Q1. The point 'z' in Argand's plane moves such that $\mathrm{Re}\left(\frac{iz+1}{iz-1}\right) = 2$, then locus of z is-

A. straight line

B. circle

C. ellipse

D. hyperbola

Ans: circle

Solution:

We have,

$\mathrm{Re}\left(\frac{iz+1}{iz-1}\right) = 2$

Let $z = x + iy$, then

$\mathrm{Re}\left(\frac{i(x+iy)+1}{i(x+iy)-1}\right) = 2$

$\Rightarrow \mathrm{Re}\left(\frac{ix+i^2y+1}{ix+i^2y-1}\right) = 2$

$\Rightarrow \mathrm{Re}\left(\frac{1-y+ix}{-1-y+ix}\right) = 2$

$\Rightarrow \mathrm{Re}\left\{\left(\frac{1-y+ix}{-1-y+ix}\right) \times \left(\frac{-1-y-ix}{-1-y-ix}\right)\right\} = 2$

$\Rightarrow \mathrm{Re}\left\{\frac{(1-y+ix)((-1-y)-ix)}{(-1-y)^2+x^2}\right\} = 2$

$\Rightarrow \left\{\frac{(1-y)(-1-y)+x^2}{(-1-y)^2+x^2}\right\} = 2$

$\Rightarrow y^2 - 1 + x^2 = 2y^2 + 2x^2 + 2 + 4y$

$\Rightarrow x^2 + y^2 + 4y + 3 = 0$

Hence, the locus is a circle.

Q2. If $z \neq i$ be any complex number such that $\frac{z-i}{z+i}$ is a purely imaginary number, then, $z+\frac{1}{z}$ is

A. any non-zero real number other than 1 .

B. a purely imaginary number.

C. 0

D. any non-zero real number

Ans: any non-zero real number

Solution: Let $z=x+iy$

$\frac{z-i}{z+i}$ is a purely imaginary number

$\Rightarrow \frac{x+i(y-1)}{x+i(y+1)} \times \frac{x-i(y+1)}{x-i(y+1)}$ is a purely imaginary

$\Rightarrow \frac{(x^2+y^2-1)-i(2x)}{x^2+(y+1)^2}$ is purely imaginary

$\Rightarrow x^2+y^2-1=0 \Rightarrow x^2+y^2=1 \ldots$(i)

$z+\frac{1}{z}=x+iy+\frac{1}{x+iy}$

$=(x+iy)+\frac{1}{(x+iy)} \times \frac{(x-iy)}{(x-iy)}$

$=(x+iy)+\frac{(x-iy)}{x^2+y^2}=2x$

$y \neq \pm 1$ so $x \neq 0$ (from (i) and since, z won't be an imaginary number)

Q3. Let $u=\frac{2z+i}{z-ki}, z=x+iy$ and $k>0$. If the curve represented by $\operatorname{Re}(u)+\operatorname{Im}(u)=1$ intersects the y-axis at points P and Q where $PQ=5$ then the value of k is

A. $\frac{3}{2}$

B. $\frac{1}{2}$

C. 4

D. 2

Ans: 2

Solution:

$u=\frac{2(x+iy)+i}{(x+iy)-ki}=\frac{2x+(2y+1)i}{x+(y-k)i}\times\frac{x-(y-k)i}{x-(y-k)i}$

Real part of $u=Re(u)=\frac{2x^2+(2y+1)(y-k)}{x^2+(y-k)^2}$

Imaginary part of $u=\operatorname{Im}(u)\frac{x(2y+1)-2x(y-k)}{x^2+(y-k)^2}$

Now $\operatorname{Re}(u)+\operatorname{Im}(u)=1$

$\frac{2x^2+(2y+1)(y-k)+x(2y+1)-2x(y-k)}{x^2+(y-k)^2}=1$

for y-axis put $x=0$

$\Rightarrow\frac{(2y+1)(y-k)}{(y-k)^2}=1$

$\Rightarrow(y-k)(y+1+k)=0$

$y=k,\ -(1+k)$

Now point $P(0,\ k),\ Q(0,\ -(1+k))$

$PQ=|2K+1|=5$

$2k+1=\pm5$

$2k=4,-6$

$k=2,-3$

Hence, $k=2 \quad (k>0)$.

Q4. The solution of the equation $|z|-z=1+2i$ is

A. $\frac{3}{2}+2i$

B. $\frac{3}{2}-2i$

C. $3-2i$

D. None of these

Ans: $\frac{3}{2}-2i$

Solution: Let $z=x+iy$

Given, $|z|-z=1+2i$

$\Rightarrow \quad \sqrt{x^2+y^2}-(x+iy)=1+2i$

$\Rightarrow \quad \sqrt{x^2+y^2}-x=1, \quad y=-2$

$\Rightarrow \quad \sqrt{x^2+4}-x=1$

$\Rightarrow \quad x^2+4=(1+x)^2$

$\Rightarrow \quad 2x=3 \Rightarrow \quad x=\frac{3}{2}$

$\therefore \quad z=\frac{3}{2}-2i$

Q5. For the complex number Z, the sum of all the solutions of $Z^2+|Z|=\left(\overline{Z}\right)^2$ is equal to

Ans: 0

Solution: $Z^2 + |Z| = \left(\overline{Z}\right)^2 \ldots (1)$

Taking conjugate,

$\left(\overline{Z}\right)^2 + |Z| = Z^2 \ldots (2)$

Adding (1) & (2), we get,

$\Rightarrow 2|Z| = 0 \Rightarrow Z = 0$

Q6. Let z be a complex number satisfying the equation $\sqrt{2}|z-1| + i + z = 0$. Find the number of such complex numbers.

