Q1. Let α and β be the roots of the equation $\mathrm{x}^2+\mathrm{ax}+1=0,\ \mathrm{a}\neq 0$. Then the equation whose roots are $-\left(\alpha+\frac{1}{\beta}\right)$ and $-\left(\frac{1}{\alpha}+\beta\right)$ is

A. $\mathrm{x}^2=0$

B. $\mathrm{x}^2+2\mathrm{ax}+4=0$

C. $\mathrm{x}^2-2\mathrm{ax}+4=0$

D. $\mathrm{x}^2-\mathrm{ax}+1=0$

Ans: $\mathrm{x}^2-2\mathrm{ax}+4=0$

Solution: $\alpha+\beta=-\mathrm{a}$ and $\alpha\beta=1$

Let S and P be the sum and product of the roots of the required equation. Then,

$\mathrm{S}=-\alpha-\frac{1}{\beta}-\frac{1}{\alpha}-\beta=-\left(\alpha+\beta\right)-\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)$

$=-\left(\alpha+\beta\right)-\left(\frac{\alpha+\beta}{\alpha\beta}\right)=-\left(-\mathrm{a}\right)-\left(\frac{-\mathrm{a}}{1}\right)=2\mathrm{a}$

$\mathrm{P}=-\left(\alpha+\frac{1}{\beta}\right)\left(-\left(\frac{1}{\alpha}+\beta\right)\right)$

$=1+\alpha\beta+\frac{1}{\alpha\beta}+1=1+1+1+1=4$

So, the required equation is

$\mathrm{x}^2-\mathrm{Sx}+\mathrm{P}=0$

i.e. $\mathrm{x}^2-2\mathrm{ax}+4=0$

Q2. If the roots of the quadratic equation $ax^2+bx+c=0$ are $\frac{k+1}{k}$ and $\frac{k+2}{k+1}$, then the value of $\left(a+b+c\right)^2$ is equal to

A. $2b^2-ac$

B. Σa^2

C. b^2-4ac

D. b^2-2ac

Ans: b^2-4ac

Solution:

We have $\frac{k+1}{k}+\frac{k+2}{k+1}=\frac{-b}{a}\ldots.(i)$

and $\frac{k+1}{k}\cdot\frac{k+2}{k+1}=\frac{c}{a}$

$\Rightarrow \frac{k+2}{k}=\frac{c}{a}$

or $\frac{2}{k}=\frac{c}{a}-1=\frac{c-a}{a}$

or $\mathrm{k}=\frac{2\mathrm{a}}{\mathrm{c}-\mathrm{a}}$ .....(ii)

Now eliminate k putting the value of k in 1st relation, we get

$\frac{c+a}{2a}+\frac{2c}{c+a}=\frac{-b}{a}$

$\Rightarrow (c+a)^2+4ac=-2b(a+c)$

$\Rightarrow (a+c)^2+2b(a+c)=-4ac$

Adding b^2 on both sides,

$(a+b+c)^2=b^2-4ac$

Q3. The possible values of n for which the equation $nx^2+(2n-1)x+(n-1)=0$ has roots of opposite sign is/are given by

A. no values of n

B. all values of n

C. $-1<n<0$

D. $0<n<1$

Ans: $0 < n < 1$

Solution: $D = (2n-1)^2 - 4n(n-1) = 4n^2 + 1 - 4n - 4n^2 + 4n = 1 > 0$

Product of roots $= \frac{n-1}{n} < 0$

$\Rightarrow n(n-1) < 0$

$\Rightarrow n \in (0, 1)$

Q4. Consider the equation $x^2 + 2x - n = 0$, where $n \in N$ and $n \in [5,\ 100]$. The number of different values of n so that the given equation has integral roots, is

Ans: 8

Solution:

$x^2 + 2x - n = 0;\ n \in [5,\ 100]$

D will be a perfect square,

$D = 4 + 4n = 4(1 + n)$

$\Rightarrow 1 + n$ is a perfect square

$\Rightarrow 1 + n = 9,\ 16,\ 25,\ 36,\ 49,\ 64,\ 81,\ 100$

$\Rightarrow n = 8,\ 15,\ 24,\ 35,\ 48,\ 63,\ 80,\ 99$

Therefore, 8 values are possible.

