Q1. The domain of definition of the function $f(x)=\sqrt{\log_{(|x|-1)}(x^2+4x+4)}$, is

A. $[-3,\ -1]\cup[1,\ 2]$

B. $(-2,\ -1)\cup[2,\ \infty)$

C. $(-\infty,\ -3]\cup(-2,\ -1)\cup(2,\ \infty)$

D. $[-2,\ -1]\cup[2,\ \infty)$

Ans: $(-\infty,\ -3]\cup(-2,\ -1)\cup(2,\ \infty)$

Solution: $\log_{(|x|-1)}\left(x^2+4x+4\right)\geq 0$

Case 1: $0<|x|-1<1$

i.e., $1<|x|<2$, then

$0<x^2+4x+4\ \leq 1\Rightarrow\quad x^2+4x+3\ \leq 0\ \&\ (x+2)^2>0$

$\Rightarrow -3\leq x\leq -1\ \&\ x\neq -2$

So, $x\in(-2,\ -1)$

Case 2: $|x|-1>1$

i.e., $|x|>2$

$x^2+4x+4\ \geq 1\Rightarrow(x+1)(x+3)\geq 0\Rightarrow x\in(-\infty,-3]\cup[-1,\infty)$

$\Rightarrow x\in(-\infty,\ -3]\cup(2,\ \infty)$

Hence, domain is $(-\infty,-3] \cup(-2,-1) \cup(2, \infty)$

Q2. If domain of $f(x)$ is $[1,3]$, then find the domain of $f\left(\log_2\left(x^2+3x-2\right)\right)$

A. $[-5,-4] \cup[1,2]$

B. $\left[-13,-2\right] \cup\left[\frac{3}{2}, 5\right]$

C. $[-4,1] \cup[2,7]$

D. $[-3,2]$

Ans: $[-5,-4] \cup[1,2]$

Solution: $1 \leq \log_2\left(x^2+3x-2\right) \leq 3 \Rightarrow 2 \leq\left(x^2+3x-2\right) \leq 8$

On solving them, we get $-5 \leq x \leq -4$ and $1 \leq x \leq 2$

Q3. If $f(x)=\tan^{-1} \sqrt{x^2+4x}+\sin^{-1} \sqrt{x^2+4x+1}$, then

A. domain of $f(x)$ contains 3 integers only

B. range of $f(x)$ has two elements only

C. $f(x)$ is a constant function $\forall x \in R$

D. $f(x)$ contains only two elements in its domain

Ans: $f(x)$ contains only two elements in its domain

Solution:

$\because\ x^2+4x \geq 0$ for the first term

and $0 \leq x^2+4x+1 \leq 1$ for the second term

$\therefore$ by both the results, there is only one possibility

$x^2+4x=0$

$\Rightarrow x=0,-4$

$\therefore\ f(x)=\tan^{-1}(0)+\sin^{-1}(1)=\frac{\pi}{2}$

Hence, $f(x)$ contains only two elements in its domain

Q4. If $f(x)=\frac{x^2-[x^2]}{1+x^2-[x^2]}$ (where $[.]$ represents the greatest integer part of x), then the range of $f(x)$ is

A. $[0,1)$

B. $(-1,1)$

C. $(0,\infty)$

D. $\left[0,\frac{1}{2}\right)$

Ans: $\left[0,\frac{1}{2}\right)$

Solution: $x^2-[x^2]=\{x^2\}$

$\therefore\ f(x)=\frac{\{x^2\}}{1+\{x^2\}}=\frac{1+\{x^2\}-1}{1+\{x^2\}}=1-\frac{1}{1+\{x^2\}}$

$\because\ 0\leq\{x^2\}<1\Rightarrow 1\leq\{x^2\}+1<2$

$\frac{1}{2}<\frac{1}{\{x^2\}+1}\leq 1\Rightarrow-\frac{1}{2}>\frac{-1}{1+\{x^2\}}\geq-1$

$\frac{1}{2}>1+\frac{(-1)}{1+\{x^2\}}\geq 0\Rightarrow$ Range of $f(x)\in\left[0,\frac{1}{2}\right)$