Ans: 0

Solution:

$\sqrt{2}\,|z-1| = -(i+z)$

$\Leftrightarrow \ \sqrt{2}\sqrt{(x-1)^2 + y^2} = -[x + i(y+1)] \qquad \ldots(i)$

$L.\ H.\ S.\ \geq 0 \Rightarrow R.\ H.\ S.$ must be real

$\Rightarrow y = -1$

(i) reduces to $\sqrt{2}\sqrt{(x-1)^2 + 1} = -x \qquad \ldots(ii)$

$L.\ H.\ S.$ of $(ii) \geq 0 \Rightarrow x \leq 0 \qquad \ldots(iii)$

Squaring (ii), we get $2\left[x^2 - 2x + 2\right] = x^2$

$\Leftrightarrow x^2 - 4x + 4 = 0$

$\Leftrightarrow x = 2$

(rejected) ...[From (iii)]

$\Rightarrow$ There is no z that satisfies the given equation. Thus, there are 0 such complex numbers.

Q7.

The locus of point z, where $z=x+iy$, satisfying the equation $\left|\frac{z-5i}{z+5i}\right|=1$, is

A. The $x-$ axis

B. The straight line $y = 5$

C. A circle passing through the origin

D. None of these

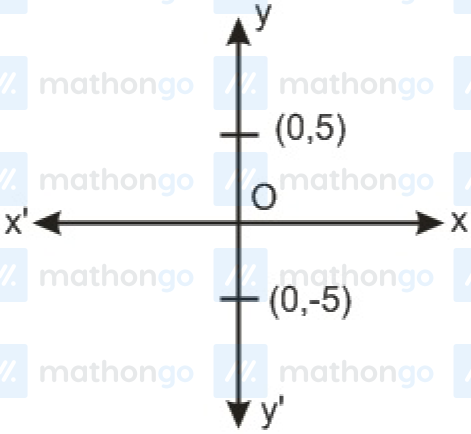
Ans: The $x-$ axis

Solution:

Given, $\left|\frac{z-5i}{z+5i}\right|=1$

$\Rightarrow|z-5i|=|z+5i|$

(if $|z-z_1|=|z-z_2|$, then it is a perpendicular bisector of the line segment joining points z_1 and z_2)



and perpendicular bisector of z_1 $(0,\ 5)$ and z_2 $(0,\ -5)$ is $x - axis$,

Therefore, z will lie on the $x -$ axis.

Q8. If $Z = \cos\phi + i\sin\phi\ \left(\forall\phi \in \left(\frac{\pi}{3}, \pi\right)\right)$, then the value of $\arg(Z^2 - Z)$ is equal to (where, $\arg(Z)$ represents the argument of the complex number Z lying in the interval $(-\pi, \pi]$ and $i^2 = -1$)

A. $\frac{3\phi+\pi}{2}$
B. $\frac{3\phi}{2}$
C. $\frac{3}{2}(\phi-\pi)$
D. $\frac{3\phi-\pi}{2}$

Ans: $\frac{3}{2}(\phi-\pi)$

Solution: $(Z^2 - Z) = Z(Z-1)$

$=(\cos\phi + i\sin\phi)(\cos\phi - 1 + i\sin\phi)$

$=(\cos\phi + i\sin\phi)\left(-2\sin^2\frac{\phi}{2} + i2\sin\frac{\phi}{2}\cos\frac{\phi}{2}\right)$

$= 2i\sin\frac{\phi}{2}\left(\cos\frac{3\phi}{2} + i\sin\frac{3\phi}{2}\right)$

$= 2\sin\frac{\phi}{2}\left(\cos\frac{3\phi+\pi}{2} + i\sin\frac{3\phi+\pi}{2}\right)$

Now, $3\phi \in(\pi, 3\pi) \Rightarrow \frac{3\phi+\pi}{2} \in(\pi, 2\pi)$

But, the argument lies in $(-\pi, \pi]$, hence

$\arg\left(Z^2 - Z\right) = \frac{3\phi+\pi}{2} - 2\pi = \frac{3}{2}(\phi - \pi)$

Q9. If z and w are two non-zero complex numbers such that $|\mathrm{zw}| = 1$ and $\arg(\mathrm{z}) - \arg(\mathrm{w}) = \frac{\pi}{2}$, then the value of $5\mathrm{i}\,\overline{\mathrm{z}}\,\mathrm{w}$ is equal to

A. -5

B. $5\mathrm{i}$

C. 5

D. $-5\mathrm{i}$

Ans: 5

Solution: $\arg\left(\frac{\mathrm{z}}{\mathrm{w}}\right) = \frac{\pi}{2} \Rightarrow \frac{\mathrm{z}}{\mathrm{w}} = \left|\frac{\mathrm{z}}{\mathrm{w}}\right| \mathrm{e}^{\mathrm{i}\pi/2}$

