Q1. If $a_1+a_5+a_{10}+a_{15}+a_{20}+a_{24}=225$, then the sum of the first 24 terms of the arithmetic progression $a_1,\ a_2,\ a_3.....$ is equal to

A. 450

B. 675

C. 900

D. 1200

Ans: 900

Solution: $S=a_1+a_2+a_3+....+a_{23}+a_{24}$

$S=\frac{24}{2}(a_1+a_{24})=12(a_1+a_{24})$

Given that, $a_1+a_5+a_{10}+a_{15}+a_{20}+a_{24}=225$

$\Rightarrow a_1+a_1+\cancel{4d}+a_1+\cancel{9d}+a_{24}-\cancel{9d}+a_{24}-\cancel{4d}+a_{24}=225$

$\Rightarrow 3(a_1+a_{24})=225$

$\Rightarrow a_1+a_{24}=75$

$\Rightarrow S=12(a_1+a_{24})=12\times 75=900$

Q2. If $2,\ 7,\ 9$ and 5 are subtracted respectively from four numbers in geometric progression, then the resulting numbers are in arithmetic progression. The smallest of the four numbers is

A. -24

B. -12

C. 6

D. 3

Ans: -24

Solution: Let the numbers be $\mathrm{a}, \mathrm{ar}, \mathrm{ar}^2, \mathrm{ar}^3$,

$\Rightarrow \mathrm{a}-2, \mathrm{ar}-7, \mathrm{ar}^2-9$ and ar^3-5 are in A. P.

$\Rightarrow(\mathrm{a}-2)+\left(\mathrm{ar}^2-9\right)=2(\mathrm{ar}-7)$ and $(\mathrm{ar}-7)+\left(\mathrm{ar}^3-5\right)=2\left(\mathrm{ar}^2-9\right)$

$\Rightarrow \mathrm{a}+\mathrm{ar}^2=2 \mathrm{ar}-3$ and $\mathrm{ar}+\mathrm{ar}^3=2 \mathrm{ar}^2-6$

$\Rightarrow 1+\mathrm{r}^2=2 \mathrm{r}-\frac{3}{\mathrm{a}}$ and $1+\mathrm{r}^2=2 \mathrm{r}-\frac{6}{\mathrm{ar}}$

$\Rightarrow \frac{3}{\mathrm{a}}=\frac{6}{\mathrm{ar}} \Rightarrow \mathrm{r}=2$

$\Rightarrow \frac{3}{\mathrm{a}}=2 \mathrm{r}-1-\mathrm{r}^2=4-1-4=-1 \Rightarrow \mathrm{a}=-3$

So, the numbers are $-3,-6,-12,-24$

Q3. If $a,\ b\ \&\ 3c$ are in arithmetic progression and $a,\ b\ \&\ 4c$ are in geometric progression, then the possible values of $\frac{a}{b}$ are

A. $\left\{\frac{2}{3}, 2\right\}$

B. $\left\{\frac{3}{2}, \frac{1}{2}\right\}$

C. $\left\{\frac{2}{3}, \frac{3}{2}\right\}$

D. $\left\{\frac{1}{2}, 2\right\}$

Ans: $\left\{\frac{3}{2}, \frac{1}{2}\right\}$

Solution: $a+3c=2b$ and $b^2=4ac$

$\Rightarrow c=\frac{2b-a}{3}=\frac{b^2}{4a}$

$\therefore 8ab-4a^2=3b^2 \Rightarrow 4a^2-8ab+3b^2=0$

$4\frac{a^2}{b^2}-8\frac{a}{b}+3=0 \Rightarrow \frac{a}{b}=\frac{8\pm\sqrt{64-4\times4\times3}}{2\times4}$

$\frac{a}{b}=\frac{2\pm\sqrt{4-3}}{2}=\frac{2\pm1}{2}=\frac{3}{2},\frac{1}{2}$.

Q4. Let a_1, a_2, a_3 be three positive numbers which are in geometric progression with common ratio r. The inequality $a_3 > a_2 + 2a_1$ holds true if r is equal to

A. 2

B. 1.5

C. 0.5

D. 2.5

Ans: 2.5

Solution: $a_2 = a_1 r, a_3 = a_1 r^2$

$a_3 > a_2 + 2a_1$

$\Rightarrow a_1 r^2 > a_1 r + 2a_1$

$\Rightarrow r^2 - r - 2 > 0$

$\Rightarrow (r-2)(r+1) > 0$

$\Rightarrow r < -1$ or $r > 2$

Q5. If $|x| < 1,\ |y| < 1$, the sum to infinity of the series $(x+y),(x^2+xy+y^2),\ (x^3+x^2y+xy^2+y^3),\ldots$ Is -