Q5. If $-\pi < \theta < \pi$, the equation $(\cos 3\theta + 1)x^2 + (2\cos 2\theta - 1)x + (1 - 2\cos\theta) = 0$ has more than two roots for

A. no value of θ

B. one value of θ

C. two value of θ

D. all values of θ

Ans: no value of θ

Solution:

Given equation has more than two roots if it is an identity

$\Rightarrow \cos 3\theta + 1 = 0$; $2\cos 2\theta - 1 = 0$ and $1 - 2\cos\theta = 0$

$\Rightarrow \cos 3\theta = -1 \Rightarrow \theta = \pm\frac{\pi}{3}$ which does not satisfy $2\cos 2\theta - 1 = 0$

Hence, no value possible

Q6. Let α and β are the roots of equation $ax^2 + bx + c = 0$ $(a \neq 0)$. If $1,\ \alpha+\beta,\ \alpha\beta$ are in arithmetic progression and $\alpha,\ 2,\ \beta$ are in harmonic progression, then the value of $\frac{\alpha^2+\beta^2-2\alpha^2\beta^2}{2(\alpha^2+\beta^2)}$ is equal to

A. 0

B. 0.5

C. 1

D. 1.5

Ans: 1.5

Solution: $1, \alpha+\beta, \alpha\beta$ are in A.P. $\Rightarrow 1, \frac{-b}{a}, \frac{c}{a}$ are in A.P.

$\Rightarrow 1 + \frac{c}{a} = \frac{-2b}{a} \Rightarrow a + c + 2b = 0 \ldots (1)$

$\frac{1}{\alpha}, \frac{1}{2}, \frac{1}{\beta}$ are in A.P. $\Rightarrow \frac{1}{\alpha} + \frac{1}{\beta} = 1 \Rightarrow \alpha + \beta = \alpha\beta$

$\Rightarrow \frac{-b}{a} = \frac{c}{a} \Rightarrow b + c = 0 \ldots (2)$

From (1) & (2) we get,

$a = -b = c$

$\Rightarrow \alpha, \beta$ are roots of equation $x^2 - x + 1 = 0$

Now, $\frac{\alpha^2+\beta^2-2\alpha^2\beta^2}{2\left(\alpha^2+\beta^2\right)}=\frac{1}{2}-\frac{(\alpha\beta)^2}{(\alpha+\beta)^2-2\alpha\beta}$

$=\frac{1}{2}-\frac{(1)^2}{(1)^2-2(1)}=\frac{1}{2}+1=1.5$

Q7. The number of quadratic equations that are unchanged by squaring their roots is

A. 2

B. 4

C. 6

D. 8

Ans: 4

Solution: $\alpha+\beta=\alpha^2+\beta^2$ & $\alpha\beta=\alpha^2\beta^2 \Rightarrow \alpha\beta=0$ or 1.

If $\alpha\beta=0$, then let, $\alpha=0 \Rightarrow \beta=0$ or 1.

If $\beta=\frac{1}{\alpha}$,

$\alpha+\frac{1}{\alpha}=\alpha^2+\frac{1}{\alpha^2}=\left(\alpha+\frac{1}{\alpha}\right)^2-2$

$\Rightarrow\left(\alpha+\frac{1}{\alpha}\right)^2-\left(\alpha+\frac{1}{\alpha}\right)-2=0 \Rightarrow \alpha+\frac{1}{\alpha}=2$ or $-1 \Rightarrow \alpha=1$ or ω, ω^2

Hence number of such equations are four $(0,\,0)$, $(0,\,1)$, $(1,\,1)$ & $\left(\omega,\,\omega^2\right)$

Q8. If α, β are roots of the equation $x^2+5(\sqrt{2})x+10=0, \alpha>\beta$ and $P_n=\alpha^n-\beta^n$ for each positive integer n, then the value of $\left(\frac{P_{17}P_{20}+5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19}+5\sqrt{2}P_{18}^2}\right)$ is equal to

Ans: 1

Solution: Given, $x^2 + 5\sqrt{2}x + 10 = 0$

and $P_n = \alpha^n - \beta^n$

Now $\frac{P_{17}P_{20}+5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19}+5\sqrt{2}P_{18}^2} = \frac{P_{17}\left(P_{20}+5\sqrt{2}P_{19}\right)}{P_{18}\left(P_{19}+5\sqrt{2}P_{18}\right)}$

$\frac{P_{17}\left(\alpha^{20}-\beta^{20}+5\sqrt{2}(\alpha^{19}-\beta^{19})\right)}{P_{18}\left(\alpha^{19}-\beta^{19}+5\sqrt{2}(\alpha^{18}-\beta^{18})\right)}$

$\frac{P_{17}\left(\alpha^{19}(\alpha+5\sqrt{2})-\beta^{19}(\beta+5\sqrt{2})\right)}{P_{18}\left(\alpha^{18}(\alpha+5\sqrt{2})-\beta^{18}(\beta+5\sqrt{2})\right)}$