$\Rightarrow \frac{\mathrm{z}}{\mathrm{w}} = \left|\frac{\mathrm{z}}{\mathrm{w}}\right| \mathrm{i}$

$\Rightarrow \mathrm{w} = \mathrm{z}\frac{|\mathrm{w}|}{|\mathrm{z}|}(-\mathrm{i})$

$\Rightarrow \mathrm{w}\overline{\mathrm{z}} = \frac{\overline{\mathrm{z}}\,\mathrm{z}\,|\mathrm{w}|}{|\mathrm{z}|}(-\mathrm{i})$

$\Rightarrow 5\mathrm{i}\,\overline{\mathrm{z}}\,\mathrm{w} = 5\left(-\mathrm{i}^2\right)\frac{|\mathrm{z}|^2|\mathrm{w}|}{|\mathrm{z}|}$

$= 5(1)|\mathrm{z}||\mathrm{w}|$

$= 5$

Q10. If z is a non-real complex number, then the minimum value of $\frac{Im\, z^5}{(Im\, z)^5}$ is ($Im\, z =$ Imaginary part of z)

A. -2

B. -4

C. -5

D. -1

Ans: -4

Solution: Let, $z=r(\cos\theta+i\sin\theta)$

$z^5=r^5(\cos 5\theta+i\sin 5\theta)$

$Im\left(z^5\right)=r^5\sin 5\theta$

and $(Im\ z)^5=r^5\sin^5\theta$

$\frac{Im\ (z^5)}{(Im\ z)^5}=\frac{\sin 5\theta}{\sin^5\theta}=$ A (Let)

$\Rightarrow \frac{dA}{d\theta}=\frac{5\sin^5\theta\cos 5\theta-5\sin 5\theta\sin^4\theta\cos\theta}{\left(\sin^5\theta\right)^2}$

$\Rightarrow \frac{dA}{d\theta}=\frac{5\sin^4\theta(\sin\theta\cos 5\theta-\sin 5\theta\cos\theta)}{\sin^{10}\theta}$

$\Rightarrow \frac{dA}{d\theta}=\frac{5}{\sin^6\theta}[\sin(-4\theta)]=0$

$\Rightarrow \theta=\frac{\pi}{4}$

The minimum value of A will be at $\theta=\frac{\pi}{4}$.

$\Rightarrow \frac{\sin 5\frac{\pi}{4}}{\left(\sin\frac{\pi}{4}\right)^5}$

$= \frac{\frac{-1}{\sqrt{2}}}{\left(\frac{1}{\sqrt{2}}\right)^5}$

$= -\left(\sqrt{2}\right)^4 = -4$

Q11. If z and w are complex numbers satisfying $\bar{z} + i\bar{w} = 0$ and $\text{amp}(zw) = \pi$, then $\text{amp}(w)$ is equal to (where, $\text{amp}(w) \in (-\pi, \pi]$)

A. $\frac{\pi}{4}$

B. $\frac{-\pi}{4}$

C. $\frac{\pi}{2}$

D. $\frac{3\pi}{4}$

Ans: $\frac{\pi}{4}$

Solution:

Given $\bar{z} + i\bar{w} = 0 \Rightarrow \bar{z} = -i\bar{w}$ or $z = iw$ or $\frac{z}{w} = i$

$\text{amp}(z) - \text{amp}(w) = \text{amp}\, i = \frac{\pi}{2} \ldots .(\text{i})$

also $\text{amp}(zw) = \pi$

$\text{amp}(z) + \text{amp}(w) = \pi \ldots ..(\text{ii})$

Adding (i) & (ii), we get,

$2\text{amp}(z) = \frac{3\pi}{2}$

$\Rightarrow \text{amp}(z) = \frac{3\pi}{4}$

Also, $\text{amp}(w) = \frac{\pi}{4}$

Q12. Let α and β be the roots of $\mathrm{x}^2+\mathrm{x}+1=0$, then the equation whose roots are α^{2020} and β^{2020} is

A. $\mathrm{x}^2+\mathrm{x}+1=0$

B. $\mathrm{x}^2-\mathrm{x}-1=0$

C. $\mathrm{x}^2+\mathrm{x}-1=0$

D. $\mathrm{x}^2-\mathrm{x}+1=0$

Ans: $\mathrm{x}^2+\mathrm{x}+1=0$

Solution: $\mathrm{x}^2+\mathrm{x}+1=0$

$\Rightarrow \mathrm{x}=\omega, \omega^2$

$\Rightarrow \alpha=\omega$ and $\Rightarrow \beta=\omega^2$

$\Rightarrow \alpha^{2020}=\omega^{2020}=\left(\omega^3\right)^{673} . \omega=\omega$

$\Rightarrow \beta^{2020}=\left(\omega^2\right)^{2020}=\left(\omega^3\right)^{2 \times 673} \times \omega^2=\omega^2$