Q5. If $f\left(x\right)=\sin^4 x+\cos^4 x-\frac{1}{2}\sin 2x$ then the range of $f(x)$ is

A. $\left[0,\frac{3}{2}\right]$

B. $\left[-\frac{1}{2},\frac{7}{2}\right]$

C. $\left[0, \frac{9}{8}\right]$

D. $\left[\frac{3}{4}, \frac{7}{8}\right]$

Ans: $\left[0, \frac{9}{8}\right]$

Solution: Let $f(x) = \sin^4 x - \sin x \cos x + \cos^4 x$

$f\left(x\right) = 1 - \frac{1}{2}\sin 2x - \frac{1}{2}\sin^2 2x$

$\Rightarrow f\left(x\right) = g\left(t\right) = 1 - \frac{1}{2}t - \frac{1}{2}t^2 \quad t \in \left[-1, 1\right]$

Hence $f\left(x\right) \in \left[0, \frac{9}{8}\right]$

Q6. Domain and range of the function $f\left(x\right) = \sqrt{\sin^{-1}\left(3x\right) + \frac{\pi}{3}}$ is $\left[\frac{a}{\sqrt{3}}, \frac{b}{3}\right]$ and $\left[c, d\sqrt{5\pi}\right]$ respectively, then $2a + b + c + 6d$ is equal to

A. 1

B. $2\sqrt{3}$

C. $\sqrt{6}$

D. none of these

Ans: $\sqrt{6}$

Solution: $\sin^{-1}(3x) + \frac{\pi}{3} \geqslant 0$ and $3x \in [-1, 1]$

$\sin^{-1}(3x) \geqslant -\frac{\pi}{3}$

$1 \geqslant 3x \geqslant -\frac{\sqrt{3}}{2}$

$\frac{1}{3} \geqslant x \geqslant \frac{-\sqrt{3}}{6}$

domain is $\left[\frac{-1}{2\sqrt{3}}, \frac{1}{3}\right] \quad \therefore a = \frac{-1}{2}$ & $b = 1.$

For Range put $x = \frac{-1}{2\sqrt{3}}$ and $x = \frac{1}{3}$

at $x = \frac{-1}{2\sqrt{3}} \quad b\left(\frac{-1}{2\sqrt{3}}\right) = 0$

at $x = \frac{1}{3} \quad b\left(\frac{1}{3}\right) = \sqrt{\frac{5\pi}{6}}$

So $C=0$ and $d=\frac{1}{\sqrt{6}}$

$2a+b+c+6d=\sqrt{6}$

Q7. The number of integral value(s) of x which satisfying the equation

$$\left|\log_4\left(2x^2-x\right)+\log_2\left(2-x^2\right)+x^2+2x+2\right|=x^2+2x+2+\left|\log_4\left(2x^2-x\right)\right|+\left|\log_2\left(2-x^2\right)\right|$$

A. 0

B. 1

C. 2

D. 4

Ans: 2

Solution:

Since $|a+b|=|a|+|b| \Rightarrow ab \geq 0$

So, $\log_4\left(2x^2-x\right) \geq 0 \Rightarrow 2x^2-x \geq 1$

$\Rightarrow \left(2x+1\right)\left(x-1\right) \geq 0 \Rightarrow x \leq -\frac{1}{2}$ or $x \geq 1$

and $\log_2\left(2-x^2\right) \geq 0 \Rightarrow 2-x^2 \geq 1$

$\Rightarrow x^2 \leq 1 \Rightarrow -1 \leq x \leq 1$

Hence, $x \in \left[-1, \frac{-1}{2}\right] \cup \{1\}$

i.e. 2 integral values.

