln y}}$

$$\Rightarrow \quad \ln x=\ln x\left(\frac{1}{\ln y}+\frac{1}{\ln z}\right)+(\ln y)\left(\frac{1}{\ln z}+\frac{1}{\ln x}\right)+\left(\frac{1}{\ln x}+\frac{1}{\ln y}\right)(\ln z)$$

Now given $\ln x+\ln y+\ln z=0$

$\therefore \quad \frac{\ln x}{\ln y}+\frac{\ln z}{\ln y}=-1$

Similarly $\frac{\ln y}{\ln x}+\frac{\ln z}{\ln x}=-1$ and

$\frac{\ln x}{\ln z}+\frac{\ln y}{\ln z}=-1$

$\therefore \quad R.H.S. \ =-3$

$\therefore \quad \ln X=-3$

$X=e^{-3}$

Q11. The solution set of $\log_{|\sin x|}\left(x^2-8x+23\right)>\frac{3}{\log_2|\sin x|}$ contains

A. $x \in(3, \pi) \cup\left(\pi, \frac{3\pi}{2}\right) \cup\left(\frac{3\pi}{2}, 5\right)$

B. $x \in(3, \pi) \cup(\pi, 5)$

C. $x \in\left(3, \frac{5\pi}{2}\right)$

D. $x \in(2, 5\pi/2)$

Solution: $x \in(2n+1)\pi/2, n\pi$ where $n \in \mathbf{I}$. The given inequality can be written as $\frac{\log_2\left(x^2-8x+23\right)}{\log_2|\sin x|}>\frac{3}{\log_2|\sin x|}$

As $\log_2|\sin x|<0$, we get

$\log_2\left(x^2-8x+23\right)<3$

$\Rightarrow x^2-8x+23<2^3=8$

$\Rightarrow x^2-8x+15<0$

$\Rightarrow(x-5)(x-3)<0 \Rightarrow 3<x<5$

For $x \in(3,5), x \in \pi, \frac{\pi}{2}, \frac{3\pi}{2}$. Hence

$x \in(3, \pi) \cup\left(\pi, \frac{3\pi}{2}\right) \cup\left(\frac{3\pi}{2}, 5\right)$

Q12. The set of all x satisfying the equation $x^{\log_3 x^2+(\log_3 x)^2-10} = 1/x^2$ is

A. $\{1, 9\}$

B. $\{1, 9, 1/81\}$

C. $\{1, 4, 1/81\}$

D. $\{9, 1/81\}$

Solution: Taking log of both the sides with base 3, we have

$\left(\log_3 x^2 + (\log_3 x)^2 - 10\right)(\log_3 x) = -2\log_3 x$

This equation is equivalent to

$\log_3 x = 0$ or $2\log_3 x + (\log_3 x)^2 - 8 = 0$

$\Rightarrow \quad x = 1, \log_3 x = -1 \pm 3$ i.e. $\log_3 x = 2, \log_3 x = -4$

Hence $x = 1, 3^2, 3^{-4} = 1, 9, 1/81$

Q13. Consider the value of x which satisfies the following relation:

$\frac{6}{5} a^{\log_{11} x \cdot \log_{10} a \cdot \log_{10} 5} = 3^{\log_{10} \frac{x}{10}} + 9^{\log_{100} x + \log_4 2}$

This value of x lies between:

A. 10 and 20

B. 30 and 40

C. 75 and 85

D. 95 and 105

Solution: We use the fact that $\log_b a = \frac{\log_c a}{\log_c b}$ to simply both the sides.

$\frac{6}{5} a^{\frac{\log x}{\log a} \cdot \frac{\log a}{\log 10} \cdot \frac{\log 5}{\log a}} = 3^{\log_{10} x - 1} + 9^{\frac{1}{\log_{10} x + \frac{1}{2}}}$ ...(1)

Consider the term on the left side:

$a^{\frac{\log x}{\log a} \cdot \frac{\log 5}{\log_{10} 0}} = a^{\log_u x \cdot \log_{10} 5} = x^{\log_{10} 5} = 5^{\log_{10} x} \quad$ (how?)

Using this in (1), along with the substitution $\log_{10} x = t$, we have

$\frac{6}{5}5^t=3^{t-1}+9^{\frac{t+1}{2}}=3^{t-1}+3^{t+1}=3^t\left(3+\frac{1}{3}\right)=10.3^{t-1}$

$\Rightarrow 5^{t-2}=3^{t-2}\Rightarrow t=2\Rightarrow x=100$

Thus, the correct option is (D).

