## Q1. If $x=\frac{1-t^2}{1+t^2}$ and $y=\frac{2t}{1+t^2}$, then $\frac{dy}{dx}$ is equal to

**A.** $-\frac{y}{x}$

**B.** $\frac{y}{x}$

**C.** $-\frac{x}{y}$

**D.** $\frac{x}{y}$

Ans: $-\frac{x}{y}$

Solution: $\because x=\frac{1-t^2}{1+t^2}$ and $y=\frac{2t}{1+t^2}$

Put $t=\tan\theta$ in both the equations,

we get $x=\frac{1-\tan^2\theta}{1+\tan^2\theta}=\cos 2\theta$

and $y=\frac{2\tan\theta}{1+\tan^2\theta}=\sin 2\theta$...(ii)

On differentiating both the eqs.(i) and (ii), we get $\frac{dx}{d\theta}=-2\sin 2\theta$ and $\frac{dy}{d\theta}=2\cos 2\theta$

Therefore, $\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=-\frac{\cos 2\theta}{\sin 2\theta}=-\frac{x}{y}$

## Q2. If $\log_{10}\left(\frac{x^3-y^3}{x^3+y^3}\right)=2$, then $\frac{dy}{dx}=$

**A.** $\frac{x}{y}$

**B.** $-\frac{y}{x}$

**C.** $-\frac{x}{y}$

**D.** $\frac{y}{x}$

Ans: $\frac{y}{x}$

Solution: Given, $\log_{10}\left(\frac{x^3-y^3}{x^3+y^3}\right)=2$

$\Rightarrow\left(\frac{x^3-y^3}{x^3+y^3}\right)=(10)^2=100$

Applying componendo and dividendo, we get, $\frac{x^3}{y^3}=-\frac{101}{99}\Rightarrow\frac{x}{y}=\left(-\frac{101}{99}\right)^{\frac{1}{3}}\ldots\ldots$ (i)

Hence, $y=\left(-\frac{99}{101}\right)^{\frac{1}{3}}x$

Differentiating, we get,

$\frac{dy}{dx}=\left(-\frac{99}{101}\right)^{\frac{1}{3}}=\frac{y}{x}$ (from (i))

**Q3. If $y=a\cos(\log x)+b\sin(\log x)$, where $a\&b$ are parameters, then $x^2y''+xy'$ is equal to**

**A.** $y$

**B.** $-y$

**C.** $2y$

**D.** $-2y$

Ans: $-y$

Solution: $y=a\cos(\log x)+b\sin(\log x)$ On differentiating w.r.t. $x$, we get $y'=\frac{-a\sin(\log x)}{x}+\frac{b\cos(\log x)}{x}$

$\Rightarrow xy'=-a\sin(\log x)+b\cos(\log x)$

Again, on differentiating w.r.t. $x$, we get

$xy''+y'=-a\cos(\log x)\frac{1}{x}-b\sin(\log x)\frac{1}{x}$

$\Rightarrow x^2y''+y'x=-[a\cos(\log x)+b\sin(\log x)]$

$\Rightarrow x^2y''+xy'=-y$

**Q4. If $\phi(x)=\log_8\log_3 x$, then $\phi'(e)$ is equal to**

**A.** $e\log 8$

**B.** $-e\log 8$

**C.** $\frac{1}{e\log 8}$

**D.** None of these

Ans: $\frac{1}{e\log 8}$

Solution: We have,

$\phi(x)=\log_8 \log_3 x=\log_8\left(\frac{\log x}{\log 3}\right)$

$=\log_8(\log x)-\log_8(\log 3)$

$=\frac{\log(\log x)}{\log 8}-\log_8(\log 3)$

$\phi'(x)=\frac{1}{\log 8}\cdot\frac{1}{\log x}\cdot\frac{1}{x}-0$

$\therefore \phi'(e)=\frac{1}{\log 8}\cdot\frac{1}{\log e}\cdot\frac{1}{e}=\frac{1}{e\log 8}$