Q8. $[x]$ and $\{x\}$ represent the greatest integer function and fractional part function respectively. Let $f\left(x\right)=\left[x\right]+\sum_{i=1}^{2020}\frac{\{x+r\}}{2020}$. Find the value of $f(-1000)$

Ans: 10.5

Solution:

$f\left(x\right)=\left[x\right]+\sum_{r=1}^{2020}\frac{\{x+r\}}{2020}$

$=\left[x\right]+\sum_{r=1}^{2020}\frac{\{x\}+r}{2020}$
$=\left[x\right]+\frac{2020}{2020}\left\{x\right\}+\frac{1+2+3+\ldots+2020}{2020}$
$=\left[x\right]+\left\{x\right\}+\frac{2020\times2021}{2\times2020}$
$=x+\frac{2021}{2}$

Q9. The function $f(x)=\sec\left[\log\left(\mathrm{x}+\sqrt{1+\mathrm{x}^2}\right)\right]$ is

A. an odd function

B. an even function

C. neither an odd nor an even function

D. a constant function

Ans: an even function

Solution: $f(-x)=\sec\left(\log\left(-x+\sqrt{1+x^2}\right)\right)$

$=\sec\left(\log\left(x+\sqrt{1+x^2}\right)^{-1}\right)$

$\left(\because\sqrt{1+x^2}-x=\frac{1}{\sqrt{1+x^2}+x}\right)$

$=\sec\left(-\log\left(x+\sqrt{1+x^2}\right)\right)$

$=\sec\left(\log\left(x+\sqrt{1+x^2}\right)\right)$ $(\because\sec(-\theta)=\sec(\theta))$

$= f(x)$

Hence $f(x)$ is an even function.

Q10. Which of the following is a function whose graph is symmetrical about the origin?

A. $f(x)=\left(2^{x}+2^{-x}\right)$

B. $f(x)=\left[\log\left(x+\sqrt{1+x^{2}}\right)\right]^{2}$

C. $f(x+y)=f(x)+f(y)\forall x, y \in R$

D. None of these

Ans: $f(x+y)=f(x)+f(y)\forall x, y \in R$

Solution: A function whose graph is symmetrical about the origin must be odd.

$\left(2^{x}+2^{-x}\right)$ is an even function.

Since, $\log\left(x+\sqrt{1+x^{2}}\right)$ is an odd function,

$\therefore\left[\log\left(x+\sqrt{1+x^{2}}\right)\right]^{2}$ is an even function.

If $f(x+y)=f(x)+f(y)\forall x, y \in R$, then

Put $x=y=0 \Rightarrow f(0)=0$

Now, put $y=-x \Rightarrow f(x)+f(-x)=0$

$\therefore f(x)$ is an odd function

Q11. If the graph of the function $f(x)=a x^{3}+x^{2}+b x+c$ is symmetric about the line $x=2$, then the value of $a+b$ is equal to

A. 10

B. -4

C. 16

D. -10

Ans: -4

Solution: $f(x)$ is symmetric about the line $x=2$

$\therefore f(2+x)=f(2-x)$

$a(2+x)^3+(2+x)^2+b(2+x)+c=a(2-x)^3+(2-x)^2+b(2-x)+c$

$a\left\{(2+x)^3-(2-x)^3\right\}+\left\{(2+x)^2-(2-x)^2\right\}+b\{(2+x)-(2-x)\}=0$

$a\left\{\left(8+12x+6x^2+x^3\right)-\left(8-12x+6x^2-x^3\right)\right\}+2(4x)+b(2x)=0$

$a\left(24x+2x^3\right)+8x+2bx=0$

$2ax^3+(24a+2b+8)x=0$

Which must be true $\forall x \in R$

$\therefore$ it is an identity

$\therefore 2a=0,\ 24a+2b+8=0$

$a=0,\ 2b+8=0$

$b=-4$

Q12. The fundamental period of the function $f(x)=|\sin x|+|\cos x|$ is

A. π

B. $\pi/2$

C. 2π

D. None of these

Ans: $\pi/2$

Solution: $f\left(\frac{\pi}{2}+x\right)=\left|\sin\left(\frac{\pi}{2}+x\right)\right|+\left|\cos\left(\frac{\pi}{2}+x\right)\right|$

$=|\cos x|+|\sin x|$ for all x.

Hence, $f(x)$ is periodic with period $\frac{\pi}{2}$.