Since $\alpha + 5\sqrt{2} = -10/\alpha$ ... (1)

and $\beta + 5\sqrt{2} = -10/\beta$ ... (2)

Now put there values in above expression

$\frac{P_{17}\left(\alpha^{19}(\alpha+5\sqrt{2})-\beta^{19}(\beta+5\sqrt{2})\right)}{P_{18}\left(\alpha^{18}(\alpha+5\sqrt{2})-\beta^{18}(\beta+5\sqrt{2})\right)} = -\frac{10P_{17}P_{18}}{-10P_{18}P_{17}} = 1$

Q9. Let α, β are the roots of the quadratic equation $2x^2 - 5x + 1 = 0$. If $S_n = (\alpha)^{2n} + (\beta)^{2n}$ then find the value of $\frac{4S_{2021}+S_{2019}}{S_{2020}}$.

Ans: 21

Solution: Given α, β are the roots of the quadratic equation $2x^2 - 5x + 1 = 0$

Let us find an equation with roots α^2 and β^2, let $y = x^2$, so $x = \sqrt{y}$

$2y - 5\sqrt{y} + 1 = 0$

$\Rightarrow 2y+1=5\sqrt{y}$

$\Rightarrow 4y^2+4y+1=25y$

$\Rightarrow 4y^2-21y+1=0 \left\langle {c \atop d} \right.$

Put $\alpha^2=c$ and $\beta^2=d$

Now, $S_n=(c)^n+(d)^n$

Consider

$4S_{2021}+S_{2019}=4\left(c^{2021}+d^{2021}\right)+c^{2019}+d^{2019}$

$=c^{2019}\left(4c^2+1\right)+d^{2019}\left(4d^2+1\right)$

$=c^{2019}(21c)+d^{2019}(21d)$

$=21S_{2020}$

Hence, $\frac{4S_{2021}+S_{2019}}{S_{2020}}=21$

Q10. If $f(x)=\prod_{k=1}^{999}\left(x^2-47x+k\right)$, then product of all real roots of $f(x)=0$ is

A. $550!$

B. $551!$

C. $552!$

D. $999!$

Ans: $552!$

Solution: Consider $x^2-47x+k=0$

For real roots, $47^2-4k\geq 0\Rightarrow k\leq 552$

$\therefore\quad k=1,2,3\ldots,552$

Product of real roots $= 1 \times 2 \times 3 \times 4 \times \ldots\ldots \times 552 = 552!$

Q11. If $-3 < \frac{x^2-\lambda x-2}{x^2+x+1} < 2$ for all $x \in R$, then the value of λ belongs to

A. $(-1,7)$

B. $(-6,2)$

C. $(-1,2)$

D. $(-6,7)$

Ans: $(-1,2)$

Solution:

$-3 < \frac{x^2-\lambda x-2}{x^2+x+1} < 2$

$\Rightarrow -3x^2 - 3x - 3 < x^2 - \lambda x - 2 < 2x^2 + 2x + 2$ $\left(\text{since } x^2 + x + 1 > 0,\ \forall x \in R\right)$

$\Rightarrow 4x^2 + x(3-\lambda)+1 > 0, x^2 + x(2+\lambda)+4 > 0$

(i) $4x^2 - x(\lambda-3)+1 > 0$

$\Rightarrow D < 0 \Rightarrow (\lambda-3)^2 - 4 \times 4 \times 1 < 0$

$\Rightarrow(\lambda-3+4)(\lambda-3-4)< 0$

$\Rightarrow(\lambda+1)(\lambda-7)< 0 \Rightarrow \lambda \in(-1,7)$

(ii) $x^2 + x(\lambda+2)+4 > 0$

$\Rightarrow D < 0 \Rightarrow (\lambda+2)^2 - 4 \times 4 < 0$

$(\lambda+2-4)(\lambda+2+4)< 0$

$(\lambda-2)(\lambda+6)< 0 \Rightarrow \lambda \in(-6,2)$

Taking the intersection of the solutions of (i) and (ii), we get,

$\lambda \in(-1,2)$

Q12. For the equation $\left|x^2-2x-3\right|= b$, which of the following statements is true?