**Q5. If $y=\log_{10} x+\log_x 10+\log_x x+\log_{10} 10$, then $\frac{dy}{dx}$ is equal to**

**A.** $\frac{1}{x\log_e 10}-\frac{\log_e 10}{x(\log_e x)^2}$

**B.** $\frac{1}{x\log_e 10}-\frac{1}{x\log_{10} e}$

**C.** $\frac{1}{x\log_e 10}-\frac{\log_e 10}{x(\log_e x)}$

**D.** None of these

Ans: $\frac{1}{x\log_e 10}-\frac{\log_e 10}{x(\log_e x)^2}$

Solution: $y=\log_{10} x+\log_x 10+\log_x x+\log_{10} 10$

$=\log_{10} x+\frac{\log_e 10}{\log_e x}+1+1$

$\Rightarrow \frac{dy}{dx}=\frac{1}{x}\log_{10} e-\frac{\log_e 10}{x(\log_e x)^2}$

**Q6. If $f: R\rightarrow R$ is a function defined as $f\left(x^3\right)=x^5, \forall x\in R-\{0\}$ and $f(x)$ is differentiable $\forall x\in R$ then the value of $\frac{1}{4}f'(27)$ is equal to (here $f'$ represents the derivative of $f$)**

Ans: 3.75

Solution: $f\left(x^3\right)=x^5$

On differentiating with respect to $x$

$f'\left(x^3\right)\cdot 3x^2=5\cdot x^4$

$f'\left(x^3\right)=\frac{5}{3}x^2$

Putting $x=3$, we get,

$f'(27)=\frac{5}{3}(9)=15$

**Q7. If $y=2+\sqrt{\sin x+2+\sqrt{\sin x+2+\sqrt{\sin x+\ldots\infty}}}$, then the value of $\frac{dy}{dx}$ at $x=0$ is**

**A.** 0

**B.** 2

**C.** $\frac{1}{2}$

**D.** $\frac{1}{3}$

Ans: $\frac{1}{3}$

Solution: Given equation can be rewritten as

$y=2+\sqrt{\sin x+y}$

$\Rightarrow(y-2)^2=\sin x+y$

$\Rightarrow y^2-4y+4=\sin x+y\ldots$ (i)

Putting $x=0$ in the equation (i), we get,

$y^2-4y+4=0+y\Rightarrow y^2-5y+4=0$

$\Rightarrow(y-1)(y-4)=0$

$y=1$ or $y=4$

$\because y>2\Rightarrow y=4$

Now differentiating equation (i) with respect to $x$, we get, $\frac{dy}{dx}=\frac{\cos x}{2y-5}$

Putting $x=0, y=4$

$\frac{dy}{dx}=\frac{\cos(0)}{2(4)-5}=\frac{1}{3}$

**Q8. If $x=3\cos t$ and $y=5\sin t$, where $t$ is a parameter, then $9\frac{d^2y}{dx^2}$ at $t=-\frac{\pi}{6}$ is equal to**

Ans: 40

Solution:

$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{5\cos t}{-3\sin t}=\frac{-5}{3}\cot(t)$

$\frac{d^2y}{dx^2}=\frac{-5}{3}\cdot\left(-\operatorname{cosec}^2 t\right)\cdot\frac{dt}{dx}=\frac{5}{3}\left(\frac{1}{\sin^2 t}\right)\left(\frac{1}{-3\sin t}\right)$

$=\frac{5}{9}(8)$

Hence, $9\frac{d^2y}{dx^2}=40$

**Q9. If $y=x^2+\frac{1}{x^2+\frac{1}{x^2+\frac{1}{x^2+\ldots\infty}}}$, then the value of $\frac{dy}{dx}$ is:**

**A.** $\frac{2\,xy}{2y-x^2}$

**B.** $\frac{xy}{y+x^2}$

**C.** $\frac{xy}{y-x^2}$

**D.** $\frac{2\,xy}{2y+x^2}$

Ans: $\frac{2\,xy}{2y-x^2}$

Solution:

$y=x^2+\frac{1}{y}$

$y^2=x^2y+1$

$2y\frac{dy}{dx}=y.2x+x^2\frac{dy}{dx}$

$\frac{dy}{dx}=\frac{2\,xy}{2y-x^2}$

**Q10. If $f(x)=\left(\frac{2+x}{1+x}\right)^{1+x}$, then $f'(0)$ is equal to**

**A.** $2\log 2$

**B.** $\log 2$

**C.** $3\log 2-1$

**D.** $2\log 2-1$

Ans: $2\log 2-1$

Solution:

By taking logarithm on both sides,

$\log(x)\ =\ (1+x)\ [\log(2+x)-\log(1+x)]$

On differentiating, we get,

$\frac{1}{f(x)} \cdot f'(x) = \frac{(1+x)}{(2+x)} + \log(2+x) - \frac{(1+x)}{(1+x)} - \log(1+x)$

Now, put $x = 0$

$f'(0) = f(0)\left\{\frac{1+\log 2}{2} - 1\right\} = f(0)\left[\log 2 - \frac{1}{2}\right]$

$= 2\log 2 - 1 (\because f(0) = 2)$

**Q11. If $x^3 + y^3 = t + \frac{4}{t}$ and $x^6 + y^6 = t^2 + \frac{16}{t^2}$ then find $\left|x^4 y^2 \frac{dy}{dx}\right|$.**

Ans: 4

Solution:

$x^3 + y^3 = t + \frac{4}{t}$

$x^6 + y^6 + 2x^3y^3 = t^2 + \frac{16}{t^2} + 8$

... [By squaring both the sides]

$\Rightarrow \left(t^2 + \frac{16}{t^2}\right) + 2x^3y^3 = t^2 + \frac{16}{t^2} + 8$

$\Rightarrow x^3y^3 = 4 \quad \ldots(i)$

Differentiating with respect to $x$ we get

$x^3\left(3y^2\frac{dy}{dx}\right) + y^3 \cdot (3x^2) = 0$

$\Rightarrow 3x^3y^2\frac{dy}{dx} = -3x^2y^3$

$\Rightarrow x^3y^2\frac{dy}{dx} = -x^2y^3$

$\Rightarrow x^4 y^2 \frac{dy}{dx} = -x^3 y^3$

$\Rightarrow x^4 y^2 \frac{dy}{dx} = -4$ ...[From (i)]

**Q12. Let $f(x)$ be a polynomial of degree $3$ such that $f(3) = 21$, $f'(3) = 30$, $f''(3) = 22$ and $f'''(3) = 6$. Find the value of $f'(2)$.**

Ans: 11

Solution:

Let $f(x) = a(x-3)^3 + b(x-3)^2 + c(x-3) + d$

$f(3) = 21 \Rightarrow d = 21$

$f'(3) = 30 \Rightarrow c = 30$

$f''(3) = 22 \Rightarrow b = 11$

$f'''(3) = 6 \Rightarrow a = 1$

$f(x) = (x-3)^3 + 11(x-3)^2 + 30(x-39) + 21$

$\Rightarrow f'(x) = 3(x-3)^2 + 22(x-3) + 30$

$\Rightarrow f'(2) = 11$

**Q13. Find the value of $(fgh)'(0)$, if $f$, $g$ and $h$ are differentiable functions with $f(0) = 1$, $g(0) = 2,\ h(0) = 3$ and the derivatives of their pair wise products at $x = 0$ are $(fg)'(0) = 6,\ (gh)'(0) = 4$ and $(hf)'(0) = 5$.**

Ans: 16

Solution:

$y = fgh$

$\frac{dy}{dx} = f'gh + fg'h + fgh'$

$= \frac{1}{2}(2f'gh + 2fg'h + 2fgh')$

$= \frac{1}{2}(h(f'g + g'f) + g(f'h + fh') + f(g'h + gh))$

$= \frac{1}{2}\left[h \cdot (fg)' + g \cdot (fh)' + f \cdot (gh)'\right]$

$(fgh)'\left(0\right) = \frac{1}{2}\left[h(0)(fg)'(0) + g(0)(fh)'(0) + f(0)(gh)'(0)\right]$