$\Rightarrow$ The required equation is $\mathrm{x}^2+\mathrm{x}+1=0$

Q13. If $z=\frac{1}{2}\left(\sqrt{3}-i\right)$ and the least positive integral value of n such that $\left(z^{101}+i^{109}\right)^{106}=z^n$ is k, then the value of $\frac{2}{5}k$ is equal to

Ans: 4

Solution: $z=\frac{-1}{2}i\left(1+i\sqrt{3}\right)=i\omega^2$

$z^{101}=i\omega$

$\left(z^{101}+i^{109}\right)^{106}=(i\omega+i)^{106}=\left(i\left(-\omega^2\right)\right)^{106}=-\omega^2$

as given that $\left(z^{101}+i^{109}\right)^{106}=z^n$

$\therefore -\omega^2 = \left(i\omega^2\right)^n = i^n\omega^{2n}$

$\omega^{2n-2}i^n = -1$

this is possible only when $n = 4r + 2$ and $2n - 2$ is a multiple of 3 i.e.,

$2(4r+2)-2$ is a multiple of 3

i.e., $8r + 2$ is a multiple of $3 \Rightarrow r = 2$

$\therefore n = 10 \quad \therefore \frac{2}{5}k = 4$

Q14. The value of $\sum_{n=0}^{100} i^{n!}$ equals (where $i = \sqrt{-1}$)

A. -1

B. i

C. $2\ i + 95$

D. $97 + i$

Ans: $2\ i + 95$

Solution:

Let, $S = \sum_{n=0}^{100} (i)^{n!}$ (where $i = \sqrt{-1}$)

$= (i)^{0!} + (i)^{1!} + (i)^{2!} + (i)^{3!} + (i)^{4!} + \ldots + (i)^{100!}$

$= (i)^1 + (i)^1 + (i)^2 + (i)^6 + (i)^{24} + \ldots. + (i)^{100!}$

(since, $i^{4n}=1$, $i^{4n+1}=i$, $i^{4n+2}=-1$ and $i^{4n+3}=-i$, where $n\in N$)

From i^{24} term onwards, every term is of the form i^{4n} which equals 1,

Simplifying above expression, we get

$=i+i-1-1+1+1+\ldots+1$

$=2i-2+97=2i+95$

Q15. If ω is the non-real cube root of unity, then the number of ordered pairs of integers (a, b), such that $|a\omega+b|=1$, is equal to

Ans: 6

Solution: We have, $|a\omega+b|^2=1$

$\Rightarrow(a\omega+b)\left(a\bar{\omega}+b\right)=1$

$\Rightarrow a^2+ab\left(\omega+\bar{\omega}\right)+b^2=1$

$\Rightarrow a^2-ab+b^2=1$

$\Rightarrow (a-b)^2+ab=1\ldots.(\mathrm{i})$

$\left(\mathrm{As,}\ 1+\omega+\omega^2=0\right)$

When $(a-b)^2=0$ and $ab=1$ then $(1,1);(-1,-1)$

When $(a-b)^2=1$ and $ab=0$ then $(0,1);(1,0);(0,-1);(-1,0)$

Hence, $(0,1);(1,0);(0,-1);(-1,0);(1,1);(-1,-1)$ i.e., 6 ordered pairs.

Q16. The number of solutions of the equation $z^3+\frac{3(\bar{z})^2}{|z|}=0$ (where, z is a complex number) are

A. 2

B. 3

C. 6

D. 5

Ans: 5

Solution: Let, $z = re^{i\theta}$

$\Rightarrow r^3 e^{i3\theta} + 3re^{-i2\theta} = 0$

$\Rightarrow r^2 e^{i5\theta} = -3$

$\Rightarrow r^2 = 3$ and $e^{i5\theta} = -1$

$\Rightarrow r = \sqrt{3}$ and $\theta = \frac{\pi}{5} + \frac{2k}{5}$ where, $k = 0, 1, 2, 3, 4$

$\Rightarrow$ 5 solutions

Q17. $\mathrm{z} \in \mathrm{C}$ **satisfies the condition** $|\mathrm{z}| \geq 3$**. Then the least value of** $\left|\mathrm{z} + \frac{1}{\mathrm{z}}\right|$ **is**

A. $\frac{3}{8}$

B. $\frac{8}{5}$

C. $\frac{8}{3}$

D. $\frac{5}{8}$

Ans: $\frac{8}{3}$

Solution: From triangle inequality we know that $|\mathrm{z}_1 + \mathrm{z}_2| \geq ||\mathrm{z}_1| - |\mathrm{z}_2||$

hence $\left|\mathrm{z} + \frac{1}{\mathrm{z}}\right| = \left|\mathrm{z} - \left(-\frac{1}{\mathrm{z}}\right)\right| \geq |\mathrm{z}| - \left|-\frac{1}{\mathrm{z}}\right| \geq\ 3 - \frac{1}{3} = \frac{8}{3}$

Hence $\frac{8}{3}$ is the correct answer.