A. $\frac{x+y-xy}{1-x-y+xy}$

B. $\frac{x+y+xy}{1-x-y+xy}$

C. $\frac{x}{1-x}+\frac{y}{1-y}$

D. $\frac{(x-y)(x+y-xy)}{1-x-y+xy}$

Ans: $\frac{x+y-xy}{1-x-y+xy}$

Solution:

As we know that $\frac{x+y}{1} \times \frac{x-y}{x-y} = \frac{x^2-y^2}{x-y}$ similarly $x^2+xy+y^2 = \frac{x^3-y^3}{x-y}$

$\Rightarrow \frac{x^2-y^2}{x-y} + \frac{x^3-y^3}{x-y} + \frac{x^4-y^4}{x-y} + \ldots \infty$

$\Rightarrow \quad \frac{1}{x-y}\left[\left(x^2+x^3+\ldots\infty\right)-\left(y^2+y^3+\ldots\infty\right)\right]$

Now we will find the sum of infinite G. P.series

$\Rightarrow \quad \frac{1}{x-y}\left[\frac{x^2}{1-x} - \frac{y^2}{1-y}\right]$

$\Rightarrow \left[\frac{x+y-xy}{1-x-y+xy}\right]$

Q6. If $|3x-1|, 3, |x-3|$ are the first three terms of an arithmetic progression, then the sum of the first five terms can be

A. 5

B. 10

C. 20

D. 30

Ans: 5

Solution: Case-I : $x < \frac{1}{3} \Rightarrow -3x+1$,3, $-x+3$ are in $A.P.$

$\Rightarrow 6 = -3x+1-x+3$

$\Rightarrow 4x = -2 \Rightarrow x = -\frac{1}{2}$

$\Rightarrow$ terms are $\frac{5}{2}$, 3 , $\frac{7}{2}$, 4 , $\frac{9}{2}$

$\Rightarrow$ sum $= \frac{35}{2}$

Case-II: $\frac{1}{3} \le x < 3$

$3x-1$,3, $-x+3$ are in A.P.

$\Rightarrow 6=3x-1-x+3=2x+2 \Rightarrow x=2$

$\Rightarrow$ terms are $5\ ,\ 3\ ,\ 1\ ,\ -1\ ,\ -3$

$\Rightarrow$ sum $=5$

Case-III: $x \geq 3$

$\Rightarrow$ terms are $3x-1$,3,$x-3$

$\Rightarrow 6=3x-1+x-3=4x-4$

$\Rightarrow 4x=10 \Rightarrow x=\frac{5}{2}$ not possible

Q7.

If $x,\ y,\ z \in R^{+}$ and $16\left(16\ x^{2}+y^{2}-4\ xy\right)=z\left(16\ x+4\ y-z\right)$, then

A.

$y,\ z,\ x$ are in $A.P.$

B.

$y,\ z,\ x$ are in $G.P.$

C.

$x,\ y,\ z$ are in $A.P.$

D. $x,\ y,\ z$ are in $G.P.$

Ans: $x,\ y,\ z$ are in $G.P.$

Solution:

$(16x)^2+(4y)^2+(z)^2-(16x)(4y)-(4y)(z)-(z)(16x)=0$

$\Rightarrow \frac{1}{2}\left[(16x-4y)^2+(4y-z)^2+(z-16x)^2\right]=0 \Rightarrow 16x=4y=z$

Q8. In a sequence of 21 terms first 11 terms are in A.P. with common difference 2 and last 11 terms are in G.P. with common ratio 2. If middle term of A.P. is equal to middle term of G.P. then, middle term in the complete sequence is

A. $\frac{10}{1-2^5}$
B. $\frac{10\left(1-2^{11}\right)}{\left(2^{10}-1\right)}$
C. $\frac{1-2^{11}}{2^{10}-1}+10$
D. $\frac{20}{2^{10}-1}$

Ans: $\frac{10}{1-2^5}$

Solution:

Let the first term of the given sequence be a

First 11 terms are: $a,\ a+2, \ldots a+20$

Since, eleventh term of the sequence is the first of last 11 terms of this sequence.

Last 11 terms are:$a+20,\ (a+20).2, \ldots (a+20)\cdot 2^{10}$

Thus, the sequence obtained is as follows:

$a,\ a+2,\ \ldots,\ a+20,\ (a+20)2,\ (a+20)2^2,\ \ldots\ldots,\ (a+20)2^{10}$

Now middle term of the AP is the $\left(\frac{(11+1)}{2}\right)^{\text{th}}$ term, i.e., 6^{th} term and the same goes for GP.