$= \frac{1}{2}(3 \times 6 + 2 \times 5 + 1 \times 4)$

$= \frac{1}{2}(18 + 10 + 4) = \frac{32}{2} = 16$

**Q14. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfy $f(x+y) = 2^x f(y) + 4^y (f(x), \forall x, y \in \mathbb{R}$. If $f(2) = 3$, then $14 \cdot \frac{f'(4)}{f'(2)}$ is equal to ______.**

Ans: 248

Solution:

Given, $f(x+y) = 2^x f(y) + 4^y (f(x)$

Put $y = 2$ we get,

$f(x+2) = 2^x \times 3 + 16 f(x)$

$f'(x+2) = 16f'(x) + 3 \times 2^x \ln 2$

Now put $x = 2$ we get,

$f'(4) = 16f'(2) + 12\ln 2 \quad \ldots(\text{i})$

Similarly, $f(y+2) = 4f(y) + 3 \times 4^y$

$f'(4) = 4f'(y) + 3 \times 4^y \ln 4$

$f'(4) = 4f'(2) + 96\ln 2 \quad \cdots(\text{ii})$

solving eq. (i) and (ii), we get

$f'(2) = 7\ln 2$

from equation (i), we get

$f'(4) = 124\ln 2$

Now $\Rightarrow 14 \times \frac{f'(4)}{f'(2)}$

$14 \times \frac{124\ln 2}{7\ln 2} = 248$

**Q15. Find the value of $f^2(4) + g^2(4)$, if $f'(x) = g(x)$ and $g'(x) = -f(x)$ for all $x$ and $f(2) = 4 = f'(2)$.**

Ans: 32

Solution:

$f^2(4) + g^2(4) = ?$

$\frac{d}{dx}\left[f^2(x) + g^2(x)\right] = 2f\left(x\right)f'\left(x\right) + 2g\left(x\right)g'\left(x\right)$

$= 2\ f(x)\ g(x) + 2\ g(x)(-f(x))$

$= 0$

$\therefore f^2(x) + g^2(x) = \text{constant}$

$\therefore f^2(4) + g^2(4) = f^2(2) + g^2(2)$

$= 4^2 + (f'(2))^2$

$= 4^2 + 4^2 = 32$

**Q16. If for $x \in (0, 1/4)$, derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x} \cdot g(x)$ then $g(x)$ is equal to.**

**A.** $\frac{3}{1+9x^3}$

**B.** $\frac{9}{1+9x^3}$

**C.** $\frac{3x\sqrt{x}}{1-9x^3}$

**D.** $\frac{3x}{1-9x^3}$

Ans: $\frac{9}{1+9x^3}$

Solution: $y = \tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x}\right)$

$= \tan^{-1}\left(\frac{2\left(3x^{\frac{3}{2}}\right)}{1-\left(3x^{\frac{3}{2}}\right)^2}\right) = 2\tan^{-1}\left(3x^{\frac{3}{2}}\right)$

$\frac{dy}{dx} = \frac{2\cdot\left(3x^{\frac{1}{2}}\right)\left(\frac{3}{2}\right)}{1+9x^2} = \frac{9}{1+9x^2}\sqrt{x}$

$\therefore \quad g(x) = \frac{9}{1+9x^3}$

**Q17. If $y = e^{nx}$, then $\left(\frac{d^2y}{dx^2}\right)\frac{d^2x}{dy^2}$ is equal to**

**A.** $ne^{nx}$

**B.** $ne^{-nx}$

**C.** 1

**D.** $-ne^{-nx}$

Ans: $-ne^{-nx}$

Solution: $y=e^{nx} \Rightarrow \operatorname{In} y=nx$

Now, $\frac{d^2y}{dx^2}=n^2e^{nx}=n^2y$

and $\frac{d^2x}{dy^2}=\frac{1}{ny^2}$

$\therefore\left(\frac{d^2y}{dx^2}\right)\left(\frac{d^2x}{dy^2}\right)=\frac{n}{y}=-ne^{-nx}$