Q18. If m and M denotes the minimum and maximum value of $|2z+1|$, where $|z-2i| \leq 1$ and $i^2=-1$, then the value of $(M-m)^2$ is equal to

A. 17

B. 34

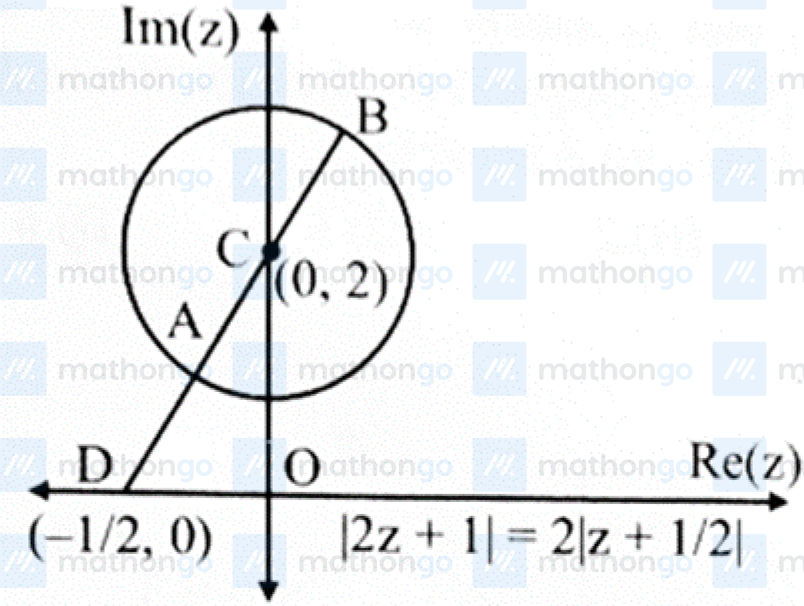
C. 51

D. 16

Ans: 16

Solution:

$z-2i=1$ represents a circle with centre at $(0,2)$ and radius 1 unit



$|2z+1|=2\left|z-\left(\frac{-1}{2}\right)\right|$ represents twice the distance from the point $\left(\frac{-1}{2},0\right)$

Hence from the diagram,

$m=2AD=2(CD-AC)$

$m=2\left(\frac{\sqrt{17}}{2}-1\right)=\sqrt{17}-2$

$M=2BD=2(CD+BC)$

$M=2\left(\frac{\sqrt{17}}{2}+1\right)=\sqrt{17}+2$

$\Rightarrow (M-m)^2=4^2=16$

Q19. For a complex number Z, if all the roots of the equation $Z^3+aZ^2+bZ+c=0$ are unimodular, then

A. $|a|>3$ and $|c|=1$

B. $|a|\leq 3$ and $|c|=3$

C. $|a|>3$ and $|c|=\frac{1}{3}$

D. $|a|\leq 3$ & $|c|=1$

Ans: $|a|\leq 3$ & $|c|=1$

Solution: Given, $|Z_1|=|Z_2|=|Z_3|=1$

Now, $|Z_1+Z_2+Z_3|\leq|Z_1|+|Z_2|+|Z_3|$

$\Rightarrow|-a|\leq 1+1+1$

$\Rightarrow|a|\leq 3$

Also, $|Z_1Z_2Z_3|=|Z_1|\times|Z_2|\times|Z_3|$

$\Rightarrow|-c|=1\times1\times1$

$\Rightarrow|c|=1$

Q20. Let z and w be non-zero complex numbers such that $\mathrm{zw}=|\mathrm{z}^2|$ and $\left|\mathrm{z}-\overline{\mathrm{z}}\right|+\left|\mathrm{w}+\overline{\mathrm{w}}\right|=4$. If w varies, then the perimeter of the locus of z is

A. $8\sqrt{2}$ units

B. $4\sqrt{2}$ units

C. 8 units

D. 4 units

Ans: $8\sqrt{2}$ units

Solution: Given, $\mathrm{zw} = |\mathrm{z}|^2 \Rightarrow \mathrm{zw} = \mathrm{z}\,\overline{\mathrm{z}}$

$\Rightarrow \mathrm{w} = \overline{\mathrm{z}}\ \{\mathrm{z} \neq 0\}$

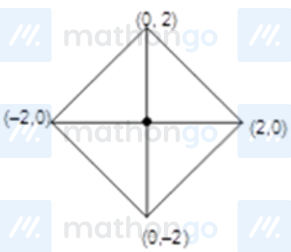
Now, $\left|\mathrm{z} - \overline{\mathrm{z}}\right| + \left|\mathrm{w} + \overline{\mathrm{w}}\right| = 4$

$\Rightarrow \left|\mathrm{z} - \overline{\mathrm{z}}\right| + \left|\mathrm{z} + \overline{\mathrm{z}}\right| = 4$

Let, $\mathrm{z} = \mathrm{x} + i\mathrm{y}$, then we get,

$|\mathrm{x}| + |\mathrm{y}| = 2$

which represents a square of side length equal to $2\sqrt{2}$



$\Rightarrow$ The perimeter of the locus is $8\sqrt{2}$ units

Q21. The straight line $(1+2i)z + (2i-1)\overline{z} = 10i$ on the complex plane, has intercept on the imaginary axis equal to

A. 5

B. $\frac{5}{2}$

C. $-\frac{5}{2}$

D. -5

E. 3

Ans: 5

Solution:

Let $z=x+iy$

$\Rightarrow(1+2i)z+(2i-1)\bar{z}=10i$

$\Rightarrow(1+2i)(x+iy)+(2i-1)(x-iy)=10i$

$\Rightarrow(x-2y)+i(2x+y)+(-x+2y)+i(2x+y)=10i$

$\Rightarrow 2i(2x+y)=10i$

Or $2x+y=5$

For interception on imaginary axis

Put $x=0$

So, we get $y=5$

Intercept on imaginary axis $=5$

Q22. If a complex number z lie on a circle of radius $\frac{1}{2}$ units, then the complex number $\omega=-1+4\mathrm{z}$ will always lie on a circle of radius k units, where k is equal to

Ans: 2

Solution: Let us assume that z lies on a circle with centre z_0 (fixed point) and radius $\frac{1}{2}$ units.