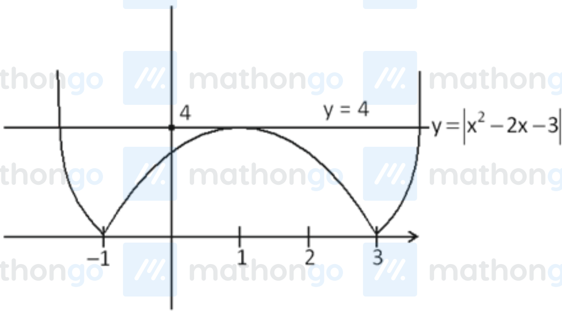
A. For $b<0$, there are no solutions

B. For $b=0$, there are three solutions

C. For $0<b<4$, there are two solutions

D. For $b=4$, there are four solutions

Ans: For $b<0$, there are no solutions



Solution:

$b<0 \rightarrow$ no solution

$b=0 \rightarrow$ two solutions

$0<b<4 \rightarrow$ four solutions

$b=4 \rightarrow$ three solutions

Q13. If a, b, c are real numbers satisfying the condition $a+b+c=0$, then the roots of the quadratic equation $3ax^2+5bx+7c=0$ are

A. Positive

B. negative

C. real and equal

D. distinct but not imaginary

Ans: distinct but not imaginary

Solution: $D=25b^2-4\times 3a\times 7c$

$=25(-a-c)^2-84ac$

$=25\left(a^2+c^2+2ac\right)-84ac$

$= 25\left(a^2 + c^2\right) - 34ac$

$= 17\left(a^2 + c^2 - 2ac\right) + 8\left(a^2 + c^2\right)$

$= 17(a - c)^2 + 8\left(a^2 + c^2\right)$

$D > 0 \Rightarrow$ Roots are real and distinct

Q14. If $\mathrm{a + b + c} > \frac{9\mathrm{c}}{4}$ and the equation $\mathrm{ax^2 + 2bx - 5c = 0}$ has non-real complex roots, then

A. $\mathrm{a} > 0,\ \mathrm{c} > 0$

B. $\mathrm{a} > 0,\ \mathrm{c} < 0$

C. $\mathrm{a} < 0,\ \mathrm{c} < 0$

D. $\mathrm{a} < 0,\ \mathrm{c} > 0$

Ans: $\mathrm{a} > 0,\ \mathrm{c} < 0$

Solution: $\mathrm{4a + 4b + 4c - 9c > 0}$

$\mathrm{4a + 4b - 5c > 0}$

$\mathrm{f(x) = ax^2 + 2bx - 5c},\ \mathrm{D} < 0$

$\mathrm{f(2) = 4a + 4b - 5c > 0}$

$\mathrm{a} > 0$ and

$\begin{array}{l}-5c>0\\ \Rightarrow c<0\end{array} \Rightarrow \boxed{\begin{array}{l}a>0\\ c<0\end{array}}$

Q15. If the graph of the function $y=(a-b)^2x^2+2(a+b-2c)x+1$ $(\forall a\neq b)$ is strictly above the x-axis, then

A. $a<b<c$

B. $a<c<b$

C. $b<a<c$

D. $c<b<a$

Ans: $a<c<b$

Solution: $D=B^2-4AC$
$=(2(a+b-2c))^2-4(a-b)^2$
$=4\left\{(a+b-2c)^2-(a-b)^2\right\}$
$=4(a+b-2c-a+b)(a+b-2c+a-b)$
$=4(2b-2c)(2a-2c)$
$=16(b-c)(a-c)$
$=16(c-b)(c-a)$
If c lies between a and b, then D is negative. Hence, the roots will be imaginary and the graph will be entirely above the x-axis as the coefficient of x^2 is positive.

Q16. The quadratic equations $x^2-6x+a=0$ and $x^2-cx+6=0$ have one root in common. The other roots of the first equation and the second equation are integers in the ratio $4:3$. Then the common root is

A. 4

B. 3

C. 2

D. 1

Ans: 2

Solution: Let, α be the common root and the other roots of the equations be 4β and 3β respectively. Then,

$\alpha + 4\beta = 6,\ \alpha(4\beta) = a$

$\alpha + 3\beta = c,\ \alpha(3\beta) = 6 \Rightarrow \frac{4}{3} = \frac{a}{6} \Rightarrow \alpha = 8$

The first equation is $x^2 - 6x + 8 = 0$

Whose roots are 2 and 4

If $\alpha = 2 \Rightarrow \beta = 1$

So, roots of first equation is $2,\ 4$ and that of second equation is $2,\ 3$

If $\alpha = 4 \Rightarrow \beta = \frac{1}{2} \Rightarrow 3\beta = \frac{3}{2}, 4\beta = 2$

Here the roots are not integers $\Rightarrow \alpha = 2$

Q17. The value of k for which both the roots of the equation $4x^2 - 20kx + \left(25k^2 + 15k - 66\right) = 0$ are less than 2, lies in

A. $\left(\frac{4}{5}, 2\right)$

B. $(0,\ 2)$

C. $\left(-1, -\frac{4}{5}\right)$

D. $(-\infty, -1)$

Ans: $(-\infty, -1)$

Solution: Let, $f(x) = 4x^2 - 20kx + \left(25k^2 + 15k - 66\right) = 0 \ldots\ldots\ldots\ldots\ldots\ldots (i)$

Let the roots of $f(x) = 0$ be $\alpha,\ \beta$

Since $\alpha,\ \beta$ are real.