## Q18. Let $\phi(x)$ be the inverse of the function $f(x)$ and $f'(x)=\frac{1}{1+x^5}$, then $\frac{d}{dx}\phi(x)$ is equal to

**A.** $\frac{1}{1+[\phi(x)]^5}$

**B.** $\frac{1}{1+[f(x)]^5}$

**C.** $1+[\phi(x)]^5$

**D.** $1+f(x)$

Ans: $1+[\phi(x)]^5$

Solution: We have, $\phi(x)=f^{-1}(x)$

$\Rightarrow x=f[\phi(x)]$

On differentiating both sides w.r.t. $x$, we get $1=f'[\phi(x)]\cdot\phi'(x) \Rightarrow \phi'(x)=\frac{1}{f'[\phi(x)]}\cdots$ (i)

Since, $f'(x)=\frac{1}{1+x^5}$ (given)

$f'(\phi(x))=\frac{1}{1+[\phi(x)]^5}$

From Equation (i),

$\phi'(x)=\frac{1}{f'[\phi(x)]}=1+[\phi(x)]^5$

## Q19. If $x^my^n=(x+y)^{m+n}$, then $\frac{dy}{dx}$ is

**A.** $\frac{x}{y}$

**B.** $\frac{y}{x}$

**C.** $\frac{x+y}{xy}$

**D.** $xy$

Ans: $\frac{y}{x}$

Solution: $m \log x + n \log y = (m+n) \log(x+y)$

$\Rightarrow \frac{m}{x} + \frac{n}{y} \frac{dy}{dx} = \frac{m+n}{x+y}\left(1 + \frac{dy}{dx}\right)$

$\Rightarrow \left(\frac{n}{y} - \frac{m+n}{x+y}\right) \frac{dy}{dx} = \frac{m+n}{x+y} - \frac{m}{y}$

$\Rightarrow \frac{nx-my}{y(x+y)} \frac{dy}{dx} = \frac{nx-my}{(x+y)x}$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$

**Q20. If $f(x) = x^3 + 3x + 1$ and $g(x)$ is the inverse function of $f(x)$, then the value of $g'(5)$ is equal to**

**A.** 3

**B.** $\frac{1}{3}$

**C.** $\frac{1}{6}$

**D.** 6

Ans: $\frac{1}{6}$

Solution: $\because f(x), g(x)$ are inverse function of each other

$\therefore g'(f(x)) = \frac{1}{f'(x)}$

Now put $x = 1; g'(f(1)) = \frac{1}{f'(1)}$

Hence, $g'(5) = \frac{1}{(3x^2+3)_{x=1}} = \frac{1}{6}$

**Q21. Let $f:(-1,1) \rightarrow \mathbf{R}$ be a differentiable function with $f(0)=-1$ and $f'(0)=1$. Let $g(x)=[f(2f(x)+2)]^2$ Then $g'(0)=$**

**A.** 0

**B.** - 2

**C.** 4

**D.** $-4$

Ans: $-4$

Solution: $g'(x)=2[f(2f(x)+2)]f'(2f(x)+2)\,(2f'(x))$

$\Rightarrow g'(0)=2[f(2f(0)+2)]f'(2f(0)+2)\,(2f'(0))$

$=4(f(0))f'(0)^2=4(-1)(1)^2=-4$

**Q22. $f(x)$ and $g(x)$ are two differential function on $[0,2]$ such that $f''(x)-g''(x)=0, f'(1)=2g'(1)=4, f(2)=3g(2)=9$ then $f(x)-g(x)$ at $x=\frac{3}{2}$ is**

**A.** 0

**B.** 2

**C.** 10

**D.** 5

Ans: 5

Solution: $f''(x)-g''(x)=0 \Rightarrow f'(x)-g'(x)=$ constant.