So, $(a+10)=(a+20)2^5$

$\Rightarrow 10-20 \cdot 2^5=a\left(2^5-1\right) \Rightarrow 10 \frac{\left(1-2^6\right)}{2^5-1}=a$

Middle term of the whole sequence $=T_{11}=a+20=10 \frac{\left(1-2^6\right)}{2^5-1}+20$

$=10\left[\frac{-1}{2^5-1}\right]=\frac{10}{1-2^5}$

Q9. If $a_1, a_2, \ldots \ldots a_{10}$ are positive numbers in an arithmetic progression such that $\frac{1}{a_1 a_2}+\frac{1}{a_2 a_3}+\ldots . .+\frac{1}{a_9 a_{10}}=\frac{9}{64}$ and $\frac{1}{a_1 a_{10}}+\frac{1}{a_2 a_9}+\ldots \ldots \frac{1}{a_{10} a_1}=\frac{1}{10}\left(\frac{1}{a_1}+\ldots .+\frac{1}{a_{10}}\right)$, then sum of digits of $\left(4\left(\frac{a_1}{a_{10}}+\frac{a_{10}}{a_1}\right)\right)$ is

Ans: 8

Solution: $\left(\frac{1}{a_1}-\frac{1}{a_2}\right)+\left(\frac{1}{a_2}-\frac{1}{a_3}\right)+\ldots\left(\frac{1}{a_9}-\frac{1}{a_{10}}\right)=\frac{9 \mathrm{~d}}{64}$

$\Rightarrow \frac{a_{10}-a_1}{a_1 a_{10}}=\frac{9 \mathrm{~d}}{64}$

$\Rightarrow a_1 a_{10}=64$

Also $\left(a_1+a_{10}\right)\left(\frac{1}{a_1 a_{10}}+\ldots \frac{1}{a_{10} a_1}\right)=\frac{a_1+a_{10}}{10}\left(\frac{1}{a_1}+\ldots \frac{1}{a_{10}}\right)$

$\Rightarrow 2\left(\frac{1}{a_1}+\ldots \frac{1}{a_{10}}\right)=\frac{a_1+a_{10}}{10}\left(\frac{1}{a_1}+\ldots \frac{1}{a_{10}}\right)$

$\Rightarrow a_1+a_{10}=20$

$a_1 a_{10}=64$ ...(1)

$a_1+a_{10}=20$ ...(2)

From (1) & (2)

$a_1=4$ & $a_{10}=16$

$4\left(\frac{a_1}{a_{10}}+\frac{a_{10}}{a_1}\right)=4\left(\frac{4}{16}+\frac{16}{4}\right)=17$

Sum of digits of $17=1+7=8$

Q10. Three numbers a, b and c are in between 2 and 18 such that $2, a, b$ are in arithmetic progression and $b, c, 18$ are in geometric progression. If $a+b+c=25$, then the value of

$c-a$ is

A. 4

B. 3

C. 7

D. 0

Ans: 7

Solution: $a+b+c=25$; $2a=2+b$; $c^2=18b$

$\Rightarrow 2a+2b+2c=50$

$\Rightarrow 2+b+2b+2c=50$

$\Rightarrow 3b+2c=48 \Rightarrow \frac{c^2}{6}+2c=48$

$\Rightarrow c^2+12c-48\times 6=0 \Rightarrow c^2+12c-24\times 12=0$

$\Rightarrow(c+24)(c-12)=0$

$\Rightarrow c=12,-24 \Rightarrow c=12$ (between 2 and 18)

$\Rightarrow b=\frac{c^2}{18}=\frac{144}{18}=8$

$\Rightarrow a=\frac{b+2}{2}=5$

$\Rightarrow a=5,\ b=8$ and $c=12$

$\Rightarrow c-a=7$

Q11. The harmonic mean of two positive numbers a and b is 4, their arithmeitc mean is A and the geometric mean is G. If $2A+G^2=27,\ a+b=\alpha$ and $|a-b|=\beta$, then the value of $\frac{\alpha}{\beta}$ is equal to

A. 1

B. 3

C. $\frac{5}{2}$

D. 5

Ans: 3

Solution:

Given, harmonic mean $H=4$

We know that, $G^2=AH$

Since, $2A+G^2=27$

$\Rightarrow 2A+AH=27$

$\Rightarrow 2A+4A=27$

$\Rightarrow A=\frac{27}{6}=\frac{9}{2}=\frac{a+b}{2}\Rightarrow a+b=9$

$G^2=AH=\frac{9}{2}\times 4=18$

$\Rightarrow ab=18$

$|a-b|=\sqrt{(a+b)^2-4ab}=\sqrt{81-4\times 18}=3$

$\Rightarrow \alpha=9,\ \beta=3$

$\Rightarrow \frac{\alpha}{\beta}=3$

Q12. If 11 arithmetic means are inserted between 28 and 10, then the number of integral arithmetic means are

A. 5

B. 6

C. 7

D. 8

Ans: 5

Solution:

Let, $A_1, A_2, A_3, \ldots\ldots, A_{11}$ be 11 A.M.'s between 28 and 10

$\Rightarrow 28, A_1, A_2, \ldots\ldots, A_{11}, 10$ are in A.P

Let, d be the common difference of A.P.

Also, the number of terms $= 13$

$\Rightarrow 10 = \mathrm{T}_{13} = \mathrm{T}_1 + 12\mathrm{d} = 28 + 12\mathrm{d}$

$\Rightarrow d = \frac{10-28}{12} = \frac{-18}{12} = \frac{-3}{2}$

$\Rightarrow$ Number of integral A.M.'s are 5

Q13. There are n sets of observations given as (1), $(2, 3)$, $(4, 5, 6)$, $(7, 8, 9, 10), \ldots\ldots$ The mean of the 13^{th} set of observations is equal to

A. 70

B. 80

C. 75

D. 85

Ans: 85

Solution:

In the n^{th} set of observation, the total observations are 'n'

and the first observation of n^{th} set is

$1 + 2 + 3 + \ldots . + (n-1) + 1 = \frac{(n-1)n}{2} + 1 = a$

Sum of all the n observations in the n^{th} set

$= \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}\left[2\left\{\frac{(n-1)n}{2} + 1\right\} + (n-1)\right]$

$= \frac{n}{2}\left[n^2 - n + 2 + n - 1\right] = n\left(\frac{n^2+1}{2}\right)$

$\text{Mean} = \frac{n\frac{\left(n^2+1\right)}{2}}{n} = \frac{n^2+1}{2}$

Hence, for $n = 13$

$\text{Mean} = \frac{13^2+1}{2} = \frac{170}{2} = 85$

Q14. The sum of the first 20 terms common between the series $3 + 7 + 11 + 15 + \ .....$ and $1 + 6 + 11 + 16 + \ ......$ is

A. 4000

B. 4200

C. 4220

D. 4020

Ans: 4020

Solution: $S_1 \equiv 3,\ 7,\ 11,\ 15,$ .... $\Rightarrow\ c.d = 4 \Rightarrow d_1 = 4$

$S_2 \equiv 1,\ 6,\ 11,\ 16,$ .... $\Rightarrow\ c.d = 5 \Rightarrow d_2 = 5$

$\Rightarrow$Every 5^{th} term of S_1 and 4^{th} term of S_2 will be same

$\Rightarrow$ Terms common to both AP will have $a = 11$ and $d = 20$

Hence, $S_{20} = \frac{20}{2}[(2 \times 11) + (20 - 1)\,20]$

$= \ 10 \times 402$

$= \ 4020$

Q15. The sum to infinity of the series $1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+\ldots$ is

A. $\frac{16}{25}$

B. $\frac{11}{5}$

C. $\frac{35}{16}$

D. $\frac{8}{11}$

Ans: $\frac{35}{16}$

Solution: $S=1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+\ldots+\infty$ ... (1)

$\frac{S}{5}=0+\frac{1}{5}+\frac{4}{5^2}+\frac{7}{5^3}+\ldots+\infty$ ... (2)

On subtracting both the eq (1) and (2), we get,

$\frac{4}{5}S=1+\frac{3}{5}+\frac{3}{5^2}+\ldots+\infty$

$S=\frac{35}{16}$

Q16. The sum (upto two decimal places) of the infinite series $\frac{7}{17}+\frac{77}{17^2}+\frac{777}{17^3}+\ldots\ldots.$ is

A. 1.06

B. 2.06

C. 3.06

D. 4.06

Ans: 1.06

Solution:

$S=\frac{7}{17}+\frac{77}{17^2}+\frac{777}{17^3}+\ldots\ldots$

$\frac{S}{17}=\frac{7}{17^2}+\frac{77}{17^3}+\frac{777}{17^4}+\ldots\ldots..$

Subtracting both the above results, we get,

$\frac{16}{17}S=\frac{7}{17}+\frac{70}{17^2}+\frac{700}{17^3}+\ldots\ldots.$
$\left(\text{Infinite G.P. with r }=\frac{10}{17}\right)$
$\frac{16}{17}S=\frac{\frac{7}{17}}{1-\frac{10}{17}}=\frac{\frac{7}{17}}{\frac{7}{17}}=1$
$S=\frac{17}{16}$