Q13. A function f is defined as $f(x)=\frac{1}{2}\left(\frac{|\sin x|}{\cos x}+\frac{\sin x}{|\cos x|}\right)$. If the fundamental period of function f is $m\pi$, then the value of m is

Ans: 2

Solution:

$f(x)=\frac{1}{2}\left(\frac{|\sin x|}{\cos x}+\frac{\sin x}{|\cos x|}\right)$

$=\frac{1}{2}\left(\frac{\sin x}{\cos x}+\frac{\sin x}{\cos x}\right),\ 0\le x<\frac{\pi}{2}$

$=\frac{1}{2}\left(\frac{\sin x}{\cos x}-\frac{\sin x}{\cos x}\right),\ \frac{\pi}{2}\le x<\pi$

$=\frac{1}{2}\left(\frac{-\sin x}{\cos x}-\frac{\sin x}{\cos x}\right),\ \pi\le x<\frac{3\pi}{2}$

$=\frac{1}{2}\left(\frac{-\sin x}{\cos x}+\frac{\sin x}{\cos x}\right),\ \frac{3\pi}{2}\le x<2\pi$

$\Rightarrow f(x)=\tan x,\ \ 0\le x<\frac{\pi}{2}$

$=0\ ,\ \frac{\pi}{2}\le x<\pi$

$=-\tan x,\ \pi\le x<\frac{3\pi}{2}$

$=0,\ \frac{3\pi}{2}\le x<2\pi$

$\Rightarrow f$ is periodic with fundamental period $2\pi\Rightarrow m=2$

Q14. If $f:R\rightarrow A$ defined as $f(x)=\tan^{-1}\left(\sqrt{4(x^2+x+1)}\right)$ is surjective, then A is equal to

A. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

B. $\left[0, \frac{\pi}{2}\right)$

C. $\left[\frac{\pi}{3}, \frac{\pi}{2}\right)$

D. $\left(0, \frac{\pi}{3}\right]$

Ans: $\left[\frac{\pi}{3}, \frac{\pi}{2}\right)$

Solution:

$\because\ x^2+x+1=\left(x+\frac{1}{2}\right)^2+\frac{3}{4}$

$\therefore\ 4\left(x^2+x+1\right) \geq 3$

$\Rightarrow \sqrt{4(x^2+x+1)} \geq \sqrt{3}$

$\Rightarrow \tan^{-1}\left(\sqrt{4(x^2+x+1)}\right) \geq \tan^{-1}\left(\sqrt{3}\right)$

$\Rightarrow f\left(x\right) \geq \frac{\pi}{3}$

$\therefore$ Range of $f(x)$ *is* $\left[\frac{\pi}{3}, \frac{\pi}{2}\right)$

Q15. Which of the following function is surjective but not injective?

A. $f: R \rightarrow R,\ f(x)=x^4+2x^3-x^2+1$

B. $f: R \rightarrow R,\ f(x)=x^3+x+1$

C. $f: R \rightarrow R^+,\ f(x)=\sqrt{1+x^2}$

D. $f: R \rightarrow R,\ f(x)=x^3+2x^2-x+1$

Ans: $f: R \rightarrow R,\ f(x)=x^3+2x^2-x+1$

Solution:

$f(x)=x^4+2x^3-x^2+1$ is polynomial of even degree hence its range can't be R. Hence not surjective.

$f(x)= x^3+x+1,\ f'(x)= 3x^2+1 \Rightarrow$ monotonic hence bijective.

$f(x)= \sqrt{1+x^2}$, neither surjective not injective

$f(x)= x^3+2x^2-x+1,\ f'(x)= 3x^2+4x-1 \Rightarrow D=16+12>0$

Hence $f(x)$ is non-monotonic cubic polynomial hence surjective but not injective.

Q16. If $f: R \rightarrow R$ be defined as $f(x)= \frac{e^{2x}-e^{-2x}}{2}$, then

A. f is many-one

B. f is into

C. $f^{-1}(x)= \frac{1}{2}\left[\log\left(x-\sqrt{x^2+1}\right)\right]$

D. $f^{-1}(x)= \frac{1}{2}\left[\log\left(x+\sqrt{x^2+1}\right)\right]$

Ans: $f^{-1}(x)= \frac{1}{2}\left[\log\left(x+\sqrt{x^2+1}\right)\right]$

Solution: Let us check for invertibility of $f(x)$

(A) one-one: we have, $f(x)= \frac{e^{2x}-e^{-2x}}{2}$

$\Rightarrow f'(x)= \frac{e^{4x}+1}{e^{2x}}$, which is strictly increasing as $e^{4x}>0$ for all x.