Q14. Solution set of the inequality $\log_x\left(2x^2+x-1\right)>\log_x(2)-1$ is

A. $(1/2,1)$

B. $(1/2,1)\cup(1,\infty)$

C. $(1,\infty)$

D. $(0,1)$

Solution: For (1) to hold, we must have

$x>0, x\neq 1$ and $2x^2+x-1>0$

$\Rightarrow x>0, x\neq 1$ and $(2x-1)(x+1)>0$

$\Rightarrow x>1/2, x\neq 1$

We can write (1) as

$\log_x\left(\frac{2x^2+x-1}{2}\right)>-1$ (2)

For $1/2<x<1$, (2) can be written as

$\frac{2x^2+x-1}{2}<\frac{1}{x}$

$\Rightarrow \quad 2x^3+x^2-x<2$

$\Rightarrow \quad 2\left(x^3-1\right)+x(x-1)<0$

$\Rightarrow \quad (x-1)\left(2x^2+3x+2\right)<0$

$\Rightarrow \quad x<1 \quad \left[\because 2x^2+3x+2>0 \forall x>0\right]$

For $x>1$, (2) can be written as

$$\frac{2x^2+x-1}{2}>\frac{1}{x}$$

$\Rightarrow \quad (x-1)\left(2x^2+3x+2\right)>0$

This is true for each $x>1$.

Thus, (1) holds for $1/2<x<1, x>1$.

Q15. Consider the equation $\log_{\sqrt{2}\sin x}\left(1+\cos x\right)=2, x\in\left[-\frac{\pi}{2},\frac{3\pi}{2}\right]$ If the sum of the roots is $\frac{p\pi}{q}$, where $GCD(p,q)=1$, then evaluate p^2+q^2

Solution: $\log_{\sqrt{2}\sin x}\left(1+\cos x\right)=2$

$\sqrt{2}\sin x\neq 1,\ \ \sqrt{2}\sin x>0,\ \ 1+\cos x>0$

$\Leftrightarrow \sin x\neq\frac{1}{\sqrt{2}}, \sin x>0$ and

$x\neq$ odd multiple of $\pi\Rightarrow x\in\left(0,\pi\right)-\left\{\frac{\pi}{4},\frac{3\pi}{4}\right\}$ (feasible region)

(i) $\Leftrightarrow\left(\sqrt{2}\sin x\right)^2=1+\cos x$

$\Leftrightarrow 2\sin^2 x=1+\cos x$

$\Leftrightarrow 2\cos^2 x+\cos x-1=0$

$\Leftrightarrow (2\cos x-1)(\cos x+1)=0$

$\Rightarrow \cos x=\frac{1}{2}\ \ \ldots\left[\cos x+1>0\right]$

$\Rightarrow x=\frac{\pi}{3}$

$\Rightarrow p=1, q=3$

$\Rightarrow p^2+q^2=10$

Q16. Solve the inequality $\frac{(x-2)^{10000}(x+1)^{253}\left(x-\frac{1}{2}\right)^{971}(x+8)^4}{x^{500}(x-3)^{75}(x+2)^{93}}\geq 0$

A. $(-\infty,-2)\cup(-1,0)\cup\left(0,\frac{1}{2}\right]\cup(3,\infty)$

B. $(-\infty,-2)\cup[-1,0)\cup\left(0,\frac{1}{2}\right]\cup(3,\infty)$

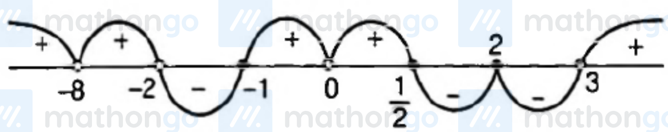
C. $(-\infty,-1]\cup\left(0,\frac{1}{2}\right]\cup(3,\infty)$

D. None of these

Solution: We have, $\frac{(x-2)^{10000}(x+1)^{253}\left(x-\frac{1}{2}\right)^{971}(x+8)^4}{x^{500}(x-3)^{75}(x+2)^{93}}\geq 0$

The critical points are $(-8)\cdot(-2),(-1),0,\frac{1}{2},2,3$

$[\because x\neq -2,0,3]$



Hence, $x \in (-\infty, -8] \cup [-8, -2) \cup [-1, 0) \cup \left(0, \frac{1}{2}\right] \cup (3, \infty)$

or $x \in (-\infty, -2) \cup [-1, 0) \cup \left(0, \frac{1}{2}\right] \cup (3, \infty)$

$\{2\}$ also satisfy the given inequality.