Q17. It is given that $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\ldots..\infty=\frac{\pi^4}{90}$ then $\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\ldots+\infty$ is equal to -

A. $\frac{\pi^4}{96}$
B. $\frac{\pi^4}{45}$
C. $\frac{89\pi^4}{90}$

D. None of these

Ans: $\frac{\pi^4}{96}$

Solution: $x=\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\ldots.\infty$

$=\left(\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\ldots\infty\right)-\left(\frac{1}{2^4}+\frac{1}{4^4}+\ldots\infty\right)$

$=\frac{\pi^4}{90}-\frac{1}{16}\left(\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\ldots..\infty\right)$

$=\frac{\pi^4}{90}-\frac{1}{16}.\frac{\pi^4}{90}=\frac{\pi^4}{96}$.

Q18. If $S=1(25)+2(24)+3(23)+\ldots\ldots+24(2)+25(1)$, then the value of $\frac{S}{900}$ is equal to

Ans: 3.25

Solution: $S=\sum_{r=1}^{25} r(26-r)=26\sum_{r=1}^{25} r-\sum_{r=1}^{25} r^2$

$=26\times\frac{25\times 26}{2}-\frac{25\times 26\times 51}{6}$

$=\frac{25\times 26}{2}\left(26-\frac{51}{3}\right)=\frac{25\times \cancel{26}13}{\cancel{2}}\times 9$

$\Rightarrow \frac{S}{900}=\frac{13}{4}=3.25$

Q19. $0.2+0.22+0.222+\ldots$ **upto** n **terms is equal to**

A. $\left(\frac{2}{9}\right)-\left(\frac{2}{81}\right)\left(1-10^{-n}\right)$

B. $n\left(\frac{1}{9}\right)\left(1-10^{-n}\right)$

C. $\left(\frac{2}{9}\right)\left[n-\left(\frac{1}{9}\right)\left(1-10^{-n}\right)\right]$

D. $\left(\frac{2}{9}\right)$

Ans: $\left(\frac{2}{9}\right)\left[n-\left(\frac{1}{9}\right)\left(1-10^{-n}\right)\right]$

Solution: $0.2+0.22+0.222+\ldots n$ terms

$=2(0.1+0.11+0.111+\ldots n\,\text{terms})$

$=2\left(\frac{1}{10}+\frac{11}{100}+\frac{111}{1000}+\ldots n\,\text{terms}\right)$

$=\frac{2}{9}\left(\frac{9}{10}+\frac{99}{100}+\frac{999}{1000}+\ldots n\text{ terms}\right)$

$=\frac{2}{9}\left(1-\frac{1}{10}+1-\frac{1}{100}+1-\frac{1}{1000}+\ldots n\text{ terms}\right)$

$=\frac{2}{9}\left[n-\left(\frac{1}{10}+\frac{1}{100}+\frac{1}{1000}+\ldots n\right)\right]$

$=\frac{2}{9}\left[n-\frac{1}{10} \frac{\left\{1-\left(\frac{1}{10}\right)^n\right\}}{\left(1-\frac{1}{10}\right)}\right]$

$=\frac{2}{9}\left[n-\frac{1}{10} \times \frac{10}{9} \cdot\left(\frac{10^n-1}{10^n}\right)\right]$

$=\frac{2}{9}\left[n-\frac{1}{9}\left(1-10^{-n}\right)\right]$

Q20. If the sum $\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+\ldots . .+$ up to 20 terms is equal to $\frac{\mathrm{k}}{21}$, then k is equal to

A. 240

B. 120

C. 60

D. 180

Ans: 120

Solution: $\mathrm{S}_{20}=\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+\ldots .+$ up to 20 terms

$=\sum_{\mathrm{r}=1}^{20} \frac{(2 \mathrm{r}+1)}{1^2+2^2+3^2+\ldots . \mathrm{r}^2}$

$=\sum_{\mathrm{r}=1}^{20} \frac{2 \mathrm{r}+1}{\frac{\mathrm{r}}{6}(\mathrm{r}+1)(2 \mathrm{r}+1)}$

$=6 \sum_{\mathrm{r}=1}^{20} \frac{1}{\mathrm{r}(\mathrm{r}+1)}$

$=6 \sum_{\mathrm{r}=1}^{20}\left(\frac{1}{\mathrm{r}}-\frac{1}{\mathrm{r}+1}\right)$