Putting $x=1$, we get $f'(1)-g'(1)=C \Rightarrow C=2g'(1)-g'(1)$

$=g'(1)=2$

So $f'(x)-g'(x)=2 \Rightarrow f(x)-g(x)=2x+C'$.

Putting $x=2$ we have $f(2)-g(2)=4+C \Rightarrow 4+$

$C'=3g(2)-g(2)=2g(2)=6 \Rightarrow C'=6-4=2$

Hence $f(x)-g(x)=2x+2$. At $x=\frac{3}{2}$

$f\left(\frac{3}{2}\right)-g\left(\frac{3}{2}\right)=2\cdot\frac{3}{2}+2=5$

## Q23. Let $y$ be an implicit function of $x$ defined by $x^{2x}-2x^{x}\cot y-1=0$. Then $y'(1)$ equals

**A.** $\log 2$

**B.** $-\log 2$

**C.** $-1$

**D.** $1$

Ans: $-1$

Solution: Putting $x=1$ in $x^{2x}-2x^{x}\cot y-1=0$

we get $1-2\cot y-1=0$

$\Rightarrow \cot y=0\Rightarrow y=\pi/2$

Using the formula,

$\frac{d}{dx}\left[f(x)^{g(x)}\right]=g(x)f(x)^{g(x)-1}f'(x)+(g'(x)\log$

$f(x))f(x)^{g(x)}$, we have

$2x\cdot x^{2x-1}(1)+(2\log x)x^{2x}-2\left[x\cdot x^{x-1}(1)+(\log x)\right.$

$\left.x^{x}\right]\cot y+2x^{x}\left(\operatorname{cosec}^{2}y\right)y'(x)=0$

Putting $x=1, y=\pi/2$, we get

$2+0-2(1)(0)+2\operatorname{cosec}^{2}(\pi/2)y'(1)=0$

$\Rightarrow y'(1)=-1$

## Q24. If $2y=\left(\cot^{-1}\left(\frac{\sqrt{3}\cos x+\sin x}{\cos x-\sqrt{3}\sin x}\right)\right)^{2}\ \forall x\in\left(0,\frac{\pi}{2}\right)$, then $\frac{dy}{dx}$ is equal to

**A.** $\frac{\pi}{6}-x$

**B.** $2x-\frac{\pi}{3}$

**C.** $x-\frac{\pi}{6}$

**D.** None of these

Ans: None of these

Solution:

$$2y=\left(\cot^{-1}\frac{\sqrt{3}\cos x+\sin x}{\cos x-\sqrt{3}\sin x}\right)^2$$

$$=\left(\cot^{-1}\left(\frac{\sqrt{3}+\tan x}{1-\sqrt{3}\tan x}\right)\right)^2$$

$$=\left(\cot^{-1}\tan\left(\frac{\pi}{3}+x\right)\right)^2$$

$$=\left(\cot^{-1}\cot\left(\frac{\pi}{2}-\left(\frac{\pi}{3}+x\right)\right)\right)^2$$

$$\Rightarrow 2y=\begin{cases}\left(\frac{\pi}{6}-x\right)^2,\ x\in\left(0,\frac{\pi}{6}\right)\\ \left(\pi+\frac{\pi}{6}-x\right)^2,\ x\in\left(\frac{\pi}{6},\frac{\pi}{2}\right)\end{cases}$$

$$\Rightarrow\ \frac{2dy}{dx}=2\left(\frac{\pi}{6}-x\right).(-1)\Rightarrow\frac{dy}{dx}=x-\frac{\pi}{6},\ x\in\left(0,\frac{\pi}{6}\right)$$

And $\frac{dy}{dx}=x-\frac{7\pi}{6},\ x\in\left(\frac{\pi}{6},\frac{\pi}{2}\right)$

Left Hand and Right Hand Derivatives are not same so function is non derivable at $x=\frac{\pi}{6}$.