Hence, answer is option 4.

Q17. Let $f(x) = \frac{(x-3)(x+2)(x+6)}{(x+1)(x-5)}$ Find where $f(x)$ is negative.

A. $(-\infty, -6) \cup (-2, -1) \cup (3, 5)$

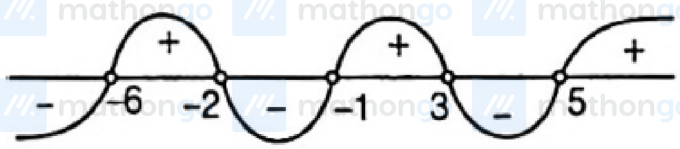
B. $(-\infty, -2) \cup (-1, 3) \cup (5, \infty)$

C. $(-\infty, -6] \cup (3, \infty)$

D. $(-\infty, -2) \cup (-1, 5)$

Solution: We have, $f(x) = \frac{(x-3)(x+2)(x+6)}{(x+1)(x-5)}$

The critical points are $(-6), (-2), (-1), 3, 5$



For $f(x) > 0, \forall x \in (-6, -2) \cup (-1, 3) \cup (5, \infty)$

For $f(x) < 0, \forall x \in (-\infty, -6) \cup (-2, -1) \cup (3, 5)$

Q18. Solve the equation $\left|\frac{x^2-8x+12}{x^2-10x+21}\right| = -\frac{x^2-8x+12}{x^2-10x+21}$

A. $[2,3)\cup[6,7]$

B. $[2,3]\cup[6,7)$

C. $[2,3)\cup[4,8)$

D. $[2,3)\cup[6,7)$

Solution: This equation has the form $|f(x)|=-f(x)$

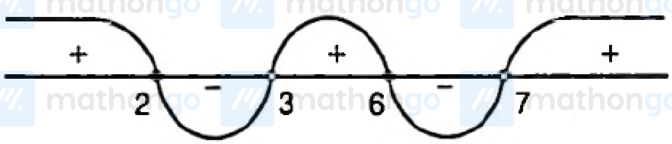
when, $f(x)=\frac{x^2-8x+12}{x^2-10x+21}$

such an equation is equivalent to the collection of systems

$$\begin{cases} f(x)=-f(x), \text{ if } f(x)\geq 0 \\ f(x)=f(x), \text{ if } f(x)<0 \end{cases}$$

The first system is equivalent to $f(x)=0$ and the second system is equivalent to $f(x)<0$ the combining both systems, we get

$$f(x)\leq 0$$
$$\therefore \frac{x^2-8x+12}{x^2-10x+21}\leq 0$$
$$\Rightarrow \frac{(x-2)(x-6)}{(x-3)(x-7)}\leq 0$$



Hence, by Wavy curve method,
$x\in[2,3)\cup[6,7)$

Q19. Solve the inequality $(x+3)(3x-2)^5(7-x)^3(5x+8)^2\geq 0$

A. $(-\infty,-3)\cup\left[\frac{2}{3},7\right)\cup\left(\frac{-8}{5}\right\}$

B. $(-\infty,-3]\cup\left[\frac{2}{3},7\right]\cup\left\{-\frac{8}{5}\right\}$

C. $\left(-\infty,\frac{2}{3}\right]\cup[7,\infty)$

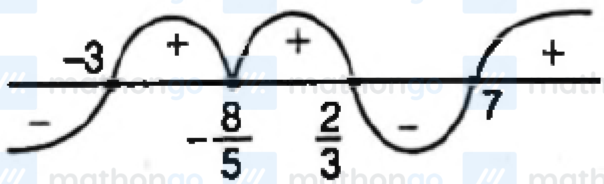
D. None of these

Solution: We have, $(x+3)(3x-2)^5(7-x)^3(5x+8)^2 \geq 0$

$\Rightarrow \quad -(x+3)(3x-2)^5(x-7)^3(5x+8)^2 \geq 0$

$\Rightarrow (x+3)(3x-2)^5(x-7)^3(5x+8)^2 \leq 0$

[take before x, $+$ ve sign in all brackets]



The critical points are (-3), $\left(-\frac{8}{5}\right)$, $\frac{2}{3}$, 7

Hence, $x \in (-\infty, -3] \cup \left[\frac{2}{3}, 7\right] \cup \left\{-\frac{8}{5}\right\}$

Q20. Find the number of integral values of x satisfying the inequation: $\frac{x}{x+2} \leq \frac{1}{|x|}$.

