Q1. If f(x) = $\begin{cases} \lambda\sqrt{2x+3}, & 0 \le x \le 3 \\ \mu x + 12, & 3 < x \le 9 \end{cases}$ is differentiable at $x = 3$, then the value of $\lambda + \mu$ is equal to

Ans: 8

Solution:

$\because f(x)$ is continuous at $x = 3$

$\therefore LHL = RHL = f(3)$

$\lambda\sqrt{2(3)+3} = \mu\left(3\right) + 12 \Rightarrow \lambda = \mu + 4$ ......(1)

Also $f(x)$ is differentiable at $x = 3$

$\therefore$ at $x = 3$ $LHD = RHD$

$\frac{\lambda}{2\sqrt{2(3)+3}} \cdot 2 = \mu \Rightarrow \lambda = 3\mu$ ......(2)

By (1) and (2)

$\mu = 2, \lambda = 6$

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Q2. Let $f(x)=\begin{cases}\dfrac{1}{x^2} & : |x| \geq 1 \\ \alpha x^2+\beta & : |x|<1\end{cases}$**. If** $f(x)$ **is continuous and differentiable at any point, then**

A. $\alpha=2, \beta=-1$

B. $\alpha=-1, \beta=2$

C. $\alpha=1, \beta=0$

D. $\alpha=-2, \beta=3$

Ans: $\alpha=-1, \beta=2$

Solution: The given function is clearly continuous at all points except possibly at $x= \pm 1$.

For $f(x)$ to be continuous at $x=1$, we must have

$$\lim_{x \rightarrow 1^-} f(x)=\lim_{x \rightarrow 1^+} f(x)=f(1)$$

$$\Rightarrow \lim_{x \rightarrow 1^-} \alpha x^2+\beta=\lim_{x \rightarrow 1^+} \frac{1}{x^2}$$

$$\Rightarrow \alpha+\beta=1 \ldots (1)$$

Now, for $f(x)$ to be differentiable at $x=1$, we must have

$$\lim_{x \rightarrow 1^-} \frac{f(x)-f(1)}{x-1}=\lim_{x \rightarrow 1^+} \frac{f(x)-f(1)}{x-1}$$

$$\Rightarrow \lim_{x \rightarrow 1^-} \frac{\alpha x^2+\beta-1}{x-1}=\lim_{x \rightarrow 1^+} \frac{\frac{1}{x^2}-1}{x-1} \ (\because \alpha+\beta=1 \therefore \beta-1=-\alpha)$$

$$\Rightarrow \lim_{x \rightarrow 1^-} \frac{\alpha x^2-\alpha}{x-1}=\lim_{x \rightarrow 1^+} \frac{\frac{1}{x^2}-1}{x-1}$$

$$\Rightarrow \lim_{x \rightarrow 1^-} \alpha(x+1)=\lim_{x \rightarrow 1^+} \frac{-2}{x^3} \Rightarrow 2\alpha=-2$$

$$\Rightarrow \alpha=-1$$

Putting $\alpha=-1$ in (1), we get $\beta=2$

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Q3. If $f(x)=\begin{cases}\dfrac{\sqrt{4+ax}-\sqrt{4-ax}}{x}, & -1\le x<0\\ \dfrac{3x+2}{x-8}, & 0\le x\le 1\end{cases}$ **is continuous in** $[-1,1]$, **then the value of** a **is**

A. 1

B. - 1

C. $\frac{1}{2}$

D. $-\frac{1}{2}$

Ans: $-\frac{1}{2}$

Solution: Given that $f(x)$ is continuous in the interval $[-1,\ 1]$, so, it will be continuous at $x=0$.

Now $f\left(0^{+}\right)=\frac{-1}{4}$

$$f\left(0^{-}\right)=\lim_{x\to 0^{-}}\frac{2ax}{x\left(\sqrt{4+ax}+\sqrt{4-ax}\right)}\ \text{(rationalizing)}$$

$$=\lim_{x\to 0^{-}}\frac{2a}{4}=\frac{a}{2}$$

Now $f\left(0^{+}\right)=f\left(0^{-}\right)$ ($\because$ $f(x)$ is continuous at point 0)

$$\Rightarrow\quad \frac{a}{2}=\frac{-1}{4}\Rightarrow\quad a=\frac{-1}{2}$$

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Q4. The value of f(0) so that the function $f(x) = \dfrac{1-\cos(1-\cos x)}{x^4}$ is continuous everywhere is k, then value of 10 k is