$\therefore\ \Delta \geq 0$

$\Rightarrow 400k^2 - 4.4\left(25k^2 + 15k - 66\right) \geq 0$

$\Rightarrow\quad -15k + 66 \geq 0 \quad \Rightarrow \quad k \leq \frac{22}{5} \ldots\ldots\ldots\ldots\ldots(ii)$

We have $\alpha,\ \beta < 2$

$\therefore\quad \alpha + \beta < 4$

$\Rightarrow\quad -\frac{(-20k)}{4} < 4 \quad \Rightarrow k < \frac{4}{5} \ldots\ldots\ldots\ldots\ldots(iii)$

$f(x) = 4(x - \alpha)(x - \beta)$

$\therefore\qquad f(2) = 4(2 - \alpha)(2 - \beta) = 4(+)(+) = +\text{ve}$

$\therefore\quad f(2) = 16 - 40k + \left(25k^2 + 15k - 66\right) > 0$

$\Rightarrow 25k^2 - 25k - 50 > 0 \quad \Rightarrow \quad k^2 - k - 2 > 0$

$\Rightarrow\quad (k+1)(k-2) > 0 \quad \Rightarrow \quad k < -1 \text{ or k} > 2 \ldots\ldots\ldots\ldots(\text{iv})$

Combining $(ii),\ (iii)\ \&\ (iv),$ we get k $\in(-\infty, -1)$

Q18. The range of a for which the equation $x^2 + ax - 4 = 0$ has its smaller root in the interval $(-1, 2)$ is

A. $(-\infty, -3)$

B. $(0, 3)$

C. $(0, \infty)$

D. $(-\infty, -3) \cup (0, \infty)$

Ans: $(-\infty, -3)$

Solution: Clearly, $f(-1) > 0,\ f(2) < 0$

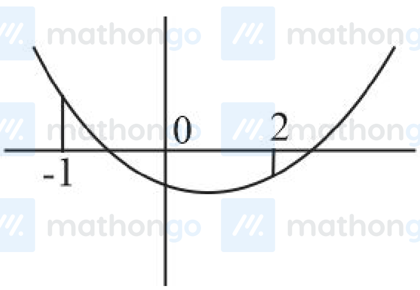
since, $f(0) = -4 < 0$

$\Rightarrow 1 - a - 4 > 0 \Rightarrow -a - 3 > 0 \Rightarrow -a > 3$

or $a < -3$ and $4 + 2a - 4 < 0$

$\Rightarrow a < 0$

$\Rightarrow a \in (-\infty, -3).$



Q19. If $f(x)$ is a polynomial of degree four with the leading coefficient one satisfying $f(1)=1$, $f(2)=2$ and $f(3)=3$, then $\left[\frac{f(-1)+f(5)}{f(0)+f(4)}\right]$ (where $[\cdot]$ represents the greatest integer function) is equal to

A. 4

B. 5

C. 6

D. 7

Ans: 5

Solution:

The roots of $f(x)-x=0$ are 1, 2 and 3.

So, we get,

$f(x)-x=(x-1)(x-2)(x-3)(x-a)$

For $x=-1$, we get,

$f(-1)+1=(-2)(-3)(-4)(-1-a)=24(1+a)$

For $x=5$, we get,

$f(5)-5=4\cdot3\cdot2(5-a)=24(5-a)$

$f(-1)+f(5)=(23+24a)+(125-24a)=148$

For $x=0$, we get,

$f(0)-0=(-1)(-2)(-3)(-a)=6a$

For $x=4$, we get,

$f(4)-4=3\cdot2\cdot1\cdot(4-a)=24-6a$

$f(0)+f(4)=28$

$\left[\frac{f(-1)+f(5)}{f(0)+f(4)}\right]=\left[\frac{148}{28}\right]$

$\left[\frac{f(-1)+f(5)}{f(0)+f(4)}\right]=5$

Q20. Sum of the squares of all integral values of a for which the inequality $x^2+ax+a^2+6a<0$ is satisfied for all $x\in(1,2)$ must be equal to