Hence, $\frac{dy}{dx}$ does not exist for all values in the given interval.

## Q25.

**If $y=y(x)$ is an implicit function of $x$ such that $\log_e(x+y)=4xy$, then $\frac{d^2y}{dx^2}$ at $x=0$ is equal to**

Ans: 40

Solution:

Given:

$\log_e(x+y)=4xy$

When $x=0$, then $y=1$

$\log_e(x+y)=4xy$

$\Rightarrow x+y=e^{4xy}$

Now differentiate w.r.t. $x$

$1+y'=e^{4xy}(4y+4xy') \quad \ldots$(i)

At $(0,1) \Rightarrow y'(0)+1=4 \Rightarrow y'(0)=3$

Now, again differentiate equation (i), we get

$y''=e^{4xy}\left(4y+4xy'\right)^2+e^{4xy}(4y'+4y'+4xy'')$

At $(0,1)$

$y''(0)=1\left(4\times1+0\right)^2+1\left(4\times3+4\times3+0\right)$

$\Rightarrow y''(0)=16+24=40$

$\Rightarrow y''(0)=40$

**Q26. Let, $f:R\to R$ be a function such that $f(x)=x^3+x^2f'(1)+xf''(2)+f'''(3)$, $\forall x\in R$. Then $f(2)$ equals**

**A.** $30$

**B.** $8$

**C.** $-4$

**D.** $-2$

Ans: $-2$

Solution:

Let, $f(x)= x^3 + ax^2 + bx + c$

$\Rightarrow f'(x)= 3x^2 + 2ax + b$

$\Rightarrow f''(x)= 6x + 2a$

$\Rightarrow f'''(x)= 6$

According to the question,

$a = f'(1)= 3 + 2a + b \Rightarrow a + b = -3 \ldots(i)$

$b = f''(2)= 12 + 2a \Rightarrow 2a - b = -12 \ldots(ii)$

$c = f'''(3)\Rightarrow c = 6$

Solving equations $(i)$ & $(ii)$, we get, $a = \ -5$ & $b = 2$

$\Rightarrow f(x)= x^3 - 5x^2 + 2x + 6$

$\therefore\ f(2)= 8 - 20 + 4 + 6 = -2.$

**Q27. Let $f$ be a differentiable function such that $8f(x) + 6f\left(\frac{1}{x}\right) - x = 5(x \neq 0)$ and $y = x^2 f(x)$, then $\frac{dy}{dx}$ at $x = -1$ is**

**A.** $\frac{15}{14}$

**B.** $-\frac{15}{14}$

**C.** $-\frac{1}{14}$

**D.** $\frac{1}{14}$

Ans: $-\frac{1}{14}$

Solution: $8 f(x)+6 f\left(\frac{1}{x}\right)-x=5$ (i)

Replace $x$ by $\frac{1}{x}$, we have $8 f\left(\frac{1}{x}\right)+6 f(x)-\frac{1}{x}=5$ (ii)

Solving (i) and (ii) for $f(x)$ and $f(1 / x)$, we have

$14 f(x)=5-\frac{3}{x}+4 x$

$\Rightarrow 14 f^{\prime}(x)=\frac{3}{x^2}+4$

So $f(-1)=\frac{1}{14}[5+3-4]=\frac{2}{7}$

and $f^{\prime}(-1)=\frac{1}{14}[3+4]=\frac{1}{2}$

Differentiating $y=x^2 f(x)$, we have $\frac{d y}{d x}=2 x f(x)+$
$x^2 f^{\prime}(x)$

$\Rightarrow \left.\frac{d y}{d x}\right|_{x=1}=-2 f(-1)+f^{\prime}(-1)=-\frac{4}{7}+\frac{1}{2}=-\frac{1}{14}$