Solution: $\frac{x|x|}{x+2} \leq 1$

$\frac{x|x|-x-2}{x+2} \leq 0$

Case I $x \in [0, \infty)$

$\frac{x^2-x-2}{x+2} \leq 0$

$\Rightarrow \quad \frac{(x-2)\ (x+1)}{x+2} \leq 0$

$\Rightarrow \quad x \leq 2$

$\Rightarrow \quad$ integral values 0, 1, 2

Case II $x \in (-\infty,\ 0)$

$\frac{-x^2-x-2}{x+2} \leq 0$

$\Rightarrow \quad x > -2$

$\Rightarrow \quad x = -1$

So 4 integral values

Q21. Solve the inequation $\sqrt{(-x^2+4x-3)} > 6-2x$

A. $\left(\frac{12}{7}, 4\right)$
B. $\left(\frac{13}{5}, 4\right)$
C. $\left(\frac{13}{5}, 3\right)$
D. $\left(\frac{12}{7}, 3\right)$

Solution: We have, $\sqrt{(-x^2+4x-3)} > 6-2x$

This inequation is equivalent to the collection of two systems, of inequations

$$\text{i.e.} \left\{\begin{array}{c} 6-2x \geq 0 \\ -x^2+4x-3 > (6-2x)^2 \end{array}\right. \text{and} \left\{\begin{array}{c} 6-2x < 0 \\ -x^2+4x-3 \geq 0 \end{array}\right.$$

$$\Rightarrow \left\{\begin{array}{c} x \leq 3 \\ (x-3)(5x-13) < 0 \end{array}\right. \text{and} \left\{\begin{array}{c} x > 3 \\ (x-1)(x-3) \leq 0 \end{array}\right.$$

$$\Rightarrow \left\{\begin{array}{c} x > 3 \\ \frac{13}{5} < x < 3 \end{array}\right. \text{and} \left\{\begin{array}{l} x > 3 \\ 1 \leq x < 3 \end{array}\right.$$

The second system has no solution and the first system has solution in the interval $\left(\frac{13}{5} < x < 3\right\}$

Hence, $x \in \left(\frac{13}{5}, 3\right)$ is the set of solution of the original inequation.

Q22. Let [a] denotes the larger integer not exceeding the real number a. If x and y satisfy the equations $y=2[x]+3$ and $y=3[x-2]$ simulaneously, determine $[x+y]$

Solution: We have, $y=2[x]+3=3[x-2] \ldots$ (i)

$\Rightarrow \quad 2[x]+3=3([x]-2) \quad$ [from property (i)]

$\Rightarrow \quad 2[x]+3=3[x]-6$

$\Rightarrow \quad [x]=9$

From Eq. (i), $y=2 \times 9+3=21$

$\therefore \quad [x+y]=[x+21]=[x]+21=9+21=30$

Hence, the value of $[x+y]$ is 30

Q23. If $\{x\}$ and $[x]$ represent fractional and integral part of x respectively, find the value of $[x]+\sum_{r=1}^{2000} \frac{\{x+r\}}{2000}$

A. x

B. $x+\{x\}$

C. $x+[x]$

D. $2x+[x]$

Solution: $[x]+\sum_{r=1}^{2000} \frac{\{x+r\}}{2000}=[x]+\sum_{r=1}^{2000} \frac{\{x\}}{2000} \quad$ [from property (i)]

$=[x]+\frac{\{x\}}{2000} \sum_{r=1}^{2000} 1=[x]+\frac{\{x\}}{2000} \times 2000=[x]+\{x\}=x$

Q24. Solve the equation $|x-|4-x||-2x=4$

A. Two solutions

B. Three solutions

C. One solution

D. No solution

Solution: This equation is equivalent to the collection of systems

$$\left\{\begin{array}{l}|x-(4-x)|-2x=4, \text{ if } 4-x\geq 0\\ |x+(4-x)|-2x=4, \text{ if } 4-x<0\end{array}\right.$$

$$\Rightarrow\left\{\begin{array}{ll}|2x-4|-2x=4, & \text{if } x\leq 4\\ \quad 4-2x=4, & \text{if } x>4\end{array}\right. \quad \ldots(\mathrm{i})$$

The second system of this collection

gives $x=0$

but $x>4$

Hence, second system has no solution.