$=6\left\{\left[\frac{1}{1}-\frac{1}{2}\right]+\left[\frac{1}{2}-\frac{1}{3}\right]+\left[\frac{1}{3}-\frac{1}{4}\right]+\ldots .+\left[\frac{1}{20}-\frac{1}{21}\right]\right\}$

$= 6\left(1 - \frac{1}{21}\right)$

$= \frac{6 \times 20}{21}$

$= \frac{120}{21}$

$= \frac{\mathrm{k}}{21}$

$\mathrm{k} = 120$

Q21. If $S = \sum_{n=1}^{9999} \frac{1}{(\sqrt{n}+\sqrt{n+1})(\sqrt[4]{n}+\sqrt[4]{n+1})}$, then the value of S is equal to

A. 9

B. 99

C. 999

D. 9999

Ans: 9

Solution: Given, $S = \sum_{n=1}^{9999} \frac{1}{(\sqrt{n}+\sqrt{n+1})(\sqrt[4]{n}+\sqrt[4]{n+1})}$

$= \sum_{n=1}^{9999} \frac{1}{(\sqrt{n}+\sqrt{n+1})(\sqrt[4]{n}+\sqrt[4]{n+1})} \times \frac{(\sqrt[4]{n+1}-\sqrt[4]{n})}{(\sqrt[4]{n+1}-\sqrt[4]{n})}$

$= \sum_{n=1}^{9999} \frac{(\sqrt[4]{n+1}-\sqrt[4]{n})}{(\sqrt{n}+\sqrt{n+1})(\sqrt{n+1}-\sqrt{n})}$

$= \sum_{n=1}^{9999} (\sqrt[4]{n+1} - \sqrt[4]{n})$

$= \left(2^{\frac{1}{4}} - 1\right) + \left(3^{\frac{1}{4}} - 2^{\frac{1}{4}}\right) + \left(4^{\frac{1}{4}} - 3^{\frac{1}{4}}\right) + \ldots + \left(10000^{\frac{1}{4}} - 9999^{\frac{1}{4}}\right)$

$= 10000^{\frac{1}{4}} - 1 = 10 - 1 = 9$

Q22. If $S = \sum_{r=1}^{80} \frac{r}{(r^4+r^2+1)}$, then the value of $\frac{6481S}{1000}$ is

Ans: 3250

Solution: $T_r=\frac{1}{2}\left\{\frac{\left(r^2+r+1\right)-\left(r^2-r+1\right)}{\left(r^2+r+1\right)\left(r^2-r+1\right)}\right\}$

$=\frac{1}{2}\left\{\frac{1}{r^2-r+1}-\frac{1}{r^2+r+1}\right\}$

$=\frac{1}{2}\left\{\frac{1}{r^2-r+1}-\frac{1}{(r+1)^2-(r+1)+1}\right\}$

$=\frac{1}{2}\{f(r)-f(r+1)\}$

$S=\Sigma T_r=\sum_{r=1}^{80}\frac{1}{2}(f(\alpha)-f(\alpha+1))$

$=\frac{1}{2}\left\{f(1)-\cancel{f(2)}+\cancel{f(2)}-\cancel{f(3)}\ldots\ldots..+\cancel{f(80)}-f(81)\right\}$

$=\frac{1}{2}(f(1)-f(81))=\frac{1}{2}\left(1-\frac{1}{81^2-81+1}\right)$

$=\frac{1}{2}\left(1-\frac{1}{6481}\right)\Rightarrow 6481S=\frac{6480}{2}=3240$

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Q23. For the series

$S=1+\frac{1}{(1+3)}(1+2)^2+\frac{1}{(1+3+5)}(1+2+3)^2+\frac{1}{(1+3+5+7)}(1+2+3+4)^2+\ldots\ldots.$, if the sum of the first 10 terms is K, then $\frac{4\mathrm{K}}{101}$ is equal to

Ans: 5

Solution: r^{th} term, $\mathrm{T}_r=\frac{(1+2+3+\ldots\ldots+r)^2}{(1+3+5+\ldots+(2r-1))}$

$=\frac{\left(\frac{r(r+1)}{2}\right)^2}{r^2}=\frac{(r+1)^2}{4}=\frac{r^2+2r+1}{4}$

$S_{10}=\mathrm{K}=\frac{1}{4}\sum_{r=1}^{10}\left(r^2+2r+1\right)$

$\Rightarrow 4\mathrm{K}=\sum_{r=1}^{10}r^2+2\sum_{r=1}^{10}r+\sum_{r=1}^{10}1$

$=\frac{10\times11\times21}{6}+2\frac{10\times11}{2}+10$

$=385+110+10=505$

$\frac{4\mathrm{K}}{101}=5$

Q24. Let the sum $\sum_{n=1}^{9} \frac{1}{n(n+1)(n+2)}$, written in the rational form be $\frac{p}{q}$ (where p and q are co-prime), then the value of $\left[\frac{q-p}{10}\right]$ is, (where [.] is the greatest integer function)