Thus, f is one-one

(B) Onto: Let $y=f(x)$

$\Rightarrow \frac{dy}{dx}=e^{2x}+e^{-2x}$, where y is strictly monotonic

Hence, the range of $f(x)=(f(-\infty), f(\infty))$

$\Rightarrow$ range of $f(x)=(-\infty, \infty)$

So, the range of $f(x)=$ co-domain

Hence, $f(x)$ is one-one and onto

(C) To find $f^{-1}: y=\frac{e^{4x}-1}{2e^{2x}}$

$\Rightarrow e^{4x}-2e^{2x}y-1=0$

$\Rightarrow e^{2x}=\frac{2y\pm\sqrt{4y^2+4}}{2}$

$\Rightarrow 2x = \log\left(y \pm \sqrt{y^2+1}\right)$

$\Rightarrow f^{-1}(y) = \frac{\log\left(y \pm \sqrt{y^2+1}\right)}{2}$

Since, $e^{f^{-1}(x)}$ is always positive, so neglecting negative sign.

Hence, $f^{-1}(x) = \frac{\log\left(x + \sqrt{x^2+1}\right)}{2}$

Q17. If $f : R \rightarrow R$ be a function such that $f(x) = x^3 + x^2 + 4x + \sin x$. Then, the function $f(x)$ is

A. one-one and onto

B. one-one and into

C. many-one and onto

D. many-one and into

Ans: one-one and onto

Solution:

$f(x) = x^3 + x^2 + 4x + \sin x$

$\Rightarrow f'(x) = 3x^2 + 2x + 4 + \cos x$

$f'(x) = 3\left[\left(x + \frac{1}{3}\right)^2 + \frac{11}{9}\right] - (-\cos x) > 0$ as $3\left[\left(x + \frac{1}{3}\right)^2 + \frac{11}{9}\right]_{\min} = \frac{11}{3}$

and $-\cos x$ has the maximum value 1.

$\Rightarrow f(x)$ is strictly increasing and hence it is one-one

Also, $\lim_{x \rightarrow \infty} f(x) \Rightarrow \infty$ and $\lim_{x \rightarrow -\infty} f(x) \Rightarrow -\infty$.

Thus, the range of $f(x)$ is R, hence it is onto.

Q18. Let $f: R \rightarrow R$ be a function defined by

$f(x)=\begin{cases} x+\frac{1}{x}, & x>0 \\ e^x, & x \leq 0 \end{cases}$ **then f is**

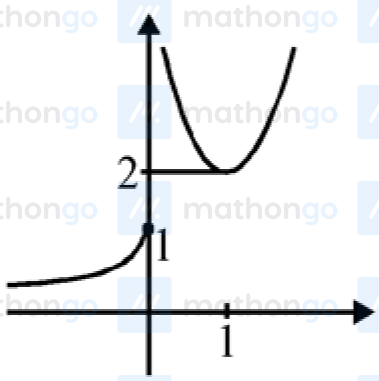
A. both one-one onto

B. one-one but not onto

C. onto but not one-one

D. neither one-one nor onto

Ans: neither one-one nor onto



Solution:

Q19. A function $f: Z \rightarrow Z$ is defined as $f(n)=\begin{cases} n+1 & n \in \text{odd integer} \\ \frac{n}{2} & n \in \text{even integer} \end{cases}$. If $k \in$ odd integer and $f(f(f(k)))=33$, then the sum of the digits of k is

A. 7

B. 5

C. 9

D. 8

Ans: 5

Solution: $\because\ k \in \text{odd integer}$

$f(k)= k+1$ which is even

$\therefore f(f(k))= \frac{k+1}{2}$

If $\frac{k+1}{2}$ is odd $\Rightarrow 33 = \frac{k+1}{2} + 1 \Rightarrow k = 63$ not possible (because $\frac{k+1}{2}$ is even)