A. 90

B. 89

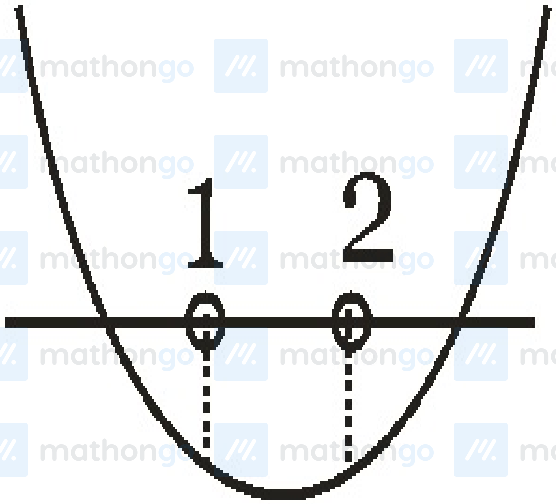
C. 88

D. 91

Ans: 91

Solution: Let, $f(x)=x^2+ax+a^2+6a$

$\therefore\ f(1)\leq 0$



$\Rightarrow \quad a^2 + 7a + 1 < 0$

or $\quad \frac{-7-3\sqrt{5}}{2} < a < \frac{-7+3\sqrt{5}}{2}$ ....(i)

$f(2) \leq 0$

$\Rightarrow \quad a^2 + 8a + 4 < 0$

or $\quad -4 - 2\sqrt{3} < a < -4 + 2\sqrt{3}$ ....(ii)

and $\quad D > 0$

$\Rightarrow a^2 - 4 \cdot 1\left(a^2 + 6a\right) > 0$

$\Rightarrow a^2 + 8a < 0$

or $\quad -8 < a < 0 \quad$ ....(iii)

From Eqs. (i), (ii) and (iii), we get,

$$\frac{-7-3\sqrt{5}}{2} \leq a \leq -4+2\sqrt{3}$$

Hence, integral values of a are $-6, -5, -4, -3, -2, -1$

Required Sum

$= (-6)^2 + (-5)^2 + (-4)^2 + (-3)^2 + (-2)^2 + (-1)^2$

$= 91.$

Q21. The equations $kx^2 + x + k = 0$ and $kx^2 + kx + 1 = 0$ have exactly one root in common for

A. $k = -\frac{1}{2}, 1$

B. $k = 1$

C. $k = -\frac{1}{2}$

D. $k = \frac{1}{2}$

Ans: $k = -\frac{1}{2}$

Solution:

Let α be the common root

$\Rightarrow k\alpha^2 + \alpha + k = 0$

$k\alpha^2 + k\alpha + 1 = 0$

On solving, we get,

$\frac{\alpha^2}{1-k^2} = \frac{\alpha}{k^2-k} = \frac{1}{k^2-k}$
$\frac{\alpha^2}{\alpha} = \frac{1-k^2}{k^2-k}$ and $\frac{\alpha}{1} = \frac{k^2-k}{k^2-k}$
$\Rightarrow \alpha = \frac{1-k^2}{k^2-k} = 1 \Rightarrow k^2 - k = 1 - k^2$

$\Rightarrow 2k^2 - k - 1 = 0 \Rightarrow k = -\frac{1}{2}, 1$

For $k = 1$, equations are identical, thus not possible

Hence, $k = -\frac{1}{2}$

Q22. If the quadratic equations $k(6x^2+3)+rx+2x^2-1=0$ and $6k(2x^2+1)+px+4x^2-2=0$ have both the roots common, then $2r-p$ is equal to

A. 0

B. 1

C. 2

D. None of these

Ans: 0

Solution:

$(6k+2)x^2 + rx + 3k - 1 = 0$
$(12k+4)x^2 + px + 6k - 2 = 0$ have
both roots common.

So,
$\frac{6k+2}{12k+4} = \frac{r}{p} = \frac{3k-1}{6k-2}$
$\Rightarrow \frac{r}{p} = \frac{1}{2} \Rightarrow 2r - p = 0$

Q23. If α, β and γ are the roots of the equation $x^3 - 13x^2 + 15x + 189 = 0$ and one root exceeds the other by 2, then the value of $|\alpha|+|\beta|+|\gamma|$ is equal to

A. 23

B. 17

C. 13

D. 19

Ans: 19

Solution: Let, the roots be $\alpha, \beta, \alpha + 2$.