Ans: 1.25

Solution: If f(x) is continuous at x = 0

$$f(x) = \lim_{x \to 0} f(x)$$

$$= \lim_{x \to 0} \frac{1-\cos(1-\cos x)}{x^4} \times \frac{1+\cos(1-\cos x)}{1+\cos(1-\cos x)}$$

$$= \lim_{x \to 0} \frac{\sin^2(1-\cos x)}{x^4 \cdot (1+\cos(1-\cos x))} \cdot \frac{(1-\cos x)^2}{(1-\cos x)^2}$$

$$= \lim_{x \to 0} \left[\frac{\sin(1-\cos x)}{(1-\cos x)}\right]^2 \times \lim_{x \to 0} \left(\frac{1-\cos x}{x^2}\right)^2 \times \lim_{x \to 0} \frac{1}{1+\cos(1-\cos x)}$$

$$= (1)^2 \times \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

$$\therefore\ k = \frac{1}{8} \Rightarrow 10\,k = \frac{5}{4} = 1.25$$

Q5. Let $f(x) = \begin{cases} \left(\dfrac{1-\cos x}{(2\pi - x)^2}\right)\left(\dfrac{\sin^2 x}{\log\left(1+4\pi^2-4\pi x+x^2\right)}\right) & : x \neq 2\pi \\ \lambda & : x = 2\pi \end{cases}$ is continuous at $x = 2\pi$, then the value of λ is equal to

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Ans: 0.5

Solution:

For the function $f(x)$ to be continuous at $x = 2\pi$

$$\lim_{x \to 2\pi} f(x) = f(2\pi)$$

Now, $\lim_{x \to 2\pi} \frac{1-\cos x}{(2\pi - x)^2} \frac{\sin^2 x}{\log\left\{1+(2\pi-x)^2\right\}} = \lambda$

Putting $x = 2\pi + t$, we get,

$$\lim_{t \to 0} \frac{1-\cos t}{t^2} \cdot \frac{\sin^2 t}{\log\left(1+t^2\right)} = \lambda$$

$$\lim_{t \to 0} \frac{1}{2} \cdot \frac{\sin^2 t}{t^2} \cdot \frac{t^2}{\log\left(1+t^2\right)} = \lambda$$

$$\frac{1}{2} \times 1 \times 1 = \lambda$$

$$\Rightarrow \lambda = \frac{1}{2}$$

Q6. If $f(x) = \begin{cases} \dfrac{e^{[x]+|x|}-1}{[x]+|x|} & : x \neq 0 \\ -1 & : x = 0 \end{cases}$ (where [.] denotes the greatest integer function), then

A. $f(x)$ is continuous at $x = 0$

B. $\lim_{x \to 0^+} f(x) = -1$

C. $\lim_{x \to 0^-} f(x) = 1$

D. $\lim_{x \to 0^+} f(x) = 1$

Ans: $\lim_{x \to 0^+} f(x) = 1$

Solution: $f(x) = \begin{cases} \dfrac{e^{[x]+|x|} - 1}{[x]+|x|} & x \neq 0 \\ -1 & x = 0 \end{cases}$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \dfrac{e^{[x]+|x|} - 1}{[x]+|x|} = \dfrac{e^{-1} - 1}{-1} = \dfrac{e-1}{e}$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \dfrac{e^{[x]+|x|} - 1}{[x]+|x|} = \lim_{x \to 0^+} \dfrac{e^x - 1}{x} = 1$

$\because$ LHL $\neq$ RHL at $x = 0 \Rightarrow f(x)$ is discontinuous at $x = 0$

Q7. Let $f(x) = \begin{cases} (1 + |\sin x|)^{\frac{l}{|\sin x|}}, & -\frac{\pi}{6} < x < 0 \\ e^{\frac{\tan 2x}{\tan 3x}}, & 0 < x < \frac{\pi}{6} \\ m, & x = 0 \end{cases}$ is continuous at $x = 0$. Then, the values of l and m are

A. $l = -\frac{2}{3}, m = e^{\frac{2}{3}}$

B. $l = \frac{2}{3}, m = e^{-\frac{2}{3}}$

C. $l = \frac{2}{3}, m = e^{\frac{2}{3}}$

D. None of these

Ans: $l=\frac{2}{3}, m=e^{\frac{2}{3}}$

Solution:

We have,

$$\lim_{h \to 0^+} f(0-h)=\lim_{h \to 0^+} [1+|\sin(-h)|]^{\frac{l}{|\sin -h|}}$$

$$=\lim_{h \to 0^+} (1+\sin h)^{\frac{l}{\sin h}}$$

$$=\lim_{h \to 0^+} \left[(1+\sin h)^{\frac{1}{\sin h}}\right]^l = e^l$$

$$\lim_{h \to 0^+} f(0+h)=\lim_{h \to 0^+} e^{\frac{\tan 2h}{\tan 3h}}$$

$$=e^{\lim_{h \to 0^+}\left(\frac{\tan 2h}{2h}\times\frac{2}{3}\times\frac{3h}{\tan 3h}\right)}=e^{1\times\frac{2}{3}\times 1}=e^{\frac{2}{3}}$$