## Q28. For $x>1$, if $(2 x)^{2 y}=4 e^{2 x-2 y}$, then $\left(1+\log_e 2 x\right)^2 \frac{d y}{d x}$ is equal to

**A.** $\log_e 2 x$

**B.** $\frac{x \log_e 2 x-\log_e 2}{x}$

**C.** $x \log_e 2 x$

**D.** $\frac{x \log_e 2 x+\log_e 2}{x}$

Ans: $\frac{x \log_e 2 x-\log_e 2}{x}$

Solution:

Given, $(2 x)^{2 y}=4. e^{2 x-2 y}$

Taking natural logarithm on both sides, we get

$2y\log_e 2x = \log_e 4 + (2x - 2y)$

$\Rightarrow 2y = \left(\frac{\log_e 4 + 2x}{1+\log_e 2x}\right)$

Differentiating both sides with respect to $x$, we get

$\frac{2dy}{dx} = \frac{(1+\log_e 2x).2-(\log_e 4+2x)\frac{1}{x}}{(1+\log_e 2x)^2}$ (Using quotient rule)

$\Rightarrow (1+\log_e 2x)^2 \frac{dy}{dx} = \left(\frac{x.\log_e 2x - \log_e 2}{x}\right).$

**Q29. Let $y = f(x)$ is an invertible function satisfying $f(1) = 5,\ f'(1) = 2,\ f''(1) = 4$, then the absolute value of $2.\left(f^{-1}\right)''(5)$ is equal to**

Ans: 1

Solution: let $b^{-1}(x) = g(x)$

$g(b(x)) = x$

$g'(f(x))b'(x) = 1$

$g'(f(x)) = \frac{1}{b'}(x)$

$g''(f(x))f'(x) = \frac{-b''(x)}{(f'(x))^2}$

If $f(x) = 5, x = 1$

$\therefore g''(5) = -\frac{f''(1)}{(f'(1))^3} = \frac{-4}{8} = \frac{-1}{2}$

$|2g''(5)| = 1$

**Q30. If $y(x) = (x^x)^x, x > 0$ then $\frac{d^2x}{dy^2} + 20$ at $x = 1$ is equal to**

Ans: 16

Solution:

Given,

$y(x)=\left(x^x\right)^x$

Taking $\log_e$ both side

$\ln y(x)=x^2 \cdot \ln x$

Now differentiating both side w.r.t $x$ we get,

$\frac{1}{y(x)} \cdot y'(x)=\frac{x^2}{x}+2x \cdot \ln x$

$y'(x)=y(x)[x+2x\ \ln x]$ .......(i)

Given $y(1)=1$, so $y'(1)=1$

Now rewriting equation (i) again we get,

$\frac{dx}{dy}=\frac{1}{x^{x^2+1}(1+2\ln x)}$

Now $\frac{d^2x}{dy^2}=\frac{d}{dx}\left(\left(x^{x^2+1}(1+2\ln x)\right)^{-1}\right)\frac{dx}{dy}$

$\Rightarrow \frac{d^2x}{dy^2}=\frac{-x^{x^2}(1+2\ln x)\left(x^2+3+2x^2\ln x\right)}{\left(x^{x^2}(1+2\ln x)\right)^3}\times 1$

$\Rightarrow \left(\frac{d^2x}{dy^2}\right)_{x=1}=-4$

So, $\left(\frac{d^2x}{dy^2}\right)_{x=1}+20=-4+20=16$

# Answer Key

| | | | |
|---|---|---|---|
| **Q1** (3) | **Q2** (4) | **Q3** (2) | **Q4** (3) |
| **Q5** (1) | **Q6** (3.75) | **Q7** (4) | **Q8** (40) |
| **Q9** (1) | **Q10** (4) | **Q11** (4) | **Q12** (11) |
| **Q13** (16) | **Q14** (248) | **Q15** (32) | **Q16** (2) |
| **Q17** (4) | **Q18** (3) | **Q19** (2) | **Q20** (3) |
| **Q21** (4) | **Q22** (4) | **Q23** (3) | **Q24** (4) |
| **Q25** (40) | **Q26** (4) | **Q27** (3) | **Q28** (2) |
| **Q29** (1) | **Q30** (16) | | |