$S_1 = \alpha + \beta + \alpha + 2 = 2\alpha + \beta + 2 = 13 \Rightarrow 2\alpha + \beta = 11 \Rightarrow \beta = 11 - 2\alpha$

$S_2 = \alpha\beta + \beta(\alpha + 2) + (\alpha + 2)\alpha = 15$

$\Rightarrow \beta(\alpha + \alpha + 2) + \alpha(\alpha + 2) = 15$

$\Rightarrow (11 - 2\alpha)(2\alpha + 2) + \alpha(\alpha + 2) = 15$

$\Rightarrow 22\alpha + 22 - 4\alpha^2 - 4\alpha + \alpha^2 + 2\alpha = 15$

$\Rightarrow 3\alpha^2 - 20\alpha - 7 = 0 \Rightarrow (\alpha - 7)(3\alpha + 1) = 0$

$\Rightarrow \alpha = 7$ or $-\frac{1}{3}$.

$\alpha = 7, \beta = 11 - 2\alpha = 11 - 14 = -3, \gamma = \alpha + 2 = 9$

$\alpha = -\frac{1}{3}, \beta = 11 - 2\alpha = 11 + \frac{2}{3} = \frac{35}{3}, \gamma = \alpha + 2 = \frac{5}{3}$.

Since, $\alpha\beta\gamma = -189$, hence we will take the first case.

$|\alpha|+|\beta|+|\gamma| = |7|+|-3|+|9| = 19$

Q24. If equations $x^2 + ax + b = 0 \left(a, b \in R\right)$ & $x^3 + 3x^2 + 5x + 3 = 0$ have two common roots, then value of $\frac{b}{a}$ is equal to

Ans: 1.50

Solution: $x^3 + 3x^2 + 5x + 3 = 0$ has one root $x = -1$

$\therefore x^3+3x^2+5x+3=(x+1)(x^2+2x+3)$

$\Rightarrow a=2,\ b=3$

Q25. If x is rational and $4\left(x^2+\frac{1}{x^2}\right)+16\left(x+\frac{1}{x}\right)-57=0$, then the product of all possible values of x is

A. 4

B. 3

C. 2

D. 1

Ans: 4

Solution: Given equation,

$4\left(x^2+\frac{1}{x^2}\right)+16\left(x+\frac{1}{x}\right)-57=0$

Let, $x+\frac{1}{x}=y\ ;\ x^2+\frac{1}{x^2}=y^2-2$

$\Rightarrow 4y^2+16y-65=0$

$\Rightarrow y=-\frac{13}{2}\ \text{ or }\ \frac{5}{2}$

When, $y=\frac{5}{2}$

$x+\frac{1}{x}=\frac{5}{2}\Rightarrow x=2\ \text{ or }\ \frac{1}{2}$

When, $y=-\frac{13}{2}$

$\Rightarrow x+\frac{1}{x}=-13/2$

$\Rightarrow 2x^2+13x+2=0$

$\Rightarrow x=\frac{-13\pm\sqrt{153}}{4}$

Since x is rational, $x=2$ or $\frac{1}{2}$

Q26. The sum of all real values of x satisfying the equation $\left(x^2-5x+5\right)^{x^2+4x-60}=1$ is

A. 6

B. 5

C. 3

D. -4

Ans: 3

Solution: The given equation will be true in 3 cases.

Case 1: when $x^2-5x+5=1$

$\Rightarrow x^2-5x+4=0$

$\Rightarrow x=1,\ 4$

Case 2: when $x^2+4x-60=0$

$\Rightarrow x=6,\ -10$

Case 3: when $x^2-5x+5=-1$ and $x^2+4x-60\in$ even integers

Now, $x^2-5x+5=-1$

$\Rightarrow x=2,\ 3$

Only $x=2$ satisfies the given condition,

Hence, sum of all real values of x is $1+4+6-10+2$= 3

Q27. If α and β are the real roots of $(\log_x 10)^3 - (\log_x 10)^2 - 6\left(\log_x 10\right) = 0$, then the value of $\left|\frac{1}{\log_{10} \alpha\beta}\right|$ is

Ans: 6

Solution:

Let $\log_x 10 = t$

$\therefore t^3 - t^2 - 6t = 0$

$\Rightarrow t\left(t^2 - t - 6\right) = 0$

$\Rightarrow t = 0, -2,\ 3$

$\Rightarrow \log_x 10 = 0,\ -2,\ 3$

$\Rightarrow 10 = x^0, x^{-2}, x^3$

$\Rightarrow x = 10^{-\frac{1}{2}},\ 10^{\frac{1}{3}}$

Let $\alpha = 10^{-\frac{1}{2}}$ and $\beta = 10^{\frac{1}{3}}$

Now, $\left|\frac{1}{\log_{10} \alpha\beta}\right| = \left|\frac{1}{\log_{10} 10^{-\frac{1}{6}}}\right|$