The first system of collection Eq. (i) is equivalent to the system of collection

$$\left\{\begin{array}{l}2x-4-2x=4, \text{ if } 2x\geq 4\\ -2x+4-2x=4, \text{ if } 2x<4\end{array}\right.$$

$$\Rightarrow \quad \left\{\begin{array}{l}-4=4, \text{ if } x\geq 2\\ -4x=0, \text{ if } x<2\end{array}\right.$$

The first system is failed and second system gives $x=0$.

Hence, $x=0$ is unique solution of the given equation.

Q25. The number of solution(s) the equation $|x-1|+|x-2|+|x-3|+|x-4|=3$ is

A. 2

B. 1

C. 0

D. 4

Solution:

As the minimum value of $|x-1|+|x-2|+|x-3|+|x-4|$ is 4.

Hence number of solutions $=0$

Q26. Find the set of all x for which $\frac{2x}{(2x^2+5x+2)}>\frac{1}{(x+1)}$

A. $\left(-2,-\frac{2}{3}\right) \cup\left(-\frac{1}{2}, \infty\right)$

B. $(-\infty,-2) \cup\left(-2,-\frac{2}{3}\right)$

C. $(-2,-1) \cup\left(-\frac{2}{3},-\frac{1}{2}\right)$

D. $\left(-2,-\frac{2}{3}\right) \cup\left(-\frac{1}{2}, 0\right)$

Solution: We have, $\frac{2x}{\left(2x^2+5x+2\right)}>\frac{1}{(x+1)}$

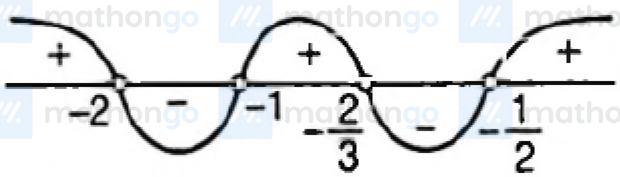
$\Rightarrow \quad \frac{2x}{(x+2)(2x+1)}-\frac{1}{(x+1)}>0$

$\Rightarrow \quad \frac{\left(2x^2+2x\right)-\left(2x^2+5x+2\right)}{(x+2)(x+1)(2x+1)}>0$

$\Rightarrow \quad -\frac{(3x+2)}{(x+2)(x+1)(2x+1)}>0$

or $\quad \frac{(3x+2)}{(x+2)(x+1)(2x+1)}<0$

The critical points are $(-2),(-1),\left(-\frac{2}{3}\right)\left(-\frac{1}{2}\right)$



Hence, $x \in(-2,-1) \cup\left(-\frac{2}{3},-\frac{1}{2}\right)$

Q27. Number of integral values of x satisfying the inequation $\frac{\left(x^2-2x+8\right)\left(e^x+2\right)(x-3)(x-8)}{\left(\log_2\left(x^2+3\right)\right)(x-5)^2} \leq 0$ are

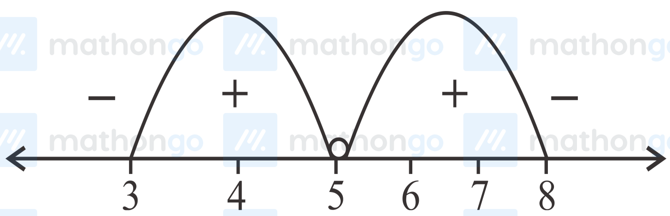
Solution:

$$\frac{\left(x^2-2x+8\right)\left(e^x+2\right)(x-3)(x-8)}{\left(\log_2\left(x^2+3\right)\right)(x-5)^2} \leq 0$$

x^2-2x+8, e^x+2 and $\log_2\left(x^2+3\right)$ are positive quantities

Next we have to find condition for $(x-3)$,$(x-5)$ and $(x-8)$

At $x=5$, the denominator $=0$. So $x=5$ is not a solution. Therefore, number of integral solutions will be between 3 and 8 excluding 5 (using wavy curve method)



Thus, we have 5 integral values possible.