Ans: 8

Solution: $\sum_{n=1}^{9} \frac{1}{2} \frac{n+2-n}{n(n+1)\ (n+2)}$

$= \frac{1}{2}\left(\left(\frac{1}{1.2}-\frac{1}{2.3}\right)+\left(\frac{1}{2.3}-\frac{1}{3.4}\right)+\ldots+\left(\frac{1}{9\times10}-\frac{1}{10\times11}\right)\right)$

$= \frac{1}{2}\left(\frac{1}{2}-\frac{1}{110}\right)= \frac{1}{2}\left(\frac{55-1}{110}\right)= \frac{27}{110}$

$\Rightarrow \quad q-p=110-27=83$

Q25.

If $S_n = (1^2-1+1)(1!)+(2^2-2+1)(2!)+\ldots+(n^2-n+1)(n!)$, then S_{50} is:

A. $52!$

B. $1+49\times51!$

C. $52!-1$

D. $50\times51!-1$

Ans: $1+49\times51!$

Solution:

Given,

$$S_n=\left(1^2-1+1\right)(1!)+\left(2^2-2+1\right)(2!)+\cdots+\left(n^2-n+1\right)(n!)$$

Its general term t_r is given as,

$\left(r^2-r+1\right) r!$

$t_r=\left[\left(r^2-1\right)-(r-2)\right](r!)$

$t_r=(r-1)(r+1)!-(r-2)(r)!$

$S_n=\sum_{r=1}^n t_r$

$S_n=(0-(-1))+(3!-0)+(2(4)!-3!)+\ldots+(n-1)(n+1)!-(n-2) n!$

$S_{50}=1+49(51)!$.

Q26. Let $a_1, a_2, \ldots \ldots, a_n$ be real numbers such that $\sqrt{a_1}+\sqrt{a_2-1}+\sqrt{a_3-2}+\ldots+\sqrt{a_n-(n-1)}=\frac{1}{2}\left(a_1+a_2+\ldots+a_n\right)-\frac{n(n-3)}{4}$. Compute the value of $\sum_{i=1}^{100} a_i$.

A. 1010

B. 505

C. 2525

D. 5050

Ans: 5050

Solution:

Let $\sqrt{a_1}=b_1$;

$\sqrt{a_2-1}=b_2$;

$\sqrt{a_3-2}=b_3$;

$\sqrt{a_n - (n-1)} = b_n$

$\therefore \text{LHS} = b_1 + b_2 + \ldots + b_n = \frac{1}{2}\left[b_1^2 + \left(b_2^2 + 1\right) + \left(b_3^2 + 2\right) + \ldots + \left(b_n^2 + (n-1)\right)\right] - \frac{n(n-3)}{4}$

$\therefore\ \Sigma b_i = \frac{1}{2}\left[\left(b_1^2 + b_2^2 + b_3^2 + \ldots + b_n^2\right) + (1 + 2 + 3 + \ldots + (n-1))\right] - \frac{n(n-3)}{4}$

$\Rightarrow 2\Sigma b_i = \Sigma b_i^2 + \frac{n(n-1)}{2} - \frac{n(n-3)}{2}$

$\Rightarrow 2\Sigma b_i = \Sigma b_i^2 + n$

$\therefore\ \Sigma b_i^2 - 2\Sigma b_i + \Sigma 1 = 0$

$\therefore \sum_{i=1}^{n} (b_i - 1)^2 = 0$

$b_1 - 1 = 0 \Rightarrow b_1^2 = a_1 = 1$

$b_2 - 1 = 0 \Rightarrow b_2^2 = a_2 - 1 = 1 \Rightarrow a_2 = 2$

$b_3 - 1 = 0 \Rightarrow b_3^2 = a3 - 2 = 1 \Rightarrow a_3 = 3$ and so on.

Hence, $a_n = n$.