Also, $f(0)=m$

For $f(x)$ to be continuous at $x=0$, we must have

$$\lim_{h \to 0^+} f(0-h)=\lim_{h \to 0^+} f(0+h)=f(0) \Rightarrow e^l=e^{\frac{2}{3}}=m$$

$$\Rightarrow l=\frac{2}{3} \text{ and } m=e^{\frac{2}{3}}$$

Q8. The function f(x) = {x} sin (π[x]), where [.] denotes the greatest integer function and {.} is the fractional part function, is discontinuous at

A. all x

B. all integer points

C. no x

D. x which is not an integer

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Ans: no x

Solution: $f(x)=\{x\}\sin(\pi[x])$

$=\{x\}\sin(\text{integral multiple of }\pi)$

$=0$

Hence, f(x) is continuous for all x.

Q9. The function $f(x)=\lim_{n\to\infty}\cos^{2n}(\pi x)+[x]$ is (where, [.] denotes the greatest integer function and $n\in N$)

A. continuous at $x=1$ but discontinuous at $x=\frac{3}{2}$

B. continuous at $x=1$ and $x=\frac{3}{2}$

C. discontinuous at $x=1$ and $x=\frac{3}{2}$

D. discontinuous at $x=1$ but continuous at $x=\frac{3}{2}$

Ans: discontinuous at $x=1$ but continuous at $x=\frac{3}{2}$

Solution:

$$f(x)=\lim_{n\to\infty}\left\{\cos^2(\pi x)\right\}^n+[x]$$

$$=\begin{cases}[x] & : \cos^2(\pi x)\in[0,1)\\ 1+[x] & : \cos^2(\pi x)=1\end{cases}$$

$$=\begin{cases}[x] & : x\notin \mathrm{I}\\ 1+[x] & : x\in \mathrm{I}\end{cases}$$

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For $x=1$,

$f\left(1^{-}\right)=\left[1^{-}\right]=0,\ f\left(1^{+}\right)=\left[1^{+}\right]=1,\ f(1)=1+[1]=2$

$\because f\left(1^{+}\right)\neq f\left(1^{-}\right)\Rightarrow f(x)$ is discontinuous at $x=1$

For $x=\frac{3}{2}$,

$f\left(\frac{3}{2}^{+}\right)=\left[\frac{3}{2}^{+}\right]=1,\ f\left(\frac{3}{2}^{-}\right)=\left[\frac{3}{2}^{-}\right]=1,\ f\left(\frac{3}{2}\right)=\left[\frac{3}{2}\right]=1$

$\because f\left(\frac{3}{2}^{-}\right)=f\left(\frac{3}{2}^{+}\right)=f\left(\frac{3}{2}\right)$

$\Rightarrow f(x)$ is continuous at $x=\frac{3}{2}$

Q10. Let $f(x)=-x^2+x+p$, where p is a real number. If $g(x)=[f(x)]$ and $g(x)$ is discontinuous at $x=\frac{1}{2}$, then p cannot be (where [.] represents the greatest integer function)

A. $\frac{1}{2}$
B. $\frac{3}{4}$
C. $\frac{7}{4}$
D. $-\frac{1}{4}$

Ans: $\frac{1}{2}$

Solution:

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$f(x) = -\left(x - \frac{1}{2}\right)^2 + \left(p + \frac{1}{4}\right)$

If $g(x)$ is discontinuous at $x = \frac{1}{2}$, then $f\left(\frac{1}{2}\right)$ should be an integer.

Hence, p cannot be $\frac{1}{2}$

Q11. Let $f(x) = [x]\left\{x^2\right\} + [x]\left[x^2\right] + \{x\}\left[x^2\right] + \{x\}\left\{x^2\right\}, \ \forall x \in [0,10]$ (where [·] and { · } are the greatest integer and fractional part functions respectively). The number of points of discontinuity of $f(x)$ is

Ans: 0

Solution: $f(x) = [x]\left(\left\{x^2\right\} + \left[x^2\right]\right) + \{x\}\left(\left[x^2\right] + \left\{x^2\right\}\right)$

$= ([x] + \{x\})\left(\left[x^2\right] + \left\{x^2\right\}\right)$

$= x . x^2 = x^3$

Hence, $f(x)$ is continuous $\forall x \in [0,10]$

Q12.