If $\frac{k+1}{2}$ is even $\Rightarrow 33 = \frac{\frac{k+1}{2}}{2} \Rightarrow 33 = \frac{k+1}{4} \Rightarrow k = 131$

Sum of digits $= 5$

Q20. If $A =\{1.\ 2,\ 3,\ 4\}$ and $f\ : A \rightarrow A$, the total number of invertible functions, 'f', such that $f(2)\neq 2,\ f(4)\neq 4,\ f(1)= 1$ is equal to

A. 1

B. 2

C. 3

D. None of these

Ans: 3

Solution: $If\ f(2)= 3 \Rightarrow f(4)= 2,\ f(3)= 2$

$If\ f(2)= 4\ \Rightarrow f(4)= 2\ ,\ f(3)= 3$

or

$f(4)=3,\ f(3)=2$

Hence there are 3 such functions

Q21. Given two real sets $A=\{a, a_2, a_3 \cdots a_{2n}\}$ and $B\{b_1, b_2, \cdots b_n\}$. If $f : A \rightarrow B$ is a function such that every element of B has an inverse image and $f(a_1) \leq f(a_2) \leq f(a_3) \leq f(a_4) \cdots \leq f(a_{2n})$, then the number of such mappings are

A. ${}^{2n}C_n$

B. ${}^{2n}C_{n-1}$

C. ${}^{2n-1}C_{n-1}$

D. ${}^{2n+1}C_n$

Ans: ${}^{2n-1}C_{n-1}$

Solution: There is no loss of generality in considering the order of bi's as $b_1 < b_2 < b_3 \cdots < b_n$ also given that $f(a_1) \leq f(a_2) \cdot f(a_{2n})$. Now suppose number of preimages of every b_i are x_i in numbers. Therefore $x_1 + x_2 + \cdots + x_n = 2n$ where $1 \leq x_i \leq n+1 \rightarrow (1)$

Number of solutions of (1) is ${}^{2n-1}C_{n-1}$ or ${}^{2n-1}C_n$

Q22. Let $A = \{0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7\}$. Then the number of bijective functions $f : A \rightarrow A$ such that $f(1) + f(2) = 3 - f(3)$ is equal to

Ans: 720

Solution:

Given the set $A=\{0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7\}$.

And, also a function $f: A \rightarrow A$ satisfying $f(1)+f(2)=3-f(3)$

$\Rightarrow\ f(1)+f(2)+f(3)=3$

Since, the range of the function is also A, hence the only possibility satisfying the given condition is: $0+1+2=3$

We know that, the number of arrangements of n objects at n places is $n!$.

Since, the given function is bijective i.e. one-one and onto, hence, the elements $1,\ 2,\ 3$ in the domain can be mapped with only $0,\ 1,\ 2$ in the co-domain in $3!$ ways and the remaining 5 elements $0,\ 4,\ 5,\ 6,\ 7$ in the domain can be mapped with any of the remaining 5 elements $3,\ 4,\ 5,\ 6,\ 7$ in $5!$ ways.

So, the number of bijective functions are $=3!\times 5!=6\times 120=720$.

Q23. If $f: R \rightarrow R, f(x)=\frac{\sin([x]\pi)}{x^2+2x+3}+2x-1+\sqrt{x(x-1)+\frac{1}{4}}$ (where $[x]$ denotes greatest integral value less than or equal to x) denotes a function, then number of real solutions of equation $f(x)=f^{-1}(x)$ is

A. 0

B. 1

C. 2

D. 3

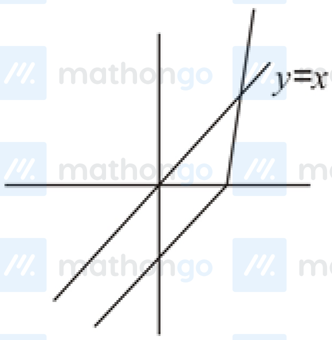
Ans: 1

Solution:

$f(x)=2x-1+\left|x-\frac{1}{2}\right|$

$$=\begin{cases} x-\frac{1}{2} & x<\frac{1}{2} \\ 3x-\frac{3}{2} & x \geq \frac{1}{2} \end{cases}$$

As $f(x)$ is increasing, solution of equation $f(x)= f^{-1}(x)$ is same as $f(x)= x$.