$\therefore \sum_{i=1}^{100} a_i = 1 + 2 + 3 + \ldots + 100 = 5050$

Q27. Let $a_1, a_2, a_3, \ldots\ldots\ldots, a_{11}$ be real numbers satisfying $a_1 = 15, 27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}\ \forall\ k = 3, 4, \ldots\ldots, 11$. If $\frac{a_1^2 + a_2^2 + \ldots + a_{11}^2}{11} = 90$, then the value of $\frac{a_1 + a_2 + \ldots + a_{11}}{11}$ is equal to

Ans: 0

Solution:

$a_k = 2a_{k-1} - a_{k-2}$

$\Rightarrow a_1, a_2, \ldots, a_{11}$ are in AP with let common difference be d

$\therefore \quad \frac{a_1^2+a_2^2+\ldots+a_{11}^2}{11} = \frac{11a^2+35\times11\mathrm{d}^2+110a\mathrm{d}}{11} = 90$

$\Rightarrow 225 + 35\mathrm{d}^2 + 150\mathrm{d} = 90$

$35\mathrm{d}^2 + 150\mathrm{d} + 135 = 0$

$\Rightarrow \mathrm{d} = -3, -\frac{9}{7}$

Given, $a_2 < \frac{27}{2} \therefore \mathrm{d} = -3$ and $\mathrm{d} \neq -\frac{9}{7}$

Hence, $a_1 + a_2 + \ldots\ldots a_{11} = \frac{11}{2}[2\times15+10(-3)] = 0$

Q28. $\frac{1.2}{1!} + \frac{2.3}{2!} + \frac{3.4}{3!} + \frac{4.5}{4!} + \ldots\infty =$

A. $2e$

B. $3e$

C. $3e - 1$

D. e

Ans: $3e$

Solution: $\frac{1.2}{1!} + \frac{2.3}{2!} + \frac{3.4}{3!} + \frac{4.5}{4!} + \ldots\infty,$

Here, $T_n = \frac{n(n+1)}{n!} = \frac{n+1}{(n-1)!} = \frac{(n-1)+2}{(n-1)!}$

$= \frac{1}{(n-2)!} + \frac{2}{(n-1)!}$

$\Rightarrow \quad S = \Sigma T_n = e + 2e = 3e$

Q29. The minimum value of sum of real numbers a^{-6}, $2a^{-4}$, $2a^{-3}$, 1 and $2a^{10}$ with $a > 0$ is equal to

A. 1

B. 2

C. 4

D. 8

Ans: 8

Solution:

$\because \ A.M. \geq G.M.$

$\Rightarrow \frac{a^{-6}+a^{-4}+a^{-4}+a^{-3}+a^{-3}+1+a^{10}+a^{10}}{8} \geq 1$

$\therefore$ Minimum value $= 8$

Q30. If $a+b+c=3$ (where $a, b, c > 0$), then the greatest value of $a^2 b^3 c^2$ is

A. $\frac{3^{10} 2^4}{7^7}$

B. $\frac{3^9 2^4}{7^7}$

C. $\frac{3^8 2^4}{7^7}$

D. $\frac{3^9 2^3}{7^6}$

Ans: $\frac{3^{10}2^4}{7^7}$

Solution: Given, $a+b+c=3$

$\Rightarrow 2 \cdot \frac{a}{2}+3 \cdot \frac{b}{3}+2 \cdot \frac{c}{2}=3$

$\frac{\frac{a}{2}+\frac{a}{2}+\frac{b}{3}+\frac{b}{3}+\frac{b}{3}+\frac{c}{2}+\frac{c}{2}}{7} \geq\left(\left(\frac{a}{2}\right)^2\left(\frac{b}{3}\right)^3\left(\frac{c}{2}\right)^2\right)^{\frac{1}{7}}$

$\frac{a+b+c}{7} \geq\left(\frac{a^2 b^3 c^2}{2^4 3^3}\right)^{\frac{1}{7}}$

$\left(\frac{3}{7}\right)^7 \geq \frac{a^2 b^3 c^2}{2^4 3^3}$

$\Rightarrow a^2 b^3 c^2 \leq \frac{3^7}{7^7} \times 2^4 \times 3^3=\frac{2^4 \times 3^{10}}{7^7}$

Hence, maximum value $=\frac{2^4 \times 3^{10}}{7^7}$

Answer Key

Q1 (3)	**Q2** (1)	**Q3** (2)	**Q4** (4)
Q5 (1)	**Q6** (1)	**Q7** (4)	**Q8** (1)
Q9 (8)	**Q10** (3)	**Q11** (2)	**Q12** (1)
Q13 (4)	**Q14** (4)	**Q15** (3)	**Q16** (1)
Q17 (1)	**Q18** (3.25)	**Q19** (3)	**Q20** (2)
Q21 (1)	**Q22** (3250)	**Q23** (5)	**Q24** (8)
Q25 (2)	**Q26** (4)	**Q27** (0)	**Q28** (2)
Q29 (4)	**Q30** (1)		