Consider function $f(x) = \begin{cases} \dfrac{\tan\{2x-3\}}{x-2}, & x \in (2, \infty) \\ \left[x^2\right] + sgn(x), & x \in (-\infty, 2] \end{cases}$, then at $x = 2$

[Note: {k} & [k] denote fractional part & greatest integer function less than or equal to k respectively and *sgn* denotes signum part of function.]

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A. $f(x)$ is continuous

B. $f(x)$ is discontinuous

C. $f(x)$ is differentiable, but $f'(x)$ is discontinuous

D. None of these

Ans: $f(x)$ is discontinuous

Solution:

$$\text{L. H. L.} = \lim_{h \to 0} \left[(2-h)^2\right] + sgn(2-h)$$

$$= 3 + 1 = 4$$

$f(2) = 4+1 = 5$

$\therefore f(x)$ is discontinuous.

Q13. Given $f\left(x\right) = \begin{cases} \sqrt{10 - x^2} & \text{if } -3 < x < 3 \\ 2 - e^{x-3} & \text{if } x \geq 3 \end{cases}$ **The graph of $f(x)$ is -**

A. continuous and differentiable at $x = 3$

B. continuous but not differentiable at $x = 3$

C. differentiable but not continuous at $x = 3$

D. neither differentiable nor continuous at $x = 3$

Ans: continuous but not differentiable at $x = 3$

Solution:

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$$f'\left(3^{+}\right)=\lim_{h\to 0}\frac{f(3+h)-f(3)}{h}$$

$$=\lim_{h\to 0}\frac{\left(2-e^{h}\right)-1}{h}=-\lim_{h\to 0}\left(\frac{e^{h}-1}{h}\right)=-1$$

$$f'\left(3^{-}\right)=\lim_{h\to 0}\frac{f(3-h)-f(3)}{-h}$$

$$=\lim_{h\to 0}\frac{\sqrt{10-(3-h)^{2}}-1}{-h}=-\lim_{h\to 0}\frac{\sqrt{1+\left(6h-h^{2}\right)}-1}{-h}$$

$$=\lim_{h\to 0}\frac{6h-h^{2}}{-h\left(\sqrt{1+6h-h^{2}}+1\right)}=\frac{-6}{2}=-3$$

Hence, $f'\left(3^{+}\right)\neq f\left(3^{-}\right)$

Q14. If $f(x)=\frac{1}{1-x}$, then the points of discontinuity of the function $f^{30}(x)$ where $f^{n}(x)=fof......of$ (n times) are

A. $x=2,1$

B. $x=0,1$

C. $x=1,2$

D. no points of discontinuity

Ans: $x=0,1$

Solution: Clearly, $x=1$ is a point of discontinuity of the function $f(x)=\frac{1}{1-x}$.

if $x\neq 1$, then

$(fof)(x)=f[f(x)]=f\left(\frac{1}{1-x}\right)=\frac{x-1}{x}$, which is discontinuous at $x=0$.

If $x\neq 0$ and $x\neq 1$, then

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$(fo\ fof)(x) = f[(fof)(x)] = f\left(\frac{x-1}{x}\right) = x$

Which is continuous everywhere.

Hence, $f^{30}(x) = x,$ which is continuous everywhere.

So, the only points of discontinuity are $x = 0$ and $x = 1$

Q15.

Let f be a composite function of x defined by

$$f(u) = \frac{1}{u^2+u-2},\ u(x) = \frac{1}{x-1}.$$

Then the number of points x where f is discontinuous is :

A. 4

B. 3

C. 2

D. 1

Ans: 3

Solution:

$\mu(x) = \frac{1}{x-1}$, which is discontinous at $x = 1$

$f(u) = \frac{1}{u^2+u-2} = \frac{1}{(u+2)(u-1)}$

which is discontinous at $u = -2,\ 1$

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when $u = -2$, then $\frac{1}{x-1} = -2 \Rightarrow x = \frac{1}{2}$

when $u = 1$, then $\frac{1}{x-1} = 1 \Rightarrow x = 2$

Hence given composite function is discontinous at three points, $x = 1, \frac{1}{2}$ and 2 .

Q16. In $(0, 2\pi)$, the total number of points where f(x) = max. {sinx, cosx, 1– cosx } is not differentiable, are equal to

A. 3

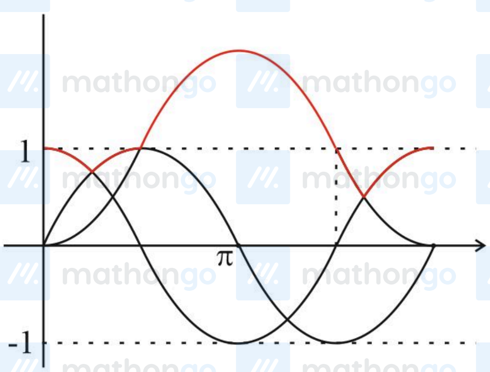
B. 4

C. 5

D. 6

Ans: 3

Solution: The graph of f(x) = max. {sinx, cosx, 1– cosx } is



$\Rightarrow$ f(x) is not differentiable at x = $\frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{3}$.