$\therefore$ No. of solutions of $f(x)= f^{-1}(x)$ is one.

Q24. Equation $|x-4|+|x+4|= ax+8$ has

A. no solution if $a \in (-\infty,-2] \cup [2,\infty)$.

B. exactly one solution if $a \in(-2,2)$.

C. exactly two solutions if $a \in(-2,0)\cup(0,2)$.

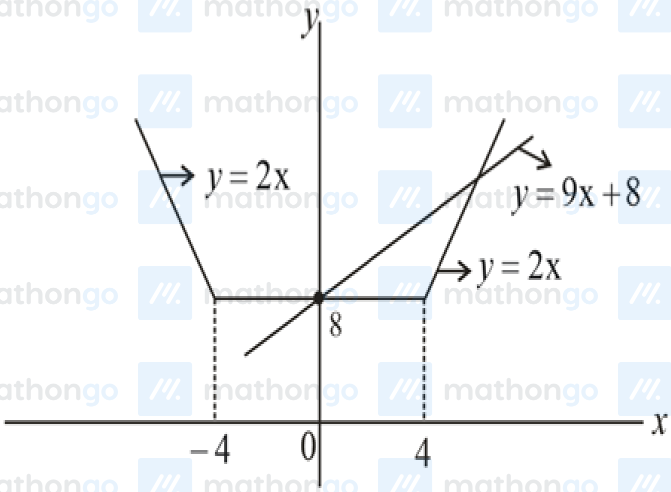
D. exactly two solutions if $a=0$$

Ans: exactly two solutions if $a \in(-2,0)\cup(0,2)$.

Solution:

We have,

$$|x+4|+|x-4|=\begin{cases} -2x & x<-4 \\ 8 & -4 \leq x \leq 4 \\ 2x & x>4 \end{cases}$$



It is clear from the above straight line $y=ax+8$ cuts the curve $y=|x+4|+|x-4|$ at exactly one point of $a\in(-\infty,-2]\cup[2,\infty)$ exactly two points of $a\in(-2,0)\cup(0,2)$ more than two points if $a=0$. Hence, option (C) is correct.

Q25. The total number of solution(s) of the equation $2x+3\tan x=\frac{5\pi}{2}$ in $x\in[0,2\pi]$ is/are equal to

A. 1

B. 2

C. 3

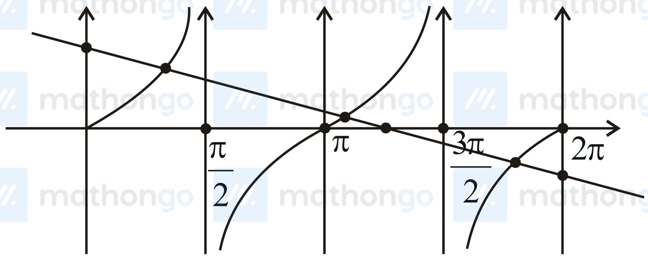
D. 4

Ans: 3

Solution:

$2x+3\tan x=\frac{5\pi}{2}\Rightarrow\tan x=-\frac{2}{3}x+\frac{5\pi}{6}$

$y=\tan x$ and $y=\frac{5\pi}{6}-\frac{2x}{3}$



Both the graphs meet exactly three times in $[0,2\pi]$.

Thus, there are 3 solutions.

Q26. If $2f(xy)=(f(x))^y+(f(y))^x$ for all $x, y \in R$ and $f(1)=3$, then the value of $\sum_{r=1}^{10} f(r)$ is equal to

A. $\frac{3}{2}\left(3^{10}-1\right)$

B. $\frac{3}{2}\left(3^{9}-1\right)$

C. $\frac{3^{10}-1}{2}$

D. $\frac{1}{2}\left(3^{10}-1\right)$

Ans: $\frac{3}{2}\left(3^{10}-1\right)$

Solution: From given functional equation, $2f(xy)=(f(x))^y+(f(y))^x$, $\forall x, y \in R$

putting $y=1$,

$2f(x)=f(x)+(f(1))^x$

$f(x)=3^x$

$\therefore \sum_{r=1}^{10} f(r)=\sum_{r=1}^{10} 3^r=\frac{3\left(3^{10}-1\right)}{3-1}=\frac{3}{2}\left(3^{10}-1\right)$