$\Rightarrow \quad |\mathrm{z}-\mathrm{z}_0|=\frac{1}{2}$

Now, $\omega=-1+4\mathrm{z} \Rightarrow \omega+1=4\mathrm{z}$

$\Rightarrow \quad \omega+1-4\mathrm{z}_0=4\mathrm{z}-4\mathrm{z}_0$

Now, taking modulus on both sides, we get,

$|\omega+1-4\mathrm{z}_0|=4|\mathrm{z}-\mathrm{z}_0| \Rightarrow |\omega+1-4\mathrm{z}_0|=2$

Locus of ω represents the circle having centre $(-1+4\mathrm{z}_0)$ and radius 2 units.

Q23. Let z be a complex number such that $\left|\frac{z-i}{z+2i}\right|=1$ and $|z|=\frac{5}{2}$. Then the value of $|z+3i|$ is

A. $\sqrt{10}$

B. $\frac{7}{2}$

C. $\frac{15}{4}$

D. $2\sqrt{3}$

Ans: $\frac{7}{2}$

Solution:

Given, $|z-i|=|z+2i|$

$\Rightarrow x^2+(y-1)^2=x^2+(y+2)^2$

$\Rightarrow -2y+1=4y+4$

$\Rightarrow 6y=-3 \Rightarrow y=-\frac{1}{2}$

From, $|z|=\frac{5}{2}$, we get,

$x^2+y^2=\frac{25}{4}\Rightarrow x^2=\frac{24}{4}=6$

$\Rightarrow z=\pm\sqrt{6}-\frac{i}{2}$

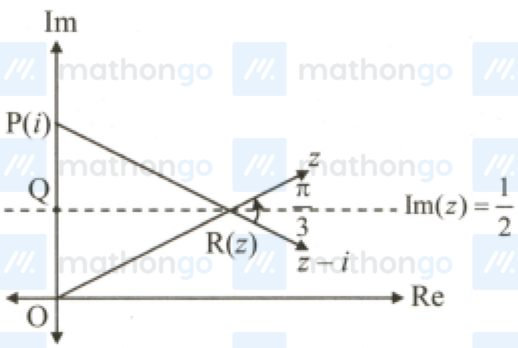
Hence, $|z+3i|=\sqrt{6+\frac{25}{4}}=\sqrt{\frac{49}{4}}$

$|z+3i|=\frac{7}{2}$

Q24. A complex number z satisfies $arg\left(\frac{z}{z-i}\right)=\frac{\pi}{3}$ and $|z|=|z-i|$, then evaluate $[Re\,(2z-i)]$ where $[\cdot]$ represents the greatest integer function.

Ans: 1

Solution:



$|z|=|z-i|$

$\Leftrightarrow|PR|=|OR|$

$arg\ \frac{z}{z-i}=\frac{\pi}{3}$

$\Leftrightarrow\angle PRO=\frac{\pi}{3}$

$\Rightarrow\Delta POR$ is an equilateral triangle.

$\Rightarrow QR$ is angle bisector as well as median.

$\Rightarrow \angle PRQ = \frac{\pi}{6}, \ |PQ| = \frac{1}{2}$

$\Rightarrow Im\ (z) = \frac{1}{2}, \ Re(z) = |QR|$

$= |PQ|\ cot\ \frac{\pi}{6}$

$= \frac{1}{2}\left(\sqrt{3}\right)$

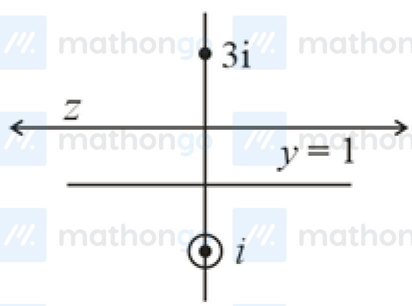
$z = \frac{\sqrt{3}}{2} + \frac{i}{2} \Rightarrow 2z - i = \sqrt{3}$

$\Rightarrow\ [Re\ (2z - i)] = [1.73] = 1$

Q25. Let z and w be two complex numbers such that $w = z\bar{z} - 2z + 2, \left|\frac{z+i}{z-3i}\right| = 1$ and $\operatorname{Re}(w)$ has minimum value. Then, the minimum value of $n \in N$ for which w^n is real, is equal to ________.