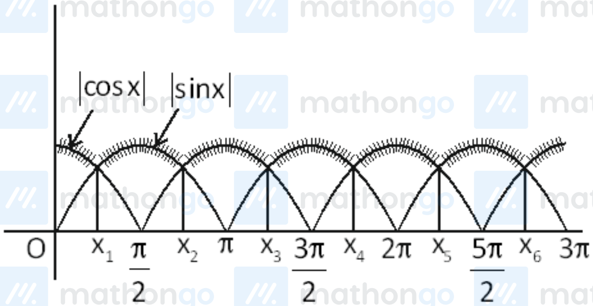
$\Rightarrow$ f(x) is not differentiable at 3 points

Q17. Consider the function $f(x) = \max\{|\sin x|, \ |\cos x|\}, \ \ \forall x \in [0,3\pi]$. If λ is the number of points at which $f(x)$ is non-differentiable, then the value of $\frac{\lambda^3}{5}$ is

Ans: 43.20

Solution: Using graph of $|\sin x| \& |\cos x|$, we get,

$f(x)$ is non differentiable at



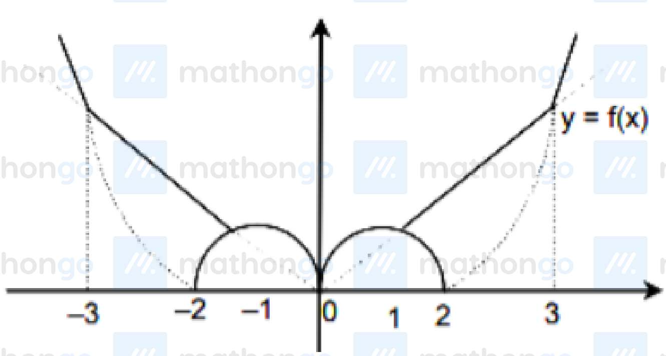
$x_1 = \frac{\pi}{4}, \ x_2 = \frac{3\pi}{4}, \ x_3 = \frac{5\pi}{4}, \ x_4 = \frac{7\pi}{4}, \ x_5 = \frac{9\pi}{4}, \ x_6 = \frac{11\pi}{4}$

Hence, $\lambda = 6$

Q18. Let $f(x) = \max.\left\{\left|x^2 - 2|x|\right|, |x|\right\}$ and $g(x) = \min.\left\{\left|x^2 - 2|x|\right|, |x|\right\}$ then if $f(x)$ is not differentiable at 'p' number of points and g(x) is non differentiable at 'q' number of points, then find |p - q|.

Ans: 2

Solution:



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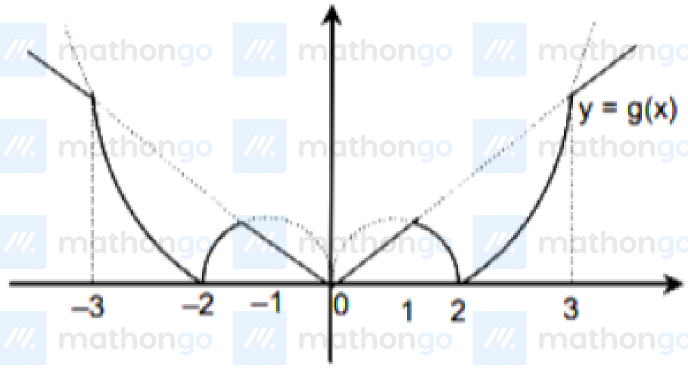
f(x) is not differentiable function at $x = -3, -1, 0, 1, 3$



$g(x)$ is not differentialble at $x = -3, -2, -1, 0, 1, 2, 3$

Q19. If $f(x) = \min\left\{\sqrt{9 - x^2}, \sqrt{1 + x^2}\right\}, \ \forall x \in [-3,3]$, then the number of point(s) where f(x) is non-differentiable is/are