Q27. Let f be a function such that $f(x)+f\left(\frac{1}{1-x}\right)=\frac{2(1-2x)}{x(1-x)}$ where $x\in R-\{0,1\}$, then the value of $f(2)$ must be

Ans: 3

Solution: Replacing x by $\frac{1}{(1-x)}$, we obtain $f\left(\frac{1}{1-x}\right)+f\left(1-\frac{1}{x}\right)=-2x+\frac{2}{x}$

Again, replacing x by $1-\frac{1}{x}$ and solve

Q28. Let $f(x)$ be a function defined as $f:R\rightarrow R$ such that $f(x+2)+f(x-2)=f(x)$ and $f(1)=3$ then the value of the expression $\sum_{r=0}^{15} f(1+12r)$ is equal to

Ans: 48.00

Solution: $\because f(x+2)+f(x-2)=f(x)$ ...(i)

Replace x by $x+2$: $f(x+4)+f(x)=f(x+2)$ ...(ii)

from eq (i) and (ii) : $f(x-2)+f(x+4)=0$

Replace x by $x+2$: $f(x)+f(x+6)=0$ ...(iii)

Replace x by x_{-2} in eq (i): $f(x)+f(x-4)=f(x-2)$ ...(iv)

from eq (i) and eq (iv): $f(x+2)+f(x-4)=0$

Replace x by $x-2$: $f(x)+f(x-6)=0$ ...(v) from (iii) and (v) : $f(x+6)=f(x-6)$

$\therefore f(x)$ is periodic with period 12.

$\therefore \sum_{r=0}^{15} f(1+12r)=f(1)+f(13)+f(25)+\ldots..$

$=16\times f(1)$

$=16\times 3$

$=48$

Q29. If $f(x+y)=f(x)+f(y)-xy-1, \forall\, x,\ y\in R$ and $f(1)=1$, then the number of solutions of $f(n)=n,\ n\in N$, is

A. one

B. two

C. no solution

D. three

E. None of these

Ans: one

Solution:

Given, $f(x+y)=f(x)+f(y)-xy-1, \forall\, x,\ y\in R$

$\therefore\ f(x+1)=f(x)+f(1)-x-1$ [putting $y=1$]

$\Rightarrow f(x+1)=f(x)-x\ \ [\because\ f(1)=1]$

$\therefore\ f(n+1)=f(n)-n<f(n)$

$\Rightarrow f(n+1)<f(n)$

So, $f(n)<f(n-1)<f(n-2)<\ldots<f(3)<f(2)<f(1)=1$

$\therefore\ f(n)=n$ holds only for $n=1$

Q30. If $f(x)$ is a real valued function such that $f(x+6)-f(x+3)+f(x)=0, \forall x \in R$, then period of $f(x)$ is

A. 6

B. 12

C. 18

D. 24

Ans: 18

Solution: In given equation, putting $x \rightarrow x+3$

$f(x+9)-f(x+6)+f(x+3)=0$

On adding both equations, we get,

$f(x+9)+f(x)=0$ …(i)

now, putting $x \rightarrow x+9$

$f(x+18)+f(x+9)=0$ …(ii)

Equation (ii) – Equation (i)

$f(x+18)=f(x)$

$\therefore$Period $=18$

Answer Key

Q1 (3)	**Q2** (1)	**Q3** (4)	**Q4** (4)
Q5 (3)	**Q6** (3)	**Q7** (3)	**Q8** (10.5)
Q9 (2)	**Q10** (3)	**Q11** (2)	**Q12** (2)
Q13 (2)	**Q14** (3)	**Q15** (4)	**Q16** (4)
Q17 (1)	**Q18** (4)	**Q19** (2)	**Q20** (3)
Q21 (3)	**Q22** (720)	**Q23** (2)	**Q24** (3)
Q25 (3)	**Q26** (1)	**Q27** (3)	**Q28** (48.00)
Q29 (1)	**Q30** (3)		