Ans: 4

Solution:



$\omega = z\bar{z} - 2z + 2$

$\left|\frac{z+i}{z-3i}\right| = 1$

$\Rightarrow |z + i| = |z - 3i|$

$\Rightarrow z=x+i,\ x \in R$

$\omega=(x+i)(x-i)-2(x+i)+2$

$=x^2+1-2x-2i+2$

$\operatorname{Re}(\omega)=x^2-2x+3$

For min $(\operatorname{Re}(\omega)), x=1$

$\Rightarrow \omega=2-2i=2(1-i)=2\sqrt{2}e^{-i\frac{\pi}{4}}$

$\omega^n=\left(2\sqrt{2}\right)^n e^{-i\frac{n\pi}{4}}$

For real and minimum value of n,

$n=4$

Q26. The complex number z, satisfying the equation $z^3=\overline{z}$ and $\arg(z+1)=\frac{\pi}{4}$ simultaneously, is (where, $i^2=-1$)

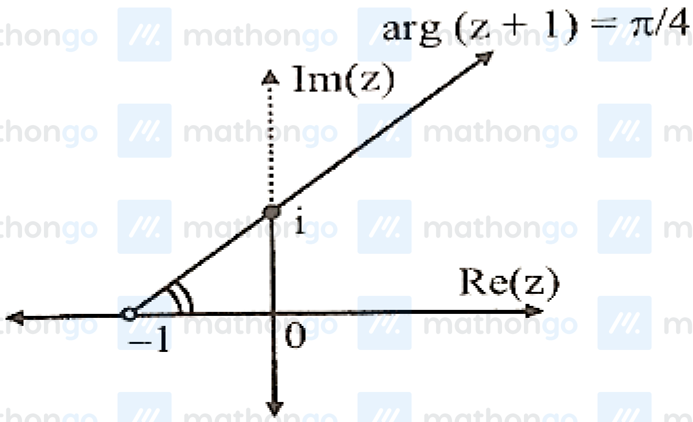
A. i

B. $1+2i$

C. $2+3i$

D. $3+4i$

Ans: i



Solution:

$z^3 = \bar{z} \Rightarrow |z| = 0$ or 1

$|z| = 0 \Rightarrow z = 0$

$|z| = 1 \Rightarrow z^4 = 1$

$\Rightarrow z = \pm 1, \pm 1$

Only $z = i$ satisfies $\arg(z + 1) = \frac{\pi}{4}$

Q27. The real part of the complex number z satisfying $|z - 1 - 2i| \leq 1$ and having the least positive argument, is

A. $\frac{4}{5}$

B. $\frac{8}{5}$

C. $\frac{6}{5}$

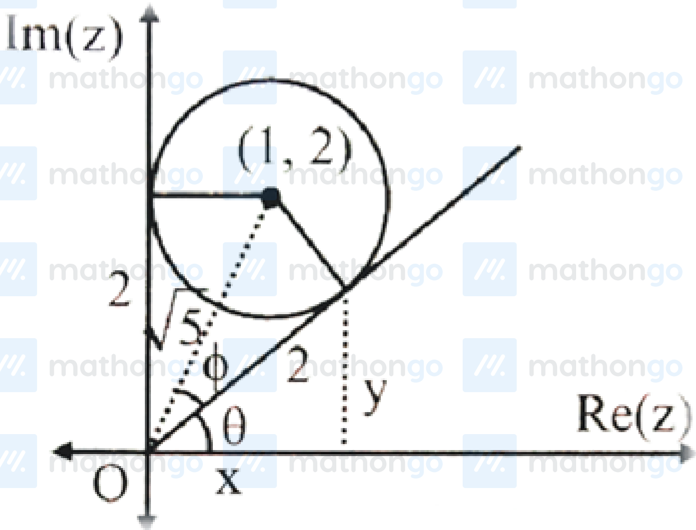
D. $\frac{7}{5}$

Ans: $\frac{8}{5}$

Solution:

Here, $|z - 1 - 2i| = 1$ represents a circle with centre $(1, 2)$ and radius 1 unit.

The complex number $z=\mathrm{x}+i\mathrm{y}$ satisfying the given inequality and having the least positive argument is the point of contact of the tangent from the origin to the circle with the least positive slope.



From the diagram,

$\tan \phi=\frac{1}{2}$
$\therefore \tan 2 \phi=\frac{2 \tan \phi}{1-\tan ^2 \phi}=\frac{4}{3}$

For the least positive argument, $\arg(z)=\theta$ (let)

$\tan \theta=\tan \left(\frac{\pi}{2}-2 \phi\right)=\cot 2 \phi=\frac{3}{4}$

Also, from the diagram,

$x^2+y^2=4$ and $\tan \theta=\frac{y}{x}=\frac{3}{4}$

i.e. $y=\frac{3}{4} x$

$\Rightarrow x^2+\frac{9 x^2}{16}=4 \Rightarrow x=\frac{8}{5}$

Hence, for the least positive argument, the real part of z is equal to $\frac{8}{5}$