A. 4

B. 3

C. 2

D. 0

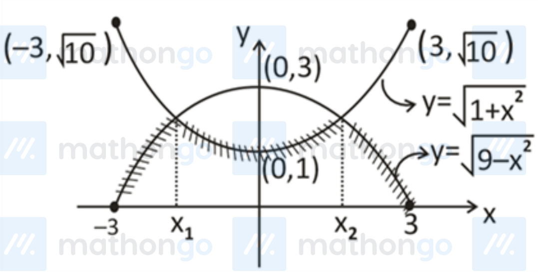
Ans: 4

Solution: Using graph of

$y = \sqrt{9 - x^2} \Rightarrow x^2 + y^2 = 9$,

$y = \sqrt{1 + x^2} \Rightarrow y^2 - x^2 = 1$

In the figure, we can see the graph of f(x)



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Clearly, f(x) is non-differentiable at

$x = -3,\ x_1,\ x_2,\ 3$ (4 points)

because there are sharp points at $x_1,\ x_2$ and at $x = 3,\ x = -3$ there are vertical tangents.

Q20. Consider a function $g(x) = f(x-2),\ \forall x \in R,$ where $f(x) = \begin{cases} \frac{1}{|x|} & : \ |x| \geq 1 \\ ax^2 + b & : \ |x| < 1 \end{cases}$. If $g(x)$ is continuous as well as differentiable for all $x,$ then

A. $a = \frac{-1}{2}, b = \frac{3}{2}$

B. $a = \frac{1}{2}, b = \frac{3}{2}$

C. $a = \frac{-1}{2}, b = \frac{-3}{2}$

D. None of these

Ans: $a = \frac{-1}{2}, b = \frac{3}{2}$

Solution: For $g(x)$ to be continuous and differentiable $\forall x \in R,\ f(x)$ must be continuous and differentiable $\forall x \in R$

Since, $f(x)$ is continuous for all $x,$ therefore, it is continuous at $x = 1$ also.

$$\therefore f(1) = \lim_{h \to 0^+} f(1-h) = 1 = \lim_{h \to 0^+} \left[a(1-h)^2 + b\right]$$

$$\Rightarrow a + b = 1 \ldots (1)$$

Also, $f(x)$ is differentiable at $x = 1 \Rightarrow f'\left(1^-\right) = f'\left(1^+\right)$

$$\Rightarrow \lim_{h \to 0^+} \frac{f(1-h)-f(1)}{-h} = \lim_{h \to 0^+} \frac{f(1+h)-f(1)}{h}$$

$$\Rightarrow \lim_{h \to 0^+} \frac{a(1-h)^2+b-1}{-h} = \lim_{h \to 0^+} \frac{\frac{1}{|1+h|}-1}{h}$$

$$\Rightarrow \lim_{h \to 0^+} \frac{(a+b-1)+\left(h^2-2h\right)a}{-h} = \lim_{h \to 0^+} \frac{1-1-h}{h(1+h)}$$

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$\Rightarrow 2a = -1$ (Using $a + b = 1$)

$\therefore a = \frac{-1}{2}$

Hence, $a + b = 1 \Rightarrow b = 1 - a = \frac{3}{2}$.

Q21. Let $f(x) = [n + p\sin x], x \in (0, \pi), n \in I$ and p is a prime number. The number of points where $f(x)$ is not differentiable is

(Here $[x]$ represents the greatest integer less than or equal to x)

A. $p - 1$

B. $p + 1$

C. $2p + 1$

D. $2p - 1$

Ans: $2p - 1$

Solution:

$f(x) = n + [p\sin x]$

As we know the greatest integer function is discontinuous at integer.

So, $f(x)$ is discontinuous when $p\sin x$ is integer.

In $\left(0, \frac{\pi}{2}\right)$, $f(x)$ is discontinuous when $p\sin x = 1, 2, \ldots\ldots. p - 1$

and in $\left(\frac{\pi}{2}, \pi\right)$, $f(x)$ is discontinuous when $p\sin x = 1, 2, \ldots\ldots. p - 1$

and at $\frac{\pi}{2}$, $p\sin x = p$

So total number of points of discontinuity is $p - 1 + p - 1 + 1 = 2p - 1$

As we know function is not differentiable at a point when function is not continuous at that point.

So total number of point of where $f(x)$ is not differentiable is $2p - 1$

Q22. The number of points at which the function $f(x) = |x - 0.5| + |x - 1| + \tan x$ is not differentiable in the interval $(0,\ 2)$ is/are

A. 1

B. 2

C. 3

D. 4

Ans: 3

Solution: The curve will have sharp corner at $x = 0.5$ and at $x = 1$ and a point of discontinuity at $x = \frac{\pi}{2}$.
Hence, total number of points where the function is not differentiable is 3.

Q23. Let $f(x) = 10 - |x - 5|, x \in R$, then the set of all values of x at which $f(f(x))$ is not differentiable is

A. $\{0,5, 10\}$

B. $\{5,10\}$

C. $\{0,5, 10,15\}$

D. $\{5,10,15\}$

Ans: $\{0,5, 10\}$

Solution: $f(f(x)) = 10 - |f(x) - 5| = 10 - |10 - |x - 5| - 5|$

$= 10 - |5 - |x - 5|| \Rightarrow 10 - ||x - 5| - 5|$

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Points where $f(f(x))$ is non-differentiable are

$x - 5 = 0$ & $|x - 5| = 5$

$x = 5$ & $x = 0, 10$

Q24. Consider $f(x) = \begin{cases} -2, & -2 \le x < 0 \\ x^2 - 2, & 0 \le x \le 2 \end{cases}$ and $g(x) = |f(x)| + f(|x|)$. Then, in the interval $(-2, 2)$, $g(x)$ is

A. not differentiable at one point

B. differentiable at all points

C. not continuous

D. not differentiable at two points

Ans: not differentiable at one point

Solution: Given, $f(x) = \begin{cases} -2, & -2 \le x < 0 \\ x^2 - 2, & 0 \le x \le 2 \end{cases}$

$$\therefore |f(x)| = \begin{cases} 2, & -2 \le x < 0 \\ 2 - x^2, & 0 \le x < \sqrt{2} \\ x^2 - 2, & \sqrt{2} \le x \le 2 \end{cases}$$

And $f(|x|) = x^2 - 2,\ x \in [-2,2]$

$$\therefore g(x) = \begin{cases} x^2, & -2 \le x < 0 \\ 0, & 0 \le x < \sqrt{2} \\ 2\left(x^2 - 2\right), & \sqrt{2} \le x \le 2 \end{cases}$$

$$g'(x) = \begin{cases} 2x & -2 < x < 0 \\ 0 & 0 < x < \sqrt{2} \\ 4x & \sqrt{2} < x < 2 \end{cases}$$

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$g'\left(0^-\right) = 0 = g'\left(0^+\right)g'\left(\sqrt{2}^-\right) = 0, g'\left(\sqrt{2}^+\right) = 4\sqrt{2}$

So, $g(x)$ is not differentiable at $x = \sqrt{2}$

Q25. Consider the function $f(x) = (x - 2)\left|x^2 - 3x + 2\right|$, then the incorrect statement is

A. $f(x)$ is continuous at $x = 1$

B. $f(x)$ is continuous at $x = 2$

C. $f(x)$ is differentiable at $x = 1$

D. $f(x)$ is differentiable at $x = 2$

Ans: $f(x)$ is differentiable at $x = 1$

Solution: $\because\ x - 2\ \&\ \left|x^2 - 3x + 2\right|$ both are continuous $\forall x \in R$

$\therefore f(x)$ is continuous $\forall x \in R$

Now, $\left|x^2 - 3x + 2\right| = |(x - 1)(x - 2)|$

$$\therefore\ f(x) = \begin{cases} (x - 2)(x - 1)(x - 2) & : \ x \in (-\infty, 1) \cup (2, \infty) \\ -(x - 2)(x - 1)(x - 2) & : \ x \in [1,2] \end{cases}$$

$$f(x) = \begin{cases} (x - 1)(x - 2)^2 & : \ x \in (-\infty, 1) \cup (2, \infty) \\ -(x - 1)(x - 2)^2 & : \ x \in [1,2] \end{cases}$$

$$f'(x) = \begin{cases} (x - 1)2(x - 2) + (x - 2)^2 & : \ x \in (-\infty, 1) \cup (2, \infty) \\ -(x - 1)2(x - 2) - (x - 2)^2 & : \ x \in (1, 2) \end{cases}$$

Clearly, $f'\left(2^-\right) = f'\left(2^+\right) = 0 \Rightarrow f(x)$ is differentiable at $x = 2$

and $f'\left(1^-\right) = 1, f'\left(1^+\right) = -1 \Rightarrow f(x)$ is non-differentiable at $x = 1$

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Q26. The number of points where the function,

$f(x) = \cos\left|2018\pi - x\right| + \sin\left|2020\pi - x\right| + (x - \pi)\left|x^2 - 3\pi x + 2\pi^2\right|$ **is non-differentiable is/are**

A. 0

B. 1

C. 2

D. 3

Ans: 2

Solution: $x = 2020\pi, 2\pi$ are points of non-differentiability

Q27.

Let $g(x) = \begin{cases} \dfrac{ax^2 + bx + c\left(\cot x\right)^n}{4 + \left(\cot x\right)^n}, & x \in \left(0, \dfrac{\pi}{4}\right) \\ 1, & \text{at } x = \dfrac{\pi}{4} \\ \dfrac{\sin x + \cos x + \left(\tan x\right)^n}{1 + c\left(\tan x\right)^n}, & x \in \left(\dfrac{\pi}{4}, \dfrac{\pi}{2}\right) \end{cases}$ **, where a, b, c are real constants and**

$f(x) = \lim\limits_{n \to \infty} g(x)$ **If** $\lim\limits_{x \to \frac{\pi}{4}} f(x)$ **exists, then c may be equal to**

A. 2

B. $\frac{1}{2}$

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C. 3

D. - 1

Ans: - 1

Solution:

$$g(x) = \begin{cases} \dfrac{ax^2+bx}{(\cot x)^n} + c \\ \dfrac{4}{1+\dfrac{4}{(\cot x)^n}} = c \quad \text{if} \quad x < \dfrac{\pi}{4} \\ \dfrac{\dfrac{\sin x+\cos x+1}{(\tan x)^n}}{\dfrac{1}{(\tan x)^n}+c} = \dfrac{1}{c} \quad \text{if} \quad x > \dfrac{\pi}{4} \end{cases}$$

limit to exist $c = \frac{1}{c} \Rightarrow c^2 = 1,\ c = \pm 1$

Q28. Let $f\left(x\right) = \cos x, g\left(x\right) = \begin{cases} \min.\ \{f(t): 0 \le t \le x\}, & x \in [0, \pi] \\ \sin x - 1, & x > \pi \end{cases}$ then

A. $g(x)$ is discontinuous at $x = \pi$

B. $g(x)$ is continuous for $x \in (0, \infty)$

C. $g(x)$ is differentiable at $x = \pi$

D. $g(x)$ is differentiable for $x \geq 0$

Ans: $g(x)$ is continuous for $x \in (0, \infty)$

Solution: $f(x) = \cos x$

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$f'(x) = -\sin x < 0$

So, function is decreasing in x ∈ $[0, \pi]$

$$g(x) = \begin{cases} f(x), & x \in [0, \pi] \\ \sin x - 1, & x > \pi \end{cases}$$

$\therefore$ L. H. L. = R. H. L. $= f(\pi) = -1$.

Q29. Let $f(x)$ is a differentiable function such that $f(x+y) = f(x) + f(y) + 2xy \; \forall x, \; y \in R$ and $\lim_{x \to 0} \frac{f(x)}{x} = 210$, then $f(2)$ is equal to

A. 20

B. 105

C. 424

D. none of these

Ans: 424

Solution:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x) + f(h) + 2xh - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \left(\frac{f(h)}{h} + 2x \right) = 210 + 2x$$

$$f(x) = 210x + x^2 + C$$

$$f(0) = 0 \Rightarrow C = 0$$

$$f(x) = 210x + x^2$$

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$\therefore f(2) = 424$

Q30. Let $f: R \rightarrow R$ be a function such that $f\left(\frac{x+y}{3}\right) = \frac{f(x)+f(y)}{3}$, $f(0) = 0$ and $f'(0) = 5$, then

A. $f(x)$ is a quadratic function

B. $f(x)$ is continuous but not differentiable

C. $f(x)$ is differentiable in R

D. $f(x)$ is bounded in R

Ans: $f(x)$ is differentiable in R

Solution: We have, $f\left(\frac{x+y}{3}\right) = \frac{f(x)+f(y)}{3}$, $f(0) = 0$ and $f'(0) = 5$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h \to 0} \frac{f\left(\frac{3x+3h}{3}\right) - f\left(\frac{3x+0}{3}\right)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{f(3x)+f(3h)}{3} - \frac{f(3x)+f(0)}{3}}{h}$$

$$= \lim_{h \to 0} \frac{f(3h)-f(0)}{3h} = f'(0) = 5$$

$\therefore f(x) = 5x + c \ \therefore f(0) = 0 \Rightarrow c = 0$

Hence, $f(x) = 5x$

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Answer Key

Q1 (8)	**Q2** (2)	**Q3** (4)	**Q4** (1.25)
Q5 (0.5)	**Q6** (4)	**Q7** (3)	**Q8** (3)
Q9 (4)	**Q10** (1)	**Q11** (0)	**Q12** (2)
Q13 (2)	**Q14** (2)	**Q15** (2)	**Q16** (1)
Q17 (43.20)	**Q18** (2)	**Q19** (1)	**Q20** (1)
Q21 (4)	**Q22** (3)	**Q23** (1)	**Q24** (1)
Q25 (3)	**Q26** (3)	**Q27** (4)	**Q28** (2)
Q29 (3)	**Q30** (3)		

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