Q28. Solution set of equation $\left|1-\log_{\frac{1}{6}} x\right|+\left|\log_2 x\right|+2=\left|3-\log_{\frac{1}{6}} x+\log_{\frac{1}{2}} x\right|$ is $\left[\frac{a}{b}, a\right], a, b \in N$, then the value of $\frac{(a+b)}{2}$ is

A. 5

B. 6

C. 7

D. 8

Solution: If $|a+b+c|=|a|+|b|+|c|$ then a, b, c have same sign

$\left|1-\log_{1/6} x\right|+\left|-\log_2 x\right|+|2|=\left|3-\log_{1/6} x-\log_2 x\right|$

$\therefore 1-\log_{1/6} x \geqslant 0$

$\frac{1}{6} \leqslant x$

$-\log_2 x \geqslant 0$

$x \leqslant 2$

$\therefore x \in\left[\frac{1}{6}, 2\right], a=2$ and $b=12$

$\frac{a+b}{2}=7$

Q29. Solve the inequation $\left|1-\frac{|x|}{1+|x|}\right| \geq \frac{1}{2}$.

A. $[-1,0]$

B. $[0,1]$

C. $[-1,1]$

D. $[-\infty,-1]$

Solution: The given inequation is equivalent to the collection of systems

$$\left\{\begin{array}{l}\left|1-\frac{x}{1+x}\right| \geq \frac{1}{2}, \text { if } x \geq 0 \\ \left|1+\frac{x}{1-x}\right| \geq \frac{1}{2}, \text { if } x<0\end{array} \Rightarrow\left\{\begin{array}{l}\frac{1}{|1+x|} \geq \frac{1}{2}, \text { if } x \geq 0 \\ \frac{1}{|1-x|} \geq \frac{1}{2}, \text { if } x<0\end{array}\right.\right.$$

$$\Rightarrow\left\{\begin{array}{l}\frac{1}{1+x} \geq \frac{1}{2}, \text { if } x \geq 0 \\ \frac{1}{1-x} \geq \frac{1}{2}, \text { if } x<0\end{array} \Rightarrow\left\{\begin{array}{l}\frac{1-x}{1+x} \geq 0, \text { if } x \geq 0 \\ \frac{1+x}{1-x} \geq 0, \text { if } x<0\end{array}\right.\right.$$

$$\Rightarrow\left\{\begin{array}{l}\frac{x-1}{x+1} \leq 0, \text { if } x \geq 0 \\ \frac{x+1}{x-1} \leq 0, \text { if } x<0\end{array}\right.$$

For $\frac{x-1}{x+1} \leq 0$, if $x \geq 0$

$\therefore \quad 0 \leq x \leq 1 \quad$ ...(i)

For $\frac{x+1}{x-1} \leq 0$, if $x<0$

$\therefore \quad -1 \leq x<0$ ...(ii)

Hence, from Eqs. (i) and (ii), the solution of the given equation is $x \in[-1,1]$

Aliter

$\left|1-\frac{|x|}{1+|x|}\right| \geq \frac{1}{2} \Rightarrow \left|\frac{1}{1+|x|}\right| \geq \frac{1}{2}$

$\Rightarrow \quad \frac{1}{1+|x|} \geq \frac{1}{2} \Rightarrow 1+|x| \leq 2$ or $|x| \leq 1$

$\therefore \quad -1 \leq x \leq 1$ or $x \Rightarrow [-1,1]$

Q30. Find the number of solution of the equation $[2x]-[x+1]=2x$ where $[\cdot]$ represent the greatest integer function.

A. 2

B. 3

C. 1

D. More than 3

Solution:

$[2x]-[x+1]=2x \quad \ldots(1)$

$-[x+1]=\{2x\},\ 0 \leq \{2x\} < 1,$

$-[x+1]=0,\ [x+1]=0$

$-1 \leq x < 0,\ -2 \leq 2x < 0$

$[2x]=\ -2,\ -1$

from equation (1)

$[2x]-0=2x,\ 2x=-2,\ -1$

$x=-1,\ -\frac{1}{2}$

Answer Key

Q1 (2)	**Q2** (4)	**Q3** (3)	**Q4** (1)
Q5 (2)	**Q6** (1)	**Q7** (2)	**Q8** (1)
Q9 (3)	**Q10** (3)	**Q11** (1)	**Q12** (2)
Q13 (4)	**Q14** (2)	**Q15** (10)	**Q16** (4)
Q17 (1)	**Q18** (4)	**Q19** (2)	**Q20** (4)
Q21 (3)	**Q22** (30)	**Q23** (1)	**Q24** (3)
Q25 (3)	**Q26** (3)	**Q27** (5)	**Q28** (3)
Q29 (3)	**Q30** (1)		