Q28. If a and b are two real numbers lying between 0 and 1, such that

$Z_1=a+i$, $Z_2=1+bi$ and $Z_3=0$ form an equilateral triangle, then

A. $a=2+\sqrt{3}$

B. $b=4-\sqrt{3}$

C. $a=b$

D. $a=2, b=\sqrt{3}$

Ans: $a=b$

Solution: If $Z_1,\ Z_2,\ Z_3$ form an equilateral triangle, then we know that,

$Z_1^2+Z_2^2+Z_3^2=Z_1Z_2+Z_2Z_3+Z_3Z_1$

$\Rightarrow a^2-1+2ai+1-b^2+2bi-a+b-i-abi=0$

$\Rightarrow(a-b)(a+b-1)+(2a+2b-ab-1)i=0$

Case-I:

$a=b\ \&\ 2a+2b-ab-1=0$

$\Rightarrow a=b\ \&\ a^2-4a+1=0\Rightarrow a=b=2-\sqrt{3}$

Case-II:

$a+b-1=0\ \&\ 2a+2b-ab-1=0$

$\Rightarrow ab=1$ (not possible because $a, b\in(0,1)$)

$\Rightarrow a=b=2-\sqrt{3}$ is the only solution

Q29. If the locus of the complex number z given by $\arg(z+i)-\arg(z-i)=\frac{2\pi}{3}$ is an arc of a circle, then the length of the arc is

A. $\frac{4\pi}{3}$

B. $\frac{4\pi}{3\sqrt{3}}$

C. $\frac{2\sqrt{3}\pi}{3}$

D. $\frac{2\pi}{3\sqrt{3}}$

Ans: $\frac{4\pi}{3\sqrt{3}}$

Solution:

Given, $\arg\left(\frac{z+i}{z-i}\right)=\frac{2\pi}{3}$

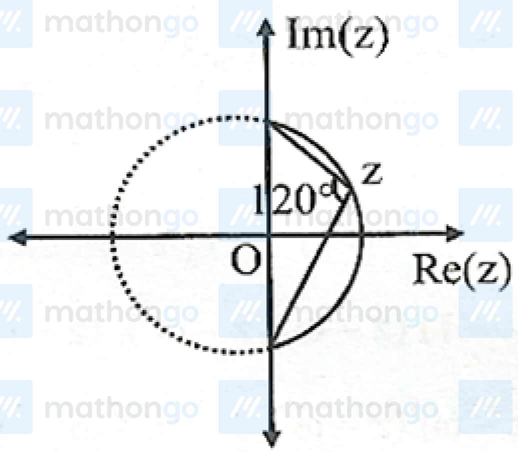
Let, $z=x+iy$

$\Rightarrow \frac{x+i(y+1)}{x+i(y-1)}=\frac{x^2+(y^2-1)+2ix}{x^2+(y-1)^2}$

$\Rightarrow \tan^{-1}\frac{2x}{x^2+y^2-1}=\frac{2\pi}{3}$

$\Rightarrow x^2+y^2+\frac{2}{\sqrt{3}}x-1=0$

Hence, the given locus is a circle with centre $\left(-\frac{1}{\sqrt{3}},0\right)$ and radius $\frac{2}{\sqrt{3}}$ units



$\Rightarrow$ Length of the arc of the circle is $\frac{2\pi}{3}\times\left(\frac{2}{\sqrt{3}}\right)=\frac{4\pi}{3\sqrt{3}}$ units

Q30. Let the locus of any point $P(z)$ in the argand plane is $\arg\left(\frac{z-5i}{z+5i}\right)=\frac{\pi}{4}$. If O is the origin, then the value of $\frac{\max.(OP)+\min.(OP)}{2}$ is

A. $5\sqrt{2}$

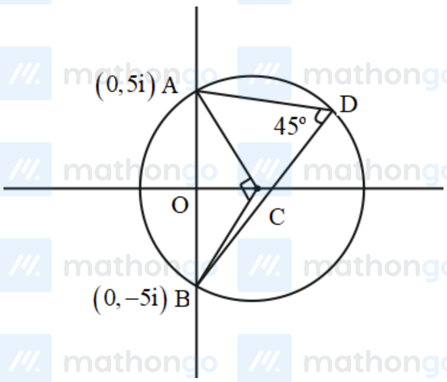
B. $5+\frac{5}{\sqrt{2}}$

C. $5+5\sqrt{2}$

D. $10-\frac{5}{\sqrt{2}}$

Ans: $5+\frac{5}{\sqrt{2}}$

Solution:



$r^2+r^2=10^2 \Rightarrow r=5\sqrt{2}$

max.(OP)= OC+ radius $=5+5\sqrt{2}$

and min.(OP)= $OA=5$

Required value $=\frac{5+5+5\sqrt{2}}{2}=5+\frac{5}{\sqrt{2}}$

Answer Key

Q1 (2)	**Q2** (4)	**Q3** (4)	**Q4** (2)
Q5 (0)	**Q6** (0)	**Q7** (1)	**Q8** (3)
Q9 (3)	**Q10** (2)	**Q11** (1)	**Q12** (1)
Q13 (4)	**Q14** (3)	**Q15** (6)	**Q16** (4)
Q17 (3)	**Q18** (4)	**Q19** (4)	**Q20** (1)
Q21 (1)	**Q22** (2)	**Q23** (2)	**Q24** (1)
Q25 (4)	**Q26** (1)	**Q27** (2)	**Q28** (3)
Q29 (2)	**Q30** (2)		