$\Rightarrow \left|\frac{1}{\log_{10} \alpha\beta}\right| = \left|\frac{-6}{\log_{10} 10}\right| = 6$

Q28. The sum of the roots of the equation $2^{(33\mathrm{x}-2)} + 2^{(11\mathrm{x}+2)} = 2^{(22\mathrm{x}+1)} + 1$ is

A. $\frac{1}{11}$

B. $\frac{2}{11}$

C. $\frac{3}{11}$

D. $\frac{4}{11}$

Ans: $\frac{2}{11}$

Solution: Let, $\mathrm{t}=2^{11\mathrm{x}}$

$\Rightarrow \frac{\left(2^{11\mathrm{x}}\right)^3}{2^2}+2^{11\mathrm{x}}.2^2=\left(2^{11\mathrm{x}}\right)^2.2+1$

$\Rightarrow \frac{\mathrm{t}^3}{4}+4\mathrm{t}=2\mathrm{t}^2+1$

$\Rightarrow \mathrm{t}^3-8\mathrm{t}^2+16\mathrm{t}-4=0$

Cubic in t has roots $\mathrm{t}_1,\mathrm{t}_2,\mathrm{t}_3$

i.e. $\mathrm{t}_1\mathrm{t}_2\mathrm{t}_3=4 \Rightarrow 2^{11\mathrm{x}_1}.2^{11\mathrm{x}_2}.2^{11\mathrm{x}_3}=4$

$\Rightarrow 2^{11\left(\mathrm{x}_1+\mathrm{x}_2+\mathrm{x}_3\right)}=2^2$

$\Rightarrow 11(\mathrm{x}_1+\mathrm{x}_2+\mathrm{x}_3)=2 \Rightarrow \mathrm{x}_1+\mathrm{x}_2+\mathrm{x}_3=\frac{2}{11}$

Q29. The number of real roots of the equation $e^{4x}-e^{3x}-4e^{2x}-e^x+1=0$ is equal to

Ans: 2

Solution:

We have, $e^{4x}-e^{3x}-4e^{2x}-e^x+1=0$

Let $e^x=t$

$t^4-t^3-4t^2-t+1=0$

$\Rightarrow t^2-t-4-\frac{1}{t}+\frac{1}{t^2}=0$

$\Rightarrow \alpha^2-\alpha-6=0, \alpha=\mathrm{t}+\frac{1}{\mathrm{t}} \geq 2$

$\Rightarrow \alpha=3,-2$ (reject)

$\Rightarrow \mathrm{t}+\frac{1}{\mathrm{t}}=3$

$\Rightarrow$ The number of real roots $= 2$.

Q30. If the equation in x given by $\left(2^{\left(\frac{1}{\cos^{-1} x}\right)}\right)^{2\pi} - \left(a + \frac{1}{2}\right)\left(2^{\left(\frac{1}{\cos^{-1} x}\right)}\right)^{\pi} - a^2 = 0$ has only one real solution then exhaustive set of values of 'a' is

A. $(-3\ ,1)$

B. $(-\infty, -3] \cup [1, \infty)$

C. $(-\infty, -3) \cup (1, \infty)$

D. $[-3, \infty)$

Ans: $(-\infty, -3] \cup [1, \infty)$

Solution:

Let $\frac{\pi}{2^{\cos^{-1} x}} = t \ \Rightarrow \ t \geq 2$

equation becomes $t^2 - \left(a + \frac{1}{2}\right)t - a^2 = 0$

has one roots 2 or greater than 2 and other root less than 2, $f(2) \leq 0$

$\Rightarrow 4 - \left(a + \frac{1}{2}\right)2 - a^2 \leq 0$

$a^2 + 2a - 3 \geq 0$

$(a + 3)(a - 1) \geq 0$

$a \leq -3$ or $a \geq 1$

Answer Key

Q1 (3)	**Q2** (3)	**Q3** (4)	**Q4** (8)
Q5 (1)	**Q6** (4)	**Q7** (2)	**Q8** (1)
Q9 (21)	**Q10** (3)	**Q11** (3)	**Q12** (1)
Q13 (4)	**Q14** (2)	**Q15** (2)	**Q16** (3)
Q17 (4)	**Q18** (1)	**Q19** (2)	**Q20** (4)
Q21 (3)	**Q22** (1)	**Q23** (4)	**Q24** (1.50)
Q25 (1)	**Q26** (3)	**Q27** (6)	**Q28** (2)
Q29 (2)	**Q